Experiment on the (Inverted) Fractal Demonstrating Micro Quantum and Macro Astronomical Observations and Conjectures

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*This paper is the update and merger of my earlier papers [1] and [2].

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Abstract

Continuing the debate on whether the universe is fractal by nature: an experiment was undertaken on the 'simple' Koch Snowflake fractal to test whether fractal geometry matches observations and conjectures. The Koch Snowflake was inverted to model observations from within an iterating fractal set it: simulating a static or 'measured' position. Converse to the fractal snowflake emergence – where triangle sizes diminish; the sizes of new triangles were held constant, and earlier triangles in the set expanded as the set iterated. Kinematic velocities and accelerations were calculated for both the area expansion of the total fractal, and the distance between points and the 'observer' within the fractal set. The inverted fractal was tested for the Hubble's Law. It was found area(s) expanded exponentially; and as a consequence, the distances between points – from arbitrary locations within the set – receded away from the 'observer' at increasing velocities and accelerations. The model was consistent with the standard ΛCDM model of cosmology and demonstrated: a singularity 'Big Bang' beginning; homogeneous isotropic expansion consistent with the observed CMB; Hubble's Law expansion – with a Hubble diagram and Hubble's constant; and accelerating expansion with a 'cosmological' constant an expansion rate consistent with, and capable of explaining the conjecture of early inflation epoch of the universe. The model predicts and matches current galaxy distribution observations – clustered nearby and smooth on large scales – and thus is inconsistence with the cosmological principle. The mechanism of expansion is consistent with quantum mechanical descriptions: the vacuum catastrophe is addressed and concluded to be as a consequence of fractal behaviour. It was concluded that the universe behaves as a general as a fractal object, where we are observing inside it.

Keywords

Fractals, Dark Energy, Inflation, Hubble's Law, Cosmological Constant, Galaxy Distribution

1 INTRODUCTION

The great unanswered and perplexing problem of modern cosmology is what is causing the accelerating expansion of the universe [3],[4] – the so-called 'dark energy'. This comes atop a list of problems – associated with observation and conjecture of the 'standard model' – including: its big bang – inflationary – beginning, the cosmic microwave background (CMB) [5], Hubble expansion, and not to mention its 'dark matter'. Add to these – assuming General Relativity is correct – the standard model doesn't match or fit with quantum mechanics – said to be the most successful theory of science – is out by a factor of 10^{120}. One thing agree upon[6] by scientists: the standard model is 'looking for a new breakthrough'.

In this paper I would like to propose and test a solution to the above. It is a different aspect or perspective of an already proposed cosmic geometry, fractal geometry. My theory is based on how fractals are produced, and the mechanics and behaviour observed when viewed within a fractal – it is an inverted fractal view. I aim to test this perspective, and prove this matches, in principle, the above large scales problems, and because of the scale invariance of the fractal, the small scale too – including quantum mechanical problems.

This is not a direct pure mathematic paper, and it does not necessarily take from current work, but is rather a basic prototype experiment – a proof of concept – and it should complement current thinking, not take from it. Its simplicity may render it trivial to some; but if the conclusions from the experiment are right, the implications will be akin to putting the Sun at the centre of the solar system – everything will fit. This investigation was an applied mathematic analysis: the growth behaviour of the complex (chaotic) fractal attractor[7] was analysed. To measure the fractal, the Koch snowflake fractal was chosen for its quantitative regularity: the snowflake was inverted, and areas recorded as the fractal iterated. Measurements were taken as from a fixed reference, perspective or position within the iterating set.

1.1 Fractals and Fractal Cosmology

Fractal attractors are in general presented as interesting computer-generated images, and as a result may easily be disregarded, but they also offer a not to be overlooked window into the mechanics of our reality, particularly the isolated, scale invariant and iterating object. Fractal geometry offers one of the best descriptions of the complexity of nature mathematics has to offer: and its insights are not lost on cosmology, and has its own field termed fractal-cosmology. Fractals are inextricably related chaos theory, and already appear in works on (eternal) inflation theory [8] and the structure and distribution of matter in the universe. [9],[10],[11],[12],[13].

Though this question of whether the universe conform to fractal geometry is not often mentioned in 'mainstream' – or even popular – science, it has indeed been debated and the claim left unfounded due to lack of (ever) larger scale observations, and by the recent findings of the WiggleZ survey [14], [15], [16],[17].

1.2 Observed Galaxy Distribution

The proponents of fractal cosmology (Luciano Pietronero, Francesco Sylos Labini, and others) have been arguing the observed hierarchies of clustering and super clustering – based on surveys similar to the 2003 2df Redshift Survey map (figure 2 below) – are direct evidence of a universe that is fractal [18],[19],[20],[21],[22],[7] and [23]. The current consensus, however, is – after even deeper cosmic surveys – the universe on large scales smoothens out to become homogenous – and is overall not (a) fractal[24],[25] and [26].

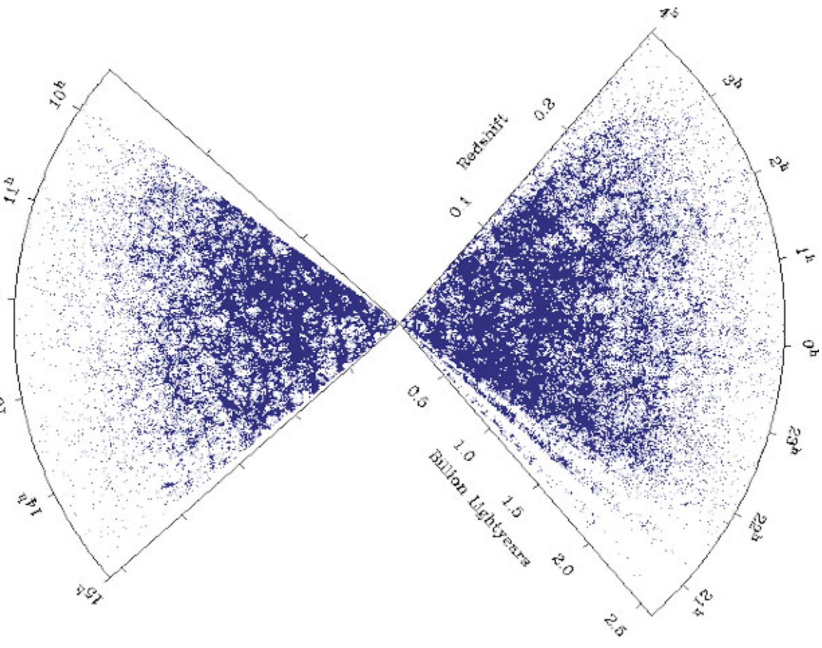


Figure 1. 2dF SDSS Galaxy Redshift Survey [27] .

It was the 2012 WiggleZ Dark Energy Survey[14] – the largest survey to date – that settled the fractal question: They concluded – with increased, but similar confidence as previous teams (namely, the 2004 Sloan Digital Sky Survey [28] and [29]) – the universe shows evidence of fractal galaxy distribution – with clustering and super-clustering – only on small scales (less than 70 to 100 Mega parsecs away) – beyond this distance, the pattern becomes a homogenous galaxy distribution.

On viewing the WiggleZ Dark Energy Survey results – particularly its figure 13 (figure 3 below) of changing galaxy distribution (from fractal at small cosmic scales to smooth at large cosmic scales) I questioned whether the inverted fractal may offer explanation and insight to the decreasing 'Scaled N (r)' transitions (and increasing smoothness) with distance.

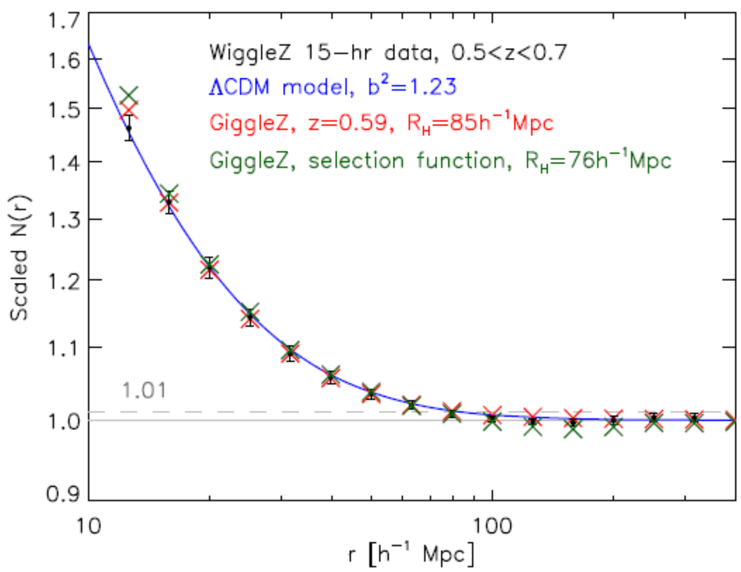


Figure 2. WiggleZ Dark Energy Survey figure 13, page 16. Revealing changing galaxy distributions from small-scale to large-scale.

In this publication I shall investigate whether the galaxy distribution results from the WiggleZ Dark Energy Survey (and its contemporises) match a view of what one would expect to see if they viewed, not at a fractal – as is implied in their studies – but from within a (growing) fractal.

To test this I shall return to the fractspansion model and analyse the occurrence of the said clustering of measurement points. For the clustering in the fractspansion model to have any significance to cosmology, it would have to demonstrate how the distribution of triangles (in the inverted Koch snowflake) changes over distance from the observer (section 4.1 and 4.2).

1.3 The Classical Fractal

Fractals – also described as L-systems – are emergent objects that develop and (or) grow with the iteration of a simple rule. They possess self-similarity at all scales and can be observed as being regular but irregular (same but different) objects. They are classically demonstrated by the original Mandelbrot Set [7], and – in one of their most simplest forms – the Koch Snowflake (Figure 3 below, A and B respectively). Familiar examples of them in reality are clouds, waves, coastlines and trees. All fractals have a defined 'fractal dimension'. Stewart in his book on chaos (and fractals) 'God does not play dice' – said: '..coastlines and Koch Snowflakes are equally rough' [30]. Indeed, the very close fractal dimension values of both the Koch Snowflake and the coastlines of islands (Great Britain) 1.26, and between 1.15 and 1.2 respectively – stands as testament of their power to best model nature and reality.

Figure 3 B below shows the classical view of fractal emergence (growth or development) – achieved by the iteration of a simple rule, by adding more (triangle) bits – of diminishing size – to the previous triangle, originating from an initial (iteration 0) bit – also known as the 'axiom' in L-system theory. The snowflake shape is formed at and around 5 or 6 iterations; from this point on – to the observer – it no longer changes shape. This equilibrium iteration count is the observable fractal distance, relative to the observer. This distance is constant irrespective of magnification.

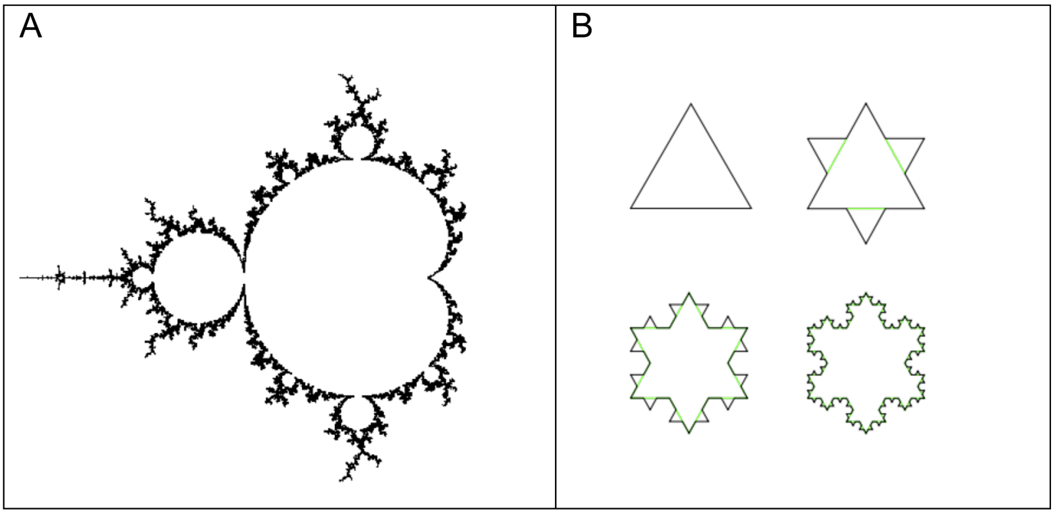


Figure 3. (Classical) Fractals. (A) boundary of the Mandelbrot set; (B) The Koch Snowflake fractal from iteration 0 to 3. Reference: (A) [31]; (B) [32].

1.4 Production, and the Inverted Fractal

To 'produce' a Lorenz curve with a fractal attractor, we first need to determine what it meant by production and thus growth of the fractal. When we attempt to do this, we find there is a paradox – there are two views that conflict with each other: one where the original triangle bit size remains constant, and the new bits diminish in size as the fractal iterates – this is termed (A) 'a consumption perspective'; and (B) where it is the new triangle bits size remain constant, and all earlier triangle bits expand and grow as the fractal iterates – termed 'a production perspective'. A is the classical view – as shown in figure 3B (and A in figure 4 below) and B the 'inverted', shown in figure 4 B. Both of these views, A and B, are relative views of the same process: both are true, but only one really describes the production and the growth from production, and for this study it is assumed to be the inverted view B.

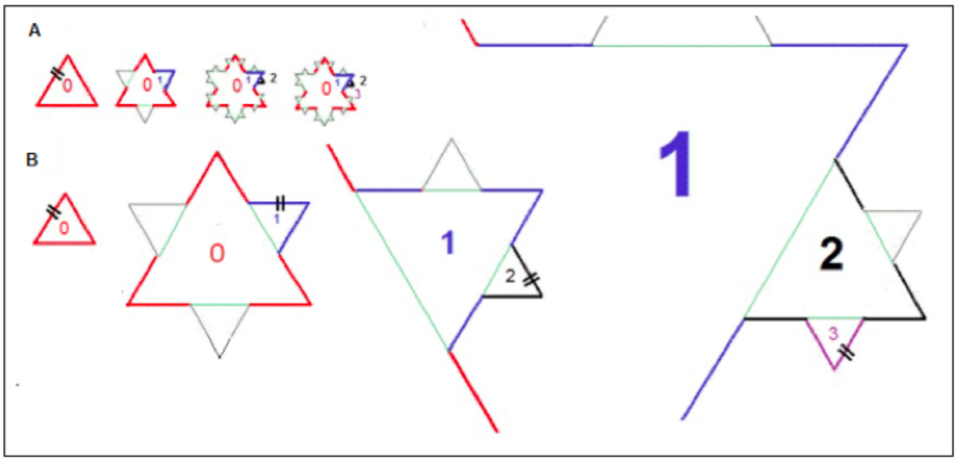


Figure 4. Expansion of the inverted Koch Snowflake fractal (fractspansion). The schematics above demonstrate fractal development by (A) the classical Snowflake perspective, where the standard sized thatched (iteration '0') is the focus, and the following triangles diminish in size from colour red iteration 0 to colour purple iteration 3; and (B) the inverted, fractspanding

perspective where the new (thatched) triangle is the focus and held at standard size while the original red iteration 0 triangle expands in area – as the fractal iterates.

There is a practical reason for this: the 'inverted' production growth can be 'observed' in real life nature fractals, trees. With trees (thus all plants – in principle) it is the trunk of the tree that grows, and the new branch size that remains constant. This is converse to 'our' static observation of trees, where the (larger) constant sized trunk is observed with diminishing sized branches 'protruding' – self similar – from it.

1.5 Time

The iterating fractal exposes the issue of time. In isolation the fractal grows with the passing of time, and in isolation this time can only be the iteration time. Iteration is: in one direction – beginning with the original triangle; discrete (bit by bit); and arbitrary in length – as there are no reference points to measure the absolute rate time. For the purposes of this investigation the iteration count was assumed to be equal to time, called: iteration time, and denoted ***i***.

1.6 The Inverted Fractal – a view from within

To simulate observations from a position or perspective within the fractal set the fractal was (simply) inverted. By doing this, the focus is placed on the newly added triangle (bit), holding its size constant, and allowing the previous bit sizes to expand – rather than diminish as with the classic fractal. The inverted fractal reveals this fractal expansion – termed fractspansion as demonstrated in Figure 4 (B). Colours (red, blue, black followed by purple) and numbers are used to demonstrate the expansion.

The size of the initial red iteration 0 triangle, with fractspansion, expands relative to the new. A practical example of this fractspansion principle is to think of the growth of a tree. Follow the first (new growth) stem size – keeping this stem/branch size at a constant size – as the rest of the tree grows. To grow more branches, the volume of the earlier/older branches must expand. Now think of sitting on one the branches of a tree that is infinitely large, infinitely growing. What would you see in front? What would you see behind?

If an observer were to remain at this constant static position (or alternatively change position by zooming forward into the structure) they would experience – according to the principles of the iterating fractal, as demonstrated in Figure 4 (A) – an infinity of self-similar Koch Snowflake like structure ahead of them, at never see triangles more than four or five iteration/sizes. There will always be (classical) fractal shape ahead, and looking back the observer would see expansion.

1.7 Spiral Propagation

The propagation of triangles in the (inverted) Koch Snowflake fractal – or bits of information on any fractal – is not linear, but rather a (logarithmic) spiral: as shown in Figure 4B (above), and in more detail in Figure 5 below. Each new bit 'branches' at an angle: in an act of rotational symmetry. The first person to describe this 'angled' process was Leonardo Da Vinci [33]. Appendix figure 3 demonstrates the wave/spiral propagation properties of 'bits' in an emergent fractal. In 'A' a triangle bit with a red dot is iterated revealing a spiral and 'superposition'; and in 'B' shown is the respective rotation of the propagating triangle bits through 360^0; and 'C' demonstrates a (non logarithmic) through time – forming a wave.

1.8 Hypothesis

Observation and behaviour of the macro universe – as described by the lambda CDM model – and the micro quantum world are inextricably linked through the geometry of the classical and (inverted) fractal.

For this to be so, the fractal will have to demonstrate not only accelerating expansion, but also temporal behaviour that matches both the observations and the theories (in terms of the shape and behaviour) of the cosmos, including:

1. a 'singularity' (Big Bang) beginning, (section 4.1);
2. an inflation epoch (section 4.5.2 and 4.6.1);
3. the presence, and dominance of a 'uniform' Cosmic Microwave Background (section 4.3);
4. a Hubble's Law [34] (section 4.4);
5. accelerating expansion (section 4.5);
6. a cosmological constant (section 4.5);
7. describe and predict the distribution of galaxies in the observable universe (section 4.10);
8. and offer insights into the nature of quantum mechanics (section 4.7).

Results from this investigation offer insights to all objects – of all scales – of a fractal nature, including:

1. the inferred emptiness of the atom;
2. the growth of trees (section 4.14.1);
3. the properties of evolution;
4. and the expansion of perceived value – with time (section 4.15).

2 METHODS

To create a quantitative data series for analysis of the inverted fractal, the classical Koch Snowflake area equations were adapted to account for this perspective, and a spreadsheet model [35] was developed to trace area expansion with iteration.

The scope of this investigation was limited to the two-dimensional; three-dimensional space or volume can be inferred from this initial assumption. Changes in the areas of triangles, and distances between points in the fractal set were measured and analysed to determine whether the fractal area and distance between points expand.

2.1 Spreadsheet Model

A data table was produced (Table 1) to calculate the area growth at each, and every iteration of a single triangle. Area was calculated from the following formula (1) measured in standard (arbitrary) centimetres (***cm***)

$$A = \frac{l^2\sqrt{3}}{4} \tag{1}$$

where (A) is the area of a single triangle, and where l is the triangle's base length. l was placed in Table 1 and was set to 1.51967128766173***cm*** so that the area of the first triangle (i_0) approximated an arbitrary area of 1 cm^2. To expand the triangle with iteration the base length was multiplied by a factor of 3. The iteration number was placed in a column, followed by the base length of the equilateral triangle, and in the final column the formula to calculate the area of the triangle. Calculations were made to the 10th iteration, and the results graphed.

2.2 Distance and Displacement

To measure and analyse the changes in position of points (the distance between points in the set after iteration) a second data table (table 2) was developed on the spreadsheet. The triangle's geometric centre points were chosen as the points to measure. Formula (2) below calculated the inscribed radius of an equilateral triangle. Distance between points was calculated by adding the inscribed radius of the first triangle ($\boldsymbol{i_0}$) to the inscribed radius of the next expanded triangle ($\boldsymbol{i_1}$) described by

$$r = \frac{\sqrt{3}}{6} l \cdot \tag{2}$$

From the radius distance measurements; displacement, displacement expansion ratio, velocity, acceleration, and expansion acceleration ratio for each and every iteration time were calculated using classical mechanics equations.

The change in distance between points was recorded, as was the change in displacement (distance from $\boldsymbol{i_0}$).

2.3 Area Expansion of the Total Inverted Fractal

With iteration, new triangles are (in discrete quantities) introduced into the set – at an exponential rate. While the areas of new triangles remain constant, the earlier triangles expand, and by this the total fractal set expands. To calculate the area change of a total inverted fractal (as it iterated), the area of the single triangle (at each iteration time) was multiplied by its corresponding quantity of triangles (at each iteration time).

Two data tables (tables 3 and 4 in the spreadsheet file) were developed. Table 3 columns were filled with the calculated triangle areas at each of the corresponding iteration time – beginning with the birth of the triangle and continuing to iteration ten.

Table 4 triangle areas of table 3 were multiplied by the number of triangles in the series corresponding with their iteration time.

Values calculated in table 3 and 4 were totalled and analysed in a new table (table 5). Analysed were: total area expansion per iteration, expansion ratio, expansion velocity, expansion acceleration, and expansion acceleration ratio. Calculations in the columns used kinematic equations developed below.

2.4 Kinematics

Classical physics equations were used to calculate velocity and acceleration of: the receding points (table 2) and the increasing area (table 5).

2.4.1 Velocity

Velocity ($\boldsymbol{v}$) was calculated by the following equation

$$v = \frac{\Delta d}{\Delta i} \tag{3}$$

where classical time was exchanged for iteration time (i). Velocity is measured in standard units per iteration $cm^{-1}\,i^{-1}$ for receding points and $cm^{-2}\,i^{-1}$ for increasing area.

2.4.2 Acceleration

Acceleration (a) was calculated by the following equation

$$a = \frac{\Delta v}{\Delta i} \tag{4}$$

Acceleration is measured in standard units per iteration $cm^{-1}\,i^{-2}$ and $cm^{-2}\,i^{-2}$.

2.4.3 Ratios

Ratios of displacement expansion and acceleration were calculated by dividing the outcome of i_1 by the outcome of i_0.

The same method of ratio calculation was used to determine change or expansion of area.

2.5 Spiral Propagation

The method given thus far assumes, and calculates the linear circumference of this spiral (the red line below in figure 5) and not the true displacement (the blue radius lines below). This method was justified by arguing the required radius (or displacement) of the logarithmic spiral calculation was too complex to calculate, (and beyond the scope of this investigation), and that expansion inferences from inverted fractal could be made from the linear circumference alone. As an aside, a spiral model was created independently, and radii measured to test whether spiral results were consistent with the linear results in the investigation. Measurements were made using TI – Nspire™ geometric software.

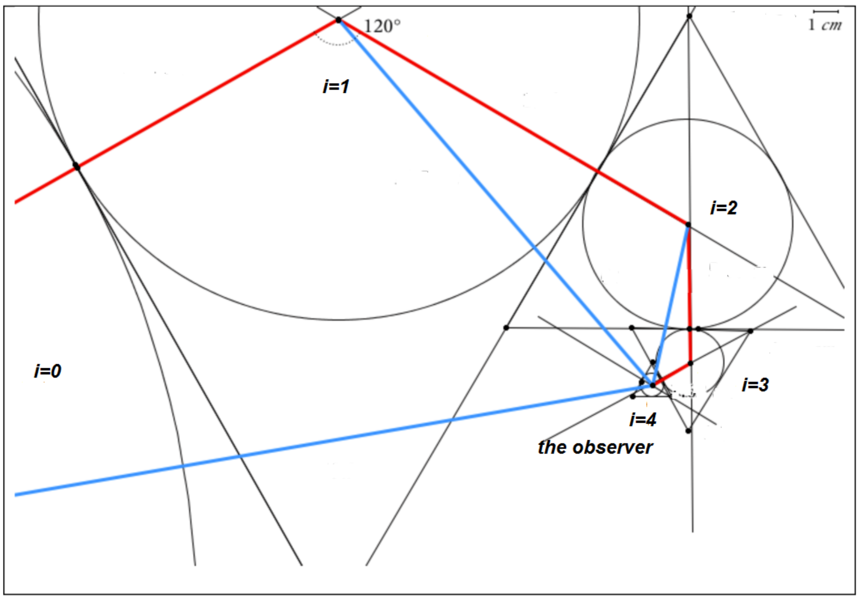


Figure 5. Spiral Observation of Points from 'the Observer'. Displacement measurements (blue) from radii on the iterating Koch Snowflake. The inverted fractal propagates triangle bits as it iterates (from i0 to i∞). The displacement is measured from an arbitrary observation point (i=4) and the previous (discrete) triangle centres to iteration 0. This displacement was used in

the calculation of the fractal/Hubble constant. The red line traces the circumference (the distance) of the fractal spiral, and the blue line the displacement of the fractal spiral from an arbitrary centre of observation. cm = centimetres.

2.6 Hubble's Law and Diagram

To test for Hubble's Law, a Hubble (like) a scatter graph titled 'The Fractal/Hubble diagram' was constructed from the results of the recession velocity and distance calculations (in table 2 of inverted fractal spread sheet file). On the x-axis was the displacement (total distance) of triangle centre points at each iteration time from t_0; and on the y-axis the expansion velocity at each iteration time. A best fitting linear regression line was calculated and a Hubble's Law equation (5) was derived

$$v = H_{i0} D \tag{5}$$

where H_{i0} the (present) Hubble constant (the gradient), and D the distance.

2.7 Acceleration vs. Distance

Using the same methods as used to develop the Hubble diagram (as described above in 2.4) an 'acceleration vs. distance' diagram was created, regressed, and an expansion constant derived.

2.8 Point/Cluster Distribution form Observation Point

To analysis whether astronomical observations match observation from within a fractal and explain the point clustering on the fractal-Hubble diagram, relevant tables in fractspansion spreadsheet model [36] were analysed – particularly the table from where the fractal-Hubble diagram was derived.

Calculations were made (listed below) and diagrams created – these may also be viewed in the spreadsheet model:

1. The quantity of triangle sizes per total distance increment on the fractal-Hubble diagram was calculated by: counting the quantity of triangle sizes (in distance column in table 2) and dividing this by the distance increments measured in the sample. See Table 2a of spreadsheet model.
2. The quantity of triangles at each increment was calculated by totalling the quantity of triangles (from table 4) for each respective iteration-distance.
3. An amended Fractal-Hubble diagram – combining (recessional) velocity with the quantity of triangles at every distance – was created. See table 7 of spreadsheet model.

3 RESULTS

Figures 6 to 14 show graphically the results of the experiment.

3.1 Expansion of Initial Triangle

The area of the initial triangle of the inverted Koch Snowflake fractal increased exponentially – shown here in Figure 6.

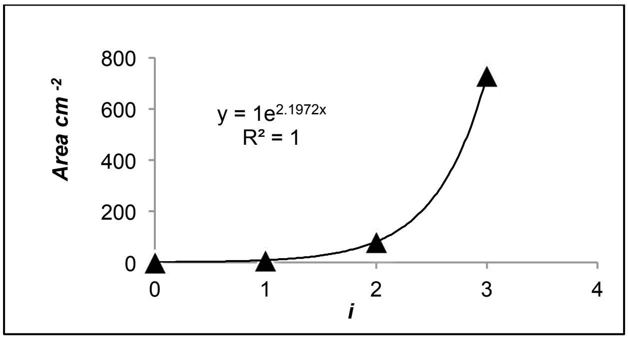


Figure 6. Area Expansion of a single triangle in the inverted Koch Snowflake fractal by iteration time (*i*). *cm* = centimetres.

This expansion with respect to iteration time is written as

$$\boldsymbol{A = 1e^{2.197i}}. \tag{6}$$

3.2 Total Fractal Expansion

The area of the total fractal (Figure 7A) and the distance between points (Figure 7B) of the inverted fractal also expanded exponentially.

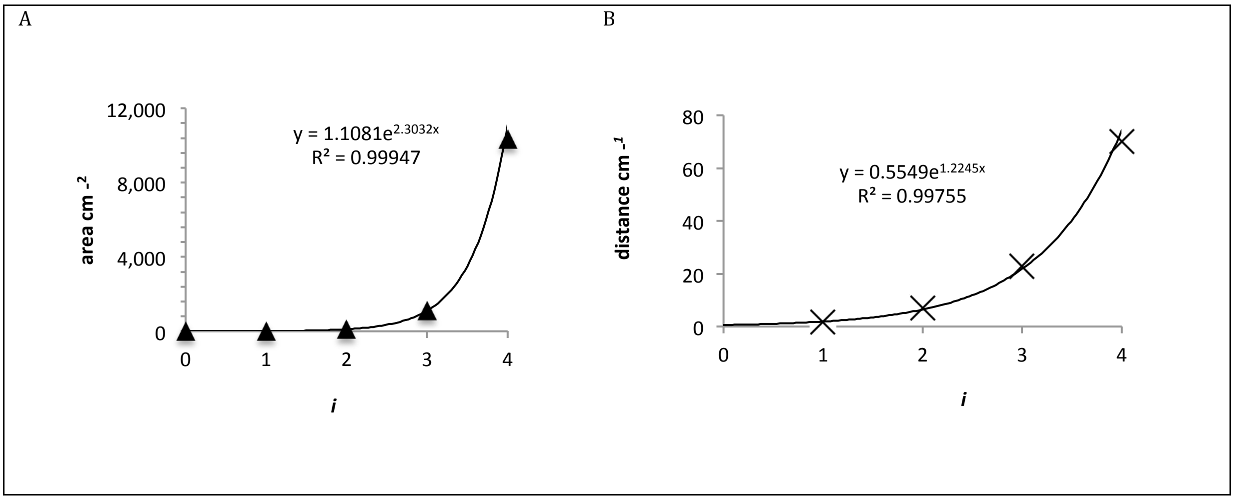


Figure 7. Inverted Koch Snowflake fractal expansion per iteration time (t). (A) total area expansion and (B) distance between points. *cm* = centimetres. *i* = iteration time.

The expansion of the total area ($\boldsymbol{A^T}$) is described as

$$A^T = 1.1081e^{2.3032i} \quad (7)$$

The expansion of distance between points (D) is described by the equation

$$D = 0.5549e^{1.2245i} \quad (8)$$

3.3 Expansion Ratios

The expansion ratios for the given sample (shown below in Figure 8A and 8B) were initially high (12 and 4 respectively), followed by a decreasing range, to settle finally at the stable ratio of expansion of 9 and 3 respectively (for the tested 10 iterations).

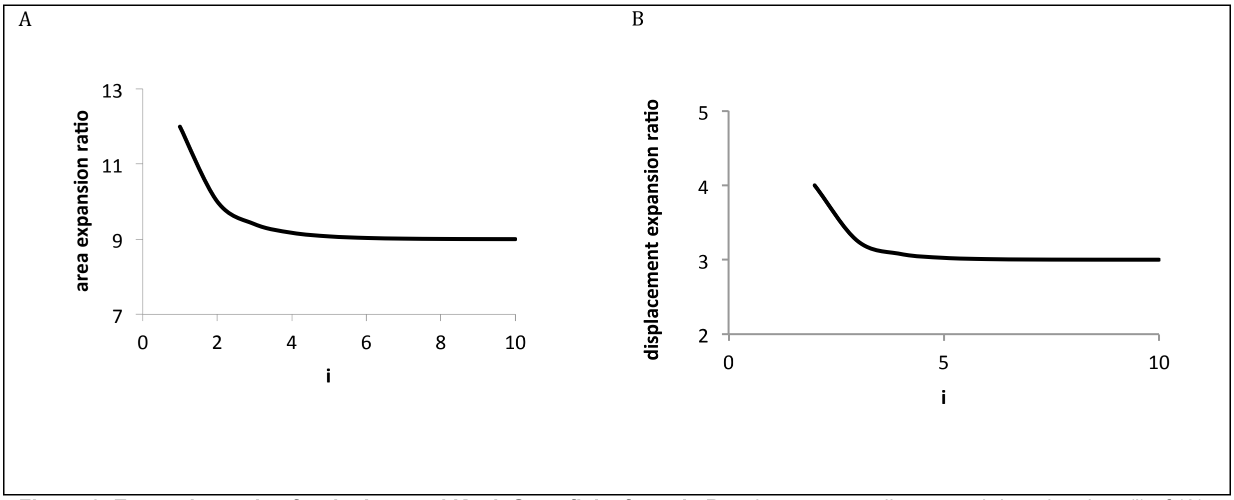


Figure 8. Expansion ratios for the Inverted Koch Snowflake fractal. Results corresponding to each iteration time (i) of (A) total area; and (B) between points. *i* = iteration time.

3.4 Velocity

The (recession) velocities for both total area and distance between points (Figures 9A and 9B respectively) increased exponentially per iteration time.

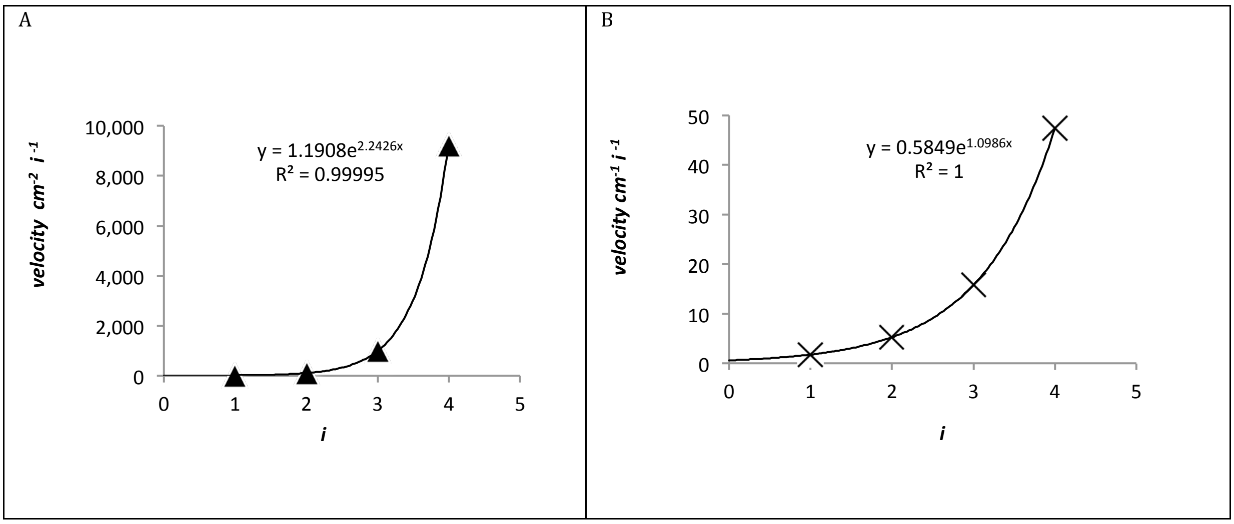


Figure 9. Inverted Koch Snowflake fractal (expansion) velocity. Expansion velocity of the inverted fractal at each corresponding iteration time (i): (A) expansion of total area, and (B) distance between points. cm = centimetres. i = iteration time.

Velocity is described by the following equations respectively

$$\boldsymbol{v = 1.1908e^{2.3032i}} \tag{9}$$

$$\boldsymbol{v^T = 0.5549e^{1.0986i}} \tag{10}$$

where $\boldsymbol{v^T}$ is the (recession) velocity of the total area; and $\boldsymbol{v}$ the (recession) velocity of distance between points.

3.5 Spiral Propagation

Displacement measurements – and the derived Hubble diagram, see Appendix figure 1 – from this radius model were – as expected produced significantly lower values than the (calculated) circumference non-vector method; but nonetheless they share the same (exponential) behaviour. Appendix Figure 10 below shows in the distance between centre points in red, and in blue the displacement.

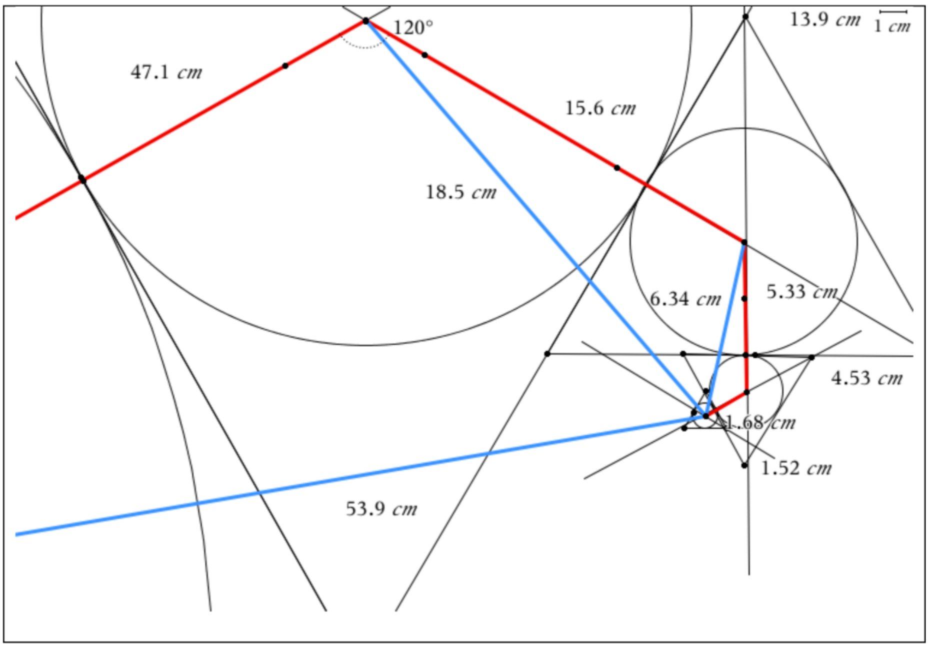


Figure 10. Displacement measurements from radii on the iterating Koch Snowflake created with TI-Nspire ™ software. Displacement is measured between (discrete) triangle centres and used in the calculation of the fractal/Hubble constant. The red line traces the circumference (the distance) of the fractal spiral, and the blue line the displacement of the fractal spiral from an arbitrary centre of observation. cm = centimetres.

3.6 The Fractal/ Hubble Diagram

As the distance between centre points increases (at each corresponding iteration time), so too does the recession velocity of the points – as shown in Figure 10 below.

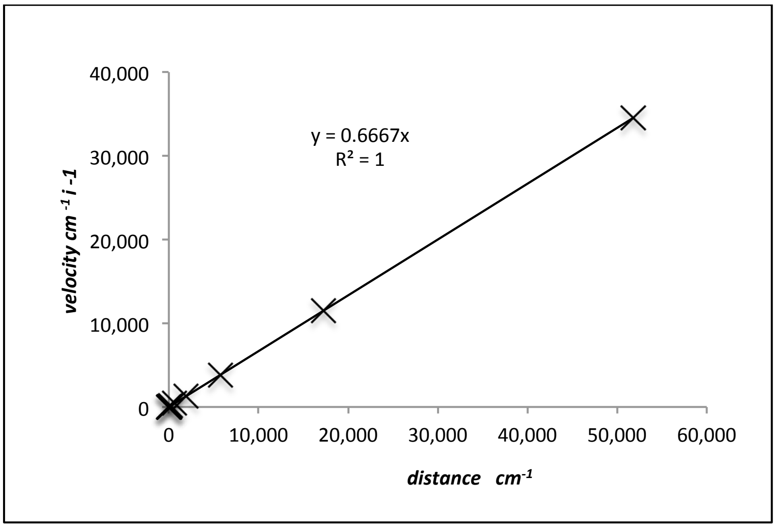


Figure 11. The Fractal Hubble diagram. As distance between triangle geometric centres increases with iteration, the recession velocity of the points increases. cm = centimetres. *i* = iteration time.

Recession velocity vs. distance of the fractal is described by the equation

$$v = \mathbf{0.6679}\boldsymbol{D} \tag{11}$$

where the constant factor is measured in units of $\boldsymbol{cm^{-1}\, i^{-1}\, cm^{-1}}$.

The spiral radius (see Appendix Figure 1 and Appendix Table 1 for details) – where the centre is the observation point – resulted in a fractal Hubble equation of

$$v = \mathbf{0.6581}\boldsymbol{D} \tag{12}$$

3.7 Acceleration of Area and Distance Between Points

The accelerations for both total area and (recession) distance between points (Figure 12A and 12B respectively) increased exponentially per iteration time.

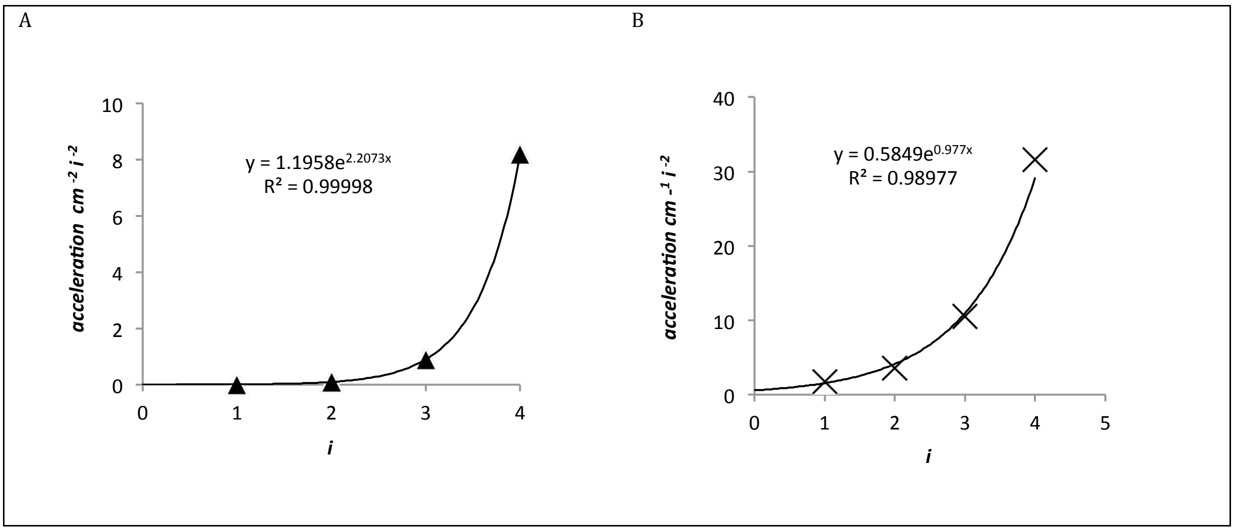


Figure 12. Inverted Koch Snowflake fractal (expansion) acceleration. Acceleration of the inverted fractal at each corresponding iteration time (i): (A) expansion of total area, and (B) distance between points. cm = centimetres. i = iteration time.

Acceleration is described by the following equations respectively

$$\boldsymbol{a^T = 1.1958e^{2.2073i}} \tag{13}$$

$$\boldsymbol{a = 0.5849e^{0.977i}} \tag{14}$$

where $\boldsymbol{a^T}$ is the (recession) acceleration of the total area, and $\boldsymbol{a}$ the (recession) acceleration of distance between points.

As function of distance from the observer, the distance of centre points increases (at each corresponding iteration time) from an observer, so does the recession acceleration of the points (expanding away) – as shown in Figure 13 below.

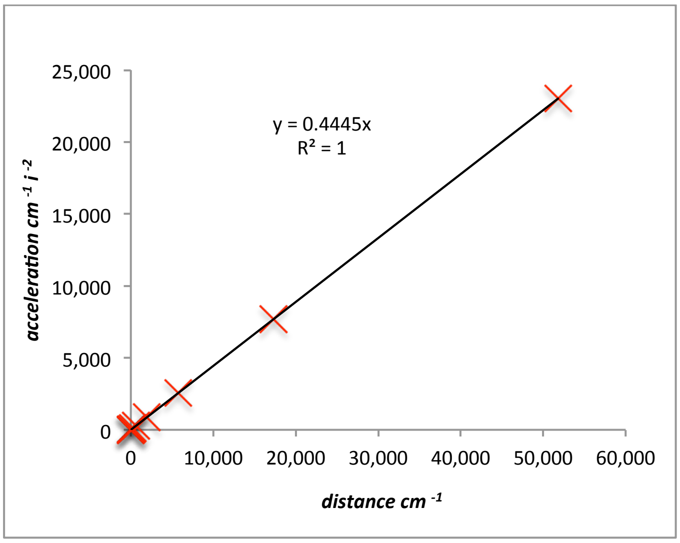


Figure 13. Recessional acceleration vs. distance on the inverted Koch Snowflake fractal. As distance between triangle geometric centres increases with iteration, the recession acceleration of the points increases. cm = centimetres. i = iteration time.

The recession acceleration of points at each iteration time at differing distances on the inverted fractal is described by the equation

$$\boldsymbol{a = 0.4447D} \tag{15}$$

where the constant factor is measured in units of $\boldsymbol{cm^{-1}\ i^{-2}\ cm^{-1}}$. $\boldsymbol{a}$ = acceleration; $\boldsymbol{D}$ = distance.

The spiral radius (see Appendix Figure 2 and Appendix Table 1 for details) – where the centre is the observation point – resulted in an acceleration equation of

The spiral radius – where the centre is the observation point – equation resulted (see Appendix Figure 3 for details)

$$a = 0.4295D. \tag{16}$$

3.8 Point (and Triangle/Bit) Distribution from an Inverted Observation Perspective

Figure 14 below shows the quantity of triangles by distance – between geometric centres – from an arbitrary observation point. The quantity of triangles decreased exponentially from 7.86E+05 bits from the observation point (distance 0), to a quantity of 1 bit at distance 51800cm^{-1}.

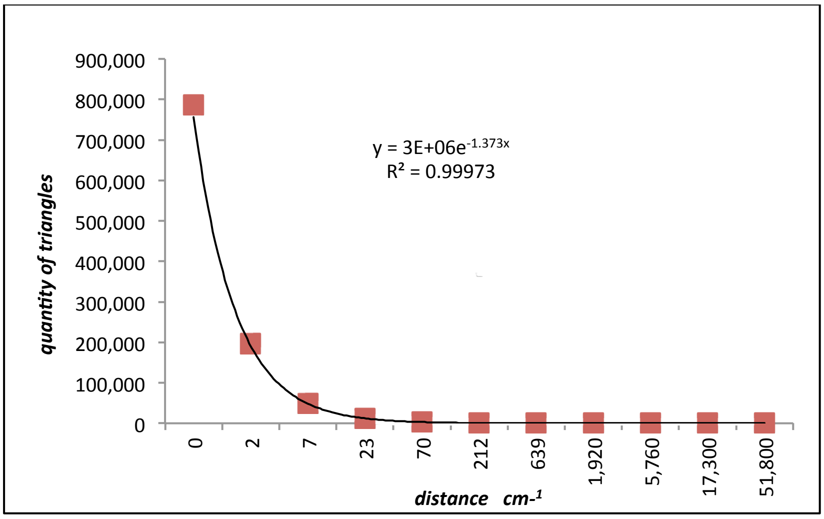


Figure 14. Quantity of triangles at each distance (point) from the observer on the inverted Koch Snowflake fractal. As the distance between triangle geometric centres increases (exponentially) with iteration, and so increasing the distance from the observer, the quantity of triangles per iteration decreases exponentially to a quantity of one – at time 0. cm = centimetre.

Eight of the ten measurement points were located inside the first (1.20E+4cm^{-1}) increment distance. The remaining 2 measurement points are outside this range.

4 DISCUSSIONS

While results from this investigation point immediately to the field of cosmology, owing to the universality of the fractal, the findings are relevant to all things fractal – able to be observed or experienced, in principle, throughout. In this discussion I shall focus on the cosmological applications of fractspansion, ending with biological (accelerating tree growth) anomalies and classical world (marginal) economics.

Fractspansion has direct relevance to problems associated with the standard ΛCMB model of cosmology, and with the de Sitter model of the universe. It demonstrated a beginning followed by (arbitrarily) rapid expansion – all as the result of the iteration of a simple rule of producing discrete bits and resulting in the wave like propagation, spiralling into infinity with increasing frequency.

4.1 Singularity

The fractal is in isolation, it is expanding into 'nothing'. The single inverted triangle expansion (Figure 6) demonstrates a 'Big Bang' singularity beginning. Its area begins arbitrary small (it could be set to any size value, one akin to the Planck area), and is followed by exponential area expansion as (iteration) time passes. It is not an explosion: it is an infinite exponential emergent expansion of area – consistent with descriptions that 'space itself that is expanding'. As the initial bit size is very small (Plank size): this is consistent with high frequency 'hot' big bang. While the model demonstrates rapid expansion (see inflation below), it does not demonstrate a singularity accumulation of all matter in one place at one time. The model does though suggest the emergence of matter through time.

4.2 Expansion in Excess of Light Speed and the Cosmological Principle

The propagation/production 'speed' (quantity of bits per unit time) of the fractal frontier is constant: its propagation is analogous to light speed where $c = f\lambda$. For the sake of the model this propagation speed is arbitrary. The frequency of bits produced per unit time (and passing through an arbitrary constant distance) increases as the wavelength (the size of the fractal bit (or triangle propagating as a spiral – see figure 5) increases. Constant light speed – and its propagation – is consistent with fractal bit propagation.

From this the inverted fractal demonstrates the cosmos's ability to expand at a speed greater than the speed of light – as proposed by Albert Einstein in his General Theory of Relativity. This expansion speed of point is consistent with and addresses issues surrounding the particle horizon problem and the cosmological principle (axiom). Points within the triangle are initially close enough to have causal contact, but this will not last – as they accelerate away from each other. Analogous to the speed of light, the fractal has a constant propagation speed (of triangle bit coming into existence): this propagation speed can be assumed to be, in principle, able to be surpassed by the speed of the (accelerating) expanding frontier of the fractal itself.

4.3 The Cosmic Microwave Background

This simplest of demonstrations is consistent with the observed very cool cosmic microwave background (CMB). To an observer anywhere in the set, this initial triangle will dominate, but will not be seen by the same observer no more than 6 or 7 iterations distant – the classical shape equilibrium iteration count, or observable fractal distance (as introduced in section 1.4). The initial triangle is both isotropic and homogeneous with expansion. The expansion of the initial triangle is due to iteration: with additional iteration time, its size and thus wavelength (due to the spiralling propagation) increases, while its frequency decreases. This is consistent with electromagnetism theory and will be the topic of further research.

4.4 Hubble's Law

Figure 11 shows the velocity of the expansion at each iteration time – for both total area and distance between points – increases with distance from the observer. The significance of this points to Edwin Hubble's observations and all the conjectures surrounding the expanding universe. It is the area between the points that is increasing. In accordance with Hubble's Law, all points (observed from any observation position in the iterating fractal set) will appear to recede (away) from an observer, and as a consequence, the observer will perceive themselves to be at the centre of the set.

When velocity ($\boldsymbol{v}$) is plotted against distance of points ($\boldsymbol{D}$) (Figure 11, and Appendix Figure 1) the inverted fractal demonstrates Hubble's Law described by the equation

$$\boldsymbol{v = F_v D} \tag{17}$$

where ($\boldsymbol{F_v}$) is the slope of the line of best fit – the fractal (Hubble) recession velocity constant.

The scale invariance of the fractal Hubble diagram concurs with the development of the original Hubble curve, from its 1929 original, to the improved 1931 'Hubble and Humason' [37]. As with any perfect fractal: however deep one looks, the (Hubble) shape will remain constant.

4.5 Accelerating Expansion and Fractal Lambda –Fractspansion:

Figure 13 (above) is consistent with the 1998 astronomical discovery (by observation) of the accelerating expanding universe and conjectures surrounding the term 'dark energy' and the cosmological constant (lambda). It can be inferred (from the fractal) that the accelerating expansion of the universe, with respect to distance (Figure 8) is a property of the fractal, a problem of geometry

$$\boldsymbol{a = F_a D} \tag{18}$$

where the expansion with respect to distance can be described by the equation

where $\boldsymbol{F_a}$ is the fractal (cosmological) recession acceleration constant measured in units of $\boldsymbol{cm^{-1}\, i^{-2}\, cm^{-1}}$.

The constant $\boldsymbol{F_a}$ (in equation 18) may be interpreted as a fractal a (cosmological constant) lambda with respect to point acceleration and distance.

The acceleration between points with respect to time (from equation 14) is described as

$$\boldsymbol{a = a_0 e^{F\lambda i}} \tag{19}$$

where the constant $\boldsymbol{F\lambda}$ may be interpreted as a fractal 'Cosmological Constant' Lambda with respect to point acceleration and iteration time.

With entry (or birth) of new triangles into the fractal set the total fractal area of the total universe (Figure 14 above), growths exponentially. The total area expansion with respect to time is described by the function

$$A^T = A_0 e^{F\Lambda i} \quad (20)$$

where $F\Lambda$ is a fractal constant with respect to total area expansion and time.

Fractspansion appears similar to (but not the same as) the theory general relativity in that it is the geometry of space-time that is curved. With general relativity the mass of objects curves space-time, and as a result we observe or experience gravity: with fractal 'fractspansion' by means of (what is described as) quantum mechanics or electromagnetism space curves and as a result of particle/bit propagation. What we observe or experience as a result is the dark energy: the micro and the macro are different aspects of the same object.

4.5.1 Changing Strength of 'Dark Energy'

From a fractspansion (inverted fractal) perspective, the strength of the expansion will increase the further from the observer the points are. This is consistent with current thinking of the strength of the dark energy. In an act of relativity, the observer will – due to lack of a frame of reference other than expanding points beyond – 'notice' no expansion, even if it can be argued they too are expanding.

4.5.2 Quintessence

While the fractal constant $F\lambda$ is in this investigation constant and relevant for only the Koch Snowflake fractal, in reality it may well be dynamic – able to change with changes of other trophic stimuli such as gravity, as posited in quintessence theory [38].

4.6 Inflation Theory and Inverted Fractal-Fractspansion

The isolated (unbounded) fractal may offer a totally alternative explanation to the early 'inflation epoch' [39] conjecture - and there is no need for an alternative 'eternal inflation' explanation. From a perspective within, if the fractal is assumed to be iterating/propagating bits at 'the speed of light', how quickly will the fractal frontier grow from an initial state? From equation 20, the initial area (the Planck area) was set to the Planck length constant (1.61619926 ×10^{-35}), and the time taken calculated by

$$i = \frac{1}{2.2073} \ln\left(2.61223 \times 10^{70}\right) \quad (21)$$

It takes the inverted fractal 72.59 (2s.f.) iteration times to expand to an area of $1cm^{-2}$. If the propagation speed of triangles on the (Koch) fractal is scaled up to be equivalent to light speed – allowing for the propagation of 6 bits per iterations cycle[40] – these 72.59 iterations may be consistent with the (small Plank) time period and rapid expansion of space conjectured during the early 'inflation epoch' of the universe. Further discussion on this issue is beyond the scope of this investigation, but suffice to say, 'inflation theory' as it stands may be redundant: no other examination is needed if this extraordinarily rapid growth is a property of the fractal on its own: the key energy that propagates the expansion is light itself, right down that the smallest of scales. This insight extends to the 'dark energy' expansion of the universe.

4.6.1 High Initial Expansion Ratios – Inflation?

Notwithstanding the discussion on inflation theory above, the first iterations of the fractal reveals an anomaly period (Figure 8A, and 8B) of high expansion ratio for both area expansion and distance between points. Though the ratio values shown are minimal in comparison to Allan Guth's inflation theory's actual predictions, the presence of this anomaly along with the above (4.6) – in the context of the other observed cosmic similarities with the fractal – may well explain the theory and cannot be over looked.

4.7 Quantum Mechanics (like) Properties of the Fractal

The assumption of observation from within the set, from a fixed position, congers fractal's uncanny resemblance to properties and problems shared with objects described only by the quantum mechanics and the electromagnetic spectrum.

When isolated, the iterating (snowflake) fractal is produced by an infinitely of discrete triangles (bits). The snowflake is a superposition of all triangles, in one place, at one time. The production of new triangles propagates in the geometry of a spiral: rotating in a arbitrary direction to form – when viewed from a side elevation – a logarithmic sinusoidal wave, comparable to the described electromagnetic spectrum. This spiralling wave like propagation is illustrated below in Figure 4 B and in Figure 10, and demonstrated in Appendix Figure 3. If the iteration time is assumed constant: the speed of propagation must be constant. This is consistent with realities constant light speed.

Location or position within this infinite set is only known when observed or measured; otherwise all positions are possible – at the same time. These 'quantum' like features of the fractal are an essential background to this investigation – one that will not be taken further in this publication, but cannot be over looked.

Put together, this single fractal attractor can explain – small-scale – quantum behaviour, while from another aspect, accelerated expansion: while it doesn't demonstrate gravity, it must be a candidate as an explanation to the great unifying question, deserving of further investigation.

4.8 Vacuum Catastrophe

The vacuum catastrophe – the 10^{120} calculation discrepancy between the quantum vacuum energy and its prediction of the (cosmological constant) expansion of space – maybe explained and resolved from a fractal perspective. If we assume (it can be demonstrated to do so and will be the topic of a coming paper) the classical view of the fractal (the Snowflake production) is behaving as a quantum system: the 'standard' iteration 0 triangle area size is akin to the assumed (Plank) 'particle' or 'bit'. If this area used to calculate the total area of the (expanded) set at any arbitrary iteration-time, the result is an extremely large value: quite within the order of the 'vacuum catastrophe' value – if given modest iteration-time. The rationale for this claim – of using the iteration 0 triangle – stems from the assumed arbitrary observation position in – what is in principle – an infinite set, presented in the introduction. This assumed measurement is akin to the quantum measurement.

To resolve this 'vacuum' discrepancy: if the total area of the inverted fractal set – at any arbitrary iteration-time – is divided by the respective (expanded) bit area 'sizes' at respective iteration times, the of the expanded triangle's area value (after their expansion at each, and every, iteration time) the will equate to a lower – and more realistic – value. The total area will equate to the total number of triangles propagated in the set. In a fractal, in principle all triangles are as identical as the iteration 0 standard triangle, and only differ in scale due to the fractspansion. In summary, if the universe is a fractal, then its area and expansion are the same as the (quantum like – micro) production of it: they are different ends of the same object.

4.9 Galaxy Distribution – Large and Small Scales

At the point of observation (the origin on the Fractal-Hubble diagram) there is a quantity of 786,432 triangles, all of which are the same size as the observers triangle viewing position. The clustering of the measurement points near the origin of the diagram is due to the location the observer within the (inverted) fractal, and the relative size of these triangles near the observer. From this, the observer will, in principle, be surrounded by these sized triangles. The observer will not see all these triangles directly – how many they will see is beyond of the scope of this investigation – but it will be many, and it may mean some of the nearby triangles on other branches could be approaching the observer: this concurs with what is observed in the cosmos, with nearby galaxies to the Milky way not all expanding away. As we view further out, the quantity of triangles decrease – while the area of the respective triangles increase. This property of clustering near the origin is scale invariant: no matter the distance, this pattern of clustering near the origin will remain. From this, the universe will not be isotropic or homogenous. If our current position were observed from the (current) far reaches of the universe, a clustering ('ball') of galaxies would be observed, after a relatively smooth section of space.

4.9.1 Clustering and the Fractal-Hubble Law

Figure 15 (below) combines – on the original (Koch snowflake) fractal-Hubble diagram – the quantity of triangles at each distance point with the (recessional) velocity at the same distance point. The diagram reveals the relationship between the clustering of measurement points close to the (low recessional velocity) origin, and the smooth distribution (high recessional velocity) at large distances.

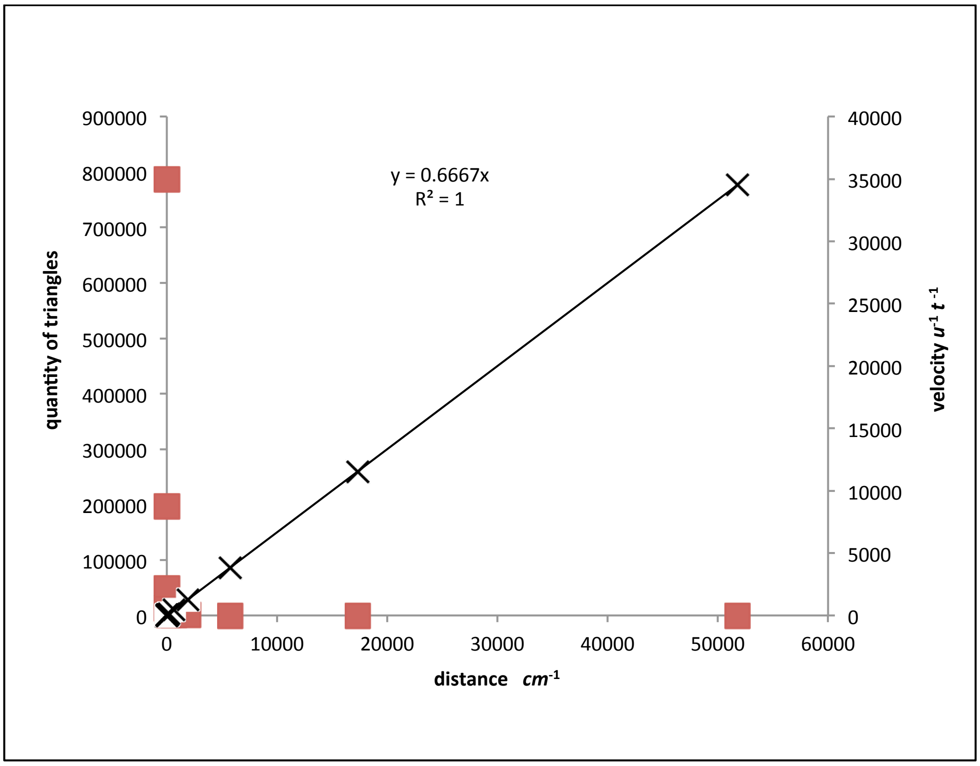


Figure 15. The inverted Fractal Hubble diagram combined with the quantity of triangles (red boxes. As distance increases with respect to iteration-time: the recession velocity of distance between geometric points increases; while the quantity of triangles at each distance decreases. *cm* = centimetre, *i* = iteration-time.

4.9.2 Tree metaphor

If the observation from deep within a snowflake fractal is substituted with observation from high within a common branching tree (also a natural fractal): the clustering of points on the Fractal-Hubble diagram would equally correspond to the clustering of self-similar (sized) branches – in the tree – surrounding the observer. If the observer were to look down, inwards from the outer branches – towards the trunk of the tree – the branch (nodes) quantity would decrease, the volume of the single branches would increase, and the branch 'clustering' would smooth out.

4.9.3 Distribution and the Universe

The distribution of measurement-point (triangle centre points) clustering along the Fractal-Hubble diagram matches the transitions from rough to smooth as revealed in (recent) galaxy surveys (figure 1 and 2 above). Observation and model diagrams of correlated in Appendix figure 3. The distribution of galaxies in the universe (figure 1) is of a nature expected if one was viewing from within a fractal universe: looking back through the universe – in terms of distance and time – to its – now expanded – origin. The smooth outer reaches of the universe – out near the CMB origin of the universe – is the 'trunk' of the universe, and the rough fractal clusters are 'the branches' of the universe. Using a tree (fractal) as a metaphor of the universe is not to say the universe is a fractal tree structure: it is to say, just as a tree is a fractal structure, the universe is a fractal structure. It should be noted trees growth have recently been found to also increase at accelerating rates [41], [42].

4.9.4 A Galaxy Distribution Prediction.

In 2016 a paper was published suggesting the large scale distribution of galaxies is as clustered on near scales, and consequently the number of galaxies in the observable universe lifted from 100 to 200 billion to 2 trillion[43]. This new figure is based on models and is at best an extrapolation. If this extrapolation is so, and the James Webb space telescope will reveal it: if it does, it will stand as a contradiction to the (inverted) fractal model distribution. I predict they will not find these galaxies and the distribution observed today will hold.

4.9.5 Super Clusters

There are many questions and issues arising from this finding – all of which, at this point, are beyond the scope of this investigation, but not beyond the scope of reason.

Proponents of fractal cosmology are expecting to see even larger galactic clusters further out into the large-scale homogeneous region. The fractspansion model would concur with this, only that the distance (in principle) to the next cluster (next larger branch or node) may be beyond the age of the universe – and or may not exist at all. If they do, smoothness will extend out beyond this point.

4.9.6 Emergent Structure

A fractal universe would imply an emergent structure – the whole made of many parts – just as the tree is made of many branches. It may force us to question the initial conditions of the big bang beginning. Namely, whether all mass (in the universe) was together in one place and at one time. It could now be argued – from the principles of fractal emergence – the universe developed/evolved mass from the bottom up, with the passing of time.

4.9.7 Growth Explanation

If the branches of the tree are akin to the small-scale galaxy clusters of the universe, we may find it profitable to search for growth explanations to the universe in the branches too. Given it is at the branches where trees grow form and the trunk and first branches are only infrastructure to the total emergent structure. This would suggest growth begins at the smallest of scales: at the sub-atomic level, the Planck scale.

The quantum nature of the fractal has been addressed in my original publication – and must be further investigated [40].

4.9.8 Dark Matter

This fractal model offers insight to the dark matter structure of the universe also. The dark matter appears to be concentrated where galaxies are concentrated conforming to the described structure in the introduction. The current mapped structure of the dark matter, and the structure of the observed universe – matching this 'inverted' fractal model – must through the fractal have a direct connection. Merger Halo Trees are where the dark matter is strongest and these structures are by their nature fractal – 'trees' – themselves. These halo have evolved from the early 'smooth' universe to the near clustered structure observed today – this is consistent the presented model.

On December 7th 2016, Sky and Telescope issued an article 'Not-So-Clumpy Dark Matter Poses Cosmological Challenge'[44]: where they outlined the "*the distribution of dark matter in the modern universe is smoother than predicted from observations of a far younger universe*".

> "..the distribution KIDS found is smoother than predicted. The apparent discrepancy poses a challenge for astronomers to explain."

This 'smoother than predicted' will be of great importance to verifying the model: the LSST survey results will be of great interest.

4.9.9 General Relativity

What is clear from the model (and observation) space is not homogenous and is not isotropic: this is in total conflict with general relativity assumptions (see below 4.10).

What a fractal universe means for the future of General Relativity theory is unclear and beyond the scope of the author – though it is conceivable it may have to be adapted to take account the geometry of the fractal. Work has already begun in this area: from noted theorist Laurent Nottale [45],[46] and others [47].

4.9.10 Fractal Dimension

Recent studies have shown fractal dimension decreases with increased *z* values [48]. This may also complement my study.

4.10 Refuting the Cosmological Principle

The inverted fractal/fractspansion model is totally inconsistent with the cosmological principle – the topology it presents is neither homogenous nor isotropic as currently assumed – but it is consistent with astronomical observation – as discussed above. The clustering of points on the fractal model near the observer, and 'smoothness' farther away (as demonstrated in figure 15 above) is consistent with non homogenous observations and investigations by SSDS and WiggleZ and others; while in terms of isotropy, the Hubble expansion will endure everywhere within the set; however, the views of the distributions will be viewed different everywhere and not the same as so claimed by the 'standard model'. A view within a (fractal) tree is different from one at its base or trunk as compared to within its branches: as stated, Earths perspective is viewed from within the newer 'branches' – viewing out to the older trunk.

4.11 Multiverse,

With some trepidation, the inverted fractal are consistent with conjectures surrounding a multiverse and as they clearly demonstrate multiple beginnings. An isolated fractal, by definition, has no arbitrary single beginning, and is an infinity of beginnings. What is important, while the inverted fractal may predict a multiverse, it does not need it to explain 'inflation' and the like.

4.12 Scale-invariance – the Atom

Fractspansion is scale-invariant; this had direct relevance to the apparent emptiness of 'Rutherford's atom', and may add strength to unparticle theory [49]. The following will be the topic of a separate investigation, suffice to say – and notwithstanding the already mentioned quantum like behaviour of fractal propagation: the frequency range of the bits and the clustering of them at the centre or observation point within the modeled fractal attractor is somewhat similar to a model of a single atom. There is a nucleus surrounded a massive expanse of space. This concurs with the work of by Renate Loll (and others) on quantum space-time [50].

"Evidently, a small object experiences space-time in a profoundly different way than a large object does. To that object, the universe has something akin to a fractal structure. A fractal is a bizarre kind of space where the concept of size simply does not exist. It is self-similar -- which means that it looks the same on all scales. This implies there are no rulers and no other objects of a characteristic size that can serve as a yardstick."

In her Perimeter Institute lecture [51] (at 01:06) Loll refers to the small scale appearing 'like a fractal' – after talking about the macro de Sitter scale.

4.13 Limitations to the Model – releasing the assumptions

The universe may by this analysis – and by the observations made – turn out to behave as a fractal, but this is not to say the universe behaves as a regular regularity fractal as the Koch snowflake. Reality seems to point to regular irregularity (roughness) as best demonstrated by the Mandelbrot diagram (Figure 3A). This irregular reality is beyond the scope of this investigation. This investigation also does not in any way suggest the universe has the shape of a tree, or a snowflake: fractspansion could have equally been demonstrated using the Sierpinski triangle. The universe shares a feature special to fractals: fractals come in many forms.

4.13.1 Deceleration

The fractspanding (regular inverted Koch snowflake) fractal does not demonstrate, or offer any insight to deceleration – whether observed post inflation epoch early universe, or conjectured pre inflation epoch.

4.14 Examples and Application of Fractspansion

If an object in our reality maybe described as a fractal structure, the same object will exhibit fractspansion and stand as an example of the large-scale universe:

4.14.1 Accelerating Tree Growth

As special as expansion at an accelerating rate may be, it is not unique to only the universe. Trees (plants) have recently been found to also grow by this behaviour with respect to time. In a recent study [41] measuring up to 80 years of tree growth, on more than 600,000 trees, over 6 continents found that 97 per cent of the trees grew at an accelerating with age. This acceleration growth with time is equally a mystery to biologists.

If a tree's growth is described by classical fractal geometry [33], so does accelerating growth of the tree reveal a property general to all things fractal? Yes.

If the productive leafy stem of the emergent tree becomes the focus of the tree growth, and held constant in size – just as with the standard triangle size is to the inverted Koch snowflake – then the older branches and the load bearing trunk of the tree will grow exponentially with iteration time. This is to say: the tree grows in terms of iteration time, and not solar time. As trees grow they lay down tree rings, these rings do not show exponential growth. Trees can generally – by counting the tree rings – age several hundreds of years old, but in terms of fractal age may only be some 4 to 7 iteration times old. One can imagine that more iteration times would result in an exponentially growing, exponentially large base trunk.

4.15 Econo-physics

First inklings of this fractspansion theory originated from the study of the fractal and its resemblance to economic theory.

4.15.1 Classical (Marginal) Economics

I had determined that the market, with its equilibrium between consumption (demand) and production (supply), is a fractal emergent, and that market supply and demand was able to be described and quantified the Koch Snowflake fractal development. When my attention was turned to the past – in terms of iteration time – I reasoned that fundamental standard events or items of the past (just like the standard triangle described in this publication) appeared larger to the observer in the present – and were thus valued more. Information value seems to grow with time. Examples maybe events such as the findings of the three great early scientists Copernicus, Kepler and Galileo: they are greater now than they possibly were in their time. Their findings were seminal, fundamental, and are metaphorically speaking the 'Big Bang' of science. As special as these 'scientists' are to us today, they were scientists of their time – just as scientists today are of our time. The same could be said of 1960's pop band The Beatles – in the context of the evolution of pop rock. Time increases value.

To demonstrate this once more, in a recent blind sound comparison testing between original Stradivarius violins and new replicas [52], testers could not discern a difference in sound quality; yet the value difference between the original and replica violins is extremely large. Fractspansion increases – distorts – the prices of the same goods.

4.15.2 Universal Lorenz Curve

The Lorenz curve shows the distribution of individuals' income (or wealth) for a given population. The Lorenz curve always falls below the line 'perfect equality' (a line of equal distribution throughout the population) and the Gini coefficient (termed 'Ginis' for short) tells the ratio between the Lorenz curve and the line of equality (area A), and the total (triangle) area under the line of equality (areas A and B).

The fractal derived Lorenz curve (figure 16 below) reveals an age order structure of the wealth distribution with growth: the oldest have the largest proportion. This agrees with real tree development and with the age of the elements in the universe on the periodic table: the trunk is the oldest 'bit'; and helium and hydrogen were the first elements formed straight after the big bang. Subatomic particles came before helium and hydrogen; and the larger – and fewer – elements like gold, came after. Whether this age order is prevalent in the economy maybe unclear to see at first, but the rule should hold on examination.

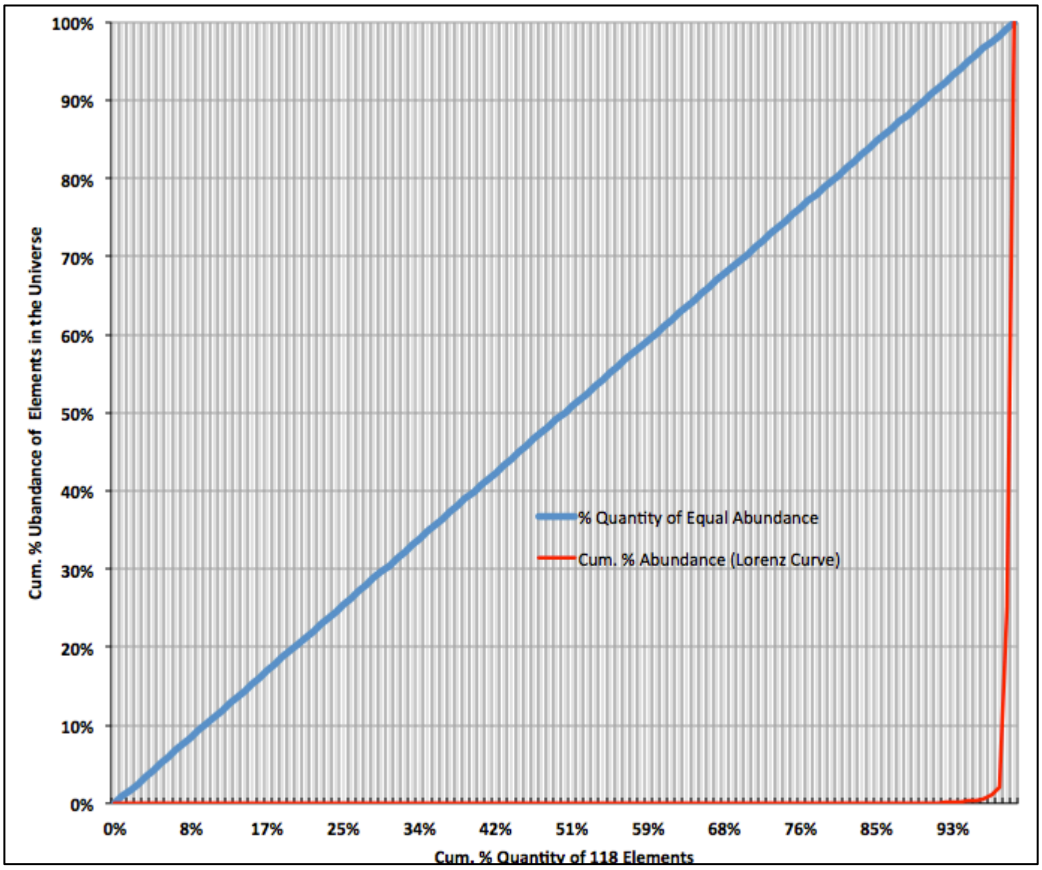


Figure 16. Lorenz Curve of the (118) Elements from the Periodic Table. Using Lorenz curve method the distribution of elements curve was produced. The Gini coefficient was calculated: it is equal to 0.9864 (4sf) [53]

5 CONCLUSIONS

This investigation it was found – when observed from a fixed (but arbitrary) location within the inverted iterating Koch snowflake fractal – areas of triangles expand exponentially, while points between triangles recede away from the observer with both with exponential velocity and acceleration. This expansion, revealed by the (unrealistic) regular, Koch snowflake – termed fractspansion – is a property unique to fractals, and is a property shared in all (irregular) fractal objects. Fractspansion demonstrates and addresses problems directly associated with the ΛCMB model – the expansion of space, and reveals directly both a Hubble's law and a cosmological constant. Fractspansion offers a geometric mechanism that explains the presence of the CMB, and deals and concurs with conjectures surrounding possible early inflation. Fractspansion explains the dark energy. The iterating fractal's quantum and electromagnetism like properties add support to this finding, and (also) opens discussion to role the geometry of the fractal has in explaining the quantum world, time, and reality itself. Fractal geometry by fractspansion explains why trees grow at an accelerating rate with age and may explain why we perceive value to increase with time.

The inverted fractal model explains the universe's galaxy distribution transition from rough (fractal) on small cosmic scales, to smooth (homogeneous) on large-scales. This demonstration can now be combined with the models original demonstrations: a single beginning; a CMB; Hubble's law; and is expansion at an accelerating rate – lambda. The results show strong agreement with the WMAP + ΛCDM models of the universe. These properties are all properties of the fractal – an inverted fractal view from within – and are inextricably linked with each other. From observations of the universe – at all scales – it can be concluded the universe is – by its nature – fractal. It looks like a fractal, and acts like a fractal – it is a fractal.

Fractspansion offers a solution to the problems facing the standard model of cosmology, in the same way the theory of plate tectonics (for example) did for earth science. It is simple and complete. It opens the door to a unified theory.

6 APPENDIX

Table 1. Displacement taken from radius measurements and calculations from the iterating Koch Snowflake fractal spiral (Appendix Figure 1).

i	Displacement:: cm	Total Displacement: cm	Expansion Ratio	Velocity: $cm\ i^{-1}$	Acceleration: $cm\ i^{-2}$	Acceleration Ratio
0			-			
1	1.68	1.68	-	1.68	1.68	
2	4.66	6.34	3.77	4.66	2.98	1.773809524
3	12.16	18.5	2.92	12.16	7.50	2.516778523
4	35.4	53.9	2.91	35.40	23.24	3.098666667

cm = centimetres. i = iteration time.

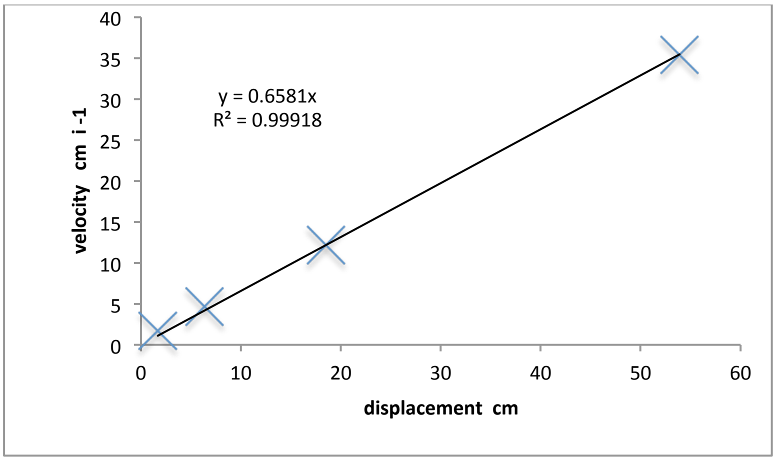


Figure 1. The Hubble Fractal Diagram (recessional velocity vs. distance) from radius measurements (Appendix Figure 1). From an arbitrary observation point on the inverted (Koch Snowflake) fractal: as the distance between triangle geometric centres points increases, the recession velocity of the points receding away increases. cm = centimetres. i = iteration time.

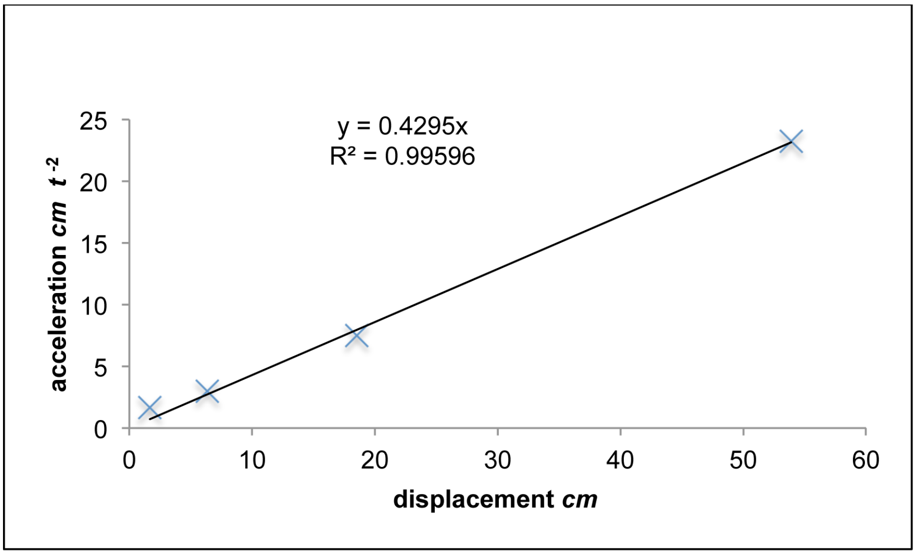
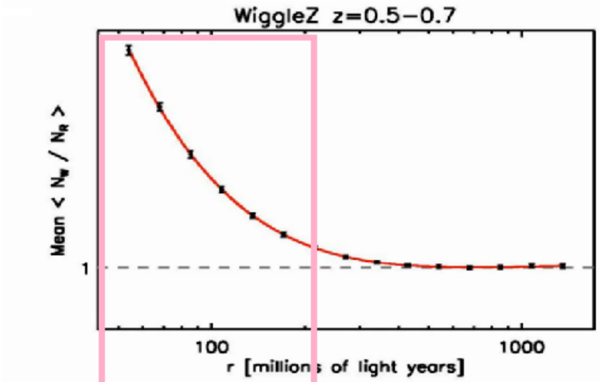
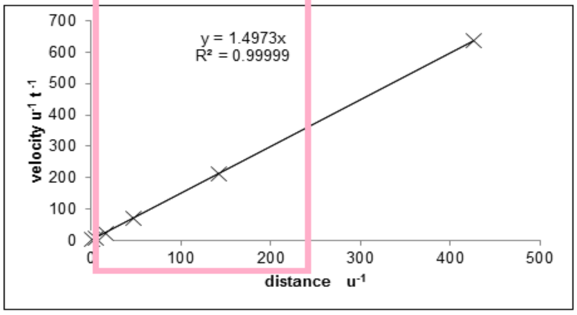


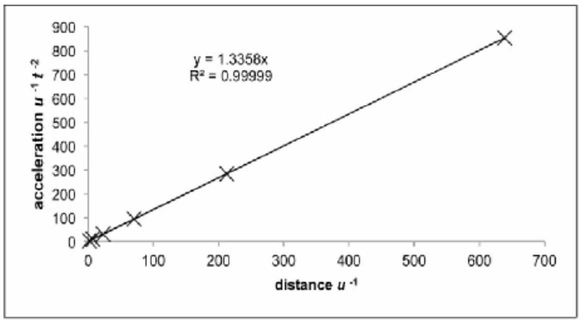
Figure 2. Recessional acceleration with distance on the inverted Koch Snowflake fractal. From a fixed central observation point. Using radius measurements (Appendix Figure 1): as the distance between triangle geometric centres points increases, the recession acceleration of the points receding away increases. cm = centimetres. t = iteration time.



https://www.youtube.com/watch?v=tVk4XRKbvqo



Fractal Hubble Diagram (figure 7. Macdonald 2014)



Fractal Acceleration Diagram (figure 9, Macdonald 2014)

Figure 3. Composite of WiggleZ Galaxy Distribution, Hubble and Acceleration Diagrams. Recessional acceleration with distance on the inverted Koch Snowflake fractal. Nearby clustering of galaxies correlates with clustering fractal centre points, Hubble (velocity) expansion, and acceleration.

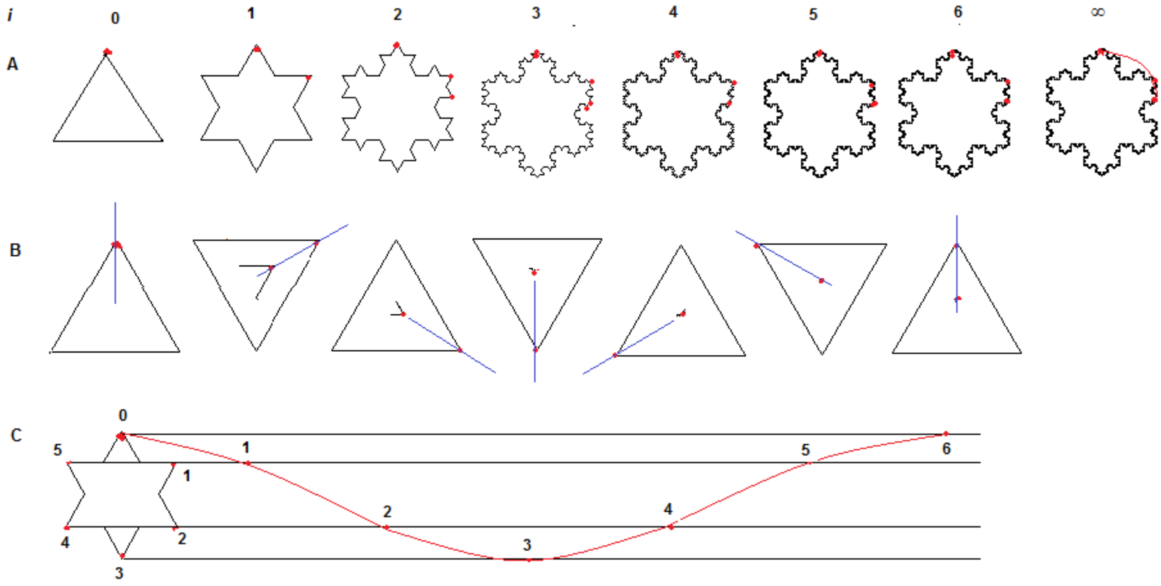


Figure 4 Fractal Spiral with Emergence. A shows the transverse wave propagation of a 'red dot' on a fractal Koch Snowflake to iteration (i) 6, and to superposition infinity (∞). B shows the rotational aspect of the triangle bits and the respective bit size; rotating clockwise through 360°. C shows the Sin wave produced at each iteration-time – assuming bit size remains constant: the real is a logarithmic sinusoidal.

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