Abstract: "A Geometric Model" A family of models in Euclidean space is developed from the following approximation.[1]

$$\frac{m_p}{m_e} \approx 4\pi \left(4\pi - \frac{1}{\pi}\right)\left(4\pi - \frac{2}{\pi}\right) = 1836.15\ldots \tag{1}$$

where m_p and m_e are the numeric values for the mass of the proton and the mass of electron, respectively. In particular, we will develop models (1) that agree with the recommended value of the mass ratio of the proton to the electron to six significant figures, (2) that explain the "shape-shifting" behavior of the proton, and (3) that are formed concisely from the sole transcendental number π. This model is solely geometric, relying on volume as the measure of mass. Claim that inclusion of quantum/relativistic properties enhance the accuracy of the model. The goal is to express the ratio of the proton mass to the electron mass in terms of (1) pure mathematical constants and (2) a quantum corrective factor.

Origin: On June 28, 1989, I discovered the following remarkable approximation:

$$\frac{m_p}{m_e} \approx 64\pi^3 - 48\pi + \frac{8}{\pi} \tag{2}$$

where m_p is the mass of the proton and m_e is the mass of the electron. At that time the recommended numerical value of the Left Hand Side (LHS) of the expression in Equation (2) was 1836.15152(70).[2] The numerical value of the Right Hand Side (RHS) is 1836.151739257.... The ellipsis[3] indicates that the RHS value is exact. The most current CODATA[4] value for the proton-electron mass ratio is 1836.15267389(17).

Assumptions: We make some assumptions. It is not unreasonable to assume that an electron is a perfect ball.[5] For convenience we set the radius of the electron equal to one. ($r_e = 1$) The volume of the electron ball is thus: $V_e = (4\pi/3) \cdot r_e^3 = (4\pi/3)$. Physicists usually minimize the importance of the leading coefficient,[6] calling it "an unimportant numerical factor like

[1] https://jointmathematicsmeetings.org/amsmtgs/2168-abstracts/1106-81-87.pdf
[2] http://physics.nist.gov/cuu/Archive/1973JPCRD.pdf
[3] http://www.thepunctuationguide.com/ellipses.html
[4] http://physics.nist.gov/cgi-bin/cuu/Value?mpsme
[5] https://www.sciencedaily.com/releases/2011/05/110525131707.htm
[6] http://math.ucr.edu/home/baez/Planck/node2.html

2π." One large assumption is that the mass is proportional to the density times the volume. Let the constant of proportionality and the density be set equal to one. We consider the environment of a fixed reference plane, without Special Relativistic (SR) effects. So if a positron is somehow able to expand its volume from $4\pi/3$ to $(4\pi/3)(4\pi)^3$, then its mass ratio with the electron will be increased to $(4\pi)^3$.

Geometric Models: The algebraic expression in the RHS of Equation (2) can be factored in several ways, giving rise to a number of possible underlying geometric objects, each with a volume equal to $(4\pi/3) \cdot 1836.15\ldots$.

$$\text{Numeric Value} = 1836.15\ldots \quad (3)$$

$$\text{Algebraic Expression} = 64\pi^3 - 48\pi + \frac{8}{\pi} \quad (4)$$

$$\text{Tri-axial (Scalene) Ellipsoid (kiwi)} = 4\pi\left(4\pi - \frac{1}{\pi}\right)\left(4\pi - \frac{2}{\pi}\right) \quad (5)$$

$$\text{Ball minus Prolate Spheroid (ball)} = \left(4\pi - \frac{1}{\pi}\right)^3 - \frac{1}{\pi^2}\left(4\pi - \frac{1}{\pi}\right) \quad (6)$$

$$\text{Ball minus Sector (donut)} = \left(4\pi - \frac{1}{\pi}\right)^3 - \frac{1}{\pi^2}\left(4\pi - \frac{1}{\pi}\right) \quad (7)$$

$$\text{Ball minus Wedge (peanut)} = \left(4\pi - \frac{1}{\pi}\right)^3 - \frac{1}{\pi^2}\left(4\pi - \frac{1}{\pi}\right) \quad (8)$$

The first thing to observe is that the numeric value in Equation (3) agrees with the CODATA to six significant figures. This makes the value of the RHS of Equation (4) an acceptable approximation to the proton-electron mass ratio.

The second thing to observe is that Equations (5), (6), (7), and (8) give rise to (spherical) geometric objects found in the Internet.[7] Rotation about the three axes and applying the *Inversion of the Spheres* derives Equation (5). Equation (6) is the ratio of a ball of radius $\tilde{R} = (4\pi - \pi^{-1})$ to a unit ball minus a prolate ellipsoid with semi-axes $\{\pi^{-1}, \pi^{-1}, (4\pi - \pi^{-1})\}$. Equations (7) and (8) give two geometric objects of equivalent mass ratio to Equation (6)—Spherical Sector[8] and Spherical Wedge[9]—and curved surface area of

[7] https://www.sciencedaily.com/releases/2003/04/030408085744.htm
[8] https://en.wikipedia.org/wiki/Spherical-sector
[9] http://mathworld.wolfram.com/SphericalWedge.html

$4\pi^{-1}$. The removed surface area affects the scattering angle which, in turn, changes the "proton shape" models.[10]

The last claim is that the volume of the model below in Figure (1) (plus the quantum corrective factor) defines the *Greatest Lower Bound* (GLB) on the experimental values for the proton-electron mass ratio. This can be validated by observing that each current, published, experimental value lies in the open interval $(1836.15, 1837.39)$, many of which are also "near" to 1836.15. The experimental values include in their intrinsic properties mass due to quantum/relativistic effects.

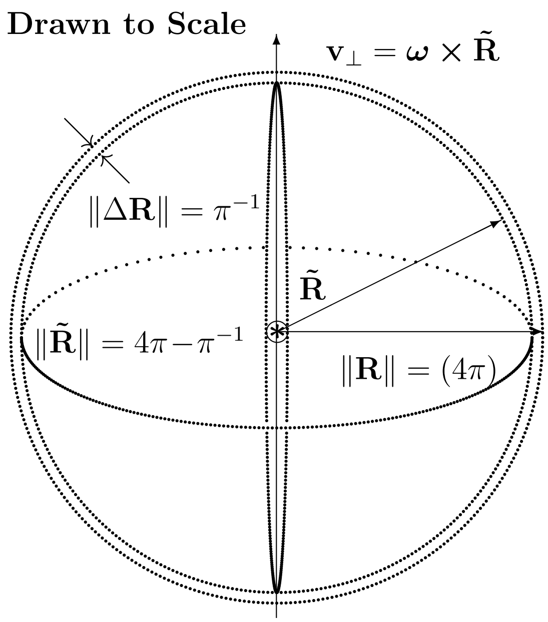

Figure (1): Proton Model

The symbol "$*$" denotes the origin, a position for a singularity. The circle centered at the origin has radius $= \pi^{-1}$.

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Circumscribed by the sphere of radius $= 4\pi - \pi^{-1}$, the ellipsoid has semi-axes $\{\pi^{-1}, \pi^{-1}, (4\pi - \pi^{-1})\}$.

[10] https://www.sciencedaily.com/releases/2003/04/030408085744.htm

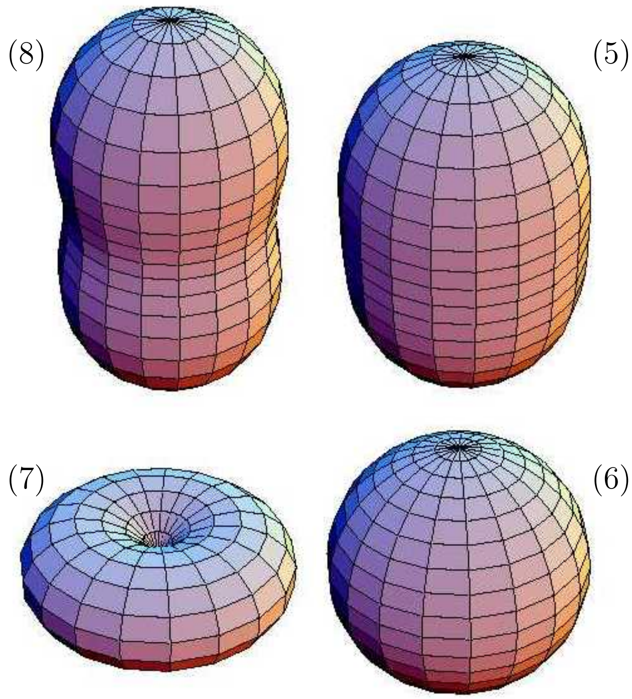

Figure (2): Four Proton Shapes

> These four pictures were copied from https://www.jlab.org /news/articles/surprise-physicists_protons-arent-always-shaped-basketball-university-washington.[11]

Remark: The *classical* view of mass of a given object $\mathcal{O}$ is the product of its volume times its density. Charge is not considered. Likewise the effects of Special Relativity (SR) are neglected. However, quantum mechanical effects cannot be so easily ignored. For an object $\mathcal{O}$ with rest mass m_0 and velocity v we have the mass m to be $m_0/\sqrt{1 - v^2/c^2}$. Subatomic particles have the intrinsic property called *spin*. It is not unreasonable to assume that, for an object $\mathcal{O}$, spin and charge affect the mass in the quantum realm. For non-relativistic velocities, $v \approx 0$ implies $m \approx m_0$. There is no simple paradigm to explain how spin and charge affect mass.

[11] https://www.jlab.org/news/articles/surprise-physicists_protons-arent-always-shaped-basketball-university-washington

The Claim: The proton-electron mass ratio is slightly larger than the mathematical constant $(4\pi(4\pi - 1/\pi)(4\pi - 2/\pi))$. Claim that this correction is explained by quantum field theory of electromagnetism.[12]

Looking solely at stationary objects, for example the volume of a sphere with a circumscribed parabola removed, fails to take into account body rotation or even how the intrinsic value of "spin" affects the mass content of an object. Assuming that r_e is the mean value of the electron radius, our approximation for the mass of an electron is given by

$$m_e \approx \frac{4\pi}{3} r_e^3 \tag{9}$$

where m_e is the electron mass.

Assume that the quantum corrective factor is ≈ 0.001. It affects the value in Equation (1).

$$4\pi \left(4\pi - \frac{1}{\pi}\right)\left(4\pi - \frac{2}{\pi}\right) + 0.001 = 1836.152739\ldots \tag{10}$$

This puts the above equation equal to eight significant figures of 1836.15267389.

01 SEP 2016 harry.watson@att.net

[12] https://www.physicsforums.com/threads/what-is-physical-significance-of-g-factor.288121/