SURAPATI PRAMANIK[1*], DURGA BANERJEE[2], B. C. GIRI[3]

1* Department of Mathematics, Nandalal Ghosh B.T. College, Panpur, P.O.-Narayanpur, District –North 24 Parganas, , West Bengal, India-743126. E-mail: sura_pati@yahoo.co.in
2 Ranaghat Yusuf Institution, P. O. Ranaghat,Dist. Nadia, India-741201. E-mail: dbanerje3@gmail.com
3 Department of Mathematics, Jadavpur University, Jadavpur, West Bengal, India-700032.
E-mail: bcgiri.jumath@gmail.com

TOPSIS Approach for Multi Attribute Group Decision Making in Refined Neutrosophic Environment

Abstract

This paper presents TOPSIS approach for multi attribute decision making in refined neutrosophic environment. The weights of each decision makers are considered as a single valued neutrosophic numbers. The attribute weights for every decision maker are also considered as a neutrosophic numbers. Aggregation operator is used to combine all decision makers' opinion into a single opinion for rating between attributes and alternatives. Euclidean distances from positive ideal solution and negative ideal solution are calculated to construct relative closeness coefficients. Lastly, an illustrative example of tablet selection is provided to show the applicability of the proposed TOPSIS approach.

Keywords

Neutrosophic set, single valued neutrosophic set, neutrosophic refined set, TOPSIS, aggregation operator.

1. Introduction

Decision making in neutrosophic environment is a developing area of research. Florentin Smarandache [1] introduced neutrosophic set which is the generalization of fuzzy set (FS) introduced by L.A. Zadeh [2] and intuitionistic fuzzy set (IFS) proposed by K. T. Atanassov [3]. Florentin Smarandache and his colleagues [4] presented an instance of single valued neutrosophic set called single valued neutrosophic set (SVNS) and their set theoretic operations. FS only considers membership function to represent imprecise data. IFS is characterized by membership and non-membership degrees, which are independent but the sum of degrees of membership and non-membership is less than unity. Both FS and IFS are unable to deal with indeterminacy in real decision making problem. Indeterminacy plays an important role in decision making situation. For example, in an application form there are three options 'YES / NO/ N. A.' for gender M / F / Others. So, different kinds of uncertainty and vagueness with indeterminacy cannot be explained by the fuzzy concept or intuitionistic fuzzy concept. Florentin Smarandache [1] first focused on indeterminacy of the imprecise data and introduced the concept of neutrosophic set consisting of three membership functions namely truth, indeterminacy and falsity membership functions which are independent.

Hawang and Yoon [5] introduced a technique for order preference by similarity to ideal solution (TOPSIS). TOPSIS for multi criteria decision making (MCDM) problem in fuzzy environment has been proposed by Chen [6]. Boran et al. [7] applied TOPSIS approach to multi attribute group decision making (MAGDM) in intuitionistic fuzzy environment. Multicriteria decision - making method using the correlation coefficient under single valued neutrosophic environment has been proposed by Ye [8]. Ye [9] further established single valued neutrosophic cross entropy for MCDM. Biswas et al. [10] presented entropy based grey relational analysis method for multi-attribute decision - making under single valued neutrosophic assessments. Biswas et al. [11] proposed MCDM with unknown weight information. Pramanik et al. [12] developed hybrid vector similarity measures and their applications to multi-attribute decision making under neutrosophic environment. Zhang et al. [13] presented interval neutrosophic MCDM. Pramanik and Mondal [14] presented interval neutrosophic multi-attribute decision-making based on grey relational analysis. Ye [15] applied aggregation operator for MCDM problem for simplified neutrosophic sets. Some important approaches in neutrosophic decision making problems can be found in [16-32]. Biswas et al. [33] proposed TOPSIS method for MAGDM for under single valued neutrosophic environment. Chai and Liu [34] applied TOPSIS method for MCDM with interval neutrosophic set. Broumi et al. [35] presented extended TOPSIS method for multiple attribute decision making based on interval neutrosophic uncertain linguistic variables. In neutrosophic hybrid environment, Pramanik et al. [36] presented TOPSIS for singled valued soft expert set based multi-attribute decision making problems. Dey et al. [37] studied generalized neutrosophic soft multi-attribute group decision making based on TOPSIS. Dey et al. [38] proposed TOPSIS for solving multi-attribute decision making problems under bi-polar neutrosophic environment. Mondal et al. [39] presented TOPSIS in rough neutrosophic environment and provided an illustrative example.

Yager [40] introduced the concept of multiset in 1986. Sebastian and Ramakrishnan [41] developed the concept of multi fuzzy set and studied some of their properties. Shinoj and John [42] presented intuitionistic fuzzy multiset. Ye and Ye [43] presented Dice similarity measure between single valued neutrosophic multisets and its application in medical diagnosis Smarandache [44] proposed n- valued refined neutrosophic logic and its application. Broumi and Smarandache [45] defined neutrosophic refined similarity measure based on cosine function. Mondal and Pramanik [46] proposed neutrosophic refined similarity measure using tangent function and applied it to multi attribute decision making. Mondal and Pramanik [47] also defined neutrosophic refined similarity measure and its application based on cotangent function. Pramanik et al. [48] recently presented MCGDM in neutrosophic refined environment and its application in teacher selection. Nadaban and Dzitac [49] discussed the general view in neutrosophic TOPSIS and presented a very brief survey on the applications of neutrosophic sets in MCDM problems.

The present paper is devoted to extend TOPSIS approach for MAGDM in refined neutrosophic environment. An aggregation operator due to Jun Ye [15] is used in refined neutrosophic environment. The relative closeness coefficients for all attributes are calculated and the alternative with least value of relative closeness coefficient is selected as the best alternative.

The rest of the paper has been framed as follows:

In section 2, we recall some relevant definitions and properties. General TOPSIS approach is discussed in section 3. TOPSIS for MAGDM is stepwise proposed in section 4. A numerical example is described and solved in section 5. Section 6 presents conclusions and future scope of research.

2. Some well established definitions and properties

In this section, we recall some established definitions and properties which are connected in the present article.

2.1.Neutrosophic set (NS)[1]

Let Y be a space of points (objects) with generic element y in Y. A neutrosophic set A in Y is denoted by

A= {<y: $T_A(y)$, $I_A(y)$, $F_A(y)$>: $y \in Y$ } where T_A, I_A, F_A represent membership, indeterminacy and non-membership function respectively. T_A, I_A, F_A are defined as follows:

$T_A : Y \to]^-0, 1^+[$

$I_A : Y \to]^-0, 1^+[$

$F_A : Y \to]^-0, 1^+[$

Here, $T_A(y)$, $I_A(y)$, $F_A(y)$ are the real standard or non-standard subset of $]^-0, 1^+[$ and

$^-0 \le T_A(y)+I_A(y)+F_A(y) \le 3^+$

2.2. Single valued neutrosophic set (SVNS) [4]

Let Y be a space of points with generic element in $y \in Y$. A single valued neutrosophic set A in Y is characterized by a truth-membership function $T_A(y)$, an indeterminacy-membership function $I_A(y)$ and a falsity-membership function $F_A(y)$, for each point y in Y, $T_A(y)$, $I_A(y)$, $F_A(y) \in [0, 1]$, when Y is continuous then single-valued neutrosophic set A can be written as

$A = \int_A < T_A(y), I_A(y), F_A(y) > / y, y \in Y$

When A is discrete, single-valued neutrosophic set can be written as $\sum_{i=1}^{n} \langle T_A(y_i), I_A(y_i), F_A(y_i) > / y_i, y_i \in Y$

2.3. Complement of neutrosophic set [1]

The complement of a neutrosophic set A is denoted by A′and defined as

A′= {<y: $T_{A'}(y)$, $I_{A'}(y)$, $F_{A'}(y)$>, $y \in Y$ }

$T_{A'}(y) = \{1^+\} - T_A(y)$

$I_{A'}(y) = 1^+\} - I_A(y)$

$F_{A'}(y) = \{1^+\} - F_A(y)$

2.4 Properties

Let A and B be two SVNSs, then the following properties [1] hold good:

$$1.\ A \oplus B = \langle T_A(x) + T_B(x) - T_A(x).T_B(x), I_A(x).I_B(x), F_A(x).F_B(x)\rangle, \forall x \in X$$

$$2.\ A \otimes B = \langle T_A(x).T_B(x), I_A(x).+I_B(x) - I_A(x).I_B(x), F_A(x) + F_B(x) - F_A(x).F_B(x)\rangle, \forall x \in X$$

$$3.\ A \cup B = \langle \max(T_A(x), T_B(x)), \min(I_A(x), I_B(x)), \min(F_A(x), F_B(x))\rangle$$

$$4.\ A \cap B = \langle \min(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \max(F_A(x), F_B(x))\rangle$$

2.5 Euclidean distance between two SVNSs [50]

Let $A = \{\langle x_i : T_A(x_i), I_A(x_i), F_A(x_i)\rangle, i = 1, 2, ..., n\}$, and $B = \{\langle x_i : T_B(x_i), I_B(x_i), F_B(x_i)\rangle, i = 1, 2, ..., n\}$ be SVNSs. Then the Euclidean distance between two SVNSs A and B can be defined as follows:

$$E(A,B) = \sqrt{\sum_{i=1}^{n}((T_A(x_i) - T_A(x_i))^2 + (I_A(x_i) - I_B(x_i))^2 + (F_A(x_i) - F_B(x_i))^2)} \tag{1}$$

The normalized Euclidean distance between two SVNSs A and B can be defined as follows:

$$E_N(A,B) = \sqrt{\frac{1}{3n}\sum_{i=1}^{n}((T_A(x_i) - T_A(x_i))^2 + (I_A(x_i) - I_B(x_i))^2 + (F_A(x_i) - F_B(x_i))^2)} \tag{2}$$

2.6 Neutrosophic refined set [44]

Let A be a neutrosophic refined set.

A= $\{<x, (T_A^1(x_i), T_A^2(x_i), ..., T_A^m(x_i)), (I_A^1(x_i), I_A^2(x_i), ..., I_A^m(x_i)), (F_A^1(x_i), F_A^2(x_i), ..., F_A^m(x_i))>: x \in X\}$ where, $T_A^j(x_i)$: $X \in [0, 1]$, $I_A^j(x_i)$: $X \in [0, 1]$, $F_A^j(x_i)$: $X \in [0, 1]$, j = 1, 2, …,m such that $0 \le \sup T_A^j(x_i) + \sup I_A^j(x_i) + \sup F_A^j(x_i) \le 3$, for j = 1, 2, …, m for any $x \in X$. Now, $(T_A^j(x_i), I_A^j(x_i), F_A^j(x_i))$ is the truth-membership sequence, indeterminacy-membership sequence and falsity-membership sequence of the element x, respectively. Also, m is called the dimension of neutrosophic refined sets A.

2.7 Crispfication of a Neutrosophic set [33]

Let $A_j = \{\langle x_i : T_{A_j}(x_i), I_{A_j}(x_i), F_{A_j}(x_i)\rangle, j = 1, 2, ..., n\}$ be n SVNSs. The equivalent crisp number of each A_j can be defined as

$$A_j^c = \frac{1 - \sqrt{((1 - T_{A_j}(x_i))^2 + (I_{A_j}(x_i))^2 + (F_{A_j}(x_i))^2)/3}}{\sum_{j=1}^{n}\left\{1 - \sqrt{((1 - T_{A_j}(x_i))^2 + (I_{A_j}(x_i))^2 + (F_{A_j}(x_i))^2)/3}\right\}}. \tag{3}$$

2.8 Aggregation operator [15]

In the present problem, there are p alternatives. The aggregation operator [15] applied to neutrosophic refined set is defined as follows:

$$F(D_1, D_2, ..., D_r) = \langle \prod_{i=1}^{r}(T_{ij}^k)^{w_i}, \prod_{i=1}^{r}(I_{ij}^k)^{w_i}, \prod_{i=1}^{r}(F_{ij}^k)^{w_i}\rangle$$

$$\widetilde{d}_{kj} = \langle \prod_{i=1}^{r}(T_{ij}^k)^{w_i}, \prod_{i=1}^{r}(I_{ij}^k)^{w_i}, \prod_{i=1}^{r}(F_{ij}^k)^{w_i}\rangle, \tag{4}$$

or $\widetilde{d}_{kj} = \langle \widetilde{T}_{kj}, \widetilde{I}_{kj}, \widetilde{F}_{kj}\rangle$ where $i = 1, 2, ..., r;\ j = 1, 2, ..., q$ and $k = 1, 2, ..., p$

Proof: For the proof see [15].

Properties

The three main properties of aggregation operator are given below:

i) Idempotency:

Let $D_1 = D_2 = \ldots = D_r = D$ where $D = \langle T, I, F\rangle$, then $F(D_1, D_2, ..., D_r) = D$

$$F(D_1, D_2, ..., D_r) = \langle \prod_{i=1}^{r} (T_{ij}^k)^{w_i}, \prod_{i=1}^{r} (I_{ij}^k)^{w_i}, \prod_{i=1}^{r} (F_{ij}^k)^{w_i} \rangle$$

$D_1=D_2=\ldots=D_r=D$ in other words $T_{ij}^k = T, I_{ij}^k = I, F_{ij}^k = F$

$$F(D_1, D_2, ..., D_r) = F(D, D, ..., D) = \langle T^{\sum_{i=1}^{r} w_i}, I^{\sum_{i=1}^{r} w_i}, F^{\sum_{i=1}^{r} w_i} \rangle = \langle T, I, F \rangle = D \text{ since, } \sum_{i=1}^{r} w_i = 1 \quad (4.1)$$

ii) Boundedness:

Since, $0 \le w_i \le 1$ and $0 \le (T_{ij}^k)^{w_i} \le 1$, $0 \le (I_{ij}^k)^{w_i} \le 1$, $0 \le (F_{ij}^k)^{w_i} \le 1$

then $0 \le \prod_{i=1}^{r} (T_{ij}^k)^{w_i} \le 1, 0 \le \prod_{i=1}^{r} (I_{ij}^k)^{w_i} \le 1, 0 \le \prod_{i=1}^{r} (F_{ij}^k)^{w_i} \le 1$

$$\text{therefore, } \langle 0,1,1 \rangle \le F(D_1, D_2, ..., D_r) \le \langle 1,0,0 \rangle \quad (4.2)$$

iii) Monotonicity:

Let us suppose, $D_j \le D_j^* \ \forall j = 1, 2, \ldots, r.$

Then $(T_{ij}^k)^{w_i} \le (T_{ij}^{*k})^{w_i}$, $(I_{ij}^k)^{w_i} \ge (I_{ij}^{*k})^{w_i}$, $(F_{ij}^k)^{w_i} \ge (F_{ij}^{*k})^{w_i}$ which implies $\prod_{i=1}^{r} (T_{ij}^k)^{w_i} \le \prod_{i=1}^{r} (T_{ij}^{*k})^{w_i}$, $\prod_{i=1}^{r} (I_{ij}^k)^{w_i} \ge \prod_{i=1}^{r} (I_{ij}^{*k})^{w_i}$ $\prod_{i=1}^{r} (F_{ij}^k)^{w_i} \ge \prod_{i=1}^{r} (F_{ij}^{*k})^{w_i}$ i.e. $F(D_1, D_2, ..., D_r) \subset F(D_1^*, D_2^*, ..., D_r^*)$ (4.3)

3. TOPSIS approach

TOPSIS approach is employed to identify the best alternative based on the concept of compromise solution. The best compromise solution reflects the shortest Euclidean distance from the positive ideal solution and the farthest Euclidean distance from the negative ideal solution. TOPSIS approach can be presented as follows:

Assume that $A = \{A_1, A_2, ..., A_m\}$ be the set of alternatives with the set C of q attributes, namely, $C = \{C_1, C_2, ..., C_q\}$, $D = (d_{ij})_{m \times q}$ be the decision matrix and $W = \{w_1, w_2, ..., w_q\}$ be the weight vector of attributes.

3.1 Normalize and weighted normalized form of decision matrix

i) For the profit matrix

Let $d_j^+ = \max_i(d_{ij})$ and $d_j^- = \min_i(d_{ij})$, then the normalized value of d_{ij} becomes $d_{ij}^N = \dfrac{d_{ij} - d_j^-}{d_j^+ - d_j^-}$ (5)

ii) For the cost matrix

Let $d_j^+ = \max_i(d_{ij})$ and $d_j^- = \min_i(d_{ij})$, then the normalized value of d_{ij} becomes $d_{ij}^N = \dfrac{d_j^+ - d_{ij}}{d_j^+ - d_j^-}$ (6)

iii) The weighted normalized decision matrix is defined as $d_{ij}^W = d_{ij}^N \times w_j$ (7)

Here, $i = 1, 2, ..., m; j = 1, 2, ..., q$, $w_j \ge 0$, and $\sum_{j=1}^{q} w_j = 1$

3.2 Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS)

i)The PIS for the profit matrix can be written as PIS = $\{d_1^{w+}, d_2^{w+}, ..., d_q^{w+}\} = \max_i d_{ij}^w$

ii) The PIS for the cost matrix can be written as PIS = $\{d_1^{w+}, d_2^{w+}, ..., d_q^{w+}\} = \min_i d_{ij}^w$

iii) The NIS for the profit matrix can be written as NIS = $\{d_1^{w-}, d_2^{w-}, \ldots, d_q^{w-}\} = \min_i d_{ij}^w$

iv)The NIS for the cost matrix can be written as NIS = $\{d_1^{w-}, d_2^{w-}, \ldots, d_q^{w-}\} = \max_i d_{ij}^w$
$i = 1,2,\ldots,m;\ j = 1,2,\ldots,q$

3.3 Euclidean distances from PIS and NIS

The deviational values from PIS and NIS can be respectively calculated as:

$$E_i^+ = \sqrt{\sum_{j=1}^{q} (d_{ij}^w - d_j^{w+})^2} \quad i = 1,2,\ldots,m \tag{8}$$

$$E_i^- = \sqrt{\sum_{j=1}^{q} (d_{ij}^w - d_j^{w-})^2} \quad i = 1,2,\ldots,m \tag{9}$$

3.4 Determination of relative closeness coefficients

The relative closeness coefficient for each alternative can be written as

$$E_i = \frac{E_i^+}{E_i^+ + E_i^-} \quad i = 1,2,\ldots,m \tag{10}$$

3.5 Ranking of alternatives

Using relative closeness coefficients, the ranking has been made in the ascending order.

4. TOPSIS approach for MAGDM with neutrosophic refined set

A systematic approach to extend the TOPSIS to the refined neutrosophic environment has been proposed in this section. This method is very suitable for solving the group decision-making problem under the refined neutrosophic environment.

Step 1:

Let us consider a group of r decision makers (D_1, $D_2 \ldots, D_r$) and q attributes (C_1, $C_2 \ldots, C_q$). The decision matrix (see Table 1) can be presented as follows:

Table 1: *Decision matrix*

	C_1	C_2	...	C_q
D_1	$\{\langle T_{11}^1, I_{11}^1, F_{11}^1\rangle, A_1$ $\langle T_{11}^2, I_{11}^2, F_{11}^2\rangle, A_2$ ...., ..., $\langle T_{11}^p, I_{11}^p, F_{11}^p\rangle A_p\}$	$\{\langle T_{12}^1, I_{12}^1, F_{12}^1\rangle, A_1$ $\langle T_{12}^2, I_{12}^2, F_{12}^2\rangle, A_2$ ...., ..., $\langle T_{12}^p, I_{12}^p, F_{12}^p\rangle A_p\}$	...	$\{\langle T_{1q}^1, I_{1q}^1, F_{1q}^1\rangle, A_1$ $\langle T_{1q}^2, I_{1q}^2, F_{1q}^2\rangle, A_2$ ...., ..., $\langle T_{1q}^p, I_{1q}^p, F_{1q}^p\rangle A_p\}$
D_2	$\{\langle T_{21}^1, I_{21}^1, F_{21}^1\rangle, A_1$ $\langle T_{21}^2, I_{21}^2, F_{21}^2\rangle, A_2$ ...., ..., $\langle T_{21}^p, I_{21}^p, F_{21}^p\rangle A_p\}$	$\{\langle T_{22}^1, I_{22}^1, F_{22}^1\rangle, A_1$ $\langle T_{22}^2, I_{22}^2, F_{22}^2\rangle, A_2$ ...., ..., $\langle T_{22}^p, I_{22}^p, F_{22}^p\rangle A_p\}$	...	$\{\langle T_{2q}^1, I_{2q}^1, F_{2q}^1\rangle, A_1$ $\langle T_{2q}^2, I_{2q}^2, F_{2q}^2\rangle, A_2$ ...., ..., $\langle T_{2q}^p, I_{2q}^p, F_{2q}^p\rangle A_p\}$
...	...	...	...	...
	$\langle T_{r1}^1, I_{r1}^1, F_{r1}^1\rangle, A_1$ $\langle T_{r1}^2, I_{r1}^2, F_{r1}^2\rangle, A_2$	$\langle T_{r2}^1, I_{r2}^1, F_{r2}^1\rangle, A_1$ $\langle T_{r2}^2, I_{r2}^2, F_{r2}^2\rangle, A_2$		$\langle T_{rq}^1, I_{rq}^1, F_{rq}^1\rangle, A_1$ $\langle T_{rq}^2, I_{rq}^2, F_{rq}^2\rangle, A_2$

$$\begin{array}{c|ccccc}
 & \left\{\begin{matrix} \ldots\ldots\ldots, \\ T_{21}^{p}, I_{21}^{p}, F_{21}^{p}\ A_p \end{matrix}\right\} & \left\{\begin{matrix} \ldots\ldots\ldots, \\ T_{22}^{p}, I_{22}^{p}, F_{22}^{p}\ A_p \end{matrix}\right\} & & \left\{\begin{matrix} \ldots\ldots\ldots, \\ T_{2q}^{p}, I_{2q}^{p}, F_{2q}^{p}\ A_p \end{matrix}\right\} \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
D_r & \left\{\begin{matrix} T_{r1}^{1}, I_{r1}^{1}, F_{r1}^{1}\ , A_1 \\ \langle T_{r1}^{2}, I_{r1}^{2}, F_{r1}^{2} \rangle, A_2 \\ \ldots\ldots\ldots, \\ \ldots\ldots\ldots, \\ \langle T_{r1}^{p}, I_{r1}^{p}, F_{r1}^{p} \rangle A_p \end{matrix}\right\} & \left\{\begin{matrix} T_{r2}^{1}, I_{r2}^{1}, F_{r2}^{1}\ , A_1 \\ \langle T_{r2}^{2}, I_{r2}^{2}, F_{r2}^{2} \rangle, A_2 \\ \ldots\ldots\ldots, \\ \ldots\ldots\ldots, \\ \langle T_{r2}^{p}, I_{r2}^{p}, F_{r2}^{p} \rangle A_p \end{matrix}\right\} & \ldots & \left\{\begin{matrix} T_{rq}^{1}, I_{rq}^{1}, F_{rq}^{1}\ , A_1 \\ \langle T_{rq}^{2}, I_{rq}^{2}, F_{rq}^{2} \rangle, A_2 \\ \ldots\ldots\ldots, \\ \ldots\ldots\ldots, \\ \langle T_{rq}^{p}, I_{rq}^{p}, F_{rq}^{p} \rangle A_p \end{matrix}\right\}
\end{array} \quad (11)$$

Step 2

Crispfication of neutrosophic weights

The r decision makers have their own neutrosophic decision weights $(w_1, w_2, \ldots w_r)$. Each $w_k = \langle T_k, I_k, F_k \rangle$ is represented by a neutrosophic number. The equivalent crisp weight can be obtained using the equation (3)

$$w_k^c = \frac{1-\sqrt{((1-T_k)^2 + (I_k)^2 + (F_k)^2))/3}}{\sum_{k=1}^{r}\left\{1-\sqrt{((1-T_k)^2 + (I_k)^2 + (F_k)^2)/3}\right\}}, \text{ and}$$

$$w_k^c \geq 0, \ \sum_{k=1}^{r} w_k^c = 1 \quad (12)$$

Step 3

Construction of aggregated decision matrix

The aggregated neutrosophic decision matrix (see Table 2) can be constructed as follows:

Table 2: *Aggregated decision matrix*

$$\begin{array}{c|cccc}
 & C_1 & C_2 & \ldots & C_q \\ \hline
A_1 & \tilde{d}_{11} & \tilde{d}_{12} & \ldots & \tilde{d}_{1q} \\
A_2 & \tilde{d}_{21} & \tilde{d}_{22} & \ldots & \tilde{d}_{2q} \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
A_p & \tilde{d}_{p1} & \tilde{d}_{p2} & \ldots & \tilde{d}_{pq}
\end{array} \quad (13)$$

Step 4

Description of weights of attributes

In decision making situation, decision makers would not like to give equal importance to all attributes. Thus each DM would have different opinion regarding the weights of attribute. For grouped opinion, all DMs' opinions need to be aggregated by the aggregation operator for a particular attribute. The weight matrix (see Table 3) can be written as follows:

Table 3: *Weight matrix of attributes*

$$\begin{array}{c|cccc}
 & C_1 & C_2 & \ldots & C_q \\ \hline
D_1 & w'_{11} & w'_{12} & \ldots & w'_{1q} \\
D_2 & w'_{21} & w'_{22} & \ldots & w'_{2q} \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
D`_r & w'_{r1} & w'_{r2} & \ldots & w'_{rq}
\end{array} \quad (14)$$

Here $w'_{ij} = \langle T'_{ij}, I'_{ij}, F'_{ij} \rangle$

The aggregated weight [15] for the attribute C_j is defined as follows:

$$\overline{w}_j = \langle \prod_{i=1}^{r} T'_{ij}, \prod_{i=1}^{r} I'_{ij}, \prod_{i=1}^{r} F'_{ij} \rangle = \langle \overline{T}_j, \overline{I}_j, \overline{F}_j \rangle \ \ j=1,2,\ldots,q \quad (15)$$

Step 5

Construction of aggregated weighted decision matrix

The aggregated weighted neutrosophic decision matrix (see Table 4) can be formed as:

Table 4: *Aggregated weighted decision matrix*

$$\begin{array}{c|cccc} & C_1 & C_2 & \dots & C_q \\ \hline A_1 & \overline{w}_1\widetilde{d}_{11} & \overline{w}_2\widetilde{d}_{12} & \dots & \overline{w}_q\widetilde{d}_{1q} \\ A_2 & \overline{w}_1\widetilde{d}_{21} & \overline{w}_2\widetilde{d}_{22} & \dots & \overline{w}_q\widetilde{d}_{2q} \\ \dots & \dots & \dots & \dots & \dots \\ A_p & \overline{w}_1\widetilde{d}_{p1} & \overline{w}_2\widetilde{d}_{p2} & \dots & \overline{w}_q\widetilde{d}_{pq} \end{array} \tag{16}$$

$$= \begin{array}{c|cccc} & C_1 & C_2 & \dots & C_q \\ \hline A_1 & \langle T^w_{11}, I^w_{11}, F^w_{11}\rangle & \langle T^w_{12}, I^w_{12}, F^w_{12}\rangle & \dots & \langle T^w_{1q}, I^w_{1q}, F^w_{1q}\rangle \\ A_2 & \langle T^w_{21}, I^w_{21}, F^w_{21}\rangle & \langle T^w_{22}, I^w_{22}, F^w_{22}\rangle & \dots & \langle T^w_{2q}, I^w_{2q}, F^w_{2q}\rangle \\ \dots & \dots & \dots & \dots & \dots \\ A_p & \langle T^w_{p1}, I^w_{p1}, F^w_{p1}\rangle & \langle T^w_{p2}, I^w_{p2}, F^w_{p2}\rangle & \dots & \langle T^w_{pq}, I^w_{pq}, F^w_{pq}\rangle \end{array} \tag{17}$$

$$\text{or } \overline{w}_j\widetilde{d}_{kj} = \langle \overline{T}_j, \overline{I}_j, \overline{F}_j\rangle \otimes \langle \widetilde{T}_{kj}, \widetilde{I}_{kj}, \widetilde{F}_{kj}\rangle = \langle \overline{T}_j.\widetilde{T}_{kj}, \overline{I}_j + \widetilde{I}_{kj} - \overline{I}_j.\widetilde{I}_{kj}, \overline{F}_j + \widetilde{F}_{kj} - \overline{F}_j.\widetilde{F}_{kj_j}\rangle = \langle T^w_{kj}, I^w_{kj}, F^w_{kj}\rangle = (d^w_{kj})_{p\times q} \tag{18}$$

where $k = 1, 2, \dots, p$ and $j = 1, 2, \dots, q$.

Step 6

Relative positive ideal solution (RPIS) and relative negative ideal solution (RNIS)

In this step, we find out relative positive ideal solution (RPIS) (S^+_N) and the relative negative ideal solution (RNIS) (S^-_N) for the above aggregated neutrosophic decision matrix. The RPIS is defined as $S^+_N = \{d^{w+}_1, d^{w+}_2, \dots, d^{w+}_q\}$, where $d^{w+}_j = \langle T^{w+}_j, I^{w+}_j, F^{w+}_j\rangle$ and

$$\langle T^{w+}_j, I^{w+}_j, F^{w+}_j\rangle = \langle \max_k T^w_{kj}, \min_k I^w_{kj}, \min_k F^w_{kj}\rangle \text{ (for profit type attribute)} \tag{19}$$

Or

$$\langle T^{w+}_j, I^{w+}_j, F^{w+}_j\rangle = \langle \min_k T^w_{kj}, \max_k I^w_{kj}, \max_k F^w_{kj}\rangle \text{ (for cost type attribute)} \tag{20}$$

The RNIS is defined as $S^-_N = \{d^{w-}_1, d^{w-}_2, \dots, d^{w-}_q\}$, where $d^{w-}_j = \langle T^{w-}_j, I^{w-}_j, F^{w-}_j\rangle$ and

$$\langle T^{w-}_j, I^{w-}_j, F^{w-}_j\rangle = \langle \min_k T^w_{kj}, \max_k I^w_{kj}, \max_k F^w_{kj}\rangle \text{ (for profit type attribute)} \tag{21}$$

Or

$$\langle T^{w-}_j, I^{w-}_j, F^{w-}_j\rangle = \langle \max_k T^w_{kj}, \min_k I^w_{kj}, \min_k F^w_{kj}\rangle \text{ (for cost type attribute)} \tag{22}$$

Step 7

Determination of distances of each alternative from the RPIS and the RNIS

The normalized Euclidean distance between $\langle T^w_{kj}, I^w_{kj}, F^w_{kj}\rangle$ and $\langle T^{w+}_j, I^{w+}_j, F^{w+}_j\rangle$ can be written as below:

$$Eu^+_k = \sqrt{\frac{1}{3q}\sum_{j=1}^{q}\left(\left(T^w_{kj} - T^{w+}_j\right)^2 + (I^w_{kj} - I^{w+}_j)^2 + (F^w_{kj} - F^{w+}_j)\right)^2} \tag{23}$$

$$Eu^-_k = \sqrt{\frac{1}{3q}\sum_{j=1}^{q}\left(\left(T^w_{kj} - T^{w-}_j\right)^2 + (I^w_{kj} - I^{w-}_j)^2 + (F^w_{kj} - F^{w-}_j)\right)^2} \tag{24}$$

Step 8

Calculation of relative closeness coefficient

The relative closeness coefficient for each alternative A_k with respect to S^+_N is defined as:

$$R_k = \frac{Eu^+_k}{Eu^+_k + Eu^-_k} \tag{25}$$

where $0 \le R_k \le 1$

Step 9

Ranking of alternatives

The alternative, for which the closeness coefficient is least, has become the best alternative.

5. Numerical Example

The stepwise description of a numerical example is presented as below:

Step 1

Suppose that the owner of a small shop wants to buy a tab. After initial screening,three tabs from three different companies A_1, A_2, A_3 remain for further evaluation. A committee comprising of four decision makers, namely, D_1, D_2, D_3, D_4, has been formed in order to buy the most suitable tablet with respect to five main attributes, C_1, C_2, C_3, C_4, C_5. The five attributes have been described below:

i. technical specifications (C_1)
ii. quality (C_2)
iii. supply chain reliability (C_3),
iv. finances (C_4)) and
v. ecology (C_5)

In the present problem, r = 4, q = 1, 2, …, 5, p = 1, 2, 3.

Step 1

The profit type decision matrix (see Table 5) can be written as:

Table 5: *Decision matrix*

	C_1	C_2	C_3	C_4	C_5
D_1	$(0.7,0.2,0.1)A_1$	$(0.8,0.3,0.3)A_1$	$(0.4,0.1,0.2)A_1$	$(0.5,0.1,0.1)A_1$	$(0.6,0.4,0.1)A_1$
	$(0.6,0.2,0.1)A_2$	$(0.7,0.4,0.2)A_2$	$(0.3,0.2,0.1)A_2$	$(0.3,0.1,0.2)A_2$	$(0.8,0.2,0.2)A_2$
	$(0.7,0.1,0.2)A_3$	$(0.6,0.2,0.2)A_3$	$(0.4,0.4,0.4)A_3$	$(0.6,0.1,0.1)A_3$	$(0.7,0.1,0.1)A_3$
D_2	$(0.8,0.2,0.1)A_1$	$(0.7,0.1,0.2)A_1$	$(0.5,0.1,0.1)A_1$	$(0.6,0.2,0.3)A_1$	$(0.5,0.6,0.1)A_1$
	$(0.7,0.3,0.2)A_2$	$(0.6,0.1,0.1)A_2$	$(0.6,0.2,0.3)A_2$	$(0.5,0.1,0.2)A_2$	$(0.4,0.5,0.2)A_2$
	$(0.6,0.2,0.2)A_3$	$(0.8,0.2,0.1)A_3$	$(0.6,0.1,0.2)A_3$	$(0.7,0.1,0.1)A_3$	$(0.5,0.5,0.1)A_3$
D_3	$(0.9,0.1,0.1)A_1$	$(0.5,0.3,0.2)A_1$	$(0.6,0.4,0.1)A_1$	$(0.2,0.5,0.3)A_1$	$(0.4,0.4,0.4)A_1$
	$(0.8,0.2,0.1)A_2$	$(0.6,0.3,0.1)A_2$	$(0.5,0.4,0.1)A_2$	$(0.4,0.2,0.1)A_2$	$(0.5,0.3,0.2)A_2$
	$(0.8,0.1,0.2)A_3$	$(0.7,0.1,0.1)A_3$	$(0.6,0.3,0.2)A_3$	$(0.4,0.1,0.1)A_3$	$(0.6,0.1,0.2)A_3$
D_4	$(0.6,0.1,0.1)A_1$	$(0.8,0.2,0.1)A_1$	$(0.9,0.2,0.3)A_1$	$(0.7,0.4,0.3)A_1$	$(0.7,0.3,0.4)A_1$
	$(0.7,0.2,0.1)A_2$	$(0.7,0.1,0.3)A_2$	$(0.7,0.3,0.1)A_2$	$(0.6,0.5,0.1)A_2$	$(0.6,0.2,0.4)A_2$
	$(0.7,0.1,0.2)A_3$	$(0.6,0.1,0.2)A_3$	$(0.6,0.2,0.1)A_3$	$(0.7,0.1,0.3)A_3$	$(0.7,0.3,0.2)A_3$

Step 2

The neutrosophic weights of decision makers are considered as {(0.8, 0.1, 0.1), (0.9, 0.2, 0.1), (0.5, 0.4, 0.1), (0.8, 0.2, 0.2)}. Using the equation (10), the equivalent crisp weights are {0.27317, 0.27317, 0.19912, 0.25453}.

Step 3

The aggregated decision matrix can be determined by applying the aggregated operator (4) and calculated as below:

Table 6: *Aggregated decision matrix*

	C_1	C_2	C_3	C_4	C_5
A_1	$(0.734, 0.146, 0.1)$	$(0.702, 0.201, 0.187)$	$(0.567, 0.157, 0.16)$	$(0.477, 0.237, 0.222)$	$(0.548, 0.415, 0.188)$
A_2	$(0.689, 0.224, 0.121)$	$(0.651, 0.182, 0.16)$	$(0.498 0.255, 0.135)$	$(0.436, 0.173, 0.146)$	$(0.5603, 0.279, 0.239)$
A_3	$(0.689, 0.121, 0.2)$	$(0.669, 0.146, 0.144)$	$(0.537, 0.217, 0.217)$	$(0.6, 0.1, 0.132)$	$(0.619, 0.205, 0.137)$

Step 4

The weight matrix (see Table 7) of attributes as described in (14) can be displayed as follows:

Table 7: *Weight matrix of attributes*

	C_1	C_2	C_3	C_4	C_5
D_1	$(0.9, 0.1, 0.2)$	$(0.8, 0.2, 0.3)$	$(0.5, 0.4, 0.3)$	$(0.5, 0.2, 0.15)$	$(0.5, 0.4, 0.4)$
D_2	$(0.8, 0.2, 0.1)$	$(0.7, 0.1, 0.3)$	$(0.6, 0.3, 0.3)$	$(0.8, 0.25, 0.1)$	$(0.6, 0.3, 0.4)$
D_3	$(0.6, 0.3, 0.2)$	$(0.5, 0.3, 0.2)$	$(0.8, 0.2, 0.1)$	$(0.7, 0.2, 0.1)$	$(0.4, 0.4, 0.4)$
D_4	$(0.6, 0.1, 0.2)$	$(0.6, 0.1, 0.2)$	$(0.6, 0.2, 0.3)$	$(0.5, 0.1, 0.2)$	$(0.3, 0.2, 0.1)$

The aggregated weights for all attributes are presented below:

$$\overline{w} = \{(0.725, 0.15, 0.166), (0.653, 0.15, 0.25), (0.604, 0.27, 0.241), (0.608, 0.178, 0.133), (0.444, 0.31, 0.281)\}.$$

Step 5

The aggregated weighted neutrosophic decision matrix (see Table 8) can be formed as:

Table 8: *The aggregated weighted neutrosophic decision matrix*

	C_1	C_2	C_3	C_4	C_5
A_1	$(0.532, 0.274, 0.249)$	$(0.458, 0.321, 0.390)$	$(0.342, 0.385, 0.362)$	$(0.29, 0.373, 0.325)$	$(0.243, 0.596, 0.416)$
A_2	$(0.4995, 0.340, 0.2669)$	$(0.425, 0.305, 0.37)$	$(0.301, 0.456, 0.343)$	$(0.265, 0.32, 0.2596)$	$(0.249, 0.502, 0.453)$
A_3	$(0.4995, 0.253, 0.333)$	$(0.437, 0.274, 0.358)$	$(0.324, 0.428, 0.406)$	$(0.365, 0.260, 0.247)$	$(0.275, 0.451, 0.3795)$

Step 6

Since the present problem is to make decision to buy a tablet, the decision matrix is profit type matrix. Using (19), the RPIS is presented below:

$$S_N^+ = \{(0.532, 0.253, 0.249), (0.45, 0.274, 0.358), (0.342, 0.385, 0.343), (0.365, 0.26, 0.247), (0.275, 0.451, 0.3795)\}.$$

Using (21) the RNIS is presented below:

$$S_N^- = \{(0.4995, 0.340, 0.333), (0.425, 0.321, 0.39), (0.301, 0.456, 0.406), (0.265, 0.373, 0.325), (0.243, 0.596, 0.453)\}.$$

Step 7

The normalized Euclidean distance from RPIS by using (22) is given below:

$Eu_1^+ = 0.0588$, $Eu_2^+ = 0.0518$, $Eu_3^+ = 0.0313$.

The normalized Euclidean distance from RNIS by using (23) is given below:

$Eu_1^- = 0.0401$, $Eu_2^- = 0.0408$, $Eu_3^- = 0.0676$.

Step 8

The relative closeness coefficient (24) for each alternative has been presented in the table 9.

Table 9: *Ranking of alternatives*

Alternatives	$R_k = \frac{Eu_k^+}{Eu_k^+ + Eu_k^-}$	Ranking
A_1	0.594	3
A_2	0.559	2
A_3	0.316	1

Step 9

Table 9 reflects that A_3 is the most suitable tablet for purchasing.

6. Conclusion

This paper presents TOPSIS approach for MAGDM for refined neutrosophic environment. This is the first attempt to propose TOPSIS in refined neutrosophic environment. The proposed approach can be applied to other real MAGDM problem in refined neutrosophic environment such as project management in IT sectors, banking system, etc. The Authors hope that this proposed approach will enlighten a new path for MAGDM in refined neutrosophic environment.

References

1. F. Smarandache. A unifying field in logics: neutrosophic logic, neutrosophy, neutrosophic set, neutrosophic probability, and neutrosophic statistics, Rehoboth: American research Press (1998).
2. L.A. Zadeh. Fuzzy sets. Information and Control 8(3) (1965), 338-3534.
3. K. T. Atanassov. Intuitionistic fuzzy sets. Fuzzy Sets and Systems 20 (1986), 87-96.
4. H. Wang, F. Smarandache, Y. Q. Zhang, R. Sunderraman. Single valued neutrosophic sets. Multispace and Multi structure 4 (2010), 410–413.
5. C. L. Hwang, K. Yoon. Multiple attribute decision making: methods and applications. Springer, New York (1981).
6. C. T. Chen. Extensions of TOPSIS for group decision making under fuzzy environment. Fuzzy Sets and Systems 114 (2000), 1- 9.
7. F. E. Boran, S. Genc, M. Kurt, D. Akay. A multui-criteria intuitionistic fuzzy group decision making for supplier selection with TOPSIS method. Expert System Application 36 (8) (2009), 11363 - 11368.
8. J. Ye. Multicriteria decision – making method using the correlation coefficient under single valued neutrosophic environment. International Journal of General Systems 42 (4) (2013), 386-394.
9. J. Ye. Single valued neutrosophic cross entropy for multi-criteria decision making problems. Applied Mathematical Modelling 38 (3) (2013), 1170- 1175.
10. P. Biswas, S. Pramanik, B.C. Giri. Entropy based grey relational analysis method for multi-attribute decision - making under single valued neutrosophic assessments. Neutrosophic Sets and Systems 2 (2014), 102 - 110.
11. P. Biswas, S. Pramanik, B.C. Giri. A new methodology for neutrosophic multi-attribute decision making with unknown weight information. Neutrosophic Sets and Systems 3 (2014), 42 - 52.
12. S. Pramanik, P. Biswas, B.C. Giri. Hybrid vector similarity measures and their applications to multi-attribute decision making under neutrosophic environment. Neural Computing and Applications (2015). DOI: 10.1007/s00521-015-2125-3.
13. H. Y. Zhang, J. Q. Wang, X. H. Chen. Interval neutrosophic sets and their application in multi – criteria decision making problems. The Scientific World Journal 2014 (2014).http://dx.doi.org/10.1155/2014/645953.
14. S. Pramanik, K. Mondal. Interval neutrosophic multi-Attributed-making based on grey relational analysis. Neutrosophic Sets and Systems 9 (2015), 13-22.

15. J. Ye. A multicriteria decision - making method using aggregation operators for simplifiedneutrosophicsets. Journal of Intelligent and Fuzzy Systems 26 (2014) 2459 – 2466.
16. Z. Tian, J. Wang, J. Wang, and H. Zhang. Simplified neutrosophic linguistic multi-criteria group decision-making approach to green product development. Group Decision and Negotiation (2016). DOI 10.1007/s10726-016-9479-5.
17. Z. Tian, J. Wang, J. Wang, and H. Zhang. An improved MULTIMOORA approach for multi-criteria decision-making based on interdependent inputs of simplified neutrosophic linguistic information. Neural Computing and Applications (2016). DOI 10.1007/s00521-016-2378-5.
18. J. Peng, J. Wang, X. Wu. An extension of the ELECTRE approach with multi-valued neutrosophic information. Neural Computing and Applications (2016). DOI:10.1007/s00521-016-2411-8.
19. P. Ji, H. Zhang, J.Wang. A projection-based TODIM method under multi-valued neutrosophic environments and its application in personnel selection. Neural Computing and Applications (2016). DOI 10.1007/s00521-016-2436-z.
20. H. Zhang, J. Wang, and X. Chen. An outranking approach for multi-criteria decision-making problems with interval-valued neutrosophic sets. Neural Computing and Applications 27(3) (2016), 615–627.
21. P. Biswas, S. Pramanik, B.C. Giri**.** Value and ambiguity index based ranking method of single-valued trapezoidal neutrosophic numbers and its application to multi-attribute decision making. Neutrosophic Sets and Systems 12 (2016), 127-138.
22. P. Biswas, S. Pramanik, B.C. Giri. Aggregation of triangular fuzzy neutrosophic set information and its application to multi-attribute decision making. Neutrosophic Sets and Systems 12 (2016), 20-40.
23. P.P. Dey, S. Pramanik, B.C. Giri. An extended grey relational analysis based multiple attribute decision making in interval neutrosophic uncertain linguistic setting. Neutrosophic Sets and Systems 11 (2016), 21-30.
24. K. Mondal,S. Pramanik. Neutrosophic decision making model for clay-brick selection in construction field based on grey relational analysis. Neutrosophic Sets and Systems 9 (2015), 64-71.
25. K. Mondal, S. Pramanik. Neutrosophic tangent similarity measure and its application to multiple attribute decision making. Neutrosophic Sets and Systems 9 (2015), 85-92.
26. P. Biswas, S. Pramanik, and B.C. Giri. Cosine similarity measure based multi-attribute decision-making with trapezoidal fuzzy neutrosophic numbers. Neutrosophic Sets and Systems 8 (2015), 47-57.
27. J. Ye. Simplified neutrosophic harmonic averaging projection-based method for multiple attribute decision-making problems. International Journal of Machine Learning and Cybernetics (2015). DOI 10.1007/s13042-015-0456-0.
28. K. Mondal, S. Pramanik. Neutrosophic decision making model of school choice. Neutrosophic Sets and Systems, 7 (2015), 62-68.
29. H. Zhang, P. Ji, J., Wang, X.Chen. An improved weighted correlation coefficient based on integrated weight for interval neutrosophic sets and its application in multi-criteria decision making problems. International Journal of Computational Intelligence Systems 8(6) (2015), 1027–1043.
30. J. J. Peng, J. Q. Wang, H. Y. Zhang, X. H. Chen. An outranking approach for multi-criteria decision-making problems with simplified neutrosophic sets. Applied Soft Computing 25 (2014), 336–346.
31. J. Ye. Bidirectional projection method for multiple attribute group decision making with neutrosophic numbers. Neural Computing and Applications (2016). DOI: 10.1007/s00521-015-2123-5.
32. K. Mondal,S. Pramanik. Multi-criteria group decision making approach for teacher recruitment in higher education under simplified neutrosophic environment. Neutrosophic Sets and Systems 6 (2014), 28-34.
33. P. Biswas, S. Pramanik, B.C. Giri.TOPSIS method for multi-attribute group decision making under single-valued neutrosophic environment. Neural Computing and Applications (2015). DOI: 10.1007/s00521-015-1891-2.
34. P. Chi, P. Liu. An extended TOPSIS method for multi-attribute decision making problems on interval neutrosophic set. Neutrosophic Sets and Systems 1 (2013), 63 -70.

35. S. Broumi, J. Ye. F. Smnarandache. An extended TOPSIS method for multiple attribute decision making based on interval neutrosophic uncertain linguistic variables. Neutrosophic Sets and Systems 8 (2015), 22-31.
36. S. Pramanik, P. P. Dey, B. C. Giri. TOPSIS for singled valued soft expert set based multi-attribute decision making problems. Neutrosophic Sets and Systems 10 (2015), 88-95.
37. P.P. Dey, S. Pramanik, B.C. Giri. Generalized neutrosophic soft multi-attribute group decision making based on TOPSIS. Critical Review 11 (2015), 41-55.
38. P. P. Dey, S. Pramanik, B. C. Giri. TOPSIS for solving multi-attribute decision making problems under bi-polar neutrosophic environment. New Trends in Neutrosophic Theories and Applications (2016). In Press.
39. K. Mondal, S. Pramanik, F. Smarandache. TOPSIS in rough neutrosophic environment. Neutrosophic Sets and Systems 13(2016). In Press.
40. R. R. Yager. On the theory of bags (multi sets). International Journal of General Systems 13(1986), 23-37.
41. S. Sebastian, T. V. Ramakrishnan. Multi fuzzy sets: an extension of fuzzy sets. Fuzzy Information Engineering 3(1) (2011), 35-43.
42. T. K. Shinoj, S. J. John. Intuitionistic fuzzy multi-sets and its application in medical diagnosis. WorldAcademy of Science, Engine Technology 61 (2012), 1178-1181.
43. S. Ye, J. Ye. Dice Similarity measure between single valued neutrosophic multisets and its application in medical diagnosis. Neutrosophic Sets and Systems 6 (2014), 50-55.
44. F. Smarandache. n-Valued refined neutrosophic logic and its applications in physics, Progress in Physics 4 (2013), 143-146.
45. S. Broumi, F. Smarandache. Neutrosophic refined similarity measure based on cosine function. Neutrosophic Sets and Systems 6 (2014), 42-48.
46. K. Mondal, S. Pramanik. Neutrosophic refined similarity measure based on tangent function and itsapplication to multi attribute decision making. Journal of New theory 8 (2015), 41-50.
47. K. Mondal, S. Pramanik. Neutrosophic refined similarity measure based on cotangent function and its application to multi-attribute decision making, Global Journal of Advanced Research 2(2) (2015), 486-494.
48. S. Pramanik, D. Banerjee, B.C. Giri. Multi – criteria group decision making model in neutrosophic refined set and its application. Global Journal of Engineering Science and Research Management 3 (6) (2016), 1- 10.
49. S. Nadaban, S. Dzitac. Neutrosophic TOPSIS: a general view. 6 th International Conference on Computers Communications and Control (ICCCC) (2016) 1- 4.
50. P. Majumder, S. K. Samanta. On similarity and entropy of neutrosophic sets. Journal of Intelligent and Fuzzy Systems 26(2014), 1245–1252.