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Hausdorff Extensions in Single Valued Neutrosophic S^* Centered Systems

Abstract

This paper explores the concept of single valued neutrosophic S^* open sets in single valued neutrosophic S^* centered system. Also the characterization of Hausdorff extensions of spaces in single valued neutrosophic S^* centered systems are established.

Keywords

Single valued neutrosophic set, single valued neutrosophic structure space, single valued neutrosophic S^* centered system, single valued neutrosophic $S^*\theta-$ homeomorphism, single valued neutrosophic $S^*\theta-$ continuous functions.

1. Introduction

Florentin Smarandache [8, 9] combined the non-standard analysis with a tri component logic/set, probability theory with philosophy and proposed the term neutrosophy which means knowledge of neutral thoughts. This neutral represents the main distinction between fuzzy and intuitionistic fuzzy logic set. In 1998, Florentin Smarandache defined the neutrosophic set [8, 9]. Florentin Smarandache and his colleagues [5] presented an instance of neutrosophic set, called single valued neutrosophic set. Alexandrov [1] developed a method of centered systems for studying compact extensions of topological spaces. The method of centered systems in topological spaces was studied by Iliadis [6] and in fuzzy topological spaces by Uma et al. [10]. We extend the same in single valued neutrosophic topological spaces.

2. Preliminaries

Definition 2.1. [5]

Let X be a space of points (objects), with a generic element in X denoted by x. A single valued neutrosophic set (SVNS) A in X is characterized by truth-membership function T_A, indeterminacy-membership function I_A and falsity-membership function F_A.

For each point x in X, $T_A(x)$, $I_A(x)$, $F_A(x) \in [0,1]$. When X is continuous, a SVNS A can be written as A, $\int_X \langle T_A(x), I_A(x), F_A(x)\rangle / x, x \in X$.

When X is discrete, a SVNS A can be written as

$$A = \sum_{i=1}^{n} \langle T(x_i), I(x_i), F(x_i)\rangle / x_i, x_i \in X$$

Definition 2.2: [10]

Let R be a fuzzy Hausdorff space. A system $p = \{\lambda_\alpha\}$ of fuzzy open sets of R is called fuzzy centered if any finite collection of the fuzzy sets of the system has a non-empty intersection. The system p is called a maximal fuzzy centered system or a fuzzy end if it cannot be included in any larger fuzzy centered system of fuzzy open sets.

Definition 2.3: [10]

Let $\theta(R)$ denote the collection of all fuzzy ends belonging to a given fuzzy Hausdorff space R. A fuzzy topology introduces into $\theta(R)$ in the following way. Let P_λ be the set of all fuzzy ends that contain λ as an element, where λ is a fuzzy open set of R. Therefore, P_λ is a fuzzy neighbourhood of each fuzzy end contained in P_λ.

3. Single valued neutrosophic S* Hausdorff extension spaces

Definition 3.1

Let X be a non-empty set and S be a collection of all single valued neutrosophic sets of X. A single valued neutrosophic S*structure on S is a collection S* of subsets of S having the following properties:

1. φ and S are in S*.
2. The union of the elements of any sub collection of S* is in S*.
3. The intersection of the elements of any finite sub collection of S* is in S*.

The collection S together with the structure S* is called single valued neutrosophic S* structure space. The members of S* are called single valued neutrosophic S* open sets. The complement of single valued neutrosophic S* open set is said to be a single valued neutrosophic S* closed set.

Example 3.2:

Let $X = \{a,b\}$, $S = \left\{ \frac{a}{\langle 0.8,0.3,0.5\rangle}, \frac{b}{\langle 0.7,0.4,0.6\rangle} \right\}$, $S^* = \{S, \phi, S_1, S_2, S_3, S_4\}$ where,

$S_1 = \left\{ \frac{a}{\langle 0.6,0.1,0.7 \rangle}, \frac{b}{\langle 0.5,0.2,0.8 \rangle} \right\}$, $S_2 = \left\{ \frac{a}{\langle 0.4,0.2,0.6 \rangle}, \frac{b}{\langle 0.5,0.3,0.9 \rangle} \right\}$, $S_3 = \left\{ \frac{a}{\langle 0.4,0.1,0.7 \rangle}, \frac{b}{\langle 0.5,0.2,0.9 \rangle} \right\}$, $S_4 = \left\{ \frac{a}{\langle 0.6,0.2,0.6 \rangle}, \frac{b}{\langle 0.5,0.3,0.8 \rangle} \right\}$.

Here (S, S^*) is a structure space.

Definition 3.3:

Let A be a member of S. A single valued neutrosophic S* open set U in (S, S^*) is said to be a single valued neutrosophic S* open neighbourhood of A if $A \in G \subset U$ for some single valued neutrosophic S* open set G in (S, S^*).

Example 3.4:

Let $X = \{a, b\}$, $S = \left\{ \frac{a}{\langle 0.8,0.3,0.5 \rangle}, \frac{b}{\langle 0.7,0.4,0.6 \rangle} \right\}$, $S^* = \{S, \phi, S_1, S_2, S_3, S_4\}$ where,

$S_1 = \left\{ \frac{a}{\langle 0.6,0.1,0.7 \rangle}, \frac{b}{\langle 0.5,0.2,0.8 \rangle} \right\}$, $S_2 = \left\{ \frac{a}{\langle 0.4,0.2,0.6 \rangle}, \frac{b}{\langle 0.5,0.3,0.9 \rangle} \right\}$,

$S_3 = \left\{ \frac{a}{\langle 0.4,0.1,0.7 \rangle}, \frac{b}{\langle 0.5,0.2,0.9 \rangle} \right\}$, $S_4 = \left\{ \frac{a}{\langle 0.6,0.2,0.6 \rangle}, \frac{b}{\langle 0.5,0.3,0.8 \rangle} \right\}$.

Let $A = \left\{ \frac{a}{\langle 0.4,0.1,0.8 \rangle}, \frac{b}{\langle 0.3,0.1,0.9 \rangle} \right\}$.

Here $A \in S_1 \subset S_4$. S_4 is the single valued neutrosophic S* open neighbourhood of A.

Definition 3.5:

Let (S, S^*) be a single valued neutrosophic S* structure space and $A = \langle x, T_A, I_A, F_A \rangle$ be a single valued neutrosophic set in X. Then the single valued neutrosophic S* closure of A (briefly $SVNS^*cl(A)$) and single valued neutrosophic S* interior of A (briefly $SVNS^*int(A)$) are respectively defined by

$SVNS^*cl(A) = \bigcap \{K: K$ is a single valued neutrosophic S* closed sets in S and $A \subseteq K\}$

$SVNS^*int(A) = \bigcup \{G: G$ is a single valued neutrosophic S^* open sets in S and $G \subseteq A\}$.

Example 3.6:

Let $X=\{a,b\}$, $S=\left\{\frac{a}{\langle 0.8,0.3,0.5\rangle},\frac{b}{\langle 0.7,0.4,0.6\rangle}\right\}$, $S^*=\{S,\phi,S_1,S_2,S_3,S_4\}$ where,

$S_1=\left\{\frac{a}{\langle 0.6,0.1,0.7\rangle},\frac{b}{\langle 0.5,0.2,0.8\rangle}\right\}$, $S_2=\left\{\frac{a}{\langle 0.4,0.2,0.6\rangle},\frac{b}{\langle 0.5,0.3,0.9\rangle}\right\}$,

$S_3=\left\{\frac{a}{\langle 0.4,0.1,0.7\rangle},\frac{b}{\langle 0.5,0.2,0.9\rangle}\right\}$, $S_4=\left\{\frac{a}{\langle 0.6,0.2,0.6\rangle},\frac{b}{\langle 0.5,0.3,0.8\rangle}\right\}$.

$S_1^c=\left\{\frac{a}{\langle 0.7,0.9,0.6\rangle},\frac{b}{\langle 0.8,0.8,0.5\rangle}\right\}$, $S_2^c=\left\{\frac{a}{\langle 0.6,0.8,0.4\rangle},\frac{b}{\langle 0.9,0.7,0.5\rangle}\right\}$,

$S_3^c=\left\{\frac{a}{\langle 0.7,0.9,0.4\rangle},\frac{b}{\langle 0.9,0.8,0.5\rangle}\right\}$, $S_4^c=\left\{\frac{a}{\langle 0.6,0.8,0.6\rangle},\frac{b}{\langle 0.8,0.7,0.5\rangle}\right\}$.

Let $A=\left\{\frac{a}{\langle 0.5,0.3,0.6\rangle},\frac{b}{\langle 0.7,0.4,0.9\rangle}\right\}$.Then $SVN\,S^*\mathrm{int}(A)=\{S_3\}$.

$SVN\,S^*cl(A)=\{S_4^c\}$.

Definition 3.7:

The ordered pair (S, S*) is called a single valued neutrosophic S* Hausdorff space if for each pair A_1, A_2 of disjoint members of S, there exist disjoint single valued neutrosophic S* open sets U_1 and U_2 such that $A_1 \subseteq U_1$ and $A_2 \subseteq U_2$.

Example 3.8:

Let $X=\{a,b\}$, $S=\left\{\frac{a}{\langle 1,1,0\rangle},\frac{b}{\langle 1,1,0\rangle}\right\}$, $S^*=\{S,\phi,S_1,S_2,S_3\}$ where,

$S_1=\left\{\frac{a}{\langle 0.5,0,1\rangle},\frac{b}{\langle 0,0.3,0.4\rangle}\right\}$, $S_2=\left\{\frac{a}{\langle 0.5,0.2,0.5\rangle},\frac{b}{\langle 0.7,0.3,0.4\rangle}\right\}$,

$S_3=\left\{\frac{a}{\langle 0,0.2,0.5\rangle},\frac{b}{\langle 0.7,0,1\rangle}\right\}$.

Let $A_1 = \left\{ \frac{a}{\langle 0.3,0,1 \rangle}, \frac{b}{\langle 0,0.1,1 \rangle} \right\}$, $A_2 = \left\{ \frac{a}{\langle 0,0.1,0.6 \rangle}, \frac{b}{\langle 0.5,0,1 \rangle} \right\}$.

Here A_1 and A_2 are disjoint members of S and S_1, S_2 are disjoint single valued neutrosophic S* open sets such that $A_1 \subseteq S_1$ *and* $A_2 \subseteq S_2$.

Hence the ordered pair (S, S^*) is a single valued neutrosophic S* Hausdorff space.

Definition 3.9:

Let (S_1, S_1^*) and (S_2, S_2^*) be any two single valued neutrosophic *S** structure spaces and let $f:(S_1, S_1^*) \to (S_2, S_2^*)$ be a function. Then f is said to be single valued neutrosophic *S** continuous iff the pre image of each single valued neutrosophic S_2^* open set in (S_2, S_2^*) is a single valued neutrosophic S_1^* open set in (S_1, S_1^*).

Definition 3.10:

Let (S_1, S_1^*) and (S_2, S_2^*) be any two single valued neutrosophic *S** structure spaces and let $f:(S_1, S_1^*) \to (S_2, S_2^*)$ be a bijective function. If both the functions f and the inverse function $f^{-1}:(S_2, S_2^*) \to (S_1, S_1^*)$ are single valued neutrosophic *S** continuous then f is called single valued neutrosophic *S** homeomorphism.

Definition 3.11:

Let f be a function from a single valued neutrosophic *S** structure space (S_1, S_1^*) into a single valued neutrosophic *S** structure space (S_2, S_2^*) with $f(A_1) = f(A_2)$ where $A_1 \in (S_1, S_1^*)$ and $A_2 \in (S_2, S_2^*)$. Then f is called a single valued neutrosophic $S^*\theta-$continuous at A_1 if for every neighbourhood O_{A_2} of A_2, there exists a neighbourhood O_{A_1} of A_1 such that $f(SVNS^*cl(O_{A_1})) \subset SVNS^*cl(O_{A_2})$. The function is called single valued neutrosophic $S^*\theta-$continuous if it is single valued neutrosophic $S^*\theta-$continuous at every member of S_1.

Definition 3.12:

A function is called a single valued neutrosophic $S^*\theta-$ homeomorphism if it is single valued neutrosophic S^* one to one and single valued neutrosophic $S^*\theta-$ continuous in both directions.

Definition 3.13:

Let (S, S^*) be a single valued neutrosophic S^* Hausdorff space. A system $p = \{U_\alpha : \alpha = 1, 2, 3, ...n\}$ of single valued neutrosophic S^* open sets is called a single valued neutrosophic S^* centered system if any finite collection of the sets of the system has a non-empty intersection.

Definition 3.14:

The single valued neutrosophic S^* centered system p is called a maximal single valued neutrosophic S^* centered system or a single valued neutrosophic S^* end if it cannot be included in any larger single valued neutrosophic S^* centered system of single valued neutrosophic S^* open sets.

Example 3.15:

In Example 3.8 let us consider the system $p_1 = \{S_\alpha, \alpha = 1,2,3\}$. p_1 is a fuzzy neutrosophic S^* centered system since S_1, S_2 has a non -empty intersection.

Let $p_2 = \{S_\alpha : \alpha = 1,2\}$ is also a fuzzy neutrosophic S^* centered system.

Here p_1 is a maximal fuzzy neutrosophic S^* centered system.

Note 3.16:

Throughout this paper $\{U_\alpha : \alpha = 1, 2, 3, ...n\}$ be a single valued neutrosophic S^* open set in (S, S^*).

Proposition 3.17:

Let (S, S^*) be a single valued neutrosophic S^* Hausdorff space and $p = \{U_\alpha\}$ is a single valued neutrosophic S^* centered system in (S, S^*).Then the following properties hold.

1.If $U_i \in p \ (i = 1,2,3,....n)$ then $\bigcap_{i=1}^{n} U_i \in p$.

2.If $\phi \neq U \subseteq H, U \in p$ and H is single valued neutrosophic S^* open set, then $H \in p$.

3.If H is single valued neutrosophic S^* open set, then $H \notin p$ iff there exists $U \in p$ such that $U \in p$ such that $U \cap H = \phi$.

4.If $U_1 \cup U_2 = U_3 \in p, U_1$ and U_2 are single valued neutrosophic S^* open sets and $U_1 \cap U_2 = \phi$, then either $U_1 \in p$ or $U_2 \in p$.

5.If $SVNS^*cl(U) = S$ then $\phi \neq U \in p$ for any single valued neutrosophic S^* end p.

Proof:

1.If $U_i \in p \ (i = 1,2,3,....n)$ then $\bigcap_{i=1}^{n} U_i \neq \phi$ .As a contrary, suppose that $\bigcap_{i=1}^{n} U_i \notin p$, then $p \cup \left\{ \bigcap_{i=1}^{n} U_i \right\}$ will be a larger single valued neutrosophic S^*end than p .This contradicts the maximality of p .Therefore $\bigcap_{i=1}^{n} U_i \in p$.

2. If $H \notin p$, then $p \cup H$ will be a larger single valued neutrosophic S^* end than p.This contradicts the maximality of p.Therefore $H \in p$.

3.Suppose that $H \notin p$.If there exists no $U \in p$ such that $H \cap U = \phi$ then by Definition 3.13 and Definition 3.14, $H \in p$.This contradicts the maximality of p, since $p \cup \{H\}$ will be a larger single valued neutrosophic S^* end than p.Conversely, suppose that there exists $U \in p$ such that $H \cap U = \phi$.If $H \in p$ then $H \cap U \neq \phi$, which is a contradiction. Hence $H \notin p$.

4.If $U_1 \notin p, U_2 \notin p$, then $U_1 \cap U_3 = U_1 \notin p$ and $U_2 \cap U_3 = U_2 \notin p$.It follows that $U_3 \notin p$, which is a contradiction. Hence either $U_1 \in p$ or $U_2 \in p$.

5.$U \cap SVNS*cl(U) = U$ and $SVNS*cl(U) = S \in p$ for all single valued neutrosophic S^* ends p.By (3) $U \cap SVNS^*cl(U) = U \neq \phi$.Therefore $\phi \neq U \in p$ for all single valued neutrosophic S^* end p.

Definition 3.18:

Let $\theta(S)$ denote the collection of all single valued neutrosophic S^*ends belonging to S.A single valued neutrosophic S^* topology is introduced into $\theta(S)$ in the following way. Let O_U be the set of all single valued neutrosophic S^*ends that contains U as an element, where U is a single valued neutrosophic S^*open set of S.Therefore O_U is a single valued neutrosophic S^* neighbourhood of each single valued neutrosophic S^*end contained in O_U.

Definition 3.19:

A subset A of a single valued neutrosophic S^*structure space (S, S^*) is said to be an everywhere single valued neutrosophic S^*dense subset in (S, S^*) if $SVNS*cl(A) = S$.

Definition 3.20:

A subset of a single valued neutrosophic S^*structure space (S, S^*) is said to be a nowhere single valued neutrosophic S^* dense subset in (S, S^*) if $X \setminus A^c$ is everywhere single valued neutrosophic S^* dense subset.

Definition 3.21:

Let (S, S^*) be a single valued neutrosophic S^*structure space and Y be a single valued neutrosophic S^* open set in (S, S^*).Then the single valued neutrosophic S^* relative topology $T_Y = \{G \cap Y : G \in S^*\}$ is called the single valued neutrosophic S^* relative (or induced or subspace) topology on Y.The ordered pair (Y, T_Y) is called a single valued neutrosophic S^*subpace of the single valued neutrosophic S^*space (S, S^*).

Definition 3.22:

Let (S,S^*) be a single valued neutrosophic S^* structure space.

1.If a family $\{U_\alpha : i \in \Lambda\}$ of single valued neutrosophic S^* open sets in (S,S^*) satisfies the condition $S = \cup\{U_\alpha : i \in \Lambda\}$, then it is called a single valued neutrosophic S^* open cover of S. A finite subfamily of the single valued neutrosophic S^* open cover $\{U_\alpha : i \in \Lambda\}$ of S, which is also a single valued neutrosophic S^* open cover of S, is called a single valued neutrosophic S^* finite subcover .

2.A single valued neutrosophic S^* structure space (S,S^*) is called single valued neutrosophic S^* compact iff every single valued neutrosophic S^* open cover of S has a single valued neutrosophic S^* finite subcover.

Definition 3.23:

A single valued neutrosophic S^* Hausdorff space $\delta(S)$ is called an extension of a single valued neutrosophic S^* Hausdorff space S is contained in $\delta(S)$ as an everywhere single valued neutrosophic S^* dense subset.

Definition 3.24:

A single valued neutrosophic S^* Hausdorff space S is called single valued neutrosophic S^*H- closed if every extension coincides with S itself.

Definition 3.25:

An extension $\delta(S)$ is called a single valued neutrosophic S^*H-closed if $\delta(S)$ is single valued neutrosophic S^*H-closed and single valued neutrosophic S^* compact if $\delta(S)$ is single valued neutrosophic S^* compact.

Definition 3.26:

Let (S,S^*) be a single valued neutrosophic S^* structure space. A system **B** of single valued neutrosophic S^* open sets of a single valued neutrosophic S^* structure space S is called a single valued neutrosophic S^*base (or basis) for (S,S^*) if each member of (S,S^*) is a union of members of **B.**A member of **B** is called a single valued neutrosophic S^* basic open set.

Definition 3.27:

Let (S,S^*) be a single valued neutrosophic S^* structure space .A system of single valued neutrosophic S^* open sets of a single valued neutrosophic S^*structure space S is called a single valued neutrosophic S^* sub base if it together with all possible finite intersections of members of the system form a base of S .

Lemma 3.28:

A single valued neutrosophic S^* structure space S is single valued neutrosophic S^*H-closed if and only if any single valued neutrosophic S^* centered system $\{U_\alpha\}$ of single valued neutrosophic S^* open sets of S satisfies the condition $\bigcap_\alpha SVNS^*cl(U_\alpha) \neq \phi$.

Proof:

Necessity: If $p = \{U_\alpha\}$ is single valued neutrosophic S^* centered system with $\bigcap_\alpha SVNS^*cl(U_\alpha) = \phi$ then it can be constructed the following single valued neutrosophic S^* extensions $\delta(S)$ which does not coincide with S and a new member p .The single valued neutrosophic S^* neighbourhoods of each member $A \in S$ in $\delta(S)$ are the same as in S .Any set U_α together with the member p is a single valued neutrosophic S^* neighbourhood of p. Because of the condition $\bigcap_\alpha SVNS^*cl(U_\alpha) = \phi$, a single valued neutrosophic S^* structure space $\delta(S)$ is single valued neutrosophic S^*Hausdorff and since $\{U_\alpha\}$ is a single valued neutrosophic S^* centered system , it contains S as an everywhere single valued neutrosophic S^*dense subset. Therefore S is not a single valued neutrosophic S^*H-closed, which is a contradiction.

Sufficiency: Let S be a proper everywhere single valued neutrosophic S^* dense subset of $\delta(S)$ .Assume that $\delta(S)$ consists of all single valued neutrosophic S^* neighbourhoods of some member $p \in \delta(S) \setminus S$ .Let this be the system $\{U_\alpha\}$.This system is single valued neutrosophic S^* centered for otherwise p would be an isolated member in $\delta(S)$ and S would not be everywhere single valued neutrosophic S^* dense subset of $\delta(S)$, since $\delta(S)$ is single valued neutrosophic S^* Hausdorff space then $\bigcap_\alpha SVNS^*cl(V_\alpha^{\delta(S)}) = p$ .But the system $\{V_\alpha = U_\alpha \cap S\}$ is single valued neutrosophic S^* centered and $\bigcap_\alpha SVNS^*cl(V_\alpha^S) = \phi$, which contradicts the condition of the Lemma.

Lemma 3.29:

A single valued neutrosophic S^* structure space S is single valued neutrosophic S^*H-closed if and only if any maximal single valued neutrosophic S^* centered system $\{U_\alpha\}$ of single valued neutrosophic S^* open sets of S contains all the single valued neutrosophic S^* neighbourhoods of some member.

The proof follows easily from Lemma 3.28.

Lemma 3.30:

The single valued neutrosophic S^* structure space S is single valued neutrosophic S^*H- closed if and only if from any single valued neutrosophic S^* cover $\{U_\alpha\}$ of S a finite subsystem $U_i\,(i=1,2,3,\ldots n)$ may be chosen such that $\bigcup_{i=1}^{n} SVNS^*cl(U_i)=S$ .The proof follows from Lemma 3.28.

4. Single valued neutrosophic S^* centered systems

Definition 4.1:

Let $\{q\}$ be a collection of single valued neutrosophic S^* centered (not necessarily maximal) systems of single valued neutrosophic S^* open sets of S .A single valued neutrosophic S^* topology may be defined on this collection.

For if U is a single valued neutrosophic S^* open set of S .Let O_U denote the collection of all single valued neutrosophic S^*centered systems $q\in\{q\}$ containing U as an element. All sets of the form O_U form a sub base.

Definition 4.2:

Let $\delta(S)$ be an arbitrary single valued neutrosophic S^* extension of S . Every member $A\in\delta(S)$ in particular. A may belong to S defines a certain single valued neutrosophic S^* centered system in S , namely $\left\{V_\alpha^A = S\cap U_\alpha^A\right\}$ where U_α^A runs through all neighbouhoods of A in $\delta(S)$.

Note 4.3:

Every extension of an arbitrary single valued neutrosophic S^* Hausdorff space S can be realized as a single valued neutrosophic S^* structure space of centered systems of single valued neutrosophic S^* open sets of S with an appropriately chosen single valued neutrosophic S^* topology.

Lemma 4.4:

For any single valued neutrosophic S^*extension $\delta(S)$, the single valued neutrosophic S^* structure space $\delta_\sigma(S)$ is a single valued neutrosophic S^* extension of S and single valued neutrosophic $S^*\theta-$ homeomorphic to $\delta(S)$, where $\delta_\sigma(S)$ denote the single valued neutrosophic S^* structure space that is obtained by introducing a single valued neutrosophic S^* topology into a set of single valued neutrosophic S^* centered systems $\left\{V_\alpha^A\right\}$ by the mentioned above.

Proof:

Since if $\{V_\alpha^{A_1}\}$ and $\{V_\alpha^{A_2}\}$ are two single valued neutrosophic S^* centered systems constructed relative to single valued neutrosophic sets A_1 and A_2, $\delta_\sigma(S)$ is single valued neutrosophic S^* Hausdorff space. Since S is a single valued neutrosophic S^* Hausdorff space, by the above procedure, they contain disjoint elements. The relation $A \in U$ and $q_A \in O_U$ are equivalent so that S is single valued neutrosophic S^* homeomorphic to a subset of $\delta_\sigma(S)$ .Since $O_U \cap O_V = O_{U \cap V}$ and since O_U contains all the q_A for which $A \in U$, it follows that S is everywhere single valued neutrosophic S^* dense in $\delta_\sigma(S)$, that is $\delta_\sigma(S)$ is a single valued neutrosophic S^* extension of S .

Next to prove that $\delta_\sigma(S)$ and $\delta(S)$ are single valued neutrosophic $S^*\theta$ -homeomorphic. There is a single valued neutrosophic S^* one-to-one correspondence between the members of $\delta(S)$ and $\delta_\sigma(S)$ which is denoted by i .Thus $i(A) = A$ if $A \in S$ .Let $A' \in \delta_\sigma(S)$, $A' \in O_V, i(A) = A'$ and let U be a single valued neutrosophic S^* neighbourhood of A in $\delta(S)$ such that $U \cap S = V$ .We prove that $i(U) = O_V$ .This shows that the function i is single valued neutrosophic S^*continuous and hence it is single valued neutrosophic $S^*\theta-$ continuous. But this is obvious because if $A_1 \in U$ then $V \in i(A_1)$ and hence $i(A_1) \in O_V$.

To prove that the inverse function is single valued neutrosophic $S^*\theta$-continuous. Let U be a single valued neutrosophic S^* neighbourhood of $i(A)$, where $V = S \cap U$ .To show that $i^{-1}(SVNS^*cl(O_V)) \subset SVNS^*cl(U)$. Let $A' \in SVNS^*cl(O_V)$. This means that an arbitrary single valued neutrosophic S^* neighbourhood O_G of A' meets O_V , that is $G \cap V \neq \phi$ and this in turns means that an arbitrary single valued neutrosophic S^* neighbourhood of $i^{-1}(A')$ meets V that is $i^{-1}(A') \in SVNS^*cl(V) = SVNS^*cl(U)$ .Thus $i^{-1}(SVNS^*cl(O_V)) \subset SVNS^*cl(U)$ and the Lemma is proved.

Definition 4.5:

A single valued neutrosophic S^* extension $\delta(S)$ is of type σ if the function i (one – to-one correspondence between the members of $\delta(S)$ and $\delta_\sigma(S)$) is a single valued neutrosophic $S^*\theta-$ homeomorphism.

Definition 4.6:

A single valued neutrosophic S^* extension $\delta(S)$ is of type τ if the set $\delta(S)\setminus S$ is discrete in the single valued neutrosophic S^* relative topology.

Proposition 4.7:

Every single valued neutrosophic S^* extension of S is a single valued neutrosophic $S^*\theta-$ homeomorphic to some extension of type σ of the same space.

Proof:

The proof follows from the fact that the single valued neutrosophic S^* extension $\delta_\sigma(S)$ in Lemma 4.4 is of type σ.

Now, let $\delta(S)$ be any single valued neutrosophic S^* extension. Let $\delta_\tau(S)$ denote the single valued neutrosophic S^* structure space obtained as follows. The members of $\delta_\tau(S)$ are those of $\delta(S)$.The single valued neutrosophic S^* neighbourhoods of members of $A \in S$ are same as in S ,but for members $A \in \delta(S) \setminus S$ the single valued neutrosophic S^* neighbourhoods are obtained from those of A in $\delta(S)$ by rejecting the set $\delta(S) \setminus S \cup A$.Clearly $\delta_\tau(S)$ is asingle valued neutrosophic S^* Hausdorff space.

Definition 4.8:

Let (S_1, S_1^*) and (S_2, S_2^*) be two single valued neutrosophic S^* structure spaces. A single valued neutrosophic S^* structure space (S_1, S_1^*) is said to be topologically embedded in another single valued neutrosophic S^* structure space (S_2, S_2^*) if (S_1, S_1^*) is a single valued neutrosophic S^* homeomorphic to a single valued neutrosophic S^* subspace of (S_2, S_2^*).

Lemma 4.9:

For any single valued neutrosophic S^* extension $\delta(S)$, the single valued neutrosophic S^* structure space $\delta_\tau(S)$ is a single valued neutrosophic S^* extension of S, single valued neutrosophic $S^*\theta-$ homeomorphic to $\delta(S)$ and of type τ.

Proof:

It is clear that S is single valued neutrosophic S^* topologically embedded in $\delta_\tau(S)$ as an everywhere single valued neutrosophic S^* dense subset , that is , $\delta_\tau(S)$ is a single valued neutrosophic S^* extension of S.

From the construction of $\delta_\tau(S)$, $\delta_\tau(S) \setminus S$ is discrete and hence $\delta_\tau(S)$ is of type τ.It remains to show that $\delta_\tau(S)$ and $\delta(S)$ are single valued neutrosophic $S^*\theta-$ homeomorphic.

This follows from the fact that if U is single valued neutrosophic S^* open set in S, then $(SVNS^*cl(U))^{\delta(S)} = (SVNS^*cl(U))^{\delta_\tau(S)}$.Then the single valued neutrosophic S^* structure space $\delta_\tau(S)$ is mapped continuously onto $\delta(S)$.

Note 4.10:

From Lemma 4.9 each single valued neutrosophic S^* extension $\delta(S)$ of S is associated with single valued neutrosophic S^* extensions $\delta_\sigma(S)$ and $\delta_\tau(S)$, of types σ and τ respectively and single valued neutrosophic $S^*\theta-$ homeomorphic to each other and also single valued neutrosophic $S^*\theta-$ homeomorphic to the original single valued neutrosophic S^* extension $\delta(S)$.

Definition 4.11:

Let **G** be a single valued neutrosophic S^* base of single valued neutrosophic S^* open sets in a single valued neutrosophic S^* structure space S and $\sigma_G(S)$, the single valued neutrosophic S^* structure space whose elements are the members of S itself and all the maximal single valued neutrosophic S^* centered systems $\{U_\alpha\}$ consisting of single valued neutrosophic S^* open sets belonging to **G** , none of which contains as a subsystem of the single valued neutrosophic S^* neighbourhoods of any single valued neutrosophic S^* open set of S belonging to **G** (Clearly this condition is equivalent to the following : $\bigcap_\alpha SVNS^*cl(U_\alpha)=\phi$).

Definition 4.12:

A single valued neutrosophic S^* topology is defined in $\sigma_G(S)$ as follows. If $U\in$**G,** O_U denotes the set of all $A\in U$ and all maximal single valued neutrosophic S^* centered system in a $\sigma_G(S)$ that contains U as an element .Since in $\sigma_G(S)$ each member $A\in S$ can be replaced by the single valued neutrosophic S^* centered system of all its single valued neutrosophic S^* neighbourhoods belonging to **G** (with the single valued neutrosophic S^* topologization : $\{U_\alpha\}\in O_U$ if $U\in\{U_\alpha\}$).It is clear that each $\sigma_G(S)$ is a single valued neutrosophic S^* Hausdorff extension of type σ of the original single valued neutrosophic S^* structure space S .

Definition 4.13:

A single valued neutrosophic S^* centered system $\{U_\alpha\}$ of single valued neutrosophic S^* open sets of **G** is called a single valued neutrosophic S^* Hausdorff system if for every $B\in S$ not belonging to $U\in\{U_\alpha\}$ there exists a $U'\in\{U_\alpha\}$ such that $B\notin SVNS^*cl(U')$.

Definition 4.14:

A maximal single valued neutrosophic S^*Hausdorff system (that is, one which cannot be extended while remaining single valued neutrosophic S^*centered system and a single valued neutrosophic S^* Hausdorff space) is called a single valued neutrosophic S^* Hausdorff end.

Note 4.15:

A single valued neutrosophic S^*structure space $\sigma_G(S)$ associated with the base containing all the single valued neutrosophic S^* open sets of S will simply be denoted by $\sigma(S)$.

Proposition 4.16:

A single valued neutrosophic S^* extension $\sigma(S)$ is a single valued neutrosophic S^* $H-$closed extension of S.

Proof:

Let $\{U_\alpha\}$ be an arbitrary single valued neutrosophic S^* centered system of single valued neutrosophic S^* open sets of $\sigma(S)$ .Let $V_\alpha = U_\alpha \cap S$.

Since S is everywhere single valued neutrosophic S^* dense in $\sigma(S)$, $\left(SVNS^* cl(V_\alpha)\right)^{\sigma(S)} = \left(SVNS^* cl(U_\alpha)\right)^{\sigma(S)}$ .Hence it is enough to show that $\bigcap_\alpha \left(SVNS^* cl(V_\alpha)\right)^{\sigma(S)} \neq \phi$.If $\bigcap_\alpha \left(SVNS^* cl(V_\alpha)\right)^{S} \neq \phi$, then by Lemma 3.28, there is nothing to prove.

If $\bigcap_\alpha \left(SVNS^* cl(U_\alpha)\right)^{S} = \phi$, then there exists a single valued neutrosophic S^* Hausdorff end p containing all the sets V_α, and hence $p \in \left(SVNS^* cl(V_\alpha)\right)^{\sigma(S)}$ for all α .

Note 4.17:

Let **G** be any single valued neutrosophic S^* base of S .If $U \in$**G** then **G** is called single valued neutrosophic S^* algebraically closed.

Remark 4.18:

If **G** is called single valued neutrosophic S^* algebraically closed base of S , then $\sigma_G(S)$ is a single valued neutrosophic S^* $H-$closed extension of S .

The proof is same as that of Proposition 4.16.

Note 4.19:

Each single valued neutrosophic S^* extension $\delta(S)$ is associated with single valued neutrosophic S^* $\theta-$homeomorphic extension $\delta_\tau(S)$ of type τ ,the single valued neutrosophic S^*structure space $\sigma_\tau(S)$ which is associated with $\sigma(S)$ is denoted by $\tau(S)$ and is called a single valued neutrosophic S^*Katetov extension of S .

Lemma 4.20:

A single valued neutrosophic $S^*\theta-$continuous image of a single valued neutrosophic $S^* H-$ closed space is a single valued neutrosophic $S^* H-$ closed.

Proof:

Let f be a single valued neutrosophic $S^*\theta-$continuous function from a single valued neutrosophic S_1^*H-closed space S_1 onto single valued neutrosophic S_2^*H-closed space S_2 .Suppose that S_2 is not a single valued neutrosophic S_2^*H-closed ,then by Lemma 3.30 there exists a single valued neutrosophic S_2^* covering $\{U_\alpha\}$ of S_2 from which a finite number of single valued neutrosophic S_2^* open sets cannot be extracted whose single valued neutrosophic S^* closures cover S_2 .Let $A_1 \in S_1$ and V_β be a single valued neutrosophic S_1^* open set of S_1 such that $A_1 \in V_\beta$ and $f(SVNS_1^* cl(V_\beta)) \subset (SVNS_1^* cl(U_\alpha))$.

Choosing such a set for each member of S_1 ,the collection $\{V_\beta\}$ of this single valued neutrosophic S_1^* structure space is obtained. A finite number of sets $V_1, V_2, ...V_n$ is picked such that $\bigcup_{i=1}^{n} SVNS_1^* cl(V_i) = S_1$.But then the union $\bigcup_{i=1}^{n} SVNS_2^* cl(U_{\alpha_i}) \supseteq \bigcup_{i=1}^{n} f(SVNS_1^* cl(V_i)) = f(S_i) = S_2$. But in general $\bigcup_{i=1}^{n} SVNS_2^* cl(U_{\alpha_i}) \subseteq S_2$ implies that $\bigcup_{i=1}^{n} f(SVNS_1^* cl(V_i)) = S_2$ is the whole of S_2 ,which is impossible by hypothesis.

Remark 4.21:

The single valued neutrosophic S^* structure space $\tau(S)$ is a single valued neutrosophic S^*H- closed extension of S .

The proof follows from Proposition 4.16 and Lemma 4.20.

Note 4.22:

A single valued neutrosophic S^* structure space $\tau(S)$ has the following maximal properties.

Proposition 4.23:

If $\delta(S)$ is any (not necessarily single valued neutrosophic S^*H- closed) single valued neutrosophic S^* extension of S then there exists a subset $\tau_\delta(S) \subseteq \tau(S)$ containing S and a single valued neutrosophic S^* continuous function f_δ of this subset onto $\delta(S)$ such that $f_\delta(A) = A$, where $A \in S$. Here if $\delta(S)$ is a single valued neutrosophic S^*H- closed extension, it may be assumed that $\tau_\delta(S) = \tau(S)$.

Proof: Let $\delta(S)$ be a single valued neutrosophic S^* extension of S .Each member $q \in \delta(S) \setminus S$ defines a single valued neutrosophic S^* centered system of single valued neutrosophic S_1^* open sets in S ,namely q defines $\{V_\alpha\} = \{U_\alpha \cap S\}$ where U_α is the set of all single valued neutrosophic S_1^* neighbourhoods of q in $\delta(S)$ .It can be further identified each member of $\delta(S)$ with the corresponding single valued neutrosophic S^* centered system

$\{V_\alpha\}$.Because $\delta(S)$ is a fuzzy neutrosophic S^* Hausdorff space the system $\{V_\alpha\}$has the property $\bigcap_\alpha (SVNS * cl(V_\alpha))^S = \phi$.

Consider in $\tau(S)$,the subset $\tau_\delta(S)$ consisting of all members of S and all single valued neutrosophic S_2^* ends containing at least one system $\{V_\alpha\}$corresponding to some $q \in \delta(S)$.The function f_δ is constructed as follows: if $A \in S$,put $f_\delta(A) = A$,while if $p \in \tau(S) \setminus S$,then p contains some q. As q is unique put $f_\delta(p) = q$.

Clearly, f is a single valued neutrosophic S^* continuity at every $A \in S$.Because S is a single valued neutrosophic S_2^* open in $\tau(S)$ (by definition of the single valued neutrosophic S^* topology of $\tau(S)$), and hence also in $\tau_\delta(S)$.Let $p \in \tau_\delta(S) \setminus S$ and $f_\delta(p) = q$.

Let U_α be a single valued neutrosophic S^*neighbourhood of q in $\delta(S)$.Then the set $V_\alpha \cup p$

is a single valued neutrosophic S^* neighbourhood of p in $\tau_\delta(S)$,where $V_\alpha = U_\alpha \cap S$ with

$f_\delta(V_\alpha \cup p) \subseteq U_\alpha$, that is, f_δ is single valued neutrosophic S^* continuous at p.

Suppose that $\delta(S)$ is a single valued neutrosophic S^*H- closed extension. Let $p \in \tau(S) \setminus S$,and let $\{U_\alpha\}$be the system of all single valued neutrosophic S_2^* neighbourhoods of p in $\tau(S)$and let $V_\alpha = U_\alpha \cap S$.Let H_α denote a single valued neutrosophic S_1^* open set in $\delta(S)$ such that $V_\alpha = H_\alpha \cap S$.

The system $\{H_\alpha\}$is a single valued neutrosophic S_1^*centered system and since $\delta(S)$ is a single valued neutrosophic S_1^*H- closed ,then by Lemma 3.28, $\bigcap_\alpha (SVNS_1^* cl(H_\alpha)) \neq \phi$ .Let $q \in \bigcap_\alpha (SVNS_1^* cl(H_\alpha))$.If G is a single valued neutrosophic S_1^* neighbourhood of q in $\delta(S)$,we have $G \cap V_\alpha \neq \phi$ for every α ,that is , $(G \cap S) \in \{V_\alpha\}$.This means that p contains the single valued neutrosophic S_1^* centered system q and $\tau(S) \subseteq \tau_\delta(S)$,that is $\tau_\delta(S) = \tau(S)$.

Remark 4.24:

$\tau_{\delta\sigma}(S)$ denotes the single valued neutrosophic S^* structure space obtained from $\tau_\delta(S)$ by

the procedure described in section 4. It is easy to see that $\tau_{\delta\sigma}(S)$ is a single valued neutrosophic $S^*\theta-$ homeomorphic to a subset of single valued neutrosophic S^* extension $\sigma(S)$.As $\tau_{\delta\sigma}(S)$ is a single valued neutrosophic $S^*\theta-$homeomorphic to $\tau_\delta(S)$, Proposition 4.23 holds if $\tau(S)$ is replaced by $\sigma(S)$ and single valued neutrosophic S^* continuity by single valued neutrosophic $S^*\theta-$continuity.

Remark 4.25:

A single valued neutrosophic S^* structure space $\sigma(S)$ can be mapped single valued neutrosophic $S^*\theta-$continuity onto any single valued neutrosophic S^*H- closed extension of S in such a way that the members of S remain fixed.

Now, the classes of single valued neutrosophic S^* Hausdorff extensions of S are discussed.

Lemma 4.26:

A single valued neutrosophic S^* open set $\tau_\delta(S)$ is the largest subset of $\tau(S)$ that can be continuously mapped onto $\delta(S)$ in such a way that the members of S remain fixed. In other words, if a set $\tau_\delta^{'}(S)$ is continuously mapped onto $\delta(S)$ in such a way that the members of S remain fixed, then $\tau_\delta^{'}(S) \subseteq \tau_\delta(S)$.

Proof:

Let $p \in \tau_\delta^{'}(S) \setminus S$ and let $f_\delta^{'}(p) = q$,where $f_\delta^{'}$ is a single valued neutrosophic S^* continuous

function of $\tau_\delta^{'}(S)$ onto $\delta(S)$ .Let U be a single valued neutrosophic S^* neighbourhood of q in $\delta(S)$ .There exists a single valued neutrosophic S^* neighbourhood H of p in $\tau(S)$ such that $f_\delta^{'}(H) \subseteq U$ .Then ,$H \cap S \subseteq U \cap S$ that is , p contains $U \cap S$ and since U is any single valued neutrosophic S^* neighbourhood of q, p contains the system q,that is, $p \in \tau_\delta(S)$.

Note 4.27:

Thus, all single valued neutrosophic S^* extensions of S fall into classes, where $\delta(S)$ and

$\delta'(S)$ are in the same class if and only if $\tau_\delta(S) = \tau_\delta^{'}(S)$. All single valued neutrosophic S^*H- closed extensions belong to the same class, by Lemma 4.20 contains only single valued neutrosophic S^*H-closed extensions.

Lemma 4.28:

If single valued neutrosophic S^* extension $\delta(S)$ and $\gamma(S)$ are single valued neutrosophic $S^*\theta-$homeomorphic, then they belong to the same class, that is $\tau_\delta(S) = \tau_\gamma(S)$.

Proof:

Let i be a single valued neutrosophic $S^*\theta-$ homeomorphism between $\delta(S)$ and $\gamma(S)$ such

that $i(A) = A$ for $A \in S$ .Let $\{U_\alpha\} = \{V_\alpha \cap S\}$,where V_α is a single valued neutrosophic S^* neighbourhood of $p \in \tau(S) \setminus S$ .Let $\{H_\alpha\} = \{G_\alpha \cap S\}$,where G_α is a single valued neutrosophic S^* neighbourhood of $i(p) = q$ in $\gamma(S)$ .If some single valued neutrosophic S^* end d of S contains all the single valued neutrosophic sets U_α,then it also contains all the H_α.Choose some H_α and a G_α such that $G_\alpha \cap S = H_\alpha$,and in $\delta(S)$ choose V_β such that $i(SVNS*cl(V_\beta)) \subseteq SVNS*cl(G_\alpha)$ .Then , $SVNS*cl(V_\beta) \cap S \subseteq SVNS*cl(G_\alpha) \cap S$ .That is , $V_\beta \cap S = U_\beta \subseteq SVNS*\text{int}(SVNS*cl(G_\alpha) \cap S)$ .Hence ,if $SVNS*\text{int}(SVNS*cl(G_\alpha) \cap S) \in d$, then $G_\alpha \cap S = V_\alpha$ as the everywhere single valued neutrosophic S^* dense subset, $SVN\,\text{int}(SVNcl(G_\alpha) \cap S)$ also belongs to *d*.

Thus, $\tau_\delta(S) \subset \tau_\gamma(S)$.Similarly, $\tau_\delta(S) \supseteq \tau_\gamma(S)$.That is $\tau_\delta(S) = \tau_\gamma(S)$.

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