KANIKA MANDAL, KAJLA BASU

Department of Mathematics, NIT Durgapur, West Bengal 713209, India.
E-mails: boson89@yahoo.com, kajla.basu@gmail.com

Multi Criteria Decision Making Method in Neutrosophic Environment Using a New Aggregation Operator, Score and Certainty Function

Abstract

Neutrosophic sets, being generalization of classic sets, fuzzy sets and intuitionistic fuzzy sets, can simultaneously represent uncertain, imprecise, incomplete, and inconsistent information existing in the real world. Neutrosophic theory has been developed in twenty first century and not much of arithmetic has been developed for this set. To solve any problem using neutrosophic data, it is desirable to have suitable operators, score function etc. Some operators like single valued neutrosophic weighted averaging (SVNWA) operator, single valued neutrosophic weighted geometric (SVNWG) operator are already defined in neutrosophic set (NS). In this paper an improved weighted average geometric (IWAG) operator which produces more meaningful results has been introduced to aggregate some real numbers and the same has been extended in neutrosophic environment. We further generalize this to include a wide range of aggregation operators for both real numbers and neutrosophic numbers. A new score function and certainty function have been defined which have some benefit compared to the existing ones. Further comparative study highlighting the benefit of this new approach of ranking in neutrosophic set has been presented. A multiple-attribute decision-making method is established on the basis of the proposed operator and newly defined score function.

Keywords

Fuzzy critical path, crashing. probability factor, neutrosophic set, aggregation Operator, score function, certainty function, and Decision Making.

1 INTRODUCTION

Neutrosophic set (NS), the generalization of classic set, fuzzy set, intutionistic fuzzy set, was first introduced by Smarandache. Smarandache [1] defined the degree of indeterminacy/neutrality as independent component in 1995 (published in 1998). NS can express uncertain, imprecise, incomplete and inconsistent information more precisely. How to aggregate information is an important problem in real management and decision process. Due to the complexity of management environments and decision problems, decision makers may provide their ratings or judgments to some certain degree, but it is possible that they are not so sure about their judgments. Namely, there may exist some uncertain, imprecise, incomplete, and inconsistent information, which are very important factors to be taken into account when trying to construct really adequate models and solutions of decision problems. Such kind of information is suitably expressed with neutrosophic fuzzy sets rather than exact numerical values, fuzzy or intuitionistic fuzzy. Thus, how to aggregate neutrosophic fuzzy information becomes an important part of multi-attribute decision-making with neutrosophic fuzzy sets.

In [17] Zhang-peng Tian et al. solved green product design selection problems using neutrosophic linguistic information. Xiao-hui Wu et al. [13] established ranking methods for simplified neutrosophic sets based on prioritized aggregation operators and cross-entropy measures to solve multi criteria decision making (MCDM) problem. Interval neutrosophic linguistic aggregation operators were developed and applied to the medical treatment selection process [16] by Yinxiang Ma et al. In [20] Hong-yu Zhang, Jian-qiang Wang and Xiao-hong Chen defined some reliable operations for interval-valued neotrosophic sets. Based on those operators they also developed two aggregation operators which were applied to solve a MCDM problem. Jun Ye introduced in [4] single-valued neutrosophic hesitant fuzzy weighted averaging *(SVNHFWA)* operator and a single-valued neutrosophic hesitant fuzzy weighted geometric *(SVNHFWG)* operator and using those operator a multiple-attribute decision-making method was established. Peide Liu, Yanchang Chu, Yanwei Li, and Yubao Chen [7] presented some operational laws for neutrosophic numbers (NNs) based on Hamacher operations and proposed several averaging operators and applied them to group decision making. Broumi, Smarandache defined operations based on the arithmetic mean, geometrical mean and harmonic mean on interval-valued neutrosophic sets in [8].

In this paper we introduce an improved aggregating operator named improved weighted averaging geometric mean *(IWAGM)* for real numbers which produces more meaningful results and extend it for single valued neutrosophic set (SVNS) as improved single valued weighted averaging geometric *(ISVWAG)* operator. We further generalize the *IWAGM* operator and introduce generalized improved weighted averaging geometric mean *(GIWAGM)* which includes a wide range of weighted average geometric operators. Also we extend the *GIWAGM* for single valued neutrosophic numbers. We introduce a new score function and certainty function which are illustrated using simple examples and applied to numerical example. A comparative study highlighting the benefit of this new approach of ranking in NS has been discussed. An algorithm has been given to find optimum solution of multi-criteria decision making problem and a numerical illustration for a network problem has been presented.

The operators developed in this paper are original and have been developed for SVNS for the first time. Very few works have been done on averaging operator in SVNS. The score function and certainty function newly introduced in this paper have some benefits compared to the existing ones. The score function and certainty function defined earlier give same score function value for different neutrosophic numbers easily. But the newly proposed ones can remove this difficulty.

The rest of the paper is structured as follows: Section 2 introduces some concepts of neutrosophic sets and simplified neutrosophic sets. Section 3 describes weighted average mean *(WAM)* and weighted geometric mean *(WGM)* for real numbers and their limitations. In section 4, we define a new *IWAGM* for real numbers and compare the results with the existing ones highlighting the improvement over the *WAM* and *WGM*. *IWAGM* has been extended in neutrosophic environment in section 5. In section 6, we generalize the *IWAGM* introduced in section 4 and extend the generalization for neutrosophic numbers. Section 7 introduces a new approach defining a new score function and certainty function to compare the neutrosophic numbers. Why the approach is more realistic and meaningful is discussed in this section. Section 8 presents the algorithm for finding optimum alternative among alternatives in a decision making problem in neutrosophic environment using the introduced operator *IWAG* in subsection 5.2 and the comparison approach defined in section 7. In section 9, a numerical example demonstrates the application and effectiveness of the proposed aggregation operator and comparison rules in decision-making problems. We conclude the paper in section 10.

2 NEUTROSOPHIC SETS

2.1 Definition

Let U be an universe of discourse then the neutrosophic set A is defined as $A = \{\langle x : T_A(x), I_A(x), F_A(x)\rangle, x \in U\}$, where the functions *T, I, F*: $U \to]^-0, 1^+[$ define respectively the degree of membership (or Truth), the degree of indeterminacy and the degree of non-membership (or falsehood) of the element $x \in U$ to the set A with the condition $^-0 \le T_A(x) + I_A(x) + F_A(x) \le 3^+$.

To apply neutrosophic set to science and technology, we consider the neutrosophic set which takes the value from the subset of $[0,1]$ instead of $]^{-}0,1^{+}[$; i.e., we consider SNS as defined by Ye in [3].

2.2 Simplified Neutrosophic Set

Let X be a space of points (objects) with generic elements in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy membership function $I_A(x)$, and a falsity-membership function $F_A(x)$, if the functions $T_A(x), I_A(x), F_A(x)$ are singleton subintervals/subsets in the real standard $[0,1]$, i.e., $T_A(x) : X \to [0,1]$, $I_A(x) : X \to [0,1]$ and $F_A(x) : X \to [0,1]$. Then a simplification of the neutrosophic set A is denoted by $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X\}$.

2.3 Simplified neutrosophic set(SVNS)

Let X be a space of points (objects) with generic elements in X denoted by x. A SVNS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy membership function $I_A(x)$ and a falsity-membership function $F_A(x)$, for each point $x \in X$, $T_A(x), I_A(x), F_A(x) \in [0,1]$. Therefore, a SVNS A can be written as $A_{SVNS} = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X\}$. For two SVNS, $A_{SVNS} = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X\}$ and $B_{SVNS} = \{\langle x, T_B(x), I_B(x), F_B(x)\rangle, x \in X\}$, the following expressions are defined in [12] as follows:
$A_{NS} \subseteq B_{NS}$ if and only if $T_A(x) \leq T_B(x)$, $I_A(x) \geq I_B(x)$, $F_A(x) \geq F_B(x)$.
$A_{NS} = B_{NS}$ if and only if $T_A(x) = T_B(x)$, $I_A(x) = I_B(x)$, $F_A(x) = F_B(x)$.
$A^c = \langle x, F_A(x), 1 - I_A(x), T_A(x)\rangle$.

For convenience, a SVNS A is denoted by $A = \langle T_A(x), I_A(x), F_A(x)\rangle$ for any x in X. For two SVNSs A and B, the operational relations (1), (2), (3) are defined by [3] and (4) by [2]
(1) $A + B = \langle\ T_A(x) + T_B(x) - T_A(x)T_B(x),\ I_A(x) + I_B(x) - I_A(x)I_B(x), F_A(x) + F_B(x) - F_A(x)F_B(x)\ \rangle$.
(2) $A.B = \langle T_A(x).T_B(x), I_A(x).I_B(x), F_A(x).F_B(x)\rangle$
(3) $A^\lambda = \left\langle T_A^\lambda(x), I_A^\lambda(x), F_A^\lambda(x)\right\rangle$.
(4) For any scalar $\lambda > 0$, $\lambda A = \langle \min(\lambda T_A(x), 1), \min(\lambda I_A(x), 1), \min(\lambda F_A(x), 1)\rangle$.

3 AGGREGATION OPERATORS

Aggregation operators are mathematical functions that are used to combine information. That is, they are used to combine N data (for example, N numerical

values) in a single datum. In classical algebra WAM and WGM are very useful to combine n real numbers $a_1, a_2, \ldots, a_n$.
The WAM of n real numbers $a_1, a_2, \ldots, a_n$ with associated weights $w_1, w_2, \ldots, w_n$ respectively, $w_i \in [0, 1]$ and $\sum w_i = 1$, is defined by $\sum_{i=1}^{n} a_i w_i$.
The WGM of n real numbers $a_1, a_2, \ldots, a_n$ with associated weights $w_1, w_2, \ldots, w_n$ respectively, $w_i \in [0, 1]$ and $\sum w_i = 1$, is defined by $\prod_{i=1}^{n} a_i^{w_i}$.

3.1 Some limitations of WAM and WGM

The result of an aggregation operator is meaningful if its value tends to one or some number(s) (among those to be combined) whose weight(s) is on the higher side. They do not correctly aggregate the information, if the aggregated value does not tend towards maximum arguments or does not lie between the maximum and minimum arguments. Let us consider some cases.
Example. Case 1: Take two real numbers 0.0001 and 1 with their weights $w_1 = 0.9$, $w_2 = 0.1$ respectively. Then $WAM = 0.10009$, $WGM = 0.000251$. Case 2: Again take 0.0001 and 1 with their weights $w_1 = 0.1$, $w_2 = 0.9$ respectively. Then $WAM = 0.90001$, $WGM = 0.398107$.
From these results we observe that from the first case the value of WGM is more close to the number whose weight is maximum than WAM. So in this case, WGM aggregates the numbers more close to the highest weighted number. On the other side in the second case the value of WAM is nearest to the maximum weighted number whereas WGM is close to 0.0001, the minimum weighted number. Here WAM value is more meaningful. The examples show that WAM and WGM operators may not simultaneously give meaningful result while aggregating the information. Now we propose a new aggregation operator that always gives a moderate value close to the maximum weighted number.

4 THE NEWLY PROPOSED WEIGHTED MEAN

Let $a_1, a_2, \ldots, a_n$ are n real numbers with associated weights $w_1, w_2, \ldots, w_n$ respectively, $w_i \in [0, 1]$ and $\sum w_i = 1$. Then we define improved weighted average geometric mean ($IWAGM$) as

$$IWAGM(a_1, a_2, \ldots, a_n) = \sum_{i=1}^{n} a_i^{\frac{1}{2}} w_i \prod_{i=1}^{n} a_i^{\frac{w_i}{2}} \tag{1}$$

4.1 Properties

Let $a_1, a_2, \ldots, a_n$ are n real numbers. Then the aggregated result of the $IWAGM$ operator clearly satisfies desired properties of an aggregation operator :

(1). **Idempotency:** let a_i $(i = 1, 2, \ldots, n)$ be a collection of real numbers. If all a_i $(i = 1, 2, \ldots, n)$ are equal, that is, $a_i = a$, for all$(i = 1, 2, \ldots, n)$, then $IWAGM(a_1, a_2, \ldots, a_n) = a^{\frac{1}{2}} \sum_{i=1}^{n} w_i \prod_{i=1}^{n} a^{\frac{w_i}{2}} = a$

(2). **Boundedness:** If $a^- = \min_i a_i$ and $a^+ = \max_i a_i$, $\sum_{i=1}^{n}(a^-)^{\frac{1}{2}} w_i \prod_{i=1}^{n}(a^-)^{\frac{w_i}{2}} \leq \sum_{i=1}^{n} a_i^{\frac{1}{2}} w_i \prod_{i=1}^{n} a_i^{\frac{w_i}{2}} \leq \sum_{i=1}^{n}(a^+)^{\frac{1}{2}} w_i \prod_{i=1}^{n}(a^+)^{\frac{w_i}{2}}$,
i.e. $a^- \leq IWAGM(a_1, a_2, \ldots, a_n) \leq a^+$.

(3). **Symmetry or commutativity:** The order of the arguments has no influence on the result. For every permutation σ of $1, 2, \ldots, n$ the operator satisfies $IWAGM(a_{\sigma(1)}, a_{\sigma(2)}, \ldots, a_{\sigma(n)}) = IWAGM(a_1, a_2, \ldots, a_n)$

(4). **Monotonicity :** If $a_i \leq a_i^*$ for all $(i = 1, 2, \ldots, n)$,
$\sum_{i=1}^{n} a_i^{\frac{1}{2}} w_i \prod_{i=1}^{n} a_i^{\frac{w_i}{2}} \leq \sum_{i=1}^{n}(a_i^*)^{\frac{1}{2}} w_i \prod_{i=1}^{n}(a_i^*)^{\frac{w_i}{2}}$.
So $IWAGM(a_1, a_2, \ldots, a_n) \leq IWAGM(a_1^*, a_2^*, \ldots, a_n^*)$.

4.2 Theorem

For n real numbers $a_1, a_2, \ldots, a_n$,
$WGM(a_1, a_2, \ldots, a_n) \leq IWAGM(a_1, a_2, \ldots, a_n) \leq WAM(a_1, a_2, \ldots, a_n)$.

Proof: We know WAM of some real numbers always greater than or equal to WGM of those real numbers.
So if we consider n numbers $a_1^{\frac{1}{2}}, a_2^{\frac{1}{2}}, \ldots, a_n^{\frac{1}{2}}$ with their weights $w_1, w_2, \ldots, w_n$ respectively, then
$\sum_{i=1}^{n} a_i^{\frac{1}{2}} w_i \geq \prod_{i=1}^{n} a_i^{\frac{w_i}{2}}$.
Now, $\frac{\sum_{i=1}^{n} a_i^{\frac{1}{2}} w_i \prod_{i=1}^{n} a_i^{\frac{w_i}{2}}}{\prod_{i=1}^{n} a_i^{w_i}} = \frac{\sum_{i=1}^{n} a_i^{\frac{1}{2}} w_i}{\prod_{i=1}^{n} a_i^{\frac{w_i}{2}}} \geq 1$.
i.e.,

$$WGM(a_1, a_2, \ldots, a_n) \leq IWAGM(a_1, a_2, \ldots, a_n) \tag{2}$$

Again we know if $a_1, a_2, \ldots, a_n$ be positive real numbers, not all equal, $w_1, w_2, \ldots, w_n$ be positive real numbers such that $\sum_{i=1}^{n} w_i = 1$ and m is rational,

lies between 0 and 1, $\sum_{i=1}^{n} w_i a_i^m \leq (\sum_{i=1}^{n} w_i a_i)^m$.
Taking $m = \frac{1}{2}$,

$$\sum_{i=1}^{n} w_i a_i^{\frac{1}{2}} \leq (\sum_{i=1}^{n} w_i a_i)^{\frac{1}{2}} \tag{3}$$

Also, $\prod_{i=1}^{n} a_i^{w_i} \leq \sum_{i=1}^{n} a_i w_i$
Taking square root in both side,

$$\prod_{i=1}^{n} a_i^{\frac{w_i}{2}} \leq (\sum_{i=1}^{n} a_i w_i)^{\frac{1}{2}} \tag{4}$$

Multiplying (3) and (4), we get

$$IWAGM(a_1, a_2, \ldots, a_n) \leq WAM(a_1, a_2, \ldots, a_n) \tag{5}$$

So combining (2) and (5), we get our proposed result.

4.3 Meaningful advantage of the proposed operator

Using the newly introduced operator, the aggregated results of the numbers with their weightage given in subsection 3.1, are given below: For case 1, $IWAGM(0.0001, 1) = 0.001728$, and for case 2, $IWAGM(0.0001, 1) = 0.5684$. So in both the cases the newly introduced operator gives a moderate value close to the maximum weighted number. WAM and WGM may not simultaneously give meaningful result for all the numbers; but result from the proposed operator is meaningful since it holds the relation (2) and (5). In fact the new operator improves both the WAM and WGM and gives a moderate, meaningful value.

5 WEIGHTED AGGREGATION OPERATORS IN NEUTROSOPHIC ENVIRONMENT

5.1 Extension of WAM and WGM of classical algebra in neutrosophic set

In neutrosophic environment $SVNWA$ and $SVNWG$, the most well known aggregation operators, are the extension of WAM and WGM of classical algebra.

5.1.1 Definition I

Let $A_i = (T_{A_i}(x), I_{A_i}(x), F_{A_i}(x))$ $(i = 1, 2, \ldots, n)$ be a collection of SVNSs. A mapping $F_w : SVNS^n \to SVNS$ is called single valued neutrosophic weighted averaging operator of dimension n if it satisfies $F_w(A_1, A_2, \ldots, A_n) = \sum_{i=1}^{n} w_i A_i$, where $w = (w_1, w_2, \ldots, w_n)^T$ is the weight vector of A_i $(i = 1, 2, \ldots, n)$, $w_i \in [0, 1]$ and $\sum w_i = 1$.

5.1.2 Definition II

Let $A_i = (T_{A_i}(x), I_{A_i}(x), F_{A_i}(x))$ $(i = 1, 2, \ldots, n)$ be a collection of SVNSs. A mapping $F_w : SVNS^n \to SVNS$ is called SVNG operator of dimension n if it satisfies $F_w(A_1, A_2, \ldots, A_n) = \prod_{i=1}^{n} A_i^{w_i}$.

5.2 Extension of proposed aggregation operator in neutrosophic set

Let $A_i = (T_A(i), I_A(i), F_A(i))$ $(i = 1, 2, \ldots, n)$ be a collection of SVNSs. Then we define improved single valued weighted averaging geometric $(ISVWAG)$ operator as

$$ISVWAG(A_1, A_2, \ldots, A_n) = \sum_{i=1}^{n} A_i^{\frac{1}{2}} w_i \prod_{i=1}^{n} A_i^{\frac{w_i}{2}} \tag{6}$$

5.2.1 Properties

Let $A_i = (T_{A_i}(x), I_{A_i}(x), F_{A_i}(x))$ $(i = 1, 2, \ldots, n)$ be a collection of SVNSs. Then the aggregated result of the $ISVNWAG$ operator is also a single valued neutrosophic number (SVNN) and satisfies the desired properties of an aggregation operator.
To prove the properties we first prove a lemma.

5.2.2 Lemma 1

Let $A_1 = (T_{A_1}(x), I_{A_1}(x), F_{A_1}(x))$, $A_2 = (T_{A_2}(x), I_{A_2}(x), F_{A_2}(x))$, $B_1 = (T_{B_1}(x), I_{B_1}(x), F_{B_1}(x))$, $B_2 = (T_{B_2}(x), I_{B_2}(x), F_{B_2}(x))$ are SVNNs such that $A_1 \supseteq B_1$, $A_2 \supseteq B_2$. Then $(A_1 + A_2) \supseteq (B_1 + B_2)$. i.e., $(T_{A_1} + T_{A_2} - T_{A_1}T_{A_2}) \geq (T_{B_1} + T_{B_2} - T_{B_1}T_{B_2})$, $(I_{A_1} + I_{A_2} - I_{A_1}I_{A_2}) \leq (I_{B_1} + I_{B_2} - I_{B_1}I_{B_2})$, $(F_{A_1} + F_{A_2} - F_{A_1}F_{A_2}) \leq (F_{B_1} + F_{B_2} - F_{B_1}F_{B_2})$.

Proof: Let X be the universe. For each point $x \in X$, $T_A(x), I_A(x), F_A(x) \in [0,1]$. Now it is given that $A_1 \supseteq B_1$, $A_2 \supseteq B_2$, i.e., for each value of $x \in X$, $T_{A_1}(x) \geq T_{B_1}(x)$ and $T_{A_2}(x) \geq T_{B_2}(x)$. Let x_1 be an arbitrary point in X. So $T_{A_1}(x_1) \geq T_{B_1}(x_1)$ and $T_{A_2}(x_1) \geq T_{B_2}(x_1)$. Also $T_{A_1}(x_1), T_{A_2}(x_1), T_{B_1}(x_1), T_{B_2}(x_1) \in [0,1]$. $\cos x$ for $x \in [0, \frac{\pi}{2}]$ is a continuous function. i.e., $\cos x$ assumes every value in $[0,1]$. So we can consider $T_{A_1}(x_1) = \cos\phi_1$, $T_{A_2}(x_1) = \cos\phi_2$, $T_{B_1}(x_1) = \cos\theta_1$, $T_{B_2}(x_1) = \cos\theta_2$, for some $\phi_1, \phi_2, \theta_1, \theta_2 \in [0, \frac{\pi}{2}]$.
Now $T_{A_1}(x_1)+T_{A_2}(x_1)-T_{A_1}(x_1)T_{A_2}(x_1) = \cos\phi_1+\cos\phi_2-\cos\phi_1\cos\phi_2 = \cos\phi_1+(1-\cos\phi_1)\cos\phi_2 = 1-2\sin^2\frac{\phi_1}{2}+2\sin^2\frac{\phi_1}{2}\cos\phi_2 = 1-2\sin^2\frac{\phi_1}{2}(1-\cos\phi_2) = 1-2\sin^2\frac{\phi_1}{2}.2\sin^2\frac{\phi_2}{2} = 1-4\sin^2\frac{\phi_1}{2}\sin^2\frac{\phi_2}{2}$.
Similarly, $T_{B_1}(x_1)+T_{B_2}(x_1)-T_{B_1}(x_1)T_{B_2}(x_1) = 1-4\sin^2\frac{\theta_1}{2}\sin^2\frac{\theta_2}{2}$.
Since $T_{A_1}(x_1) \geq T_{B_1}(x_1), T_{A_2}(x_1) \geq T_{B_2}(x_1)$,
$\cos\phi_1 \geq \cos\theta_1$. i.e., $-\cos\phi_1 \leq -\cos\theta_1$, $1-\cos\phi_1 \leq 1-\cos\theta_1$, i.e., $2\sin^2\frac{\phi_1}{2} \leq 2\sin^2\frac{\theta_1}{2}$.
Similarly, $2\sin^2\frac{\phi_2}{2} \leq 2\sin^2\frac{\theta_2}{2}$.
i.e., $4\sin^2\frac{\phi_1}{2}\sin^2\frac{\phi_2}{2} \leq 4\sin^2\frac{\theta_1}{2}\sin^2\frac{\theta_2}{2}$.
i.e., $1-4\sin^2\frac{\phi_1}{2}\sin^2\frac{\phi_2}{2} \geq 1-4\sin^2\frac{\theta_1}{2}\sin^2\frac{\theta_2}{2}$.
i.e.,

$$T_{A_1}(x_1)+T_{A_2}(x_1)-T_{A_1}(x_1)T_{A_2}(x_1) \geq T_{B_1}(x_1)+T_{B_2}(x_1)-T_{B_1}(x_1)T_{B_2}(x_1) \quad (7)$$

Since (7) is true for any $x_1 \in X$, $T_{A_1}(x)+T_{A_2}(x)-T_{A_1}(x)T_{A_2}(x) \geq T_{B_1}(x)+T_{B_2}(x)-T_{B_1}(x)T_{B_2}(x)$.
So it has been shown that $T_{A_1} \geq T_{B_1}$ and $T_{A_2} \geq T_{B_2}$ imply $[T_{A_1}+T_{A_2}-T_{A_1}T_{A_2}] \geq [T_{B_1}+T_{B_2}-T_{B_1}T_{B_2}]$. In the same way,
$I_{A_1} \leq I_{B_1}$ and $I_{A_2} \leq I_{B_2}$ imply $(I_{A_1}+I_{A_2}-I_{A_1}I_{A_2}) \leq (I_{B_1}+I_{B_2}-I_{B_1}I_{B_2})$ also $F_{A_1} \leq F_{B_1}$ and $F_{A_2} \leq F_{B_2}$ imply $(F_{A_1}+F_{A_2}-F_{A_1}F_{A_2}) \leq (F_{B_1}+F_{B_2}-F_{B_1}F_{B_2})$. The proof is generic as it is true for each and every value of the truth, indeterminacy and falsity membership functions and does not depend on the types of the functions (triangular, trapezoidal, piecewise linear or Gaussian).

On the basis of the basic operations of SVNSs described in subsection 2.3, the value of the truth, indeterminacy and falsity membership function in aggregated result belongs to $[0,1]$. So the aggregated operator is also a SVNN. We will prove that the $ISVNWAG$ operator has the following desired properties:

(1) **Idempotency:** let A_i $(i = 1,2,\ldots,n)$ be a collection of SVNNs. If all A_i $(i = 1,2,\ldots,n)$ are equal, that is, $A_i = A$, for all$(i = 1,2,\ldots,n)$, then $ISVWAG(A_1, A_2, \ldots, A_n) = A$

(2) **Boundedness:** Let $A^- = (T_{A^-}(x), I_{A^-}(x), F_{A^-}(x))$, and $A^+ = (T_{A^+}(x), I_{A^+}(x), F_{A^+}(x))$, where $T_{A^-}(x) = \min_i T_{A_i}(x)$, $I_{A^-}(x) = \max_i I_{A_i}(x)$, $F_{A^-}(x) = \max_i F_{A_i}(x)$ and $T_{A^+}(x) = \max_i T_{A_i}(x)$, $I_{A^+}(x) = \min_i I_{A_i}(x)$, $F_{A^+}(x) = \min_i F_{A_i}(x)$. So $A^- \subseteq A_i \subseteq A^+$ for all $(i = 1, 2, \ldots, n)$. Also $(T_{A^-}(x))^{0.5} w_i \leq (T_{A_i}(x))^{0.5} w_i \leq (T_{A^+}(x))^{0.5} w_i$, $(I_{A^-}(x))^{0.5} w_i \geq (I_{A_i}(x))^{0.5} w_i \geq (I_{A^+}(x))^{0.5} w_i$ and $(F_{A^-}(x))^{0.5} w_i \geq (F_{A_i}(x))^{0.5} w_i \geq (F_{A^+}(x))^{0.5} w_i$. So $(A^-)^{0.5} w_i \subseteq (A_i)^{0.5} w_i \subseteq (A^+)^{0.5} w_i$. By using lemma 1, $\sum_{i=1}^n (A^-)^{0.5} w_i \subseteq \sum_{i=1}^n A_i^{\frac{1}{2}} w_i \subseteq \sum_{i=1}^n (A^+)^{0.5} w_i$, i.e.,

$$(A^-)^{\frac{1}{2}} \subseteq \sum_{i=1}^n A_i^{\frac{1}{2}} w_i \subseteq (A^+)^{\frac{1}{2}} \tag{8}$$

Again similarly, $(A^-)^{\frac{w_i}{2}} \subseteq A_i^{\frac{w_i}{2}} \subseteq (A^+)^{\frac{w_i}{2}}$, i.e.,

$$(A^-)^{\frac{1}{2}} \subseteq \prod_{i=1}^n A_i^{\frac{w_i}{2}} \subseteq (A^+)^{\frac{1}{2}} \tag{9}$$

From (8)

$$T_{(A^-)^{\frac{1}{2}}} \leq T_{\sum_{i=1}^n A_i^{\frac{1}{2}} w_i} \leq T_{(A^+)^{\frac{1}{2}}} \tag{10}$$

From (9)

$$T_{(A^-)^{\frac{1}{2}}} \leq T_{\prod_{i=1}^n A_i^{\frac{w_i}{2}}} \leq T_{(A^+)^{\frac{1}{2}}} \tag{11}$$

Multiplying (10) and (11) we get, $T_{A^-} \leq T_{ISVWAG(A_1, A_2, \ldots, A_n)} \leq T_{A^+}$.
similarly, $I_{A^-} \geq I_{ISVWAG(A_1, A_2, \ldots, A_n)} \geq I_{A^+}$ and $F_{A^-} \geq F_{ISVWAG(A_1, A_2, \ldots, A_n)} \geq F_{A^+}$.
Thus $A^- \subseteq ISVWAG(A_1, A_2, \ldots, A_n) \subseteq A^+$.

(3) **Symmetry or commutativity:** The order of the arguments has no influence on the result. For every permutation σ of $1, 2, \ldots, n$ the operator satisfies $ISVWAG(A_{\sigma(1)}, A_{\sigma(2)}, \ldots, A_{\sigma(n)}) = ISVWAG(A_1, A_2, \ldots, A_n)$

(4) **Monotonicity :**Let $A_i \subseteq A_i^*$ for all $(i = 1, 2, \ldots, n)$, then $T_{A_i}(x) \leq T_{A_i^*}(x)$, $I_{A_i}(x) \geq I_{A_i^*}(x)$ and $F_{A_i}(x) \geq F_{A_i^*}(x)$. i.e., $(T_{A_i}(x))^{0.5} w_i \leq (T_{A_i^*}(x))^{0.5} w_i$, $(I_{A_i}(x))^{0.5} w_i \geq (I_{A_i^*}(x))^{0.5} w_i$ and $(F_{A_i}(x))^{0.5} w_i \geq (F_{A_i^*}(x))^{0.5} w_i$. So $(A_i)^{0.5} w_i \subseteq (A_i^*)^{0.5} w_i$. Therefore from the lemma 1 $\sum_{i=1}^n A_i^{0.5} w_i \subseteq \sum_{i=1}^n (A_i^*)^{0.5} w_i$ and also since $\prod_{i=1}^n A_i^{\frac{w_i}{2}} \subseteq \prod_{i=1}^n A_i^{*\frac{w_i}{2}}$, $ISVWAG(A_1, A_2, \ldots, A_n)$
$\subseteq ISVWAG(A_1^*, A_2^*, \ldots, A_n^*)$.

6 GENERALIZATION OF $IWAGM$ AND ITS EXTENSION IN NEUTROSOPHIC ENVIRONMENT

We formulate a general operator in case of real numbers and extend it to neutrosophic set also.
Let $a_1, a_2, \ldots, a_n$ are n real numbers with associated weights $w_1, w_2, \ldots, w_n$ respectively, $w_i \in [0,1]$ and $\sum w_i = 1$. Then we define generalized improved weighted averaging geometric mean ($GIWAGM$) as

$$GIWAGM(a_1, a_2, \ldots, a_n) = (\sum_{i=1}^{n} a_i^{\frac{1}{k}} w_i \prod_{i=1}^{n} a_i^{\frac{w_i}{k}})^{k/2} \tag{12}$$

where k is any real number. The equation (12) satisfy the desired properties of aggregation operator:

(1) **Idempotency:** let a_i $(i = 1, 2, \ldots, n)$ be a collection of real numbers. If all a_i $(i = 1, 2, \ldots, n)$ are equal, that is, $a_i = a$, for all$(i = 1, 2, \ldots, n)$, then $GIWAGM(a_1, a_2, \ldots, a_n) = (a^{\frac{1}{k}} \sum_{i=1}^{n} w_i \prod_{i=1}^{n} a^{\frac{w_i}{k}})^{k/2} = a$

(2) **Boundedness:** If $a^- = \min_i a_i$ and $a^+ = \max_i a_i$,
$(\sum_{i=1}^{n} (a^-)^{\frac{1}{k}} w_i \prod_{i=1}^{n} (a^-)^{\frac{w_i}{k}})^{k/2} \leq (\sum_{i=1}^{n} a_i^{\frac{1}{k}} w_i \prod_{i=1}^{n} a_i^{\frac{w_i}{k}})^{k/2} \leq (\sum_{i=1}^{n} (a^+)^{\frac{1}{k}}$ $w_i \prod_{i=1}^{n} (a^+)^{\frac{w_i}{k}})^{k/2}$, i.e. $a^- \leq GIWAGM(a_1, a_2, \ldots, a_n) \leq a^+$.

(3) **Symmetry or commutativity:** The order of the arguments has no influence on the result. For every permutation σ of $1, 2, \ldots, n$ the operator satisfies $GIWAGM(a_{\sigma(1)}, a_{\sigma(2)}, \ldots, a_{\sigma(n)}) = GIWAGM(a_1, a_2, \ldots, a_n)$

(4) **Monotonicity :**If $a_i \leq a_i^*$ for all $(i = 1, 2, \ldots, n)$,
$(\sum_{i=1}^{n} a_i^{\frac{1}{k}} w_i \prod_{i=1}^{n} a_i^{\frac{w_i}{k}})^{k/2} \leq (\sum_{i=1}^{n} (a_i^*)^{\frac{1}{k}} w_i \prod_{i=1}^{n} (a_i^*)^{\frac{w_i}{k}})^{k/2}$.
So $GIWAGM(a_1, a_2, \ldots, a_n) \leq GIWAGM(a_1^*, a_2^*, \ldots, a_n^*)$.
And for neutrosophic sets let $A_i = (T_A(i), I_A(i), F_A(i))$ $(i = 1, 2, \ldots, n)$ be a collection of SVNSs with associated weights $w_1, w_2, \ldots, w_n$ respectively, $w_i \in [0,1]$ and $\sum w_i = 1$.. Then we define generalized improved single valued weighted averaging geometric (GISVWAG) operator as

$$GISVWAG(A_1, A_2, \ldots, A_n) = (\sum_{i=1}^{n} A_i^{\frac{1}{k}} w_i \prod_{i=1}^{n} A_i^{\frac{w_i}{k}})^{k/2} \tag{13}$$

where k is any real number. The equation (13) satisfy the desired properties of aggregation operator:

(1) **Idempotency:** let A_i $(i = 1, 2, \ldots, n)$ be a collection of SVNNs. If all A_i $(i = 1, 2, \ldots, n)$ are equal, that is, $A_i = A$, for all$(i = 1, 2, \ldots, n)$, then $ISVWAG(A_1, A_2, \ldots, A_n) = A$

(2) **Boundedness:** Let $A^- = (T_{A^-}(x), I_{A^-}(x), F_{A^-}(x))$, and $A^+ = (T_{A^+}(x), I_{A^+}(x), F_{A^+}(x))$, where $T_{A^-}(x) = \min_i T_{A_i}(x)$, $I_{A^-}(x) = \max_i I_{A_i}(x)$, $F_{A^-}(x) = \max_i F_{A_i}(x)$ and $T_{A^+}(x) = \max_i T_{A_i}(x)$, $I_{A^+}(x) = \min_i I_{A_i}(x)$, $F_{A^+}(x) = \min_i F_{A_i}(x)$. So $A^- \subseteq A_i \subseteq A^+$ for all $(i = 1, 2, \ldots, n)$. Also $(T_{A^-}(x))^{1/k} w_i \leq (T_{A_i}(x))^{1/k} w_i \leq (T_{A^+}(x))^{1/k} w_i$, $(I_{A^-}(x))^{1/k} w_i \geq (I_{A_i}(x))^{1/k} w_i \geq (I_{A^+}(x))^{1/k} w_i$ and $(F_{A^-}(x))^{1/k} w_i \geq (F_{A_i}(x))^{1/k} w_i \geq (F_{A^+}(x))^{1/k} w_i$. So $(A^-)^{1/k} w_i \subseteq (A_i)^{1/k} w_i \subseteq (A^+)^{1/k} w_i$. By using lemma 1, $\sum_{i=1}^n (A^-)^{1/k} w_i \subseteq \sum_{i=1}^n A_i^{1/k} w_i \subseteq \sum_{i=1}^n (A^+)^{1/k} w_i$, i.e.,

$$(A^-)^{1/k} \subseteq \sum_{i=1}^{n} A_i^{1/k} w_i \subseteq (A^+)^{1/k} \tag{14}$$

Again similarly, $(A^-)^{\frac{w_i}{k}} \subseteq A_i^{\frac{w_i}{k}} \subseteq (A^+)^{\frac{w_i}{k}}$, i.e.,

$$(A^-)^{\frac{1}{k}} \subseteq \prod_{i=1}^{n} A_i^{\frac{w_i}{k}} \subseteq (A^+)^{\frac{1}{k}} \tag{15}$$

From (14)

$$T_{(A^-)^{\frac{1}{k}}} \leq T_{\sum_{i=1}^n A_i^{\frac{1}{k}} w_i} \leq T_{(A^+)^{\frac{1}{k}}} \tag{16}$$

From (15)

$$T_{(A^-)^{\frac{1}{k}}} \leq T_{\prod_{i=1}^n A_i^{\frac{w_i}{k}}} \leq T_{(A^+)^{\frac{1}{k}}} \tag{17}$$

Multiplying (16) and (17) we get, $T_{(A^-)^{\frac{2}{k}}} \leq T_{\sum_{i=1}^n A_i^{\frac{1}{k}} w_i} T_{\prod_{i=1}^n A_i^{\frac{w_i}{k}}} \leq T_{(A^+)^{\frac{2}{k}}}$.
i.e., $T_{A^-} \leq T_{GISVWAG(A_1, A_2, \ldots, A_n)} \leq T_{A^+}$.
similarly, $I_{A^-} \geq I_{GISVWAG(A_1, A_2, \ldots, A_n)} \geq I_{A^+}$ and $F_{A^-} \geq F_{GISVWAG(A_1, A_2, \ldots, A_n)} \geq F_{A^+}$. So $A^- \subseteq GISVWAG(A_1, A_2, \ldots, A_n) \subseteq A^+$.

(3) **Symmetry or commutativity:** The order of the arguments has no influence on the result. For every permutation σ of $1, 2, \ldots, n$ the operator satisfies $GISVWAG(A_{\sigma(1)}, A_{\sigma(2)}, \ldots, A_{\sigma(n)}) = GISVWAG(A_1, A_2, \ldots, A_n)$

(4) **Monotonicity :**Let $A_i \subseteq A_i^*$ for all $(i = 1, 2, \ldots, n)$, then $T_{A_i}(x) \leq T_{A_i^*}(x)$, $I_{A_i}(x) \geq I_{A_i^*}(x)$ and $F_{A_i}(x) \geq F_{A_i^*}(x)$. i.e., $(T_{A_i}(x))^{1/k}w_i \leq (T_{A_i^*}(x))^{1/k}w_i$, $(I_{A_i}(x))^{1/k}w_i \geq (I_{A_i^*}(x))^{1/k}w_i$ and $(F_{A_i}(x))^{1/k}w_i \geq (F_{A_i^*}(x))^{1/k}w_i$. So $(A_i)^{1/k}w_i \subseteq (A_i^*)^{1/k}w_i$. Therefore from the lemma 1, $\sum_{i=1}^{n} A_i^{1/k}w_i \subseteq \sum_{i=1}^{n}(A_i^*)^{1/k}w_i$ and also since $\prod_{i=1}^{n} A_i^{\frac{w_i}{k}} \subseteq \prod_{i=1}^{n} A_i^{*\frac{w_i}{k}}$, $GISVWAG(A_1, A_2, \ldots, A_n) \subseteq$ $GISVWAG(A_1^*, A_2^*, \ldots, A_n^*)$.

Now if we put $k = 2$, (12) and (13) reduce to (1) and (6) respectively, i.e., the newly proposed operator for real numbers given in (1) is one of the particular cases of generalized operator (12) and similar for the case (6) and (13) also. For different values of k it is possible to study these families individually.

7 COMPARISON APPROACH

7.1 Definition [20], [7]

Let A and B are two SVNN. Then the comparison approach based on score function (s), accuracy function (a) and certainty function (c) is given as follows:
(1) If $s(A) > s(B)$, then $A > B$.
(2) If $s(A) = s(B)$ and $a(A) > a(B)$, then $A > B$.
(3) If $s(A) = s(B)$ also $a(A) = a(B)$, but $c(A) > c(B)$, then $A > B$.
(4) If $s(A) = s(B)$, $a(A) = a(B)$ and $c(A) = c(B)$, then $A = B$.

7.2 Proposed score and certainty function

We introduce a new score function, accuracy function and certainty function to compare neutrosophic fuzzy numbers. According to the definition of score function as defined in [20], the larger the T_A is, the greater the neutrosophic number is; the smaller the I_A is, the greater the neutrosophic number is and the same holds for F_A also. Based on the definition we give a new score function. Let $A = (T_A(x), I_A(x), F_A(x))$ be a neutrosophic number. The **score function of** A is given by $s(A) = T_A(x)(1 + \sin(T_A(x)\frac{\pi}{2})) + \frac{1}{2(1+I_A(x))}(\cos(I_A(x)\frac{\pi}{2})) + \frac{1}{1+F_A(x)}(\cos(F_A(x)\frac{\pi}{2}))$. The accuracy function as defined in [7] is $a(A) = T_A(x) - F_A(x)$.
We define a new **certainty function** $c(A) = \frac{|cosT_A(x)\pi| + |cosI_A(x)\pi| + |cosF_A(x)\pi|}{3}$. In [7] Liu et al. gave the formula of score, accuracy function for a SVNN, A, as follows: Score function $s_1(A) = 2 + T_A(x) - I_A(x) - F_A(x)$, accuracy function $a_1(A) = T_A(x) - F_A(x)$. With these formulas in [20] Hong-yu Zhang, Jian-qiang

Wang, and Xiao-hong Chen added certainty function as $c_1(A) = T_A(x)$. The score function in [7] gives same value when $I_A(x)$ and $F_A(x)$ of a neutrosophic number is interchanged, i.e., different neutrosophic numbers can exist easily for which the given score function gives same value. For example $(0.2, 0.9, 0.1)$ and $(0.2, 0.1, 0.9)$ have same score function according [7] and [20]. But the newly introduced score function based on trigonometric function does not give same value for these neutrosophic sets. From another point of view as discussed in [5] the uncertainty is maximum (=1) at $(0.5, 0.5, 0.5)$, i.e., the certainty should be minimum (=0) at $(0.5, 0.5, 0.5)$ and the value of certainty increases if we increase or decrease any of truth, indeterminacy and falsity membership grade. But this property is not satisfied by the certainty function given in [20], whereas the newly proposed certainty function in sec 7 gives realistic result.

7.3 Comparison analysis using different examples

Table 1: Comparison analysis using different examples

Neutrosophic numbers	*Method*	*Score value*	*Accuracy value*	*Certainty Value*	*Ranking order*
$A = (0.4, 0.3, 0.2)$	*Existing*	$s_1(A) = 1.9$			$A > B$
$B = (0.4, 0.5, 0.6)$		$s_1(B) = 1.3$			
	Proposed	$s(A) = 1.77$			$A > B$
		$s(B) = 1.23$			
$A = (1, 0, 1)$	*Existing*	$s_1(A) = 2$			$A > B$
$B = (1, 1, 0.473)$		$s_1(B) = 1.527$			
	Proposed	$s(A) = 2.5$	$-$		$A < B$
		$s(B) = 2.5$	$a(B) = 0.527$		
$A = (0.0867, 0.2867, 0.0867)$	*Existing*	$s_1(A) = 1.7133$			$A > B$
$B = (0.3096, 0.5096, 0.0.3096)$		$s_1(B) = 1.4904$			
	Proposed	$s(A) = 1.3599$	$-$	$c(A) = 0.8491$	$A > B$
		$s(B) = 1.3599$	$-$	$c(B) = 0.38548$	

8 ALGORITHM FOR FINDING OPTIMUM ALTERNATIVE IN A MULTI-CRITERIA DECISION MAKING PROBLEM

Let $A_i, (i = 1, 2, \ldots, m)$ be m alternatives and $C_j, (j = 1, 2, \ldots, n)$ are n criteria. Assume that the weight of the criteria $C_j (j = 1, 2, \ldots, n)$, given by the decision-maker, is w_j, $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$. The m options according to the n criterion are given below:

	C_1	C_2	C_3	$\ldots$	C_n
A_1	C_1^1	C_2^1	C_3^1	$\ldots$	C_n^1
A_2	C_1^2	C_2^2	C_3^2	$\ldots$	C_n^2
A_3	C_1^3	C_2^3	C_3^3	$\ldots$	C_n^3
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
A_m	C_1^m	C_2^m	C_3^m	$\ldots$	C_n^m

where each C_j^i, $(i = 1, 2, \ldots, m)$ and $(j = 1, 2, \ldots, n)$ are in neutrosophic form and $C_j^i = \left\{T_{C_j}^i, I_{C_j}^i, F_{C_j}^i\right\}$ We propose a method to derive optimum alternative among the given alternatives through the algorithm given below:

Step 1: use the $ISVWAG$ operator given in (6) to combine n criteria for each alternative.

Step 2: calculate the score, accuracy and certainty function to compare the neutrosophic number as defined in section 7.

Step 3: Rank the alternatives.

9 NUMERICAL EXAMPLE

In a certain network, there are four options to go from one node to the other. Which path to be followed will be impacted by two benefit criteria C_1, C_2 and one cost criteria C_3 and the weight vectors are 0.35, 0.25 and 0.40 respectively. A decision maker evaluates the four options according to the three criteria mentioned above. We compare the proposed method with the existing methods in table 3 using the newly introduced approach to obtain the most desirable alternative from the decision matrix given in table 2.

Table 2: Decision matrix (information given by DM)

	c_1	c_2	c_3
A_1	(0.4,0.2,0.3)	(0.4,0.2,0.3)	(0.2,0.2,0.5)
A_2	(0.6,0.1,0.2)	(0.6,0.1,0.2)	(0.5,0.2,0.2)
A_3	(0.3,0.2,0.3)	(0.5,0.2,0.3)	(0.5,0.3,0.2)
A_4	(0.7,0,0.1)	(0.6,0.1,0.2)	(0.4,0.3,0.2)

9.1 Comparison of aggregation operators using cosine similarity measure

To measure the similarity between two neutrosophic numbers we consider the cosine similarity measure as discussed by Jun Ye in [9] as follows:

Table 3: Result Comparison: the proposed method with the existing methods

Aggregation Operator	*Aggregated Result*	*Score using existing method*	*Score using proposed formula*	*Ranking order in both approach*
Single valued weighted average	$SVWA(C_1^{(1)}, C_2^{(1)}, C_3^{(1)}) = (0.287, 0.187, 0.337)$	$s_1(A_1) = 1.76$	$s(A_1) = 1.46$	$A_4 > A_2 > A_3 > A_1$
	$SVWA(C_1^{(2)}, C_2^{(2)}, C_3^{(2)}) = (0.462, 0.134, 0.187)$	$s_1(A_2) = 2.14$	$s(A_2) = 2.007$	
	$SVWA(C_1^{(3)}, C_2^{(3)}, C_3^{(3)}) = (0.373, 0.222, 0.238)$	$s_1(A_3) = 1.912$	$s(A_3) = 1.716$	
	$SVWA(C_1^{(4)}, C_2^{(4)}, C_3^{(4)}) = (0.460, 0.142, 0.156)$	$s_1(A_4) = 2.16$	$s(A_4) = 2.03$	
Single valued weighted geometric	$SVWG(C_1^{(1)}, C_2^{(1)}, C_3^{(1)}) = (0.303143, 0.2, 0.368011)$	$s_1(A_1) = 1.735$	$s(A_1) = 1.450532$	$A_4 > A_2 > A_3 > A_1$
	$SVWG(C_1^{(2)}, C_2^{(2)}, C_3^{(2)}) = (0.5578, 0.131951, 0.2)$	$s_1(A_2) = 2.22$	$s(A_2) = 2.211256$	
	$SVWG(C_1^{(3)}, C_2^{(3)}, C_3^{(3)}) = (0.418141, 0.235216, 0.255085)$	$s_1(A_3) = 1.92$	$s(A_3) = 1.7845$	
	$SVWG(C_1^{(4)}, C_2^{(4)}, C_3^{(4)}) = (0.538451, 0, 0.156917)$	$s_1(A_4) = 2.38$	$s(A_4) = 2.2798$	
Improved single valued weighted average geometric	$ISVWAG(C_1^{(1)}, C_2^{(1)}, C_3^{(1)}) = (0.254226, 0.172108, 0.303)$	$s(A_1) = 1.77$	$s(A_1) = 1.44$	$A_4 > A_2 > A_3 > A_1$
	$ISVWAG(C_1^{(2)}, C_2^{(2)}, C_3^{(2)}) = (0.432056, 0.118963, 0.17)$	$s_1(A_2) = 2.14$	$s(A_2) = 1.96$	
	$ISVWAG(C_1^{(3)}, C_2^{(3)}, C_3^{(3)}) = (0.338061, 0.201253, 0.21)$	$s_1(A_3) = 1.92$	$s(A_3) = 1.68$	
	$ISVWAG(C_1^{(4)}, C_2^{(4)}, C_3^{(4)}) = (0.421219, 0, 0.13)$	$s_1(A_4) = 2.28$	$s(A_4) = 2.03$	

Let X be the universe and $A = \{\langle x_i, T_A(x_i), I_A(x_i), F_A(x_i)\rangle / x_i \in X\}$ and $B = \{\langle x_i, T_B(x_i), I_B(x_i), F_B(x_i)\rangle / x_i \in X\}$ are two SVNSs, then cosine similarity measure between A and B is
$C(A,B) = \frac{1}{n}\sum_{i=1}^{n} \frac{T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)+F_A(x_i)F_B(x_i)}{\sqrt{(T_A(x_i))^2+I_A(x_i))^2+F_A(x_i))^2}\sqrt{(T_B(x_i))^2+I_B(x_i))^2+F_B(x_i))^2}}$
Using the similarity measure formula comparison of aggregation operators are given in table 4:

9.2 Result discussion

The results given in table 4 show that all the aggregated results are more or less close to the corresponding maximum weighted neutrosophic number as similarity measure values are nearer to 1. Also it is observed that the proposed method gives almost same similarity measure value as the other existing methods as discussed

Table 4: Comparison of aggregation operators using similarity measure

Alternative	*Aggregation operator*	*Aggregated result*	*Corresponding maximum weighted number*	*Similarity measure value*
	$SVWA$	$(0.287, 0.187, 0.337)$		0.978
A_1	$SVWG$	$(0.303143, 0.2, 0.368011)$	$(0.4, 0.2, 0.3)$	0.975
	$ISVWAG$	$(0.254226, 0.172108, 0.303)$		0.977
	$SVWA$	$(0.46, 0.13, 0.18)$		0.993
A_2	$SVWG$	$(0.55, 0.13, 0.2)$	$(0.6, 0.1, 0.2)$	0.997
	$ISVWAG$	$(0.43, 0.11, 0.17)$		0.995
	$SVWA$	$(0.373, 0.222, 0.238)$		0.9806
A_3	$SVWG$	$(0.418, 0.23, 0.25)$	$(0.3, 0.2, 0.3)$	0.974
	$ISVWAG$	$(0.33, 0.2, 0.21)$		0.9803
	$SVWA$	$(0.46, 0.14, 0.15)$		0.946
A_2	$SVWG$	$(0.54, 0, 0.16)$	$(0.7, 0, 0.1)$	0.989
	$ISVWAG$	$(0.42, 0, 0.13)$		0.987

in table 4. In other words, the newly introduced operator gives moderate and meaningful value similar to existing methods and close to the maximum weighted neutrosophic number.

10 CONCLUSION

At first we introduced a new aggregation operator ($IWAGM$) to combine n real numbers. We proved that the result using this operator always lies between WAM and WGM operator and the result will be meaningful in all the cases. Then we extended the operator in neutrosophic environment and it has also been shown that the extended operator ($ISVWAG$) gives meaningful result in neutrosophy. Next we introduced a trigonometric function based score function. Further we proposed a certainty function as well which gives realistic results comparison to the existing ones. A numerical problem has been solved using the proposed operator and the newly defined score function.

ACKNOWLEDGMENT

The authors are very grateful to Florentin Smarandache, Surapati Pramanik and Pranab Biswas for their insightful and constructive comments and suggestions, which have been very helpful in improving the paper.

References

[1] Florentin Smarandache, Neutrosophy/Neutrosophic Probability, Set and Logic, *Amer. Res.Press, Rehoboth, USA*, (1998).

[2] Haibin Wang, Florentin Smarandache, Yanqing Zhang and Rajshekhar Sunderraman, Interval Neutrosophic Sets and Logic: Theory and Application in Computing, *Hexis, Neutrosophic Book Series*, (2005).

[3] Jun Ye, A Multi-criteria Decision Making Method Using Aggregation Operators for Simplified Neutrosophic Sets, *Journal of Intelligent & Fuzzy Systems*, **26** (2014), 2459-2466, doi 10.3233/IFS-130916.

[4] Jun Ye, Multiple-attribute Decision Making Method under a Single Valued Neutrosophic Hesitant Fuzzy Environment, *Journal of Intelligent Systems*, **24** (2014), 23-36, doi 10.1515/jisys-2014-0001.

[5] Pinaki Majumdar and S.K. Samanta, On Similarity and Entropy of Neutrosophic Sets, *Journal of Intelligent and Fuzzy Systems*, **26** (2014), 1245–1252, doi 10.3233/IFS-130810.

[6] Lotfi Zadeh, Fuzzy Entropy and Conditioning, *Information Sciences*, **40** (1986), 165–174.

[7] Peide Liu, Yanchang Chu, Yanwei Li and Yubao Chen, Some Generalized Neutrosophic Number Hamacher Aggregation Operators and Their Application to Group Decision Making, *International Journal of Fuzzy systems*, **16** (2014), 242–255.

[8] Said Broumi and Florentin Smarandache, New Operations on Interval Neutrosophic Sets, *Journal of New Theory*, **1** (2015), 24–27.

[9] Jun Ye, Vector Similarity Measures of Simplified Neutrosophic Sets and Their Application in Multicriteria Decision Making, *International Journal of Fuzzy Systems*, **16** (2014), 204–215.

[10] Wenkai Zhang, Xia Li and Yanbing Ju, Some Aggregation Operators Based on Einstein Operations under Interval-Valued Dual Hesitant Fuzzy Setting and Their Application, *Mathematical Problems in Engineering*, **2014** (2014), 958927–9589248, doi 10.1155/2014/958927.

[11] Said Broumi and Florentin Smarandache, Cosine Similarity Measure of Interval Valued Neutrosophic Sets, *Neutrosophic Sets and Systems*, **5** (2014), 15–20.

[12] Haibin Wang, Florentin Smarandache, Yanqing Zhang and Rajshekhar Sunderraman, Single Valued Neutrosophic Sets, *in Multispace and Multistructure*, **4**, (2010), 410–413. http://fs.gallup.unm.edu/SingleValuedNeutrosophicSets.pdf.

[13] Zhang-peng Tian, Jing Wang, Hong-yu Zhang and Jian-qiang Wang, Multi-criteria Decision Making Based on Generalized Prioritized Aggregation Operators under Simplified Neutrosophic Uncertain Linguistic Environment, *Int. J. Mach. Learn. & Cyber*, (2016), 1–17, doi 10.1007/s13042-016-0552-9.

[14] Juan-juan Peng, Jian-qiang Wang, Hong-yu Zhang and Xiao-hong Chen, An Outranking Approach for Multi-criteria Decision Making Problems with Simplified Neutrosophic Sets, *Applied Soft Computing*, **25** (2014), 336–346.

[15] Zhang-peng Tian, Hong-yu Zhang, Jing Wang, Jian-qiang Wang and Xiao-hong Chen, Multi-criteria Decision Making Method Based on a Cross-entropy with Interval Neutrosophic Sets, *International Journal of Systems Science*, **47** (2016), 3598–3608, doi 10.1080/00207721.2015.1102359.

[16] Yin-xiang Ma, Jian-qiang Wang, Jing Wang and Xiaohui Wu, An Interval Neutrosophic Linguistic Multi-criteria Group Decision Making Method and Its Application in Selecting Medical Treatment Options, *Neural Computer & Application*, (2016), 1–21, doi 10.1007/s00521-016-2203-1.

[17] Zhang-peng Tian, Jing Wang, Jian-qiang Wang and Hong-yu Zhang, Simplified Neutrosophic Linguistic Multicriteria Group Decision-Making Approach to Green Product Development, *Group Decision and Negotiation*, **4** (2016), doi 10.1007/s10726-016-9479-5.

[18] Hong-yu Zhang, Pu Ji, Jian-qiang Wang and Xiao-hong Chen, A Neutrosophic Normal Cloud and Its Application in Decision Making, *Cognitive Computation*, **8** (2016), 649–669, doi 10.1007/s12559-016-9394-8,2016.

[19] Zhang-peng Tian, Jing Wang, Jian-qiang Wang and Hong-yu Zhang, A Likelyhood-based Qualitative Flexible Approach with Hesitant Fuzzy Linguistic Information, *Cognitive Computation*, **8** (2016), 670–683, doi 10.1007/s12559-016- 9400-1,2016.

[20] Hong-yu Zhang, Jian-qiang Wang and Xiao-hong Chen, Interval Neutrosophic Sets and Their Application in Multi-criteria Decision Making Problems, *The Scientic World Journal*, **2014** (2014), 645953-645967, doi 10.1155/2014/645953.