# The quintic: $z^5 + z^4 + z - 1 = 0$

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**abstract**

The quintic: $p(z) = z^5 + z^4 + z - 1 = 0$ , the number pi , and fractals.

## 1. Introducción.

La ecuación :

$$p(z) = z^5 + z^4 + z - 1 = 0 \tag{1}$$

posee una raíz real y cuatro complejas $\left( i = \sqrt{-1} \right)$ :

$$r = 0.66796070 \ldots \tag{2}$$

$$s = 0.32731953 \ldots + i\, 0.84977785 \ldots \tag{3}$$

$$s^* = 0.32731953 \ldots - i\, 0.84977785 \ldots \tag{4}$$

$$t = -1.16129988 \ldots + i\, 0.67580978 \ldots \tag{5}$$

$$t^* = -1.16129988 \ldots - i\, 0.67580978 \ldots \tag{6}$$

poniendo : $u + i\, v = p(x + i\, y)$ , se tiene :

$$u = \mathrm{Re}(p(x + i\, y)) = -1 + x + x^4 + x^5 - 6\, x^2\, y^2 - 10\, x^3\, y^2 + y^4 + 5\, x\, y^4 \tag{7}$$

$$v = \mathrm{Im}(p(x + i\, y)) = y + 4\, x^3\, y + 5\, x^4\, y - 4\, x\, y^3 - 10\, x^2\, y^3 + y^5 \tag{8}$$

poniendo : $z = w - a$ , $a = 1/5$ , $2/5$ , $3/5$ , se tiene :

$$3125\, w^5 - 1250\, w^3 + 500\, w^2 + 3050\, w - 3746 = 0 \tag{9}$$

$$3125\, w^5 - 3125\, w^4 + 1000\, w^2 + 2725\, w - 4327 = 0 \tag{10}$$

$$3125\, w^5 - 6250\, w^4 + 3750\, w^3 + 2450\, w - 4838 = 0 \tag{11}$$

sea $x_n$ , $n \in \mathbb{N}$, la sucesión definida por :

$$x_{n+1} = \frac{1 + 3\, x_n^4 + 4\, x_n^5}{1 + 4\, x_n^3 + 5\, x_n^4} \; , \; x_1 = . .. \tag{12}$$

se tiene :

$$x_1 = 1 \Longrightarrow x_n \longrightarrow r \tag{13}$$

$$x_1 = \frac{1}{2} + i \Longrightarrow x_n \longrightarrow s \tag{14}$$

$$x_1 = \frac{1}{2} - i \Longrightarrow x_n \longrightarrow s^* \tag{15}$$

$$x_1 = -1 + \frac{i}{2} \Longrightarrow x_n \longrightarrow t \tag{16}$$

$$x_1 = -1 - \frac{i}{2} \Longrightarrow x_n \longrightarrow t^* \tag{17}$$

otras recurrencias para $r$ , $s$ , $s^*$ , $t$ , $t^*$ :

$$x_{n+1} = \frac{1 + 2\,x_n - x_n^4 - x_n^5}{3} \;,\; x_1 = 0 \Longrightarrow x_n \longrightarrow r \tag{18}$$

$$x_{n+1} = \frac{(2 + 4\,i)\,x_n - 1 + x_n^4 + x_n^5}{1 + 4\,i} \;,\; x_1 = 0 \Longrightarrow x_n \longrightarrow s \tag{19}$$

$$x_{n+1} = \frac{(2 - 4\,i)\,x_n - 1 + x_n^4 + x_n^5}{1 - 4\,i} \;,\; x_1 = 0 \Longrightarrow x_n \longrightarrow s^* \tag{20}$$

$$x_{n+1} = \frac{(8 + 4\,i)\,x_n - 1 + x_n^4 + x_n^5}{7 + 4\,i} \;,\; x_1 = 0 \Longrightarrow x_n \longrightarrow t \tag{21}$$

$$x_{n+1} = \frac{(8 - 4\,i)\,x_n - 1 + x_n^4 + x_n^5}{7 - 4\,i} \;,\; x_1 = 0 \Longrightarrow x_n \longrightarrow t^* \tag{22}$$

fórmulas para pi :

$$\pi = 4\tan^{-1}(r) + 4\tan^{-1}\left(r^4\right) \tag{23}$$

$$\pi = 4\tan^{-1}(s) + 4\tan^{-1}\left(s^4\right) \tag{24}$$

$$\pi = -4\tan^{-1}(1/t) - 4\tan^{-1}\left(1\big/t^4\right) \tag{25}$$

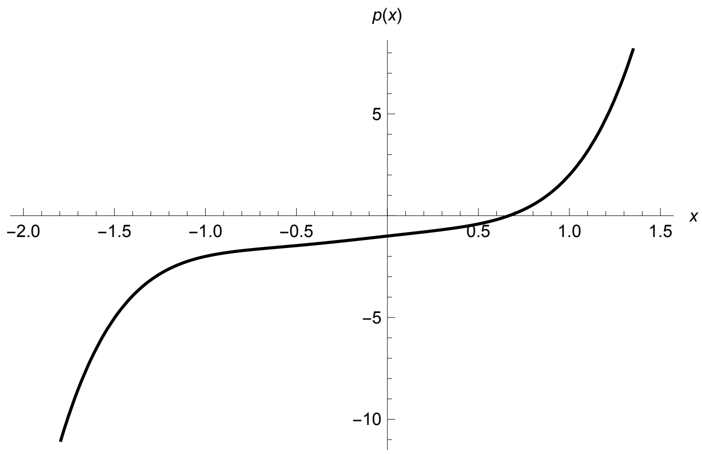
$$\pi = 4\sum_{n=0}^{\infty} \frac{c_n}{n+1}\, r^{n+1} = 4\sum_{n=0}^{\infty} \frac{c_n}{n+1}\, s^{n+1} = -4\sum_{n=0}^{\infty} \frac{c_n}{n+1}\, (1/t)^{n+1} \tag{26}$$

donde

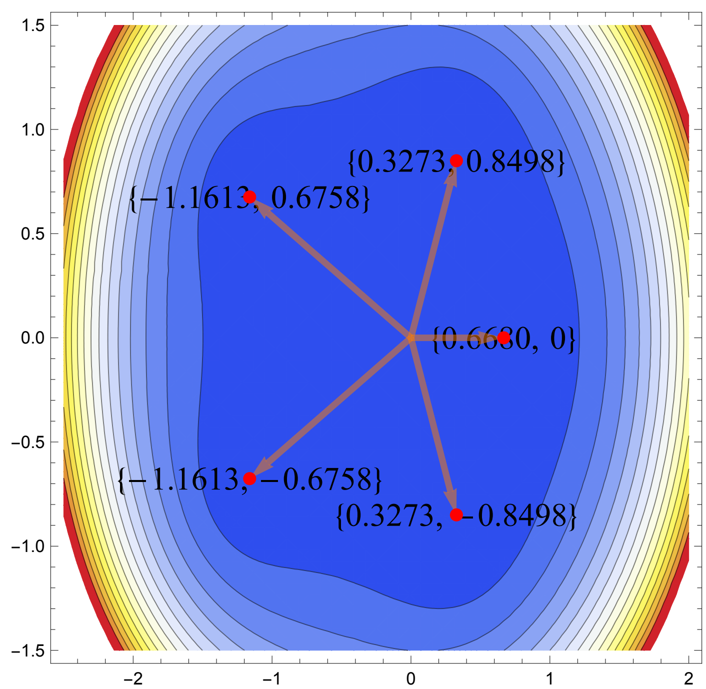
$$c_{n+10} = -(c_{n+8} + c_{n+2} + c_n) \tag{27}$$

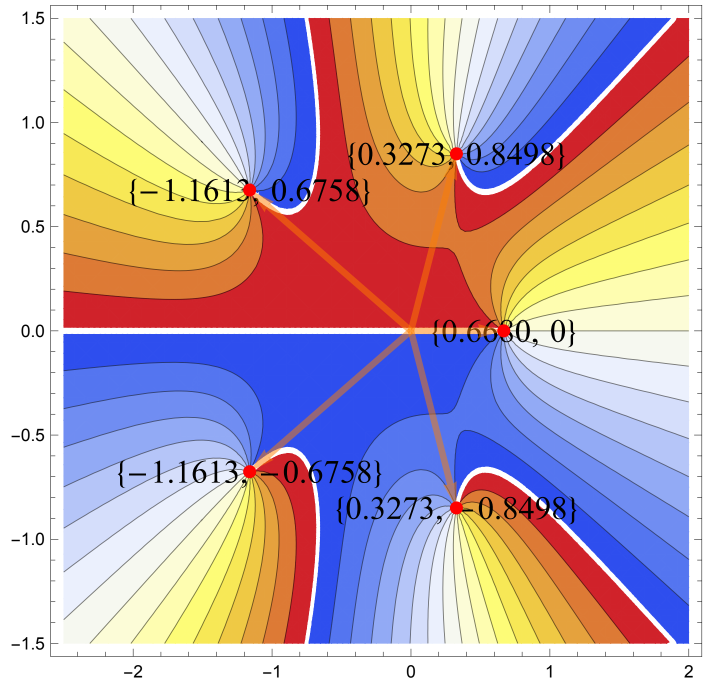
$$c_0 = 1 \;,\; c_1 = 0 \;,\; c_2 = -1 \;,\; c_3 = 4 \;,\; c_4 = 1 \;,\; c_5 = 0 \;,\; c_6 = -1 \;,\; c_7 = 0 \;,\; c_8 = 1 \;,\; c_9 = 0 \tag{28}$$

# 2. Gráfico de p(x).

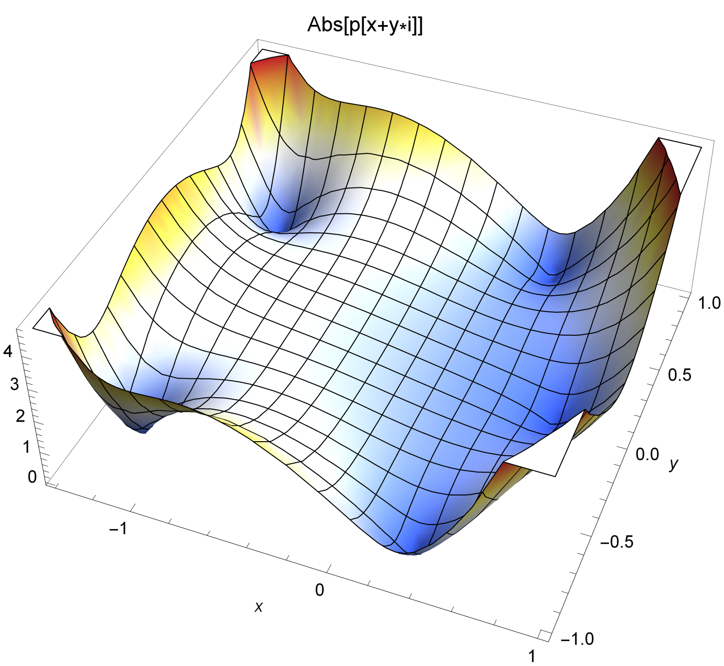


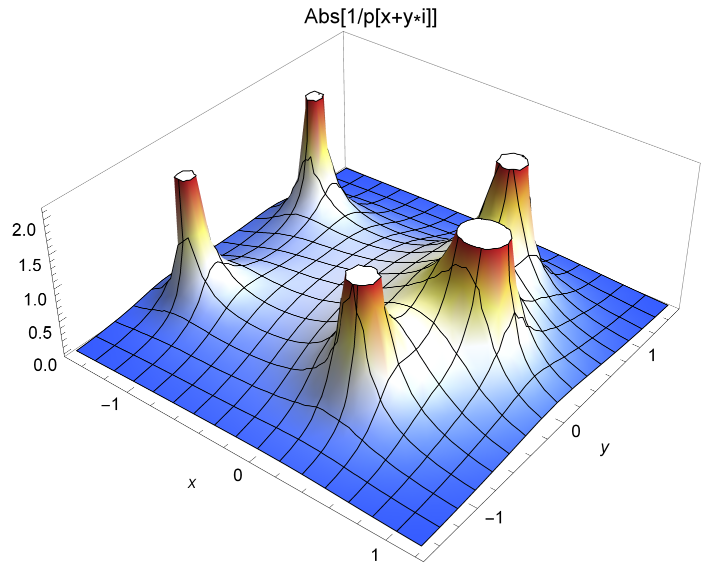

## 3. Gráfico de raíces complejas.






# 4. Gráficos para p(x).




Abs[1/p[x+y*i]]
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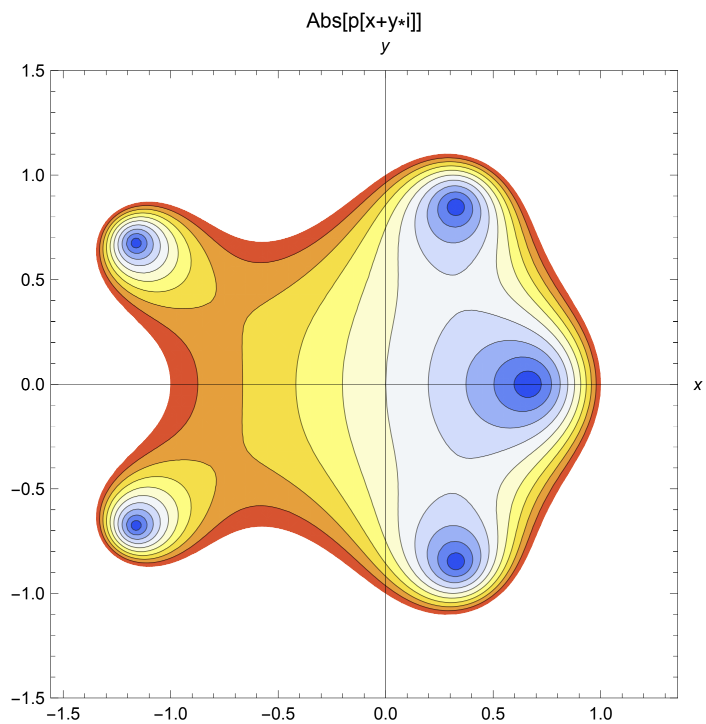
1.5
1.0
0.5
0.0
−1
x
0
1
1
0
y
−1


Abs[p[x+y*i]]
y
1.5
1.0
0.5
0.0
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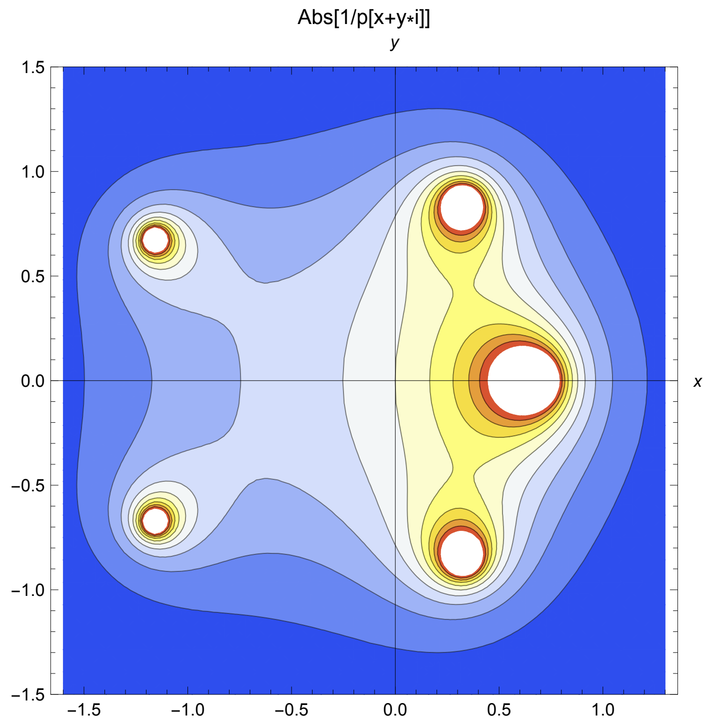
−1.0
−1.5
x
−1.5
−1.0
−0.5
0.0
0.5
1.0


Abs[1/p[x+y*i]]
y
1.5
1.0
0.5
0.0
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−1.0
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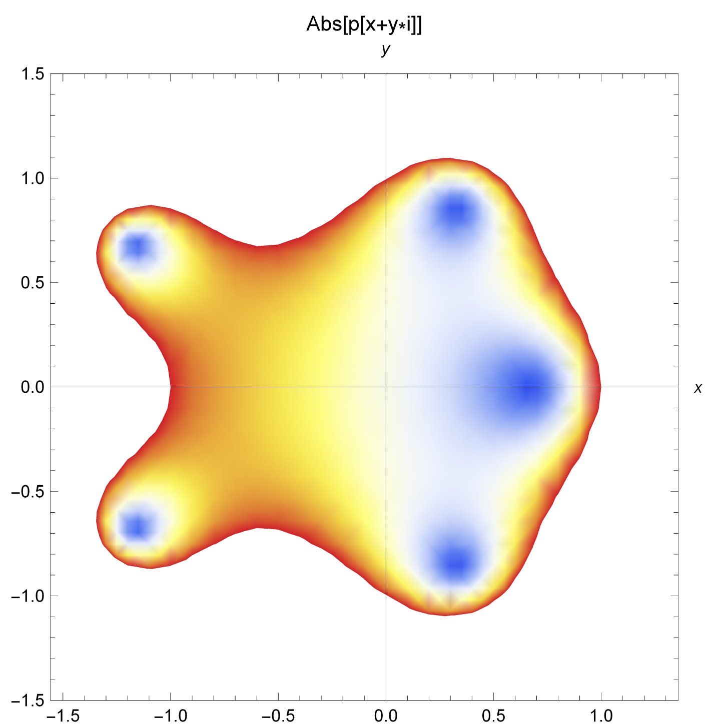
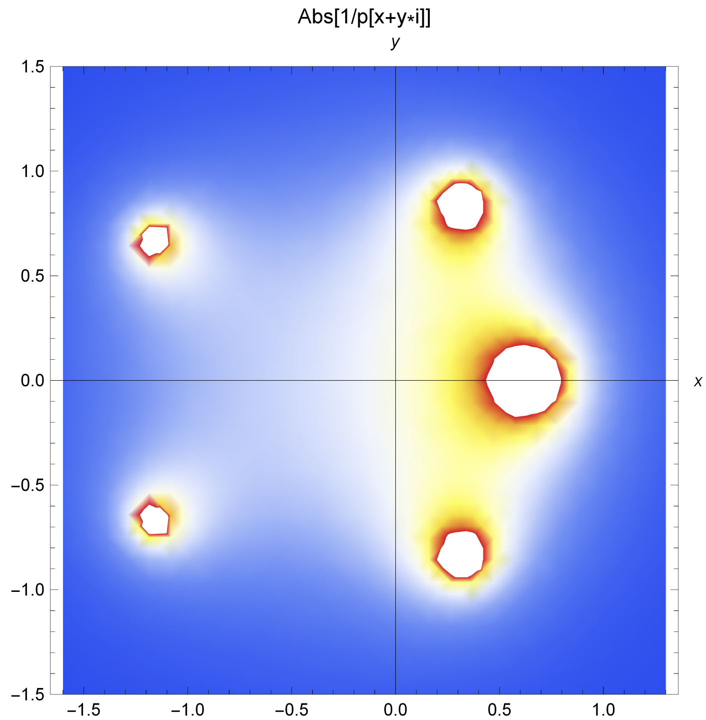
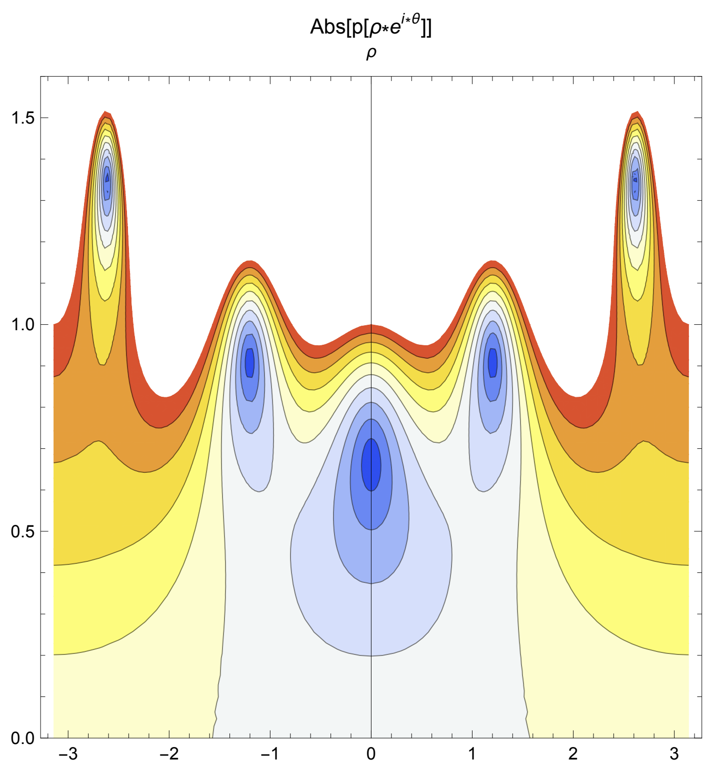
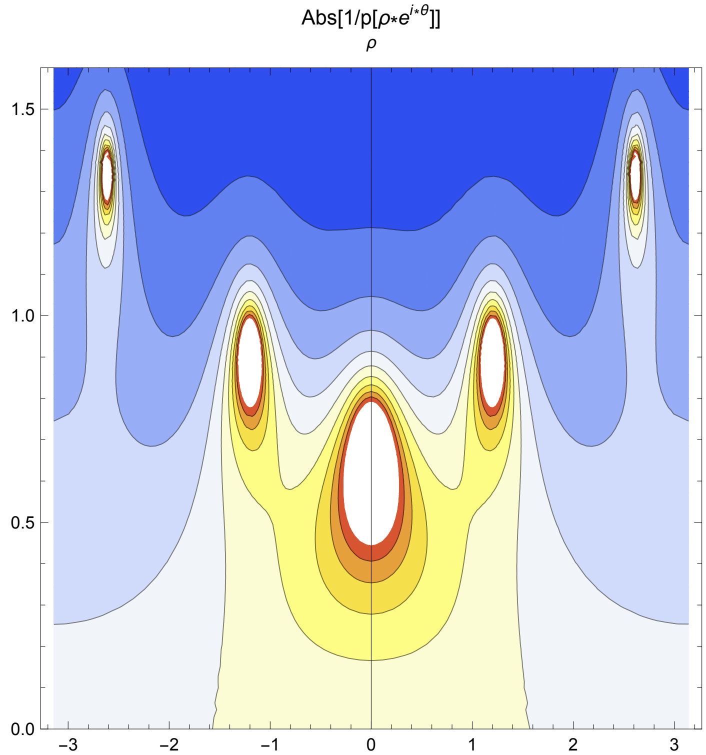
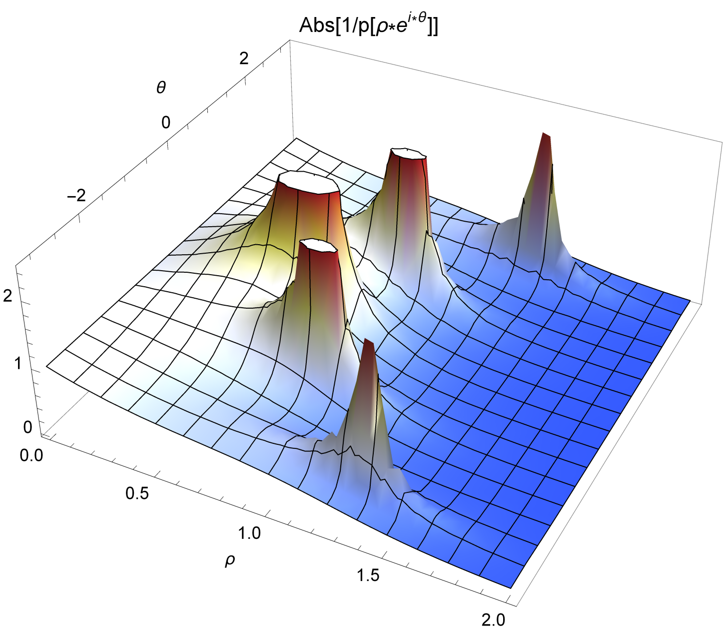
x
−1.5
−1.0
−0.5
0.0
0.5
1.0


Abs[p[x+y*i]]
y
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
x
−1.5
−1.0
−0.5
0.0
0.5
1.0

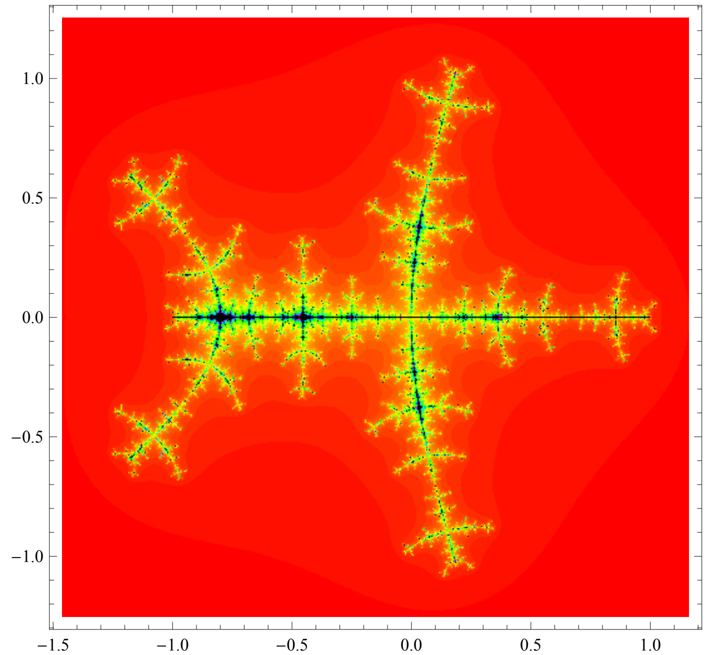

Abs[1/p[x+y*i]]
y
x
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0


Abs[p[ρ*e^{i*θ}]]
ρ
1.5
1.0
0.5
0.0
−3
−2
−1
0
1
2
3




## 5. Fractales relacionados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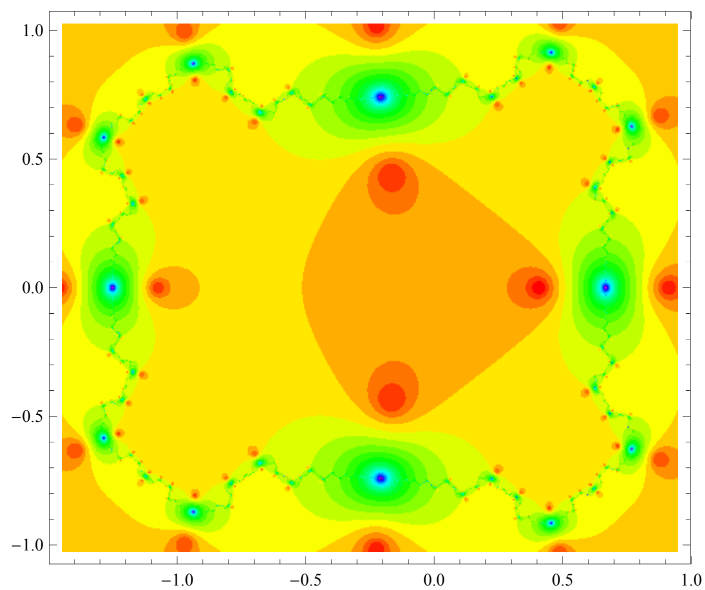
0.0
0.5
1.0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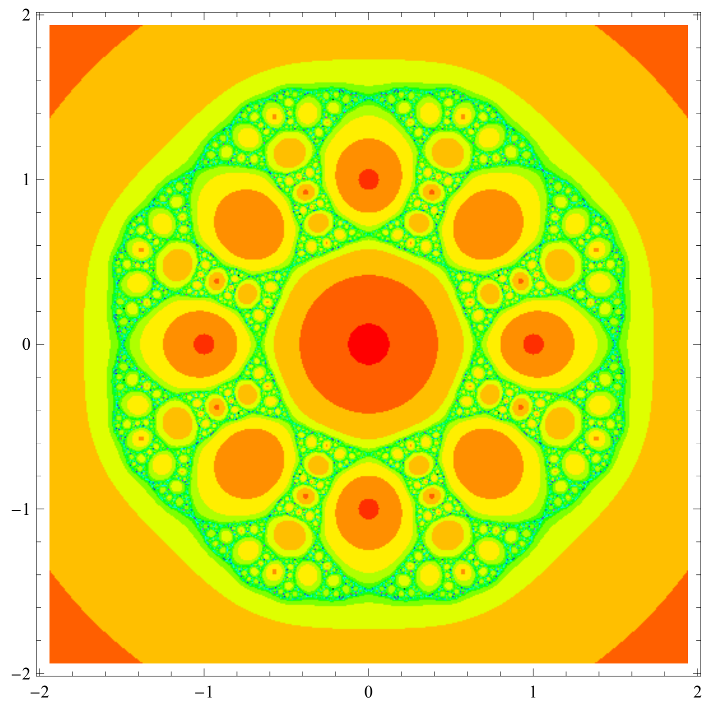
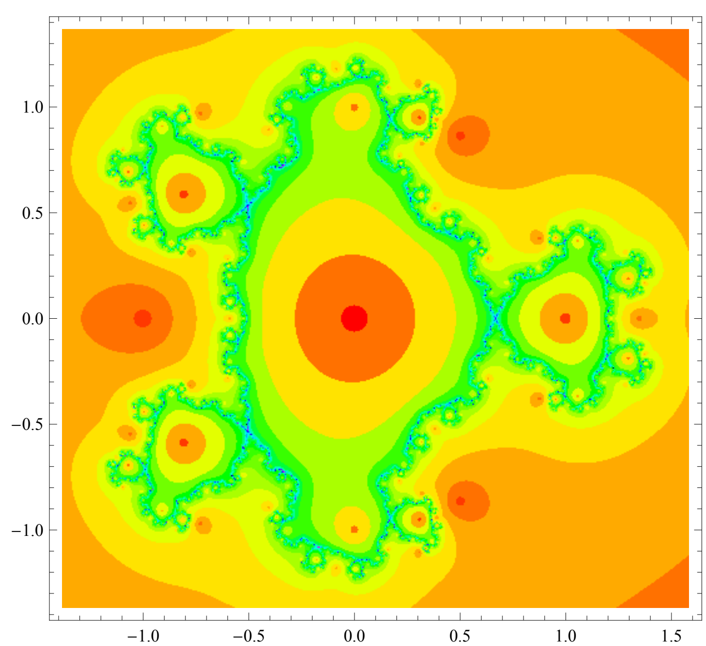
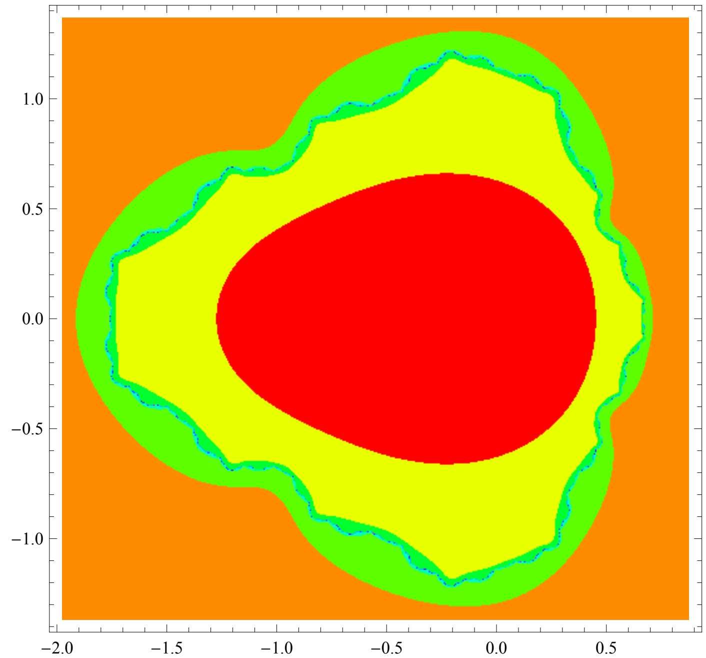
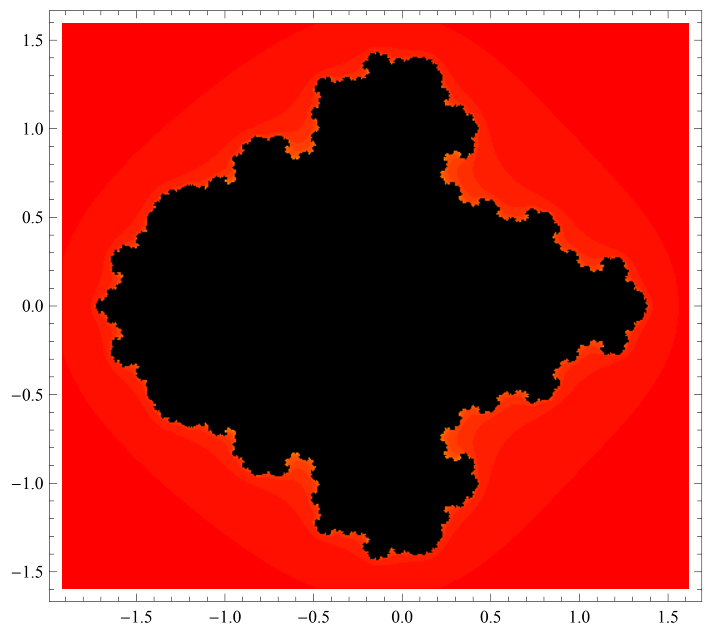
−1.0
−0.5
0.0
0.5
1.0


2
1
0
−1
−2
−2
−1
0
1
2


1.0
0.5
0.0
−0.5
−1.0
−1.0
−0.5
0.0
0.5
1.0
1.5



1.0
0.5
0.0
−0.5
−1.0
−2.0
−1.5
−1.0
−0.5
0.0
0.5

1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
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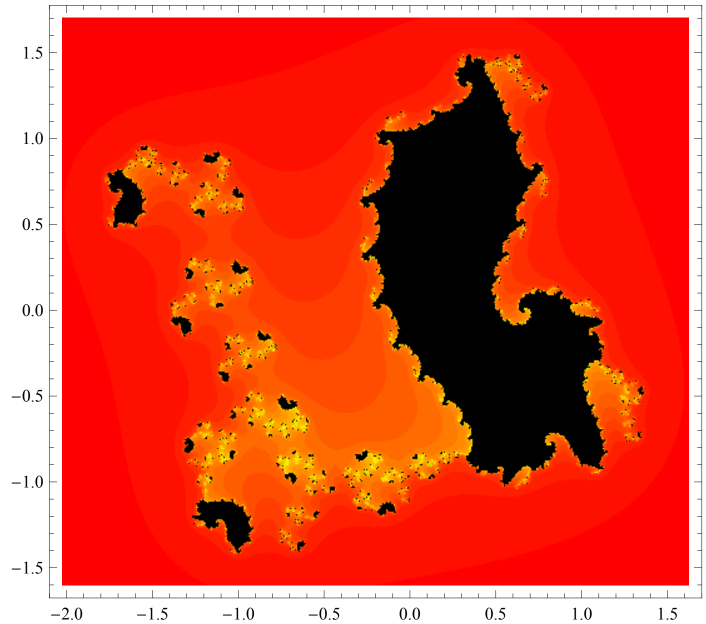
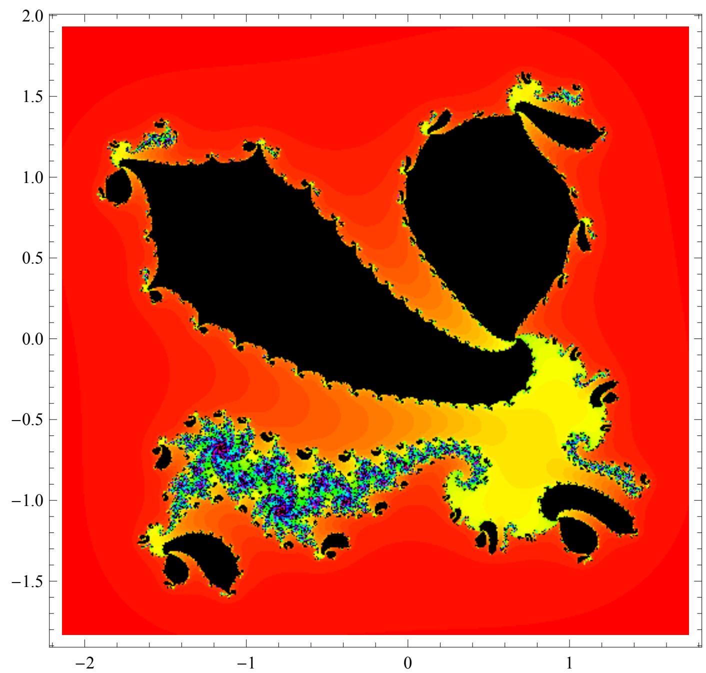
0.5
1.0
1.5




# Referencia

A. Valdebenito , E.: The quintic $z^5 + z^4 + z - 1 = 0$ , and the pi number, unpublished note , 2016.