How the mechanical clock slows down in high velocity

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Abstract

In this paper, the slowdown mechanism of the moving mechanical clock is derived. The whole deductive process is based on velocity transformation and length contraction of the special relativity. The result shows that when in moving the mechanical clock runs slower in the exactly same proportion as the light clock does.

Introduction

In the theory of special relativity, time dilation is a difference of elapsed time between two events as measured by observers either moving relative to each other. To explain how the time/clock runs slower in motion, a simple light clock is used as a demonstration to explain the time slowdown mechanism. Based on constancy of light speed, the time dilation mechanism of the light clock is well derived.[1,2,3,4] When it comes to the clocks with other principles, such as mechanic clock, there have not been seen a similar derivation to explain how these types clocks to run slower when in moving. It is only simply claimed that they must run slower in the same proportion as the light clock does as long as the theory of special relativity is right. If not, it will against the principle of relativity of the first postulate of the special relativity. For non-light clocks, only this explanation is not satisfying.

In this paper, two types of mechanical clocks are used as an example. The whole process is based on velocity transformation and length contraction of the special relativity. By accurate mathematical deduction, the slowdown mechanism of the mechanical clock is successfully derived. The result shows that when in moving the mechanical clock runs slower in the exactly same proportion as the light clock does.

Structure of the mechanical clocks

Let's consider the two types of mechanical clocks. The first is very similar to that of a light clock. It is a both ends closed straight tube. There is a particle moving back and forth within it. The collision between the particle and the ends of the tube is perfectly elastic, so the particle keeps going back and forth, making a click every time it comes back, like a standard ticking clock. It is shown in figure 1.

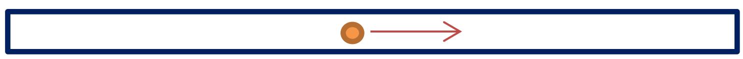

Figure 1. A particle moves back and forth in a tube, which works like a simple mechanical clock.

Concerning the second type clock, it is a simple pulley system as shown in figure 2. A long rope is driven by the pulley system. There is a mark on the rope, so the mark keeps going back and forth between the two ends, making a record every time it finishes a round trip. It is also like a standard ticking clock.

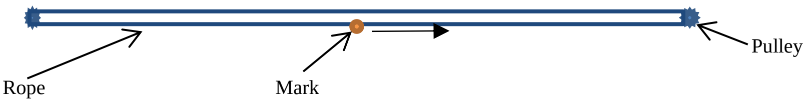

Figure 2. A kind of pulley system works like a simple mechanical clock

Time dilation derivation for the two types of the clocks

Now, let's look what happens to the moving clock. The direction of the clock should make no different to the time dilation, so we need to investigate two exactly the same clocks in the moving frame. One is parallel with the x' axis and the other is perpendicular to the x' axis. See figure 3.

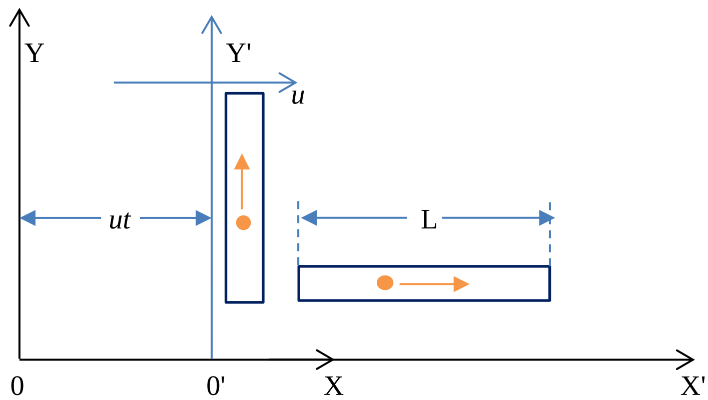

Figure 3. The S frame is a stationary frame. The S' frame is moving to left in the velocity of u. The two mechanical clocks are at rest in S' frame.

The two identical clocks are at rest in the moving frame S', so they are in the same pace to an observer in S' frame. In the S' frame, assume the length of the tube is L and the velocity of the particle is v, then the time takes for a round trip of the particle will be,

$$\Delta t' = 2\mathrm{L}/v$$

For an observer in S frame, the clocks are moving to left in the speed of u. Now, let's investigate how long it takes for a round trip of the particle to an observer in S frame.

According to the theory of special relativity, the transformation of coordinates follows Lorentz Transformation[1,2]:

$$x = \frac{x' + ut'}{\sqrt{1 - u^2/c^2}}$$

$$y = y'$$

$$z = z'$$

$$\mathrm{t} = \frac{\mathrm{t}' + u\mathrm{x}'/c^2}{\sqrt{1-u^2/c^2}}$$

Transformation of velocities is:

$$v_x = \frac{u + v_{x\prime}}{1 + \mathrm{u}v_{x\prime}/c^2}$$

$$v_y = v_{y\prime}\sqrt{1-u^2/c^2} \qquad (\text{when } v_{x\prime} = 0)$$

First, let’s see the situation of the clock which is perpendicular to the x' axis. when observed in S frame the time takes for a round trip of the particle will be,

$$\Delta t = \frac{2\mathrm{L}}{v_y} = \frac{2\mathrm{L}}{v_{y\prime}\sqrt{1-u^2/c^2}} = \frac{\Delta t'}{\sqrt{1-u^2/c^2}}$$

The result shows that the clock runs slower in the exactly same proportion as a light clock does. The time interval gets longer $1/\sqrt{1-u^2/c^2}$ times when the clock in motion.

Now, let’s see the situation of the clock which is parallel to the x' axis. When the particle moves to right the velocity in S frame is,

$$v_{x+} = \frac{u+v}{1+\mathrm{u}v/c^2}$$

When the particle moves to the left the velocity in S frame is,

$$v_{x-} = \frac{v-u}{1-\mathrm{u}v/c^2} \qquad (when\ v \geq u)$$

or

$$v_{x-} = \frac{u-v}{1-\mathrm{u}v/c^2} \qquad (when\ v \leq u)$$

Let’s assume that t_+ is the time needed for the particle running from left end to right end of the tube and t_- is the time needed for the particle running from right end to left end of the tube. Thus, the time for a round trip will be,

$$\Delta t = t_+ + t_-$$

In the situation of $v \geq u$,

$$v_{x+}t_+ = \mathrm{L} + ut_+ \ \text{(As shown in figure 4)}$$

$$t_+ = \frac{L}{v_{x+} - u} = \frac{L}{\frac{u+v}{1+\mathrm{u}v/c^2} - u} = \frac{L(1+\frac{\mathrm{u}v}{c^2})}{v(1-\frac{u^2}{c^2})}$$

$$v_{x-}t_{-} = \mathrm{L} - ut_{-} \quad \text{(As shown in figure 5)}$$

$$t_{-} = \frac{L}{v_{x-} + u} = \frac{L}{\frac{v-u}{1-\mathrm{u}v/c^2} + u} = \frac{L(1-\frac{\mathrm{u}v}{c^2})}{v(1-\frac{u^2}{c^2})}$$

$$\Delta t = t_{+} + t_{-} = \frac{L(1+\frac{\mathrm{u}v}{c^2})}{v(1-\frac{u^2}{c^2})} + \frac{L(1-\frac{\mathrm{u}v}{c^2})}{v(1-\frac{u^2}{c^2})} = \frac{2L}{v(1-\frac{u^2}{c^2})}$$

In the above, the Lorentz length contraction in the moving direction is not considered. If the contraction is considered into,

$$\Delta t = \frac{2L\sqrt{1-u^2/c^2}}{v(1-\frac{u^2}{c^2})} = \frac{\Delta t'}{\sqrt{1-u^2/c^2}}$$

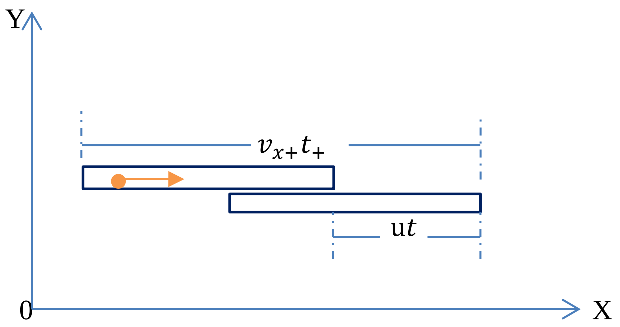

Figure 4. The particle moves from left end to right end of the tube

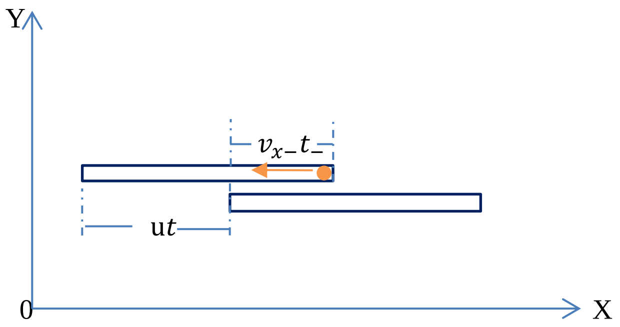

Figure 5. The particle moves back from right end to left end of the tube

In the situation of $v \leq u$ we still have,

$$v_{x+}t_{+} = \mathrm{L} + ut_{+}$$

$$t_+ = \frac{L}{v_{x+} - u} = \frac{L(1+\frac{uv}{c^2})}{v(1-\frac{u^2}{c^2})}$$

But for the particle moves back from right to left end of the tube in S' frame, because of $v \leq u$ the net speed of the particle observed in S frame is in the opposite direction (from left to right). So we will have (as explained in figure 6),

$$v_{x-}t_- + \mathrm{L} = ut_-$$

$$t_- = \frac{L}{u - v_{x-}} = \frac{L}{u - \frac{u-v}{1-uv/c^2}} = \frac{L(1-\frac{uv}{c^2})}{v(1-\frac{u^2}{c^2})}$$

$$\Delta t = t_+ + t_- = \frac{L(1+\frac{uv}{c^2})}{v(1-\frac{u^2}{c^2})} + \frac{L(1-\frac{uv}{c^2})}{v(1-\frac{u^2}{c^2})} = \frac{2L}{v(1-\frac{u^2}{c^2})}$$

Considering the length contraction in x direction we have,

$$\Delta t = \frac{2L\sqrt{1-u^2/c^2}}{v(1-\frac{u^2}{c^2})} = \frac{\Delta t'}{\sqrt{1-u^2/c^2}}$$

Now we get that the mechanical clock, which is parallel to the x' axis, also runs slower in the exactly same proportion as a light clock does. The time interval gets longer $1/\sqrt{1-u^2/c^2}$ times when the clock in motion.

By replacing the moving particle with the moving mark, this derivation process is also valid to the pulley system clock. So, for the pulley system clock, the exactly same time dilation will be observed, too.

Now, the whole derivation completed. The results show that the mechanical clock runs slower in the exactly same proportion as a light clock does when in motion.

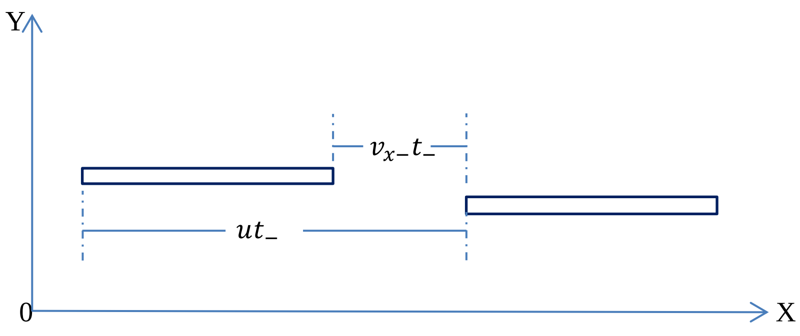

Figure 6. When $v \leq u$, although the particle moves from right to left in S' frame, it moves from left to right in S frame.

Conclusions

Based on the velocity transformation and length contraction of the special relativity, the slowdown mechanism of the mechanical clock is well derived. The result shows that when in moving the mechanical clock runs slower in the exactly same proportion as the light clock does.

References

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