PROOF OF TWIN PRIME CONJECTURE

SAFAA ABDALLAH MOALLIM MUHAMMAD

ABSTRACT. In this paper we prove that there exist infinitely many twin prime numbers by studying n when $6n \pm 1$ are primes. By studying n we show that for every n that generates a twin prime number, there has to be $m > n$ that generates a twin prime number too.

1. INTRODUCTION

Considering that every twin prime can be written as $6n \pm 1$ except for 3 and 5. By studying the properties of n we make sure that there's $m > n$ that generates a twin prime which means $6m \pm 1$ are primes. First values of n that generate twin primes are $\{1, 2, 3, 5, 7, 10, 12, 17, 18, 23 \ldots\}$. Let's name our n a twin prime generator.

Theorem 1.1.

If $k_1 < k_2$ where $k_1 = p(n_1) + rem_1$ or $k_1 mod\ p = rem_1$, $k_2 = p(n_2) + rem_2$, $k_1 \neq k_2$ and $rem_1 \neq rem_2$ then $6k_1\ mod\ p \neq 6k_2\ mod\ p$.

Where $n \in \mathbb{N}$ and p is a prime number.

Proof 1.1.

Let $k_1 = pn_1 + rem_1$, and $k_2 = pn_2 + rem_2$

Let $rem_1 \neq rem_2$

$6k_1 = 6(pn_1 + rem_1)$

$6k_2 = 6(pn_2 + rem_2)$

$6k_1 = P(6n_1) + 6rem_1$ (1)

$6k_2 = P(6n_2) + 6rem_2$ (2)

If $6rem_1$ and $6rem_1$ are bigger than p, then we divide it to values $6rem_3 + p(L_1)$ where n can be zero if $6rem_1$ is not bigger than p.

$6k_1 = P(6n_1) + rem_3 + p(L_1)$ (3)

$6k_2 = P(6n_2) + rem_4 + p(L_2)$ (4)

$6k_1 = P(6n_1 + L_1) + rem_3$

$6k_2 = P(6n_2 + L_2) + rem_4$

Let $rem_3 = rem_4$

Then from 3 and 4, $6k_1 - P(6n_1 + L_1) = 6k_2 - P(6n_2 + L_2)$ (5)

From 1, $6rem_1 = 6k_1 - P(6n_1)$ (6)

From 2, $6rem_2 = 6k_2 - P(6n_2)$ (7)

From 5, 6, and 7, $6rem_1 + pL_1 = 6rem_2 + pL_2$

Since $6rem_1 = rem_3 + p(L_1)$ and $6rem_2 = rem_4 + p(L_2)$

$rem_3 + pL_1 + pL_1 = rem_4 + pL_2 + pL_2$

$L_1 = L_2$

From 5, $6k_1 - P(6n_1) = 6k_2 - P(6n_2)$

$rem_1 = rem_2$ which is a contradiction.

What we conclude from theorem 1 is that for every two numbers have not the same remainder from the division by a prime number, then after multiplying by 6 they can't have the same remainder too. In brief, every distinct remainder from the division by a prime number after multiplying by 6 will have a distinct remainder from the division by the same prime number.

Theorem 1.2.

If 6m $\pm$ 1 is divisible by $(6n+1)$ or $(6n-1)$ then $m \ mod \ (6n+1) = ((6n+1) \pm n) \ mod \ (6n+1)$ or $m \ mod \ (6n-1) = ((6n-1) \pm n) \ mod \ (6n-1)$, where $(6n+1)$ and $(6n-1)$ are primes.

Proof 1.2.

We know that $((6n+1)+n) \ mod \ (6n+1) = n$, $((6n+1)-n) \ mod \ (6n+1) = \mathbf{5}n+1$, $((6n-1)+n) \ mod \ (6n-1) = n$ and $((6n-1)-n) \ mod \ (6n-1) = \mathbf{5}n-1$

Let $x = k(6n+1)+n$, Then $6x = 36kn + 6k + 6n$

$6x + 1 = 36kn + 6k + 6n + 1$

$6x + 1 = 6k(6n+1) + (6n+1)$

$6x + 1 = (6k+1)(6n+1)$ which is divisible by $(6n+1)$.

$6x \ mod \ (6n+1) = 6n$

From theorem 1.1, the remainder n is the only reminder that can lead to the remainder $6n$ where the next number is divisible by $6n+1$ when it's multiplied by 6.

Let $x = k(6n+1) - n$ that $x \ mod \ (6n+1) = 5n+1$, Then $6x = 36kn + 6k - 6n$

$6x - 1 = 36kn + 6k - 6n - 1$

$6x - 1 = 6k(6n+1) - (6n+1)$

$6x - 1 = (6k-1)(6n+1)$ which is divisible by $(6n+1)$.

$6x \ mod \ (6n+1) = 1$

From theorem 1.1, the remainder $5n+1$ is the only remainder that can lead to the remainder 1 where the behind number is divisible by $6n+1$ when it's multiplied by 6.

Let $x = k(6n-1)+n$, Then $6x = 36kn - 6k + 6n$

$6x - 1 = 36kn - 6k + 6n - 1$

$6x - 1 = 6k(6n-1) + (6n-1)$

$6x - 1 = (6k + 1)(6n - 1)$ which is divisible by $(6n - 1)$.

$6x \; mod \; (6n - 1) = 1$

From theorem 1.1, the remainder n is the only reminder that can lead to the remainder 1 where the behind number is divisible by $6n - 1$ when it's multiplied by 6.

Let $x = k(6n - 1) - n$ that $x \; mod \; (6n - 1) = \mathbf{5}n - 1$, Then $6x = 36kn - 6k - 6n$

$6x + 1 = 36kn - 6k - 6n + 1$

$6x + 1 = 6k(6n - 1) - (6n - 1)$

$6x + 1 = (6k - 1)(6n - 1)$ which is divisible by $(6n - 1)$.

$6x \; mod \; (6n - 1) = 6n$

From theorem 1.1, the remainder $\mathbf{5}n - 1$ is the only remainder that can lead to the remainder $6n$ where the next number is divisible by $6n - 1$ when it's multiplied by 6.

Lemma 1.1.

For m to be a twin prime generator, it has to fulfill the condition that $m \; mod \; (6n + 1) \neq ((6n + 1) \pm n) \; mod \; (6n + 1)$ and $m \; mod \; (6n - 1) \neq ((6n - 1) \pm n) \; mod \; (6n - 1)$, where $n \in \mathbb{N}$ and $n \neq 0$.

We know that m to be a twin prime generator, $6m \pm 1$ shouldn't be divisible by 5 or 7, $6m \pm 1$ shouldn't be divisible by 11 or 13, $6m \pm 1$ shouldn't be divisible by 17 or 19, $6m \pm 1$ shouldn't be divisible by 23 or 25, and so on.

From theorem 1.2, m to be a twin prime generator, $m \pm 1$ shouldn't be divisible by 5 or 7, $m \pm 2$ shouldn't be divisible by 11 or 13, $m \pm 3$ shouldn't be divisible by 17 or 19, $m \pm 4$ shouldn't be divisible by 23 or 25, and so on. From here we can say that m to be a twin prime generator $m \; mod \; (6n + 1) \neq ((6n + 1) \pm n) \; mod \; (6n + 1)$ and $m \; mod \; (6n - 1) \neq ((6n - 1) \pm n) \; mod \; (6n - 1)$.

Theorem 1.3.

The longest interval of integers covered by the union of $4n$ arithmetic progressions $\pm k \; mod(6k - 1)$ and $\pm k \; mod(6k + 1)$ is less than $4n^2$ where $1 \leq k \leq n$ and $n \in \mathbb{Z}$.

Proof 1.3.

The number of integers that are covered at most in the interval $4n^2$ by $-k \; mod(6k - 1)$ equal $\left\lfloor \frac{4n^2}{6k-1} \right\rfloor + 1$ and by $+k \; mod(6k - 1)$ equal $\left\lfloor \frac{4n^2}{6k-1} \right\rfloor + 1$. Then, number of integers that are covered by $\pm k \; mod(6k - 1)$ equal $2\left\lfloor \frac{4n^2}{6k-1} \right\rfloor + 2$ and by $\pm k \; mod(6k + 1)$ equal $2\left\lfloor \frac{4n^2}{6k+1} \right\rfloor + 2$..

Maximum integers covered, keeping in mind integers that get overlapped, by the $4n$ progressions equal $2 + 2\left(1 - \frac{2}{5}\right) + 2\left(1 - \frac{2}{5} - \frac{6}{35}\right) + \cdots + 4n^2\left(\frac{2}{5} + \frac{2}{7} - \frac{2}{7}\left(\frac{2}{5}\right) + \frac{2}{11} - \frac{2}{11}\left(\frac{2}{5}\right) - \frac{2}{11}\left(\frac{2}{7}\right) + \frac{2}{11}\left(\frac{2}{7}\right)\left(\frac{2}{5}\right) + \frac{2}{13} - \frac{2}{13}\left(\frac{2}{5}\right) - \frac{2}{13}\left(\frac{2}{7}\right) - \frac{2}{13}\left(\frac{2}{11}\right) + \cdots + \frac{2}{(6(n)+1)} - \frac{2}{(6(n)+1)}\left(\frac{2}{6(1)-1}\right) - \cdots\right) = 2 + 2\left(1 - \frac{2}{5}\right) + 2\left(1 - \frac{2}{5} - \frac{6}{35}\right) + \cdots + 4n^2\left(\frac{2}{5} + \frac{6}{35} + \frac{6}{77} + \frac{54}{1001} + \cdots + \frac{2}{(6(n)+1)}\left(1 - \frac{2}{5} + \frac{6}{35} + \frac{6}{77} + \frac{54}{1001} + \cdots\right)\right)$. Thus 5 covers $\frac{2}{5}$, 7 covers $\frac{6}{35}$ and so on.

5 covers	$r_1 = \frac{2}{5}$
7 covers	$r_2 = (1 - r_1)\left(\frac{2}{7}\right) = \frac{6}{35}$
11 covers	$r_3 = (1 - r_1 - r_2)\left(\frac{2}{11}\right) = \frac{6}{77}$
13 covers	$r_4 = (1 - r_1 - r_2 - r_3)\left(\frac{2}{13}\right) = \frac{54}{1001}$
17 covers	$r_5 = (1 - r_1 - r_2 - r_3 - r_4)\left(\frac{2}{17}\right) = \frac{54}{1547}$
19 covers	$r_6 = (1 - r_1 - r_2 - r_3 - r_4 - r_5)\left(\frac{2}{19}\right) = \frac{810}{29393}$

Number of integers that aren't covered equal $4(n)^2(1 - r_1 - \cdots - r_{n2}) - (2 + 2(1 - r_1) + \cdots + 2(1 - r_1 - \cdots - r_{n1}))$.

If we assume that $1 - (r_1 + r_2 + r_3 + r_4 + \cdots + r_{n1} + r_{n2}) = 0$. Then $1 - (r_1 + r_2 + r_3 + r_4 + \cdots + r_{n1}) = r_{n2}$ which results in a contradiction because $r_{n2} = \big(1 - (r_1 + r_2 + r_3 + r_4 + \cdots + r_{n1})\big)\left(\frac{2}{6n+1}\right)$. Thus for every n, $1 - (r_1 + r_2 + r_3 + r_4 + \cdots + r_{n1} + r_{n2}) > 0$, and this is the inequality number (1).

In the case $n = 3$, we have the inequality number (2) that is

$$4(3)^2(1 - r_1 - r_2 - r_3 - r_4 - r_5 - r_6) > 2 + 2(1 - r_1) + 2(1 - r_1 - r_2) + 2(1 - r_1 - r_2 - r_3) + 2(1 - r_1 - r_2 - r_3 - r_4) + 2(1 - r_1 - r_2 - r_3 - r_4 - r_5)$$

$$4(3)^2(1 - r_1 - r_2 - r_3 - r_4 - r_5 - r_6) > 2\big(2(3)\big) - 2r_1(2(3) - 1) - 2r_2(2(3) - 2) - 2r_3(2(3) - 3) - 2r_4(2(3) - 4) - 2r_5(2(3) - 5)$$

$$4(3)^2(1 - r_1 - r_2 - r_3 - r_4 - r_5 - r_6) > 2\big(2(3)\big) - 2r_1\big(2(3)\big) + 2r_1 - 2r_2\big(2(3)\big) + 2r_2(2) - 2r_3\big(2(3)\big) + 2r_3(3) - 2r_4\big(2(3)\big) + 2r_4(4) - 2r_5\big(2(3)\big) + 2r_5(5)$$

And this inequality holds true because it results in $8 + \frac{748}{1729} > 5.875442205$.

In the case $n > 3$, $n = 3 + k$, we got the inequality number (3)

$$4(3 + k)^2\big(1 - r_1 - r_2 - r_3 - r_4 - r_5 - r_6 - \cdots - r_{(3+k)1} - r_{(3+k)2}\big) > 2 + 2(1 - r_1) + 2(1 - r_1 - r_2) + 2(1 - r_1 - r_2 - r_3) + 2(1 - r_1 - r_2 - r_3 - r_4) + 2(1 - r_1 - r_2 - r_3 - r_4 - r_5) + \cdots + 2\big(1 - r_1 - r_2 - r_3 - r_4 - r_5 - \cdots - r_{(3+k)1}\big)$$

$$4(3^2 + 6k + k^2)\big(1 - r_1 - r_2 - r_3 - r_4 - r_5 - r_6 - \cdots - r_{(3+k)1} - r_{(3+k)2}\big) > 2\big(2(3 + k)\big) - 2r_1(2(3 + k) - 1) - 2r_2(2(3 + k) - 2) - 2r_3(2(3 + k) - 3) - 2r_4(2(3 + k) - 4) - 2r_5(2(3 + k) - 5) - \cdots - 2r_{(3+k)1}\big(2(3 + k) - (2(3 + k) - 1)\big)$$

$$4(3)^2\big(1 - r_1 - r_2 - r_3 - r_4 - r_5 - r_6 - \cdots - r_{(3+k)1} - r_{(3+k)2}\big) + 4(6k)\big(1 - r_1 - r_2 - r_3 - r_4 - r_5 - r_6 - \cdots - r_{(3+k)1} - r_{(3+k)2}\big) + 4k^2\big(1 - r_1 - r_2 - r_3 - r_4 - r_5 - r_6 - \cdots - r_{(3+k)1} - r_{(3+k)2}\big) > 2\big(2(3 + k)\big) - 2r_1\big(2(3 + k)\big) + 2r_1 - 2r_2\big(2(3 + k)\big) + 2r_2(2) - 2r_3\big(2(3 + k)\big) + 2r_3(3) - 2r_4\big(2(3 + k)\big) + 2r_4(4) - 2r_5\big(2(3 + k)\big) + 2r_5(5) - \cdots - 2r_{(3+k)1}\big(2(3 + k)\big) + 2r_{(3+k)1}(2(3 + k) - 1)$$

$$4(3)^2\big(1 - r_1 - r_2 - r_3 - r_4 - r_5 - r_6 - \cdots - r_{(3+k)1} - r_{(3+k)2}\big) + 4(6k)\big(1 - r_1 - r_2 - r_3 - r_4 - r_5 - r_6 - \cdots - r_{(3+k)1} - r_{(3+k)2}\big) + 4k^2\big(1 - r_1 - r_2 - r_3 - r_4 - r_5 - r_6 - \cdots - r_{(3+k)1} - r_{(3+k)2}\big) > 2\big(2(3)\big) + 2(2k) - 2r_1\big(2(3)\big) - 2r_1(2k) + 2r_1 - 2r_2\big(2(3)\big) - 2r_2(2k) + 2r_2(2) - 2r_3\big(2(3)\big) - 2r_3(2k) + 2r_3(3) - 2r_4\big(2(3)\big) - 2r_4(2k) + 2r_4(4) - 2r_5\big(2(3)\big) - 2r_5(2k) + 2r_5(5) - \cdots - 2r_{(3+k)1}\big(2(3)\big) - 2r_{(3+k)1}(2k) + 2r_{(3+k)1}(2(3 + k) - 1).$$

Subtracting inequality number (2) from inequality number (3) we get

$$4(3)^2\big(-r_7 - \cdots - r_{(3+k)1} - r_{(3+k)2}\big) + 4(6k)\big(1 - r_1 - r_2 - r_3 - r_4 - r_5 - r_6 - \cdots - r_{(3+k)1} - r_{(3+k)2}\big) + 4k^2\big(1 - r_1 - r_2 - r_3 - r_4 - r_5 - r_6 - \cdots - r_{(3+k)1} - r_{(3+k)2}\big) > 2(2k) - 2r_1(2k) - 2r_2(2k) - 2r_3(2k) - 2r_4(2k) - 2r_5(2k) - \cdots - 2r_{(3+k)1}\big(2(3)\big) - 2r_{(3+k)1}(2k) + 2r_{(3+k)1}(2(3 + k) - 1).$$

$4(3)^2(-r_7-\cdots-r_{(3+k)1}-r_{(3+k)2})+4(6k)(1-r_1-r_2-r_3-r_4-r_5-r_6-\cdots-r_{(3+k)1}-r_{(3+k)2})+4k^2(1-r_1-r_2-r_3-r_4-r_5-r_6-\cdots-r_{(3+k)1}-r_{(3+k)2})>2(2k)(1-r_1-r_2-r_3-r_4-r_5-r_6-\cdots-r_{(3+k)1})+2(2(3))(-r_6-\cdots-r_{(3+k)1})+2(2(3))(r_6+\cdots+r_{(3+k)1})+2(2k)(r_6+\cdots+r_{(3+k)1})+2(-2(3+k)r_6-\cdots-3r_{(3+k-1)1}-2r_{(3+k-1)2}-r_{(3+k)1})$.

$36(-r_7-\cdots-r_{(3+k)1}-r_{(3+k)2})+24k(1-r_1-r_2-r_3-r_4-r_5-r_6-\cdots-r_{(3+k)1}-r_{(3+k)2})+4k^2(1-r_1-r_2-r_3-r_4-r_5-r_6-\cdots-r_{(3+k)1}-r_{(3+k)2})>4k(1-r_1-r_2-r_3-r_4-r_5-r_6-\cdots-r_{(3+k)1})+4k(r_6+\cdots+r_{(3+k)1})+2(-2(3+k)r_6-\cdots-3r_{(3+k-1)1}-2r_{(3+k-1)2}-r_{(3+k)1})$.

$2(2(3+k)r_6+\cdots+3r_{(3+k-1)1}+2r_{(3+k-1)2}+r_{(3+k)1})+4k(1-r_1-r_2-r_3-r_4-r_5-r_6-\cdots-r_{(3+k)1}-r_{(3+k)2})+20k(1-r_1-r_2-r_3-r_4-r_5-r_6-\cdots-r_{(3+k)1}-r_{(3+k)2})+4k^2(1-r_1-r_2-r_3-r_4-r_5-r_6-\cdots-r_{(3+k)1}-r_{(3+k)2})>4k(1-r_1-r_2-r_3-r_4-r_5-r_6-\cdots-r_{(3+k)1})+4k(r_6+\cdots+r_{(3+k)1})+36(r_7+\cdots+r_{(3+k)1}+r_{(3+k)2})$

$2(2(3+k)r_6+\cdots+3r_{(3+k-1)1}+2r_{(3+k-1)2}+r_{(3+k)1})+20k(1-r_1-r_2-r_3-r_4-r_5-r_6-\cdots-r_{(3+k)1}-r_{(3+k)2})+4k^2(1-r_1-r_2-r_3-r_4-r_5-r_6-\cdots-r_{(3+k)1}-r_{(3+k)2})>4k(r_6+\cdots+r_{(3+k)1}+r_{(3+k)2})+36(r_7+\cdots+r_{(3+k)1}+r_{(3+k)2})$.

Partitioning the inequality into smaller inequalities, we have

$$4k^2(1-r_1-r_2-r_3-r_4-r_5-r_6-\cdots-r_{(3+k)1}-r_{(3+k)2})>4k(r_6+\cdots+r_{(3+k)1}+r_{(3+k)2})$$

$$k(1-r_1-r_2-r_3-r_4-r_5-r_6-\cdots-r_{(3+k)1}-r_{(3+k)2})>(r_6+\cdots+r_{(3+k)1}+r_{(3+k)2})$$

From inequality number (1), we know that $(r_6+\cdots+r_{(3+k)1}+r_{(3+k)2})<1-r_1-r_2-r_3-r_4-r_5$, $(r_6+\cdots+r_{(3+k)1}+r_{(3+k)2})<0.261797$. In the case $k=2$, $k(1-r_1-r_2-r_3-r_4-r_5-r_6-\cdots-r_{(3+k)1}-r_{(3+k)2})>0.3$.

Comparing between k and $k+1$ to know if $k(1-r_1-r_2-r_3-r_4-r_5-r_6-\cdots-r_{(3+k)1}-r_{(3+k)2})$ is increasing.

$$(k+1)(1-r_1-r_2-r_3-r_4-r_5-r_6-\cdots-r_{(3+k)1}-r_{(3+k)2}-r_{(3+k+1)1}-r_{(3+k+1)2}) > k(1-r_1-r_2-r_3-r_4-r_5-r_6-\cdots-r_{(3+k)1}-r_{(3+k)2})$$

$$k(1-r_1-r_2-r_3-r_4-r_5-r_6-\cdots-r_{(3+k)1}-r_{(3+k)2})+k(-r_{(3+k+1)1}-r_{(3+k+1)2}) + (1-r_1-r_2-r_3-r_4-r_5-r_6-\cdots-r_{(3+k)1}-r_{(3+k)2}-r_{(3+k+1)1}-r_{(3+k+1)2}) > k(1-r_1-r_2-r_3-r_4-r_5-r_6-\cdots-r_{(3+k)1}-r_{(3+k)2})$$

$$(1-r_1-r_2-r_3-r_4-r_5-r_6-\cdots-r_{(3+k)1}-r_{(3+k)2}-r_{(3+k+1)1}-r_{(3+k+1)2}) > k(r_{(3+k+1)1}+r_{(3+k+1)2})$$

$$(1-r_1-r_2-r_3-r_4-r_5-r_6-\cdots-r_{(3+k)1}-r_{(3+k)2})>(k+1)(r_{(3+k+1)1}+r_{(3+k+1)2})$$

$$\begin{aligned}(k+1)(r_{(3+k+1)1}+r_{(3+k+1)2}) &= (1-r_1-r_2-r_3-r_4-r_5-r_6-\cdots-r_{(3+k)1}-r_{(3+k)2})\left(\frac{2k+2}{6(3+k)-1}\right) \\ &+ (1-r_1-r_2-r_3-r_4-r_5-r_6-\cdots-r_{(3+k)1}-r_{(3+k)2} \\ &- r_{(3+k+1)1})\left(\frac{2k+2}{6(3+k)+1}\right) \\ &= (1-r_1-r_2-r_3-r_4-r_5-r_6-\cdots-r_{(3+k)1}-r_{(3+k)2})\left(\frac{24k^2+96k+72}{36k^2+216k+323}\right) \\ &- \left(\frac{2k+2}{6(3+k)+1}\right)(1-r_1-r_2-r_3-r_4-r_5-r_6-\cdots-r_{(3+k)1} \\ &- r_{(3+k)2})\left(\frac{2k+2}{6(3+k)-1}\right) \\ &= (1-r_1-r_2-r_3-r_4-r_5-r_6-\cdots-r_{(3+k)2})\left(\frac{20k^2+88k+68}{36k^2+216k+323}\right)\end{aligned}$$

And this means that $(1 - r_1 - r_2 - r_3 - r_4 - r_5 - r_6 - \cdots - r_{(3+k)2})\left(\frac{\mathbf{20k^2+88k+68}}{\mathbf{36k^2+216k+323}}\right) < (1 - r_1 - r_2 - r_3 - r_4 - r_5 - r_6 - \cdots - r_{(3+k)1} - r_{(3+k)2})$ holds true. This inequality is inequality number (4).

Taking another part from the inequality, we get $20k(1 - r_1 - r_2 - r_3 - r_4 - r_5 - r_6 - \cdots - r_{(3+k)2}) > 36(r_7 + \cdots + r_{(3+k)1} + r_{(3+k)2})$, with n holding any value $36(r_7 + \cdots + r_{(3+k)1} + r_{(3+k)2}) < 8.432620012$.

In the case $k = 3$, $20k(1 - r_1 - r_2 - r_3 - r_4 - r_5 - r_6 - \cdots - r_{(3+k)2}) > 10$. From inequality number (4), we know that $20k(1 - r_1 - r_2 - r_3 - r_4 - r_5 - r_6 - \cdots - r_{(3+k)2})$ is increasing.

So we conclude that $36(-r_7 - \cdots - r_{(3+k)1} - r_{(3+k)2}) + 20k(1 - r_1 - r_2 - r_3 - r_4 - r_5 - r_6 - \cdots - r_{(3+k)1} - r_{(3+k)2}) + 4k^2(1 - r_1 - r_2 - r_3 - r_4 - r_5 - r_6 - \cdots - r_{(3+k)1} - r_{(3+k)2}) > 4k(r_6 + \cdots + r_{(3+k)1} + r_{(3+k)2}) + 2(-2(3+k)r_6 - \cdots - 3r_{(3+k-1)1} - 2r_{(3+k-1)2} - r_{(3+k)1})$ is correct. Thus when $n > 3$, there're integers in the interval $4n^2$ that are not covered.

Theorem 1.4.

Let m be a twin prime generator and x is the next twin prime generator where $x > m$ then $x < m + 4\left\lfloor\frac{m}{6}\right\rfloor^2 + 1$.

Proof 1.4.

From theorem 1.3 we know that the longest interval of integers covered by the union of $4n$ arithmetic progressions $\pm l \; mod \; (6(l) - 1), (6(l) + 1)$ where $l \leq n$ and $n = \left\lfloor\frac{m}{6}\right\rfloor$ is less than $4n^2$.

Let p be a prime number greater than m, then the integers covered by p equal $k_p = k(6s \pm 1) \pm s$ where $s > \lfloor\frac{m}{6}\rfloor$.

If $k < s$, then $k_p = k(6s \pm 1) \pm s = k6s \pm k \pm s = s(6k \pm 1) \pm k$ which means that they're already covered by primes less than m (by one of $\pm l \; mod \; (6(l) - 1), (6(l) + 1)$). We conclude from here that k has to be greater than or equal n to cover an integer that's not covered already.

$s(6s + 1) + s > s(6s + 1) - s > s(6s - 1) - s > s(6s - 1) - s > m + 4\left\lfloor\frac{m}{6}\right\rfloor^2 + 1$ which means that for primes greater than m, they can't cover integers between m and $m + 4\left\lfloor\frac{m}{6}\right\rfloor^2$. Thus, that leads to the fact that there has to be an integer that isn't covered (a twin prime generator) x where $m < x < m + 4\left\lfloor\frac{m}{6}\right\rfloor^2 + 1$.

2. NEXT TWIN PRIME

Definition 2.1.

Let $o_1 = (6n + 1) + n$, $o_2 = (6n + 1) - n$, $o_3 = (6n - 1) + n$, $o_4 = (6n - 1) - n$.

PROOF OF TWIN PRIME CONJECTURE

Let c be a twin prime generator.

Let $k_1 = c\ mod\ (6n+1)$ and $k_2 = c\ mod\ (6n-1)$.

Let $f_1 = |k_1 - o_1 - (6n+1)|\ mod\ (6n+1)$, $f_2 = |k_1 - o_2 - (6n+1)|\ mod\ (6n+1),$, $f_3 = |\ k_2 - o_3 - (6n-1)|\ mod\ (6n-1),$, $f_4 = |\ k_2 - o_4 - (6n-1)|\ mod\ (6n-1)$.

Let $F = \{x \in \mathbb{N}: x \le \left(c + 4\left\lfloor\frac{c}{6}\right\rfloor^2 + 1\right)\ and\ x = f_1\ or\ x = f_2\ or\ x = f_3\ or\ x = f_4\}$.

Let $T = \{x \in \mathbb{N}: x \le \left(c + 4\left\lfloor\frac{c}{6}\right\rfloor^2 + 1\right)\ and\ x \notin F\}$.

Then $next\ twin\ prime\ generator = c + MIN[T]$

Example 1.

We know that 12 is a twin prime generator, then

$c = 12$

The next twin prime generator is definitely within the next 7 numbers

We calculate just when $n \le \left\lfloor\frac{c}{6}\right\rfloor$ or $n \le 2$.

$f_1(1) = |k_1 - o_1 - (6(1)+1)|\ mod\ (6(1)+1) = |5 - 8 - 7|\ mod\ 7 = 10\ mod\ 7 = 3$

$f_1(2) = |k_1 - o_1 - (6(2)+1)|\ mod\ (6(2)+1) = |12 - 15 - 13|\ mod\ 13 = 16\ mod\ 13 = 3$

$f_2(1) = |k_1 - o_2 - (6(1)+1)|\ mod\ (6(1)+1) = |5 - 6 - 7|\ mod\ 7 = 8\ mod\ 7 = 1$

$f_2(2) = |k_1 - o_2 - (6(2)+1)|\ mod\ (6(2)+1) = |12 - 11 - 13|\ mod\ 13 = 12\ mod\ 13 = 12$

$f_3(1) = |\ k_2 - o_3 - (6(1)-1)|\ mod\ (6(1)-1) = |2 - 6 - 5|\ mod\ 5 = 9\ mod\ 5 = 4$

$f_3(2) = |\ k_2 - o_3 - (6(2)-1)|\ mod\ (6(2)-1) = |1 - 13 - 11|\ mod\ 11 = 23\ mod\ 11 = 1$

$f_4(1) = |\ k_2 - o_4 - (6(1)-1)|\ mod\ (6(1)-1) = |2 - 4 - 5|\ mod\ 5 = 7\ mod\ 5 = 2$

$F = x \in \mathbb{N}: x \le \left(c + 4\left\lfloor\frac{c}{6}\right\rfloor^2 + 1\right)\ and\ x = f_1\ or\ x = f_2\ or\ x = f_3\ or\ x = f_4)\} = \{1,2,3,4,12\}$

$T == \{x \in \mathbb{N}: x \le \left(c + 4\left\lfloor\frac{c}{6}\right\rfloor^2 + 1\right)\ and\ x \notin F\} = \{5,6,7,8,9,10,11,13,14,15,16,17,18,19,20,21,22\}$

$next\ twin\ prime\ generator = c + MIN[T] = 12 + MIN[5,6,7] = 12 + 5 = 17$

3. REFERENCES

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Qassim University, Safa Abdallah Moallim Muhammad

E-mail address: safofoh.100@gmail.com