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Neutrosophic Crisp Closed Region and Neutrosophic Crisp Continuous Functions

Abstract

In this paper, we introduce and study the concept of "neutrosophic crisp closed set "and "neutrosophic crisp continuous function. Possible application to GIS topology rules are touched upon.

Keywords

Neutrosophic crisp closed set, neutrosophic crisp set; neutrosophic crisp topology; neutrosophic crisp continuous function.

1. Introduction

The idea of "neutrosophic crisp set" was first given by Salama and Smarandache [8]. Neutrosophic crisp operations have been investigated by Salama and Alblowi [4, 5], Salama [6], Salama and Smarandache [7, 8], Salama, and Elagamy [9], Salama et al. [10]. Neutrosophy has laid the foundation for a whole family of new mathematical theories, generalizing both their crisp and fuzzy counterparts [13]. Here we shall present the neutrosophic crisp version of these concepts. In this paper, we introduce and study the concept of "neutrosophic crisp closed set "and "neutrosophic crisp continuous function".

2. Terminologies

We recollect some relevant basic preliminaries, and in particular the work of Smarandache in [11,12], and Salama and Alblowi [4, 5], Salama [6], . Salama and Smarandache [7, 8], Salama, and Elagamy [9], Salama et al. [10].

2.1 Definition:

Let X be a non-empty fixed set. A generalized neutrosophic crisp set $(GNCS)$ A is an object having the form A = <A ,A$_2$,A$_3$>, where $A_1, A_2, A_3 \subseteq X$ and $A_1 \cap A_2 \cap A_3 = \phi$.

2.2 Definition [8,10]

As defined in [10] a neutrosophic crisp topology (NCT) on a non-empty set X is a family, τ, of neutrosophic crisp subsets of X satisfying the following axioms:

(NCT_1) $\emptyset_N$, $X_N \in \tau$,

(NCT_2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

(NCT_3) $\cup G_i \in \tau \, \forall \{G_i : i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called a neutrosophic crisp topological space (NCTS) and the elements of τ are called neutrosophic crisp open sets, (NCOS). A neutrosophic crisp set F is said to be neutrosophic crisp closed if and only if its complement, F^c, is neutrosophic crisp open.

2.3 Definition [7]

Let (X, Γ) be $NCTS$ and $A = \langle A_1, A_2, A_3 \rangle$ be a NCS in X. Then the neutrosophic crisp closure of A ($NCcl(A)$) and neutrosophic interior crisp ($NCint(A)$) of A are defined by

$NCcl(A) = \cap \{K: K$ is an $NCCS$ in X and $A \subseteq K\}$

$NCint(A) = \cup \{G: G$ is an $NCOS$ in X and $G \subseteq A\}$,

where NCS is a neutrosophic crisp set and $NCOS$ is a neutrosophic crisp open set. It can be also shown that $NCcl(A)$ is a neutrosophic crisp closed set($NCCS$) and $NCint(A)$ is a neutrosophic crisp open set ($NCOS$) in X.

3. Neutrosophic Crisp Co-Topology

3.1 Definition

Let (X,T) be a neutrosophic crisp topological space, a neutrosophic crisp set A in (X,T) is said to be neutrosophic crisp closed (NC-closed), if NCcl(A) $\subseteq$ G whenever A $\subseteq$ G and G is neutrosophic crisp open set.

3.2 Proposition

If A and B are neutrosophic crisp closed sets, then A$\cup$B is neutrosophic crisp closed set.

3.3 Remark

The intersection of two neutrosophic crisp closed (NC-closed) sets need not be neutrosophic crisp closed set.

3.4 Example

Let X = {a,b,c,d,e,f,g} and that A = <{a,b},{b,c},{b,d}>, B = <{a,c},{d,c},{a,c}> are two neutrosophic crisp sets on X. Then $T = \{\phi_N, \chi_N, A, B\}$ is a neutrosophic crisp topology on X. Define the two neutrosophic crisp sets A_1 and A_2 as follows,

A_1 = <{b,d},{a,d,e,f,g},{a,b}>

A_2 = <{a,c},{a,b,e,f,g},{a,c}>

A_1 and A_2 are neutrosophic crisp closed set but $A_1 \cap A_2$ is not a neutrosophic crisp closed set.

3.5 Proposition

Let (X,T) be a neutrosophic crisp topological space. If B is neutrosophic crisp closed set and B $\subseteq$ A $\subseteq$ NCcl (B), then A is NC-closed.

Definition

(Defining NC topology by closed sets). A NC topology on a set X is given by defining " NC open set" of X. Since NC closed sets are just exactly the complement of NC open sets, it is possible to define NC topology by giving a collection of NC closed sets. Let K be a collection of NC subsets of X satisfying

(NCT_1) $\emptyset_N$, $X_N \in$ K,

(NCT_2) $G_1 \cup G_2 \in K$ for any $G_1, G_2 \in K$,

(NCT_3) $\bigcap G_i \in K \, \forall \{G_i : i \in J\} \subseteq K$.

Then define T by: T:={X-C |C∈K}

Is a NC topology, i.e. it satisfyies Definition(2.2). On the other hand, if T is a NC topology, i.e. the collection of NC-open sets, then K:={X-U|U∈T}.

In this case the pair (X, K) is called a neutrosophic crisp Co-topological space $(NCKS)$ and the elements of K are called neutrosophic crisp closed sets, ($NCCS$ for short).

3.6 Proposition

In a neutrosophic crisp topological space (X,T), T=$\Im$ (the family of all neutrosophic crisp closed sets) iff every neutrosophic crisp subset of (X,T) is a neutrosophic crisp closed set.

Proof.

Suppose that every neutrosophic crisp set of (X,T) is NC-closed, and let A$\in$T. Since A $\subseteq$ A and A is NC-closed, NCcl(A) $\subseteq$ A. However, we have that A $\subseteq$NCcl(A), for each set A. Hence, NCcl (A) = A. thus, A$\in\Im$. Therefore, T$\subseteq\Im$. Now, consider $B \in \Im$, then $B^c \in$T $\subseteq \Im$. Hence B$\in$T, That is, $\Im \subseteq$ T. Therefore T=$\Im$.

Conversely, suppose that A be a neutrosophic crisp set in (X,T), and B is a neutrosophic crisp open set in (X,T) such that A $\subseteq$ B. By hypothesis, B is NC-closed. By definition of any neutrosophic crisp closure set, we have that NCcl(A) $\subseteq$ B. Therefore A is NC-closed set.

3.7 Proposition

Let (X,T) be a neutrosophic crisp topological space. A neutrosophic crisp set A is neutrosophic crisp open iff B $\subset$ NCInt (A), whenever B is neutrosophic crisp closed and B $\subset$ A.

Proof

Let A a neutrosophic crisp open set and B be a NC-closed, such that B $\subset$ A. Now, $B \subset A \Rightarrow A^c \subset B^c$ and A^c is a neutrosophic crisp closed set $\Rightarrow NCcl(A^c) \subset B^c$. That is, B = $(B^c)^c \subset (NCcl(A^c))^c$. But $(NCcl(A^c))^c$ = NCint (A). Thus, B $\subset$ NCint (A). Conversely, suppose that A be a neutrosophic crisp set, such that

$B \subset NC\,\mathrm{int}(A)$ whenever B is neutrosophic crisp closed and B $\subset$ A. Let $A^c \subset$ B $\Rightarrow$ $B^c \subset$ A. Hence by assumption $B^c \subset NC\,\mathrm{int}(A)$. that is, $(NC\,\mathrm{int}(A))^c \subset B$. But $(NC\,\mathrm{int}(A))^c = NCcl(A^c)$.

Hence $NCcl(A^c) \subset B$. That is A^c is neutrosophic crisp closed set. Therefore, A is neutrosophic crisp open set

3.8 Proposition

If (A) $\subseteq$ B $\subseteq$ NCcl(A) and if A is neutrosophic crisp closed set then B is also neutrosophic crisp closed set.

4. Neutrosophic Crisp Continuous Functions

4.1 Definition

(c) If $A = \langle A_1, A_2, A_3 \rangle$ is a NCS in X, then the neutrosophic crisp image of A under f, denoted by $f(A)$, is the a NCS in Y defined by $f(A) = \langle f(A_1), f(A_2), f(A_3) \rangle$.

(d) If f is a bijective map then f^{-1}: $Y \longrightarrow X$ is a map defined such that:
for any NCS $B = \langle B_1, B_2, B_3 \rangle$ in Y, the neutrosophic crisp preimage of B, denoted by $f^{-1}(B)$, is a NCS in X defined by $f^{-1}(B) = \langle f^{-1}(B_1), f^{-1}(B_2), f^{-1}(B_3) \rangle$.

Here we introduce the properties of images and preimages some of which we shall frequently use in the following sections .

4.2 Corollary

Consider, the two families of neutrosophic crisp sets;

A= $\{A_i$: i$\in$I, $A_i \subseteq$X$\}$ and B = $\{B_j$: j$\in$J, $B_j \subseteq$Y$\}$; and let f be a function such that f : X $\rightarrow$ Y.

(a) $A_1 \subseteq A_2 \Leftrightarrow f(A_1) \subseteq f(A_2),\ B_1 \subseteq B_2 \Leftrightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$,

(b) $A \subseteq f^{-1}(f(A))$ and if f is injective, then $A = f^{-1}(f(A))$.

(c) $f(f^{-1}(B)) \subseteq B$ and if f is surjective, then $f(f^{-1}(B)) = B$, .

(d) $f^{-1}(\cup B_i) = \cup f^{-1}(B_i),\ f^{-1}(\cap B_i) = \cap f^{-1}(B_i)$,

(e) $f(\cup A_i) = \cup f(A_i);\ f(\cap A_i) \subseteq \cap f(A_i)$;and if f is injective, then $f(\cap A_i) = \cap f(A_i)$;

(f) $f^{-1}(Y_N) = X_N\ f^{-1}(\phi_N) = \phi_N$.

(g) $f(\phi_N) = \phi_N,\ f(X_N) = Y_N$ if f is subjective.

Proof

Obvious.

4.3 Proposition

Consider the function $f: X \to Y$, then f is said to be neutrosophic crisp continuous iff the preimage of each neutrosophic crisp closed set in Y is a neutrosophic crisp closed set in X.

4.4 Proposition

Consider the function $f: X \to Y$, then f is said to be neutrosophic crisp continuous iff the image of each neutrosophic crisp closed set in X is a neutrosophic crisp closed set in Y.

4.5 Proposition

The following are equivalent to each other:

(a) $f:(X,\Gamma_1)\to(Y,\Gamma_2)$ is neutrosophic crisp continuous .

(b) $f^{-1}(NCInt(B)) \subseteq NCInt(f^{-1}(B))$ for each *NCCS* B in Y.

(c) $NCCl(f^{-1}(B)) \subseteq f^{-1}(NCCl(B))$ for each *N NCCS* B in Y.

4.6 Example

Let (Y,Γ_2) be a *NCTS* and $f: X \to Y$ be a function. In this case $\Gamma_1 = \{f^{-1}(H): H \in \Gamma_2\}$is a *NCT* on X. Indeed, it is the coarsest *NCT* on X which makes the function $f: X \to Y$ neutrosophic crisp continuous. One may call it the initial neutrosophic crisp topology with respect to f.

4.7 Definition

Let (X,T) and (Y,S) be two neutrosophic crisp topological space, then

(a) A bijective map *f*:(X,T)→ (Y,S) is called neutrosophic crisp irresolute if the inverse image of every neutrosophic crisp closed set in (Y,S) is neutrosophic crisp closed in (X,T). Equivalently if the inverse image of every neutrosophic crisp open set in (Y,S) is neutrosophic crisp open in (X,T).

(b) A map *f*:(X,T)→ (Y,S) is said to be strongly neutrosophic crisp continuous if *f*(A) is both neutrosophic crisp open and neutrosophic crisp closed in (Y,S) for each neutrosophic crisp set A in (X,T).

(c) A map *f* : (X,T) → (Y,S) is said to be perfectly neutrosophic crisp continuous if f^{-1}(B) is both neutrosophic crisp open and neutrosophic crisp closed in (X,T) for each neutrosophic crisp open set B in (Y,S).

4.8 Proposition

Let (X,T) and (Y,S) be any two neutrosophic crisp topological spaces. Let *f* : (X,T) → (Y,S) be neutrosophic crisp continuous. Then for every neutrosophic crisp set A in X, *f*(NCcl(A)) ⊆ NCcl(*f*(A)).

4.9 Proposition

Let (X,T) and (Y,S) be any two neutrosophic crisp topological spaces. Let *f* : (X,T) → (Y,S) be neutrosophic crisp continuous. Then for every neutrosophic crisp set A in Y, NCcl(f^{-1}(A)) ⊆ f^{-1}(NCcl(A)).

Definition

Let (X, Γ_1) and (Y, Γ_2) be two NCTSs and let $f : X \to Y$ be a function. Then f is said to be open iff the neutrosophic crisp image of each NCS in Γ_1 is a NCS in Γ_2.

Definition

Consider the two neutrosophic crisp co-topologios (X, K_1), (Y, K_2) and function $f : X \to Y$ the function f is said to be neutrosophic crisp closed iff $f(A) \in K_2$, $\forall$ $A \in K_1$.

Or equivalently, $f^{-1}(B) \in K_1$, $\forall$ $B \in K_2$.

Definition

Consider the two neutrosophic crisp topologios (X, T_1), (Y, T_2) and function $f : X \to Y$ the function f is said to be neutrosophic crisp closed iff $f(A) \in T_2$, $\forall$ $A \in T_1$.

Or equivalently, $f^{-1}(B) \in T_1$, $\forall$ $B \in T_2$.

4.10 Proposition

Let (X,T) and (Y,S) be any two neutrosophic crisp topological spaces. If A is a neutrosophic crisp closed set in (X,T) and if f : (X,T) $\to$ (Y,S) is neutrosophic crisp continuous and neutrosophic crisp closed then f(A) is neutrosophic crisp closed in (Y,S).

Proof.

Let G be a neutrosophic crisp- open in (Y,S). If $f(A) \subseteq G$, then $A \subseteq f^{-1}(G)$ in (X,T). Since A is neutrosophic crisp closed and $f^{-1}(G)$ is neutrosophic crisp open in (X,T), $NCcl(A) \subset f^{-1}(G)$, (i.e) $f(NCcl(A) \subset G$. Now by assumption, $f(NCcl(A))$ is neutrosophic crisp closed and $NCcl(f(A)) \subseteq Ncl(f(NCcl(A))) = f(NCcl(A)) \subset G$. Hence, f(A) is NC-closed.

4.11 Proposition

If the function $f : X \to Y$ is neutrosophic crisp continuous, then it is neutrosophic crsip closed. Whereas, the converse need not be true, as shown in Example 4.12.

4.12 Example

Let X ={a,b,c,d,e,f,g} and Y ={a,b,c}. Define neutrosophic crisp sets A and B as follows:

A = <{d,a} ,{f,g}, {c,b}>

B = <{f,a}, {e,f},{d,c} >

Then the family T = { ϕ_N, X_N, A} is a neutrosophic crisp topology on X and S = { ϕ_N, X_N, B} is a neutrosophic crisp topology on Y. Thus (X,T) and (Y,S) are neutrosophic crisp topological spaces. Define

f : (X,T) $\to$ (Y,S) as f(a) = b , f(b) = a, f(c) = c. Clearly f is NC-closed continuous. Now f is not neutrosophic crisp continuous, since $f^{-1}(B) \notin T$ for $B \in S$.

Definition

If the function $f : X \to Y$ *is neutrosophic crisp continuous, then it is* neutrosophic crisp open. *Whereas, the converse need not* be true, as shown in Example 4.13.

4.13 Example

Let X = {a,b,c,d,e,f,g}. Define the neutrosophic crisp sets A and B as follows.

A = <{f,g}, {d,a} , {c,b}>

B = <{f,a}, {d,c} , {e,f}> and

C = <{b,d}, {c,d} , {d,a}>

T = { ϕ_N, X_N, A ,B} and

S = { ϕ_N, X_N, C}

are neutrosophic crisp topologies on X. Thus (X,T) and (X,S) are neutrosophic crisp topological spaces. Define f: (X,T) → (X,S) as follows f(a) = b,

f(b) = b, f(c) = c. Clearly f is NC-continuous. Since

D = <{d,a}, {c,f} , {g,e}>

is neutrosophic crisp open in (X,S), f^{-1}(D) is not neutrosophic crisp open in (X,T).

4.14 Proposition

Let (X,T) and (Y,S) be any two neutrosophic crisp topological space. If f : (X,T) → (Y,S) is strongly NC-continuous then f is neutrosophic crisp continuous.

The converse of Proposition 4.16 is not true. See Example 4.17

4.15 Example

Let X ={a,b,c}. Define the neutrosophic crisp sets A and B as follows.

A = <{d,e}, {f,h} , {c,a}>

B = <{g,e}, {q,z} , {b,a}>and

C = <{a,c}, {f,a} , {h,d}>

T = { ϕ_N, X_N, A ,B} and S = { ϕ_N, X_N, C} are neutrosophic crisp topologies on X. Thus (X,T) and (X,S) are neutrosophic crisp topological spaces. Also define f :(X,T)→ (X,S) as follows f(a) = a, f(b) = c, f(c) = b. Clearly f is neutrosophic crisp continuous. But f is not strongly NC-continuous. Since D = <{c,f},{e,c},{b,g,d}>

Is a neutrosophic crisp open set in (X,S), f^{-1}(D) is not neutrosophic crisp open in (X,T).

4.16 Proposition

Let (X,T) and (Y,S) be any two neutrosophic crisp topological spaces. If f: (X,T) → (Y,S) is perfectly NC-continuous then f is strongly NC-continuous.

The converse of Proposition 4.16 is not true. See Example 4.17

4.17 Example

Let X = {a,b,c,d,e,f,g}. Define the neutrosophic crisp sets A and B as follows.

A = <{f,g}, {d,a} , {c,b}>, B = <{f,a}, {d,c} , {e,f}> and

C = <{b,b}, {c,d} , {d,a}>

T = {ϕ_N, X_N, A ,B} and S = {ϕ_N ,X_N, C} are neutrosophic crisp topologies space on X. Thus (X,T) and (X,S) are neutrosophic crisp topological spaces. Also define f : (X,T) → (X,S) as follows f(a) = a, f(b) = f(c) = b. Clearly f is strongly NC-continuous. But f is not perfectly NC continuous. Since D = <{d,a},{b,b}, {c,d}> is aneutrosophic crisp open set in (X,S), f^{-1}(D) is neutrosophic crisp open and not neutrosophic crisp closed in (X,T).

4.18 Proposition

Let (X,T) and (Y,S) be any neutrosophic crisp topological spaces. If f: (X,T) → (Y,S) is strongly neutrosophic crisp continuous then f is strongly NC-continuous.

The converse of proposition 4.20 is not true. See Example 4.21

4.19 Example

Let X = {a,b,c,d,e,f,g} and define the neutrosophic crisp sets A and B as follows.

A = <{sb,b}, {d,a} , {c,d}>

B = <{e,f }, {d,c} , {f,a}> and

C = <{f,g}, {c,b} , {d,a}>

T = {ϕ_N, X_N, A ,B} and S = {ϕ_N, X_N, C} are neutrosophic crisp topologies on X. Thus (X,T) and (X,S) are neutrosophic crisp topological spaces. Also define f : (X,T) → (X,S) as follows: f(a) = a, f(b) = f(c) = b. Clearly f is strongly NC-continuous. But f is not strongly neutrosophic crisp continuous. Since

D = <{d,a}, {f,g} , {c,b}>

be a neutrosophic crisp set in (X,S), f^{-1}(D) is neutrosophic crisp open and not neutrosophic crisp closed in (X,T).

4.20 Proposition

Let (X,T),(Y,S) and (Z,R) be any three neutrosophic crisp topological spaces. Suppose f : (X,T) → (Y,S), g : (Y,S) → (Z,R) be maps. Assume f is neutrosophic crisp irresolute and g is NC-continuous then g o f is NC-continuous.

4.21 Proposition

Let (X,T), (Y,S) and (Z,R) be any three neutrosophic crisp topological spaces. Let f : (X,T) → (Y,S), g : (Y,S) → (Z,R) be map, such that f is strongly NC-continuous and g is NC-continuous. Then the composition g o f is neutrosophic crisp continuous.

4.22 Definition

A neutrosophic crisp topological space (X,T) is said to be neutrosophic crisp $T_{1/2}$ if every neutrosophic crisp closed set in (X,T) is neutrosophic crisp closed in (X,T).

4.23 Proposition

Let (X,T),(Y,S) and (Z,R) be any neutrosophic crisp topological spaces. Let f : (X,T) → (Y,S) and g : (Y,S) → (Z,R) be mapping and (Y,S) be neutrosophic crisp $T_{1/2}$ if f and g are NC-continuous then the composition g o f is NC-continuous.

The proposition 4.11 is not valid if (Y,S) is not neutrosophic crisp $T_{1/2}$.

4.24 Example

Let X = {a,b,c,d,e,f,g}. Define the neutrosophic crisp sets A,B and C as follows.

A = <{d,c},{d,a} , {c,b}>

B = <{f,g},{b,b},{e,f}> and

C = < {f,a}, {c,d},{d,a}>

Then the family T={ϕ_N, X_N, A}, S={ϕ_N, X_N, B} and R={ϕ_N,X_N,C} are neutrosophic crisp topologies on X. Thus (X,T),(X,S) and (X,R) are neutrosophic crisp topological spaces. Also define f : (X,T) $\to$ (X,S) as f(a) = b, f(b) = a, f(c) = c and g : (X,S) $\to$ (X,R) as g(a) = b, g(b) = c, g(c) = b. Clearly f and g are NC-continuous function. But g o f is not NC-continuous. For C^c is neutrosophic crisp closed in (X,R). $f^{-1}(g^{-1} C^c)$ is not NC closed in (X,T).

G o f is not NC-continuous.

5. Conclusion

In this paper, we presented a generalization of the neutrosophic crisp topological space. The basic definitions of neutrosophic crisp closed set "and "neutrosophic crisp continuous function. with some of their characterizations were deduced. Furthermore, we constructed a neutrosophic crisp open and closed functions, with a study of a number its properties.

References

1. S.A. Alblowi, A.A. Salama and Mohmed Eisa, New Concepts of Neutrosophic Sets, International Journal of Mathematics and Computer Applications Research (IJMCAR), Vol. 3, Issue 3, Oct, 95-102, (2013).
2. A. Hanafy, A.A. Salama and K. Mahfouz, Correlation of neutrosophic Data, International Refereed Journal of Engineering and Science (IRJES), Vol. (1), Issue 2, PP.39-33. (2012).
3. I.M. Hanafy, A.A. Salama and K.M. Mahfouz," Neutrosophic Classical Events and Its Probability" International Journal of Mathematics and Computer Applications Research (IJMCAR) Vol. (3), Issue 1, pp171-178, Mar (2013).
4. A.A. Salama and S.A. Alblowi, "Generalized Neutrosophic Set and Generalized Neutrosophic Topological Spaces,"Journal Computer Sci. Engineering, Vol. (2) No. (7) pp 129-132(2012) .
5. A.A. Salama and S.A. Alblowi, Neutrosophic Set and Neutrosophic Topological Spaces, ISORJ. Mathematics, Vol. (3), Issue (3), pp-31-35(2012).
6. A. A. Salama, "Neutrosophic Crisp Points & Neutrosophic Crisp Ideals", Neutrosophic Sets and Systems, Vol.1, No. 1, pp. 50-54(2013).
7. A. A. Salama and F. Smarandache, "Filters via Neutrosophic Crisp Sets", Neutrosophic Sets and Systems, Vol.1, No. 1, pp. 34-38 (2013).
8. A.A. Salama and F. Smarandache, Neutrosophic crisp set theory, Educational Publisher, Columbus,USA, (2015).
9. A.A. Salama, and H. Elagamy, "Neutrosophic Filters" International Journal of Computer Science Engineering and Information Technology Reseearch (IJCSEITR), Vol.3, Issue (1), pp 307-312, Mar (2013).
10. A. A. Salama, F. Smarandache and Valeri Kroumov "Neutrosophic Crisp Sets & Neutrosophic Crisp Topological Spaces" Bulletin of the Research Institute of Technology (Okayama University of Science, Japan), in January-February (2014).

11. F. Smarandache, Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy , Neutrosophic Logic , Set, Probability, and Statistics University of New Mexico, Gallup, NM 87301, USA(2002).
12. F. Smarandache. A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability. American Research Press, Rehoboth, NM, (1999).
13. L.A. Zadeh, Fuzzy Sets, Inform and Control 8, 338-353, (1965).