On the Lorentz transformation and time dilation

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Abstract. The special theory of relativity (STR) describes the time dilation between two reference frames moving relative to each other at a constant speed. The Lorentz transformation provides the magnitude of this time dilation. The present work focuses on the fact that the time observed on the 'other' system will depend on the location of the clocks used for time comparisons, and we refer to positional (location specific) time. The paper points to the various 'observational principles', *i.e*. the specification of which clocks to apply, and we present a unified framework for these principles. It is argued that the total picture of the observed time dilations is more informative than the usual approach of focusing on one specific expression for time dilation, apparently based on a specific observational principle and a somewhat arbitrary definition of simultaneity. The motivation of the paper is to challenge the current narrative regarding time dilation occurring under the conditions of the STR.

Key words: Lorentz transformation; time dilation; symmetry; positional time; observational principle; length contraction.

1. Introduction

The Lorentz transformation provides the mathematical description of space-time for two reference frames moving relative to each other at constant speed; *i.e.* the situation described in the special theory of relativity (STR). This paper strives to explore the full potential of the Lorentz transformation, and thereby the interpretation of time dilation.

We specify the various expressions for the time dilation following from the Lorentz transformation. In doing so, we introduce the concept of observational principle; that is, the specification of which clocks to use for the required time comparisons. A unified framework for these observational principles is given, stressing that time measured for 'the other system' is given by the location where the time reading/comparison is carried out. Thus, we will refer to positional (location specific) time.

So rather than specifying *one* generic time dilation formula – which is typically based on a somewhat arbitrary definition of simultaneity – we look at the total picture of all expressions for time dilation.

Regarding *simultaneity,* we will in this paper restrict to consider events which occur at the same location *and* time. We will assume that each reference frame has a set of calibrated clocks, located at virtually any position. So in principle we can at any position compare the clocks of the two reference frames. Thus, any convention to define simultaneity across reference frames by use of light rays is not part of the considerations in the present paper.

A vast literature exists on the STR. As a basis for our discussions we will in Chapter 2 shortly comment on some aspects from a small sample of these references. Next we present a list of assumptions for the further discussion. In particular we assume a strict symmetry between the two reference frames. Thus, the choice of observational principle represents the only possible departure from symmetry.

The various observational principles based on the Lorentz transformation are thoroughly discussed in chapters 3-5. The provided results are well-known, but the presentation is in some respect believed to be original, presenting a common, consistent framework for all the time dilation expressions. A chapter

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on bidirectional rays is also included. Further, an Annex presents a simple derivation of the Lorentz transformation for one space co-ordinate (x) and also provides a discussion on length contraction.

Obviously, a substantial part of the present work presents standard material; for many readers providing well-known results. However, the main objective is not tutorial but to give a consistent description of time dilation, as obtained from the Lorentz transformation. This leads to views that in some respect seem to challenge the prevailing understanding of time dilation. We note that the present work is a report of an observer from outside the physical community. The considerations are essentially mathematical, but with necessity we also consider the physical interpretation of time dilation, (*cf.* Annex C).

2. The Lorentz transformation

This chapter provides a background for the subsequent discussions.

2.1 Review of some literature

The basis for the discussions is the standard theoretical experiment, with two co-ordinate systems (reference frames), K and K' moving relative to each other at speed, v. We investigate the relation between space and time parameters, (x, t) on system K and the corresponding parameters (x', t') on system K'. The relation is provided by the Lorentz transformation, (*e.g.* [1] - [4]).

A vast amount of literature exists on this topic. As a background we consider a small sample, authored by experienced scientists in the field: First two older books ('classics'), Einstein's introduction to the STR, [1], and Chapters 3 and 4 of the Feynman lectures, [2]. Further, the more recent and insightful books by Giulini, [3] and Mermin, [4], which are frequently referred on the topic. Finally we consider some web pages; in particular Andrew Hamilton, [5] and Pössel, ('Einstein Online'), [6]. These references mainly address non-experts. But it is of interest to see how the main ideas of the STR are presented, and we review a few aspects of these works.

Definition of *simultaneity* becomes crucial as clocks are moving relative to each other. It seems to be a common understanding that no unique definition of simultaneity exists *across systems* (reference frames); these definitions being based on utilizing light rays. The arguments on time dilation are frequently based on such a definition of simultaneity. This tends to introduce some asymmetry between the two systems; resulting in an asymmetric solution to a symmetric situation.

The question of *symmetry* is interesting. The STR essentially describes a symmetric situation for the two systems/observers moving relative to each other. And for instance the reference [5] specifies an experiment of complete symmetry, referring to two spaceships moving relative to each other. Other references, however, are not found that explicit. Some will for instance include examples, like the 'travelling twin', *e.g.* [4], which clearly involves asymmetry. But by claiming that the slower aging of the 'travelling twin' is not restricted to the acceleration/deceleration periods; this actually seems to be taken as an example of a 'true time dilation' occurring also under the conditions of the STR.

Regarding the basic concept of *time dilation*, I firstly miss a more precise discussion of the multitude of (time) solutions offered by the Lorentz transformation. It is treated by some authors, *e.g.* in [4], but in my opinion not in sufficient depth. More generally, it is to me not completely clear whether the sources are fully consistent; *i.e.* telling 'the same story' regarding time dilation. In particular, to what extent do they agree regarding the physical reality of time dilation?

So, how should we interpret the common statement: 'Moving clock goes slower'? Many authors apply the expression 'as seen' by the observer on the other reference system, perhaps indicating that it is an apparent effect, not a physical reality, without elaborating on the interpretation of 'as seen'. However, others stress that 'everything goes slower' on the 'moving system', not only the clocks; truly stating that the time dilation represents a physical reality also under the conditions of STR, (no gravitation *etc*.).

Giulini [3] in Section 3.3 of his book states: ‘Moving clocks slow down’ is ‘potentially misleading and should not be taken too literally’. However, he does not follow up on this and explicitly state whether time dilation is just an apparent effect without physical reality. I do not find his expression ‘not be taken too literally’ to be a very strong (or precise) statement.

As pointed out *e.g.* by Pössel [6] the phenomenon of time dilation stems from the fact that clocks of the two systems have to be compared at least twice, so it cannot be the same two clocks being compared. Since movement is relative, however, an interesting question is how to decide which system (clock) is moving. Mermin [4] states that what 'moves' is decided by which clocks are chosen to be synchronized. This seems to be is in line with the views of the present work: the procedure of clock synchronization and clock comparison decides which reference system has the time moving faster/slower.

In conclusion, I find the sources somewhat ambiguous regarding the very interpretation of time dilation. What is actually the message of the physical community? In what sense – and under which precise conditions – is time dilation to be considered a true physical phenomenon? In other words: When do we have a ‘true time dilation’? In a *completely symmetric* situation – *e.g.* the case of two free floating spaceships moving relative to each other – one would hardly consider time dilation as a physical reality, even if it is an 'observational reality'.

2.2 Basic assumptions

The main focus of this paper is the Lorentz transformation, describing two reference frames, K and K' passing each other at a relative speed, v. We restrict to consider just one space co-ordinate, (x-axis), and the discussions of the present paper will be based on the following assumptions/specifications:

- *Speed of light* will be measured to be constant in both directions and equal to c, independent both of the speed of the observer and speed of the light source.
- *Length contraction*. There is observed a length contraction, k_x along the x- axis of ‘the other’ reference frame. When we from a specific location on K observe the passing of a measure stick of length, x', (as measured on K'), then the time observed between the passing of its two endpoints, will correspond to the stick (apparently) having a length $k_x x'$; (*cf.* Annex B).
- There is a complete *symmetry* between the two co-ordinate systems, K and K'. This symmetry will include the past history of the systems; (how they came into this state of relative movement). Thus, in our idealized experiment we consider the systems to be identical in all respects.
- On both reference frames there is an arbitrary number of identical, synchronized clocks, located at various positions where it is required to measure time. Thus, simultaneity of events on a specific system is confirmed by comparing clock readings.
- At time $t = t' = 0$, clocks at the location $x = 0$ on K and location $x' = 0'$ on K' are synchronized. This represents the the definite starting point, from which all events are measured; it is the ‘point of initiation’, playing an essential role in the description of subsequent events.
- When events are related to two reference system, simultaneity of events will mean that they occur at the same time *and* at the same location. The events are just measured in different bodies of reference. The clock located on K and the clock on K' will usually provide different time readings. The measurements are, however, objective, and the observers will agree on both the time readings at the specified location.
- We choose the *perspective* of one of the systems, say K, and refer to this as the *primary* system. This will simply mean that events are here described by classical physics by parameters (x, t). In particular, the time on this system for any position, x is given as a constant, $t(x) \equiv t$; and consequently the time t' observed on the 'other' system at a specific instant will depend on the location, x.
- We apply ‘Newtonian/classical’ arguments for events relative to a specific system. Further, we assume a completely *idealized situation*: systems are 'free floating', moving without disturbance, neglecting gravitational forces; all data can be gathered precisely, without measurement errors; *etc.*

2.3 Formulation of the Lorentz transformation

The Lorentz transformation represents the fundament for our discussion of time dilation (*cf.* Annex A). We introduce the so-called length contraction,

$$k_x = \sqrt{1 - (\frac{v}{c})^2} \tag{1}$$

Using the notation of Section 2.2, the Lorentz transformation for one space co-ordinate is now given by

$$x' = \frac{x - vt}{\sqrt{1-(\frac{v}{c})^2}} \tag{2}$$

$$t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1-(\frac{v}{c})^2}} \tag{3}$$

This relates simultaneous time readings, t and t' performed at identical locations x on K and x' on K'. So observers (observational equipment) at different locations on K will agree regarding the current time on K. Similarly, all observers (clocks) on K' will agree regarding the time on K'. However, an observer at K and an observer at K', which at an instant in time are at the same location - actually passing each other at the moment in question - will (usually) observe $t \neq t'$, but they will agree both on the time t at K and on the time t' at K'; these observed values being specified by the above Lorentz transformation.

3 Two observational principles regarding time dilation

We now point out some direct consequences of the Lorentz transformation presented in Section 2.3. Due to the relative movement of the two reference frames, one should specify exactly how the comparisons of time and length are carried out. So now we present two different observational principles for observing time on 'the other system'; taking the perspective of this system, *cf.* Section 2.2.

- *Principle A: Following a fixed clock on 'the other system', K'*. Here a set of clocks and observational instruments are located along the x-axis of K, allowing observers to follow and compare time readings with a fixed clock on K'. At the moment when this clock on K' passes a position x we can observe both the time t' on K' and the time t on K. The fixed clock on K' is located at position, $x' \equiv 0'$, and the first time comparison is carried out at $t = t' = 0$.The second time comparison at time t on K is carried out at the location $x = vt$ on K. The Lorentz transformation, (3) now gives the following relation between t and t' at this location:

$$t' = t\sqrt{1 - (\frac{v}{c})^2} \tag{4}$$

  This equals the 'standard' time dilation formula; *e.g.* see [1].

- *Principle B: Observing various clocks on 'the other system', K' from a fixed location on K*. Now there is a single clock on K at a fixed location on the x-axis, and from this position we make registrations of clocks on K' as they pass along. Now choosing the location $x \equiv 0$ on K, the Lorentz transformation directly gives that at time t on K the time observed on K' at this location equals

$$t' = \frac{1}{\sqrt{1-(\frac{v}{c})^2}}t \tag{5}$$

In conclusion, both $t' = t\sqrt{1 - (\frac{v}{c})^2}$ and $t' = t\,/\sqrt{1 - (\frac{v}{c})^2}$ are valid results for the 'simultaneous' time observed on K'. The first expression being valid at position, $x = vt$, ($x' = 0'$); the second for position $x =$

0, $(x' = -\frac{vt}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = -vt')$.[2] As observed, all results being a direct consequences of the Lorentz transformation.

Actually, the perspective (choice of 'primary system') does not matter here, but rather the specification of which clocks are used for the comparisons. The question is: Which reference system applies a single clock, and which reference system utilizes at least two clocks for the time comparisons. Thus, it could be more informative to summarize the above results as follows:

Let A be the reference frame where there are used at least two clocks for time comparison, letting x_A and t_A be the position and time measurements on this system. Thus, there at least two clocks on A, located at $x_A = 0$ and $x_A = vt_A$, respectively.

Let B be the reference frame with just one clock, letting x_B and t_B be the length and time measurements on this system. Thus, system B has one clock located in $x_B = 0$.

Then, identical time measurements, (*i.e.* clock readings at same location and same time), will be related by the formula:

$$t_B = t_A\sqrt{1 - (\frac{v}{c})^2} \tag{6}$$

Given this observational set-up, we stress that observers on both reference frames agree on this. Thus, I find it misleading in this situation to use the phrase 'as seen'[3] (by the observer on the other system); an expression used by several authors. Time readings are objective, and all observers on the location 'see' the same thing. The point is that observers (observational equipment) at different locations will disagree.

Now the result could be summarized as follows: An observer at rest at a specific location, seeing clocks passing by, will see these clocks going slower than his own clock. So, this confirms – in a rather narrow sense – the standard phrase: 'moving clock goes slower'. However, in such a narrow sense, I find the statement of limited interest. It hardly follows that this choice of observational set-up - which may be interchanged at random - will cause a 'true time dilation'; *i.e.* affect the relative *time* (including aging) on the two systems.

In the next chapter we explore further implications of the Lorentz transformation; providing results within the framework suggested by the above discussion.

4 Use of light rays and a generalization

Now we consider a third observational principle and a generalization..

4.1 Unidirectional flashes. A third observational principle based on light rays

In Chapter 3 we compared time readings when a clock at a fixed location on one of the reference frames is compared with clock measurements on the other reference frame passing by. Now consider the result obtained by inser ting $x = ct$, and thus also $x' = ct'$ in (3). Thus, at time $t = t' = 0$ there is emitted a flash of light at location $x = 0$ (or $x' = 0'$) along the positive x-axis, and we compare the times at the location of this flash at a later time, t on K. The Lorentz transformation directly gives the following relation between the clock readings at the identical locations $x = ct$, and $x' = ct'$:

$$t' = \frac{1-v/c}{\sqrt{1-(\frac{v}{c})^2}}t = \frac{\sqrt{1-v/c}}{\sqrt{1+v/c}}t; \quad (x = ct) \tag{7}$$

[2] Note that the result, (5), (independent of x) is valid at any position, x, provided t and t' are interpreted as time differences.

[3] Here we talk about clock readings. Length contraction, on the other side, implies that observers on different systems will 'see' (*i.e.* observe) different lengths of the same object, *cf.* Annex B.

So here we apply (at least) two clocks on both system: one at $x = 0$ and one at $x = ct$ on K; and similarly, one at $x' = 0'$ and one at $x' = ct'$ on K'. We may refer to this approach for time measurement/comparison as observational principle C. So it utilizes the constancy of speed of light to give the relation between time, t' on K' and time, t on K at a specific position along the positive x-axis, (*i.e.* at locations, $x = ct$).

So eq. (7) is valid when the light ray is emitted in the positive direction ($x > 0$); *i.e.* c having the same direction as the velocity v, as seen from K. In the negative direction, (choosing $x = - ct$) we similarly get

$$t' = \frac{1+v/c}{\sqrt{1-(\frac{v}{c})^2}}t = \frac{\sqrt{1+v/c}}{\sqrt{1-v/c}}t; \quad (x = -ct) \tag{8}$$

We refer to this result as being obtained using observational principle C*. So the difference between the observational principles C and C* is not so much the different use of clocks, but rather the direction of the light flash. Also the results (7) and (8) are of course well-known, *e.g.* see [3], [4]. Again these relations demonstrate that we may observe both $t' < t$, and $t' > t$; now depending on whether we observe a light flash having the same or the opposite direction of the relative movement, v.

Note that the time measurements of (7)-(8) are based on unidirectional flashes of light. In Chapter 6 below we will also consider taking averages of these time measurements; thus, applying bidirectional rays. However, we first present a generalization of observational principles, A, B and C.

4.2 The generalized observational principle

We have seen that different observational principles, A, B and C give different relations between t and t'. At an instant when the time all over K is measured to equal t, one will at different locations on K observe different times, t' on K'.

Now consider a generalization of these principles A, B and C. Taking the perspective of K, we may at time t choose an 'observational position' equal to $x = wt$, (for an arbitrary w). By inserting $x = wt$ in (3) we directly get that time on K' at this position equals:

$$t' = \frac{1-\frac{vw}{c^2}}{\sqrt{1-(\frac{v}{c})^2}}t \tag{9}$$

Further, we specify a velocity u so that position $x' = ut'$, at that time corresponds to (has the same location as) $x = wt$. Now, also inserting $x' = ut'$ in (2), we will after some manipulations obtain[4]

$$t' = \frac{\sqrt{1-(\frac{v}{c})^2}}{1+\frac{uv}{c^2}}t \tag{10}$$

These are seen as two fundamental relations regarding time dilation. As a by-product we can use (9) and (10) to obtain the well-known result regarding the speed of the 'moving object relative to K:

$$w = \frac{x}{t} = \frac{u+v}{1+\frac{uv}{c^2}} \tag{11}$$

Further, we see that the results for the observational principles A, B and C directly follows from (9) by choosing $w = 0$, $w = v$ and $w = c$, respectively. And they follow from (10) by choosing $u = -v$, $u = 0$ and $u = c$, respectively. Thus, equations (9) and (10) provide the relation between clock readings at identical locations if the location at K is given by $x = wt$, and the location at K' is given by $x' = ut'$. Now *eq.* (9) for instance tells that, given time t on K, the time t' on K' is a linear, decreasing function in w.

In addition, now having these general expressions, (9), (10), we could ask which value of u and w would results in $t = t'$. It is easily derived that this equality is obtained by choosing

[4] Or we could apply the 'transformed' Lorentz transformation (taking the perspective of K'); replacing v by $-v$.

$$w = -u = w_0 = \frac{c^2}{v}\left(1 - \sqrt{1 - \left(\frac{v}{c}\right)^2}\right) = \frac{v}{1+\sqrt{1-(\frac{v}{c})^2}} \qquad (12)$$

This means that if we consistently consider the positions where simultaneously $x = w_0t$ and $x' = -w_0t'$, then no time dilation will be observed in these positions. Thus, at such locations we have $t' = t$ and $x' = -x$, providing a nice symmetry. Note that we also obtain (12) directly from the Lorentz transformation, by requiring $t = t'$.

We refer to this choice, $x = w_0t$, as observational principle D. Now observe that when we choose this observational principle - which is symmetric with respect to the two reference frames - then absolutely everything is symmetric, and consequently we get $t' = t$. Thus, it could seem that departure from this equality of time is caused by applying a non-symmetric observational principle!

Actually, it now also seems possible to specify some kind of simultaneity for events occurring on moving reference frames. At location $x = w_0t$ on K and $x' = -w_0t'$ on K' there is a point of simultaneity across the systems. The two clocks at this position show the same time, $t = t'$. Further, all clocks on K are synchronized to show the same time, t (on K), and similarly all clocks on K' show the time t' (on K'). So the symmetry of this situation might suggest $t = t'$ as a candidate for defining one kind of simultaneity.

5 Summing up on time dilation and observational principles

We now sum up the main findings of the previous chapters regarding time dilation. A number of equations have been obtained, all expressing the time dilation between the two systems K and K'. This could seem confusing, but the different results have been related to various observational principles. Table 1 summarizes these principles, providing reference to the corresponding equation, relating t' and t. In particular it points out which expression for x (and x') we insert in the Lorentz transformation in order to obtain this specific result for t'.

Table 1 Review of observational principles; (time on *K* equals *t*).

Principle	Expression for t'	Description of time reading, t', at a specific location on K
A	*Eq.* (4)	Obtained at location $x=vt$; (following fixed point, $x' = 0$ on K').
B	*Eq.* (5)	Obtained at fixed location, $x = 0$.
C	*Eq.* (7)	Obtained at location $x = ct$, (and $x' = ct'$); using light flash.
C*	*Eq.* (8)	Obtained at location $x = - ct$, (and $x' = - ct'$); using light flash.
D	$t' = t$	Obtained at location $x = w_0t$ (and $x' = -w_0t'$); w_0 given by *eq.* (12).

The various results of Table 1, are also illustrated in Figure 1; giving the observed time, t' at various positions on K, when time on K equals t. The figure demonstrates that the observational principles A, B, C, C* and D are just special cases of a general 'principle', specifying the time t' on K' observed from position $x = wt$ at time t on K.

In the figure the values of w are ranging from $-c$ to c. The principles C and C*, based on light rays, providing the range of t' values to equal $(\frac{\sqrt{1-v/c}}{\sqrt{1+v/c}}t, \frac{\sqrt{1+v/c}}{\sqrt{1-v/c}}t)$. Observational principles A and B represent a considerable narrowing of the range of t' values: $(t\sqrt{1-(\frac{v}{c})^2}, t/\sqrt{1-\left(\frac{v}{c}\right)^2})$. We note that principle A, which provides the lower limit of this last interval, represents the 'standard' time dilation formula. However, we mention that the geometric mean of the end points of both these two intervals equals t. The result, $t' = t$, is also obtained by principle D.

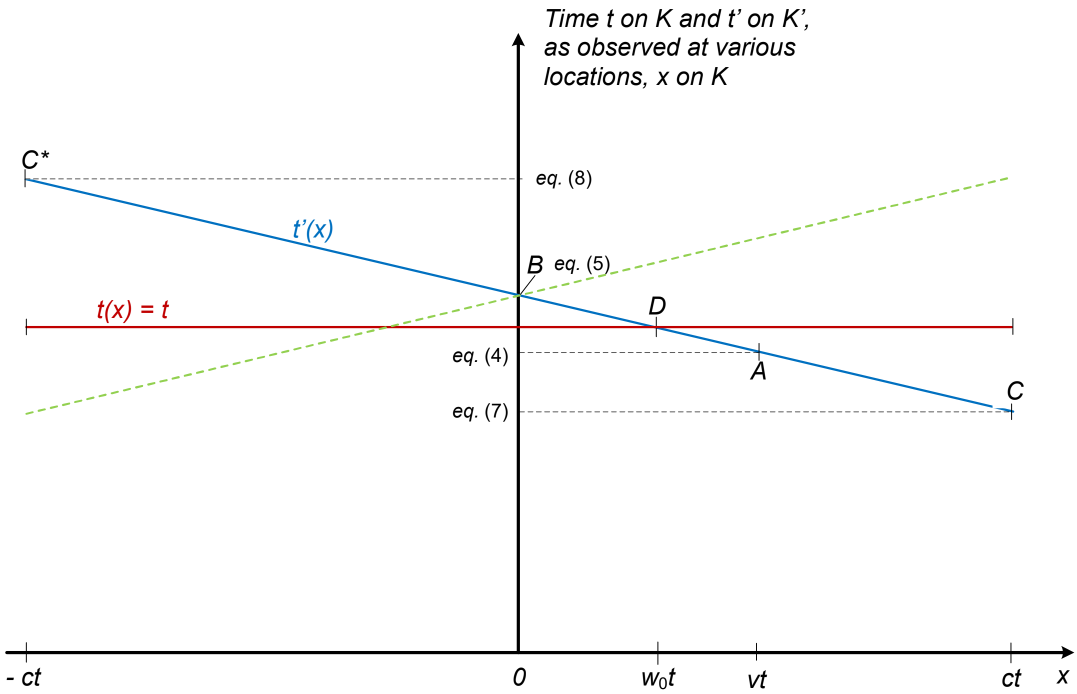

Figure 1 Positional time: Time, $t' = t'(x)$, (blue) on K' at different locations, $x = wt$, on K, when the time on K equals t, (red); *i.e.* perspective of K. Various observational principles specified along the line, $t'(x)$. The dotted green line gives values of, $t' = t'(x)$, when v is replaced by $-v$.

Figure 1 is actually just a presentation of eq. (3), giving t' as a linear, decreasing function of x, when t is fixed. It presents a total picture of the relation between t and t', as obtained by the Lorentz transformation. Thus, time, t' is uniquely given by the position, x on K where it is observed. Each t'-value is the one directly being observed at the specified position, x on K, and we might thus refer to *positional* (*i.e.* location specific) time.

As a further illustration we have in Figure 1 also included a dotted green line, representing time, t' - as measured from position x on K - on a system moving at speed, $-v$. We see that the blue and green lines corresponding to v and $-v$, respectively, form a 'bow tie' with a knot in observational principle B. This point is 'shifted upwards', relative to the red line, t, with a factor $1/\sqrt{1-\left(\frac{v}{c}\right)^2} > 1$. So the blue line (represented by the point B) moves faster away from $t = 0$ than does the red line. Actually, if we from this figure should suggest an 'overall time dilation factor' for the 'moving system', then this factor could seem quite appropriate; which would rather tell that on average 'time on moving system goes faster'.

In summary the relations presented in Figure 1 seem rather fundamental for the interpretation of relative time. Being a direct consequence of the Lorentz transformation, it is of course well-known; being pointed out *e.g.* in Feynmann, [2], *p* 175. Further, Mermin [4] gives a thorough discussion on this relation, focusing on how the exact expression for $t_2' - t_1'$ depends on $x_2 - x_1$. In spite of this, also these authors seem to accept the common formulation that the time dilation is basically given by equation (4); representing a rather narrow description of the phenomenon of time dilation.

An alternative illustration of the various relations between t and t' is given in Figure 2. The lines of this figure present the relation between t and t', given specific observational principles, (specific locations for time comparisons).

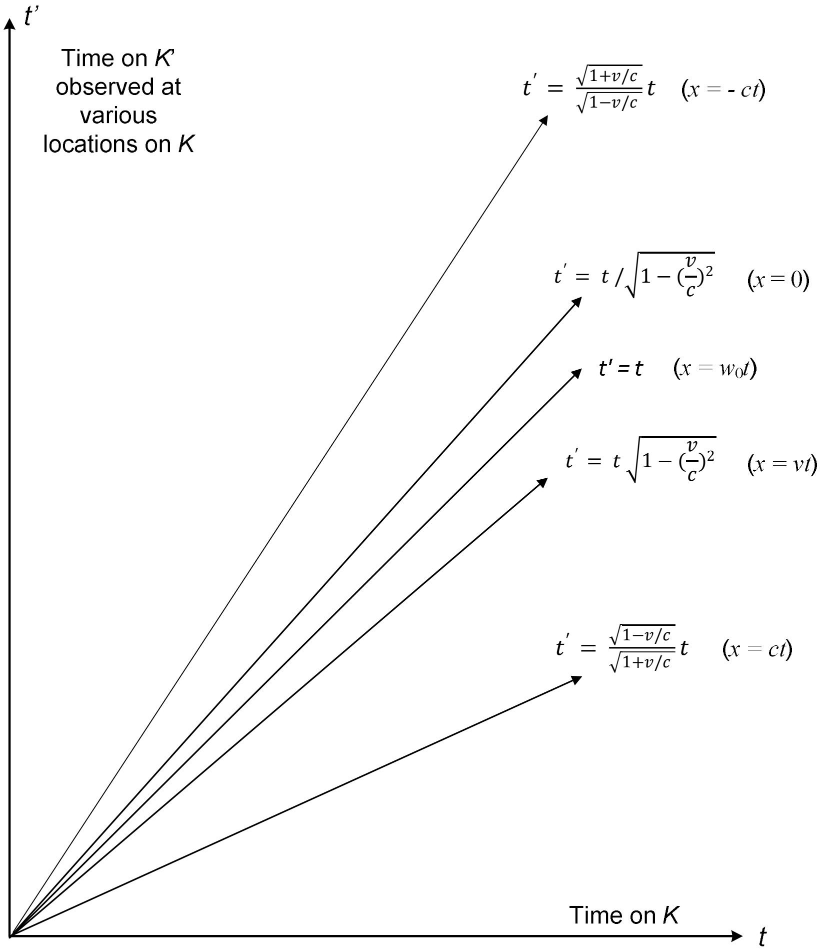

Figure 2 Values for time *t′* on *K′* as a function of time *t* on *K*. Various observational principles, (*i.e.* observational positions).

6 Bidirectional rays.

For completeness we now return to the investigation of time determined by light rays. Section 4.1 presented the approach of light flashes emitted along the x-axis. It was, however, observed that the result depended on the direction of the flashes. So when comparing time measurements on K and K' it is common to consider a 'round trip'; *i.e.* a flash going from one location, then being reflected, and finally returning to the 'same' location. Such a light flash can also be seen to represent an atomic clock, the time of one round trip representing the time unit.

So at time $t = t' = 0$ a ray is emitted from the origin, $x = 0$, $x' = 0'$. Further, at time t_1 on K and t_1' on K', the ray is reflected back in the opposite direction at the location $x = ct_1$ (corresponding to $x' = ct_1'$). Finally they return (simultaneously) to the location of emission at times t_2 and t_2', respectively. Obviously the point of return must be specified, as location $x = 0$ no longer coincides with $x' = 0'$.

If the flash starts in the positive direction, *eq*. (7) is valid until the reflection occurs, thereafter eq. (8) applies. Thus, we have

$$t_1' = \frac{\sqrt{1-v/c}}{\sqrt{1+v/c}}\; t_1 = \frac{1-v/c}{\sqrt{1-(\frac{v}{c})^2}}\,t_1 \tag{13a}$$

$$t_2' - t_1' = \frac{\sqrt{1+v/c}}{\sqrt{1-v/c}}\;(t_2 - t_1) = \frac{1+v/c}{\sqrt{1-(\frac{v}{c})^2}}\,(t_2 - t_1) \tag{13b}$$

So, to decide on the time dilation it is common to compare the times of return (total time of a round trip), that is t_2' and t_2. But at this point, we have to decide on the 'point of return'; whether it is 0 (located on *K*) or 0' (located on *K'*). At the time of emission these points where located at the same place, but by the return they have moved relative to each other. If we choose $x = 0$ as the point of return, the distance in negative direction becomes longer than if we chose $x' = 0'$, and so the weighting of equations (7) and (8); *i.e.* (13a) and (13b) will differ.

First, if we follow the ray on *K'*, (*i.e.* return to 0'). Then the light will on *K'* passes the distance $x' = ct_1'$ in both directions, and obviously $t_2' - t_1' = t_1'$. Then equations (13a), (13b) directly give

$$t_2 = (t_2 - t_1) + t_1 = \left(\frac{\sqrt{1-v/c}}{\sqrt{1+v/c}} + \frac{\sqrt{1+v/c}}{\sqrt{1-v/c}}\right) t_1'$$

By also using $t_2' = 2t_1'$ it directly follows that

$$t_2 = \frac{1}{\sqrt{1-(\frac{v}{c})^2}} t_2' \qquad (14)$$

This means that if we consider times of return only, then the relation between time measured on *K'*, (t') and time measured on *K*, (t) is given by, (14), being equivalent to (4). Now using subscript AvL for t' to indicate Average Low, we will in this case get

$$t'_{AvL} = t\sqrt{1-(\frac{v}{c})^2} \qquad (15)$$

Secondly, if we choose to follow the ray on *K*, that is return to 0, we have $t_2 - t_1 = t_1$, and (13a) and (13b) directly give

$$t_2' = \frac{1}{\sqrt{1-(\frac{v}{c})^2}} t_2 \qquad (16)$$

So rather than (15) and (4) we now get the 'opposite' result, (16), being identical to (5). Applying subscript AvH for t' to indicate Average High we can write this as:

$$t'_{AvH} = t/\sqrt{1-\left(\frac{v}{c}\right)^2} \qquad (17)$$

In conclusion, we have that the two formulas (4) and (5) of Chapter 3 can also be interpreted as the times for a round trip, *i.e.* time measurement when we restrict to record the time to the instants of the return of 'round trip' flashes. Now the subsequent round trips are performed relative to the reference frame of the return; the flash on *K* returning to $x = 0$, and the flash on *K'* returning to $x' = 0'$.

Figure 3 provides an illustration; similar to Figure 2. First, it gives the relation between t and various times, t', when using unidirectional light rays, *i.e.* measuring time at position $x = ct$, (principle C, *eq.* (13a)), and at position $x = -ct$, (principle C*, *eq.* (13b)); see the solid black lines.

Next, the two dotted colored lines indicate examples of time for bidirectional (reflected) light flashes. These are alternatively parallel with the C and C* line, and are referred to as oscillating times. They correspond to the observational position moving back and forth along the *x*-axis at speed, *c*; returning to and being reflected again either at x=0 on *K* or at x'=0' on *K'*.

Finally, by considering only the points of return, we arrive at the time measurements $t'_{AvL} = t\sqrt{1-(\frac{v}{c})^2}$ and $t'_{AvH} = t/\sqrt{1-\left(\frac{v}{c}\right)^2}$, respectively; see the solid colored lines. Which of these two result are obtained will depend on whether we follow a light ray returning to *K'*, or a light ray returning to *K*. However, this result just corresponds to specifying a specific type of clocks to be used in the time comparisons. These clocks consist of light rays performing a roundtrips, and the two results correspond to having a clock at fixed location; either at K or at K'. Thus, it is no surprise that we get the results of observational principle A, (*eq*. (4)), and principle B, (*eq*. (5)), respectively. In this sense the above results on t'_{AvL} and t'_{AvH} are special cases of the general result given in Figure 1, and provide no essential new information.

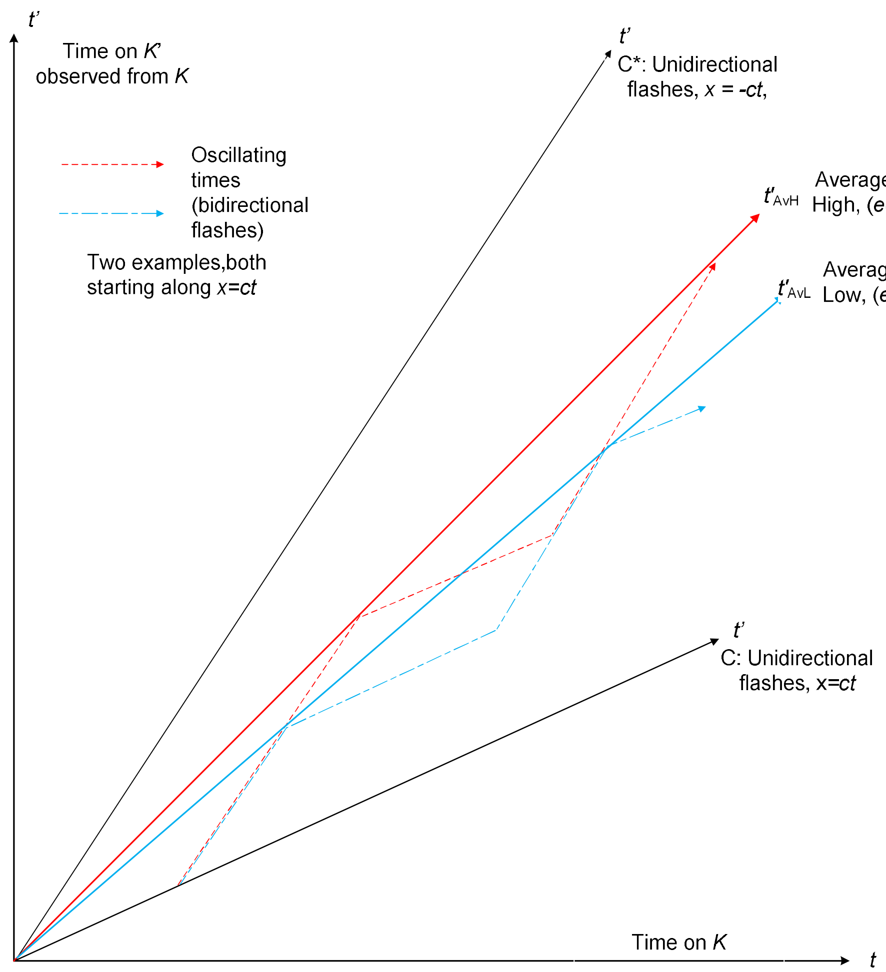

Figure 3 Illustration of time readings, utilizing light rays for time comparison.

However, if we do not restrict to define time (clock reading) by the instants of flash return, we could in principle use this approach to define an *oscillating time*. In the figure we have exaggerated the length of the flashes between reflections. Usually the round trips are very short, and the oscillating times are oscillating very closely around t'_{AvL}, (or t'_{AvH}). Thus, the oscillating time depends on the length of the reflection distance, L. When $L \rightarrow 0$, we get the average time(s), (15), (17), and when $L \rightarrow \pm\infty$ we get the times of principles C and C*, respectively.

These oscillating lines of Figure 3 present a generalization not included in Figure 2. Figure 2 presents only those cases where the observational position is given by $x = wt$, and not the situations where w is changing with time, as in the oscillating case illustrated in Figure 3.

As a final comment on this, we mention another possible use of bidirectional rays. We could have an observer located at $x = 0$, also having two observers (equipment) located along the x-axis; one at location, x, and another at location $-x$. Emitting a ray in both directions from $x = 0$, the equipment at both positions observe the clock reading on K' when the ray arrives (after time $t = x/c$). These clock readings are given by (7) and (8), respectively, which give the arithmetic mean, t'_{AvH}, and the geometric mean, t, neither being equal to the standard time dilation expression; again demonstrating that it may be hard to point to one generic time dilation factor.

7 Conclusions

This paper investigates the phenomenon of time dilation between two reference frames moving relative to each other at speed *v*; assuming that the conditions of the special theory of relativity (STR) holds. The Lorentz transformation is utilized to present a number of – essentially well-known – results within a structured framework.

An overall conclusion is that there is a multitude of time dilation formulas to be obtained by a systematic use of the Lorentz transformation. To elaborate on this we introduce the concept of observational principle for comparing the time t' on a clock at *K',* corresponding to time *t* on a clock at *K;* both having the same location. We always start with location $x = 0$ on *K* being identical to location $x' = 0'$ on K' at time $t = t' = 0$, (point of initiation), and then the observational principle decides which clocks are compared when the time on *K* equals *t*.

So the choice of clocks used for the time comparisons is crucial. Relying on just one observational principle may not give a trustworthy result, and I argue that one should look at the total picture, taking all information into account; *i.e.* the overall solutions, as given by the Lorentz transformation. The framework suggested here has the following characteristics:

- There is a complete *symmetry* between the two reference frames.
- We do not utilize any definition of *simultaneity* across systems. The approach restricts to explore direct comparisons of clocks being at the same location at the same time.
- All clocks on the same reference frame are synchronized; and in addition we have the common 'point of initiation'.
- We do not use the expression '*as seen'* (from the other reference frame). Observers (observational equipment) on both reference frames agree on all the time readings (clock comparisons); as they are carried out 'on location', *i.e.* comparing the same clocks.
- We specify the applied *observational principle*, *i.e.* specify the location of the clocks that are used for time comparisons.
- We choose the *perspective* of one of the reference frames (the 'primary'). To take the perspective of a specific reference system, implies that time on this system is given as $t(x) \equiv t$, all *x*.
- We focus on how observed time, t' (on the 'other' system) depends on the position, *x* on the 'primary' reference frame; leading to the concept of *positional time*. This provides a framework for all the observational principles.

Thus, we stress the fact that at a given time, *t* on *K*, the time t' observed on K' will depend on the position, *x* on *K*. The general framework can be specified as follows: At any time *t* on *K*, we chose to observe/compare the time reading of the clocks (on *K* and K') which at this instant are located at $x = wt$.

There are some interesting special cases. As the first observational principle we may choose $w = v$, meaning that we consequently perform the clock readings/comparisons at position $x = vt$. Thus, on K' we only apply the clock located at $x' = 0'$ (as it passes along the *x*-axis). This gives the standard result $t' = t\sqrt{1-\left(\frac{v}{c}\right)^2}$; corresponding to the expression 'moving clock goes slower'; then, referring to the clock located at $x' = 0'$ on *K',* (which is moving relative to *K*).

Secondly, we may choose $w = 0$, meaning that we consequently apply the clock on *K* located at $x = 0$, and compare this with the passing clock on *K'.* This observational principle gives $t' = t\ /\sqrt{1-(\frac{v}{c})^2}$, or if we prefer, $t = t'\sqrt{1-\left(\frac{v}{c}\right)^2}$, which could perhaps be described by the opposite result: the stationary clock (on *K*) goes slower! So 'moving' or 'stationary' may not be the fundamental characteristics here. The decisive factor is which of the two systems applies a single clock for the time comparison, and this single clock is of course moving with respect to one system and stationary with respect to the other.

Further, we have the interesting result that if we choose the value of w so that the position $x = wt$ at any time t is located at the midpoint between $x = 0$ and $x' = 0'$, then we will at this location observe $t' = t$. This choice, of course, represents an observational principle being symmetric with respect to the two reference frames. So when we here assume everything to be symmetric, and in addition let the observational principle be symmetric, then (of course) also the result is symmetric, *i.e.* $t = t'$. This should imply that when we observe $t \neq t'$, it is caused by the asymmetry of the chosen observational principle; everything else being identical.

From this discussion I would actually conclude that under the strict symmetry conditions there is no 'true' time dilation, *e.g.* in the sense that it would cause different ageing on the two systems. The various time dilations observed at different locations, could rather be seen as an 'observational deficiency'.

This suggests that when we investigate occurrences of time dilation, it is essential to identify any asymmetry between the two reference systems, and clarify whether this could possibly cause a 'true' time dilation to occur. In particular, it should be a most interesting task to identify the precise conditions – *e.g* departures from symmetry - which would cause time dilation to represent a physical reality. To me it is not clear that this task is currently achieved.

An additional conclusion might be that a specific observer moving relative to the reference frame where the event takes place, is a rather unreliable observer regarding time. The different observational principles will give different results. So one should be careful to let such an observer define the phenomenon. Even if a phenomenon appears in a particular way for this observer, it does not need to the 'correct' answer; one should rather realize the imperfection of a single observer to comprehend in full depth a phenomenon occurring on another reference frame. This might further have some implications for the interpretation of well-known example as 'the travelling twin' and µ-meson, (*e.g.* [4]), *cf* Annex C.

In summary, the present work utilizes the Lorentz transformation to present a narrative on time dilation, which seems to deviate somewhat from the prevailing views of the current literature.

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Annex A. A derivation of the Lorentz transformation

Various derivations of the Lorentz transformation exist. The following – valid for one space co-ordinate (x-axis) - is based on three assumptions, (*cf.* the list of assumptions given in Section 2.2):

1. *Speed of light* will be measured to be constant in both directions and equal to c, independent both of the speed of the observer and speed of the light source.
2. *Length contraction*. There is observed a length contraction, k_x along the x- axis of 'the other' reference frame. When we from a specific location on K observe the passing of a measure stick of length, x', (as measured on K'), then the time observed between the passing of its two endpoints, will correspond to the stick (apparently) having a length $k_x x'$; *cf.* Annex B below.
3. There is a complete *symmetry* between the two co-ordinate systems, K and K', and we consider the systems to be identical in all respects.

We consider the 'standard situation': The reference frame K' moves relative to K with the velocity, v. Initially at time $t = t' = 0$, the origins $x = 0$ on K and $x' = 0'$ on K' have the same location. At any later instant, any location, x on K is positioned at the same location as x' on K', and at this position time is measured to equal t on K and t' on K'.

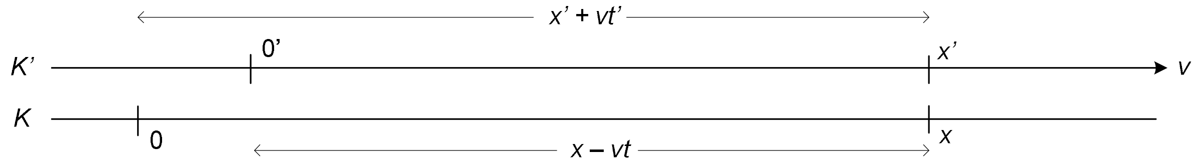

Figure 4 Identical positions, x and x' at time t on K and time t' on K', (measured at this location).

Observed on K the origin 0' has at time t moved a distance vt, (all clocks on K are synchronized). Thus $x - vt$ corresponds to the length x' on K', (Figure 4). Utilizing the assumption of contraction, we will from K observe the distance, x' to have length $k_x \cdot x'$. Thus, as observed from K:

$$x - vt = k_x \cdot x' \tag{A1}$$

In exactly the same way we have (*cf.* symmetry) that observed from K':

$$x' + vt' = k_x \cdot x \tag{A2}$$

Next, we consider the case that a flash of light is emitted from the origin at time $t = t' = 0$. We now utilize the constancy of speed of light; *i.e.* if $x = ct$ then also $x' = ct'$, and vice versa. That is

$$(x = ct) <=> (x' = ct') \tag{A3}$$

So as a special case we insert $x = ct$ and $x' = ct'$ in (A1) and (A2) and get, respectively

$$t' = \frac{1-\frac{v}{c}}{k_x} t \tag{A4a}$$

$$t' = \frac{k_x}{1+\frac{v}{c}} t \tag{A4b}$$

By combining these two expressions we determine the length contraction:

$$k_x = \sqrt{1 - (\frac{v}{c})^2} \tag{A5}$$

Thus, the requirement $x = ct$ iff $x' = ct'$ is sufficient to determine k_x. Next inserting this result, (A5) into (A1) and (A2), we easily obtain the Lorentz transformation

$$x' = \frac{x - vt}{\sqrt{1-(\frac{v}{c})^2}} \tag{A6}$$

$$t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1-(\frac{v}{c})^2}} \tag{A7}$$

Thus, the Lorentz transformation, (A6) - (A7) in a simple way follows from the three basic relations (A1) - (A3). Observe that the relations (A4) apply for the special case, $x = ct$, $x' = ct'$; while (A6), (A7) are general relations of simultaneous time readings, t and t' performed at identical locations x and x'.

The expressions (A6), (A7) are also valid for negative x and x'. They are, however, derived under the assumptions that K' moves in the direction of the positive x-axis, as seen from K. By changing the direction of the relative movement, one should either let v be negative (replace v by $-v$), or, alternatively, interchange x and x', and t and t' in the formulas.

Annex B. Length contraction

We include a short discussion on length contraction. The common approach is to place a rod of length x_0 along the x'-axis of K'. We locate one end in the origin, O', and the other in a point, C' along the negative x'-axis. At time $t = t' = 0$, we have that the location of the origin, O', ($x' = 0'$) on K' coincides with the origin, O, ($x = 0$) on K. The other end of the rod, located in C' with coordinate $x' = - x_0$, corresponds to a point C on K. According to *eq.* (2) this coordinate equals $x = x'\sqrt{1-(\frac{v}{c})^2} = - x_0\sqrt{1-(\frac{v}{c})^2}$. This directly gives the distance x_0 as measured from K. We simply observe that at time $t = 0$ the distance OC equals $x = - x_0\sqrt{1-(\frac{v}{c})^2}$. So since OC at this instant 'corresponds to' $O'C'$, and $O'C'$ has length x_0, it will of course follow that the length contraction is given by *eq.* (1).

So when the length of a rod on 'the other' reference frame (K') is measured by performing simultaneous position measurements within the reference frame, K, then we will observe the specified length contraction. We could, however, ask whether there exist observational principles, possibly giving other results. Above the observer utilized two observational positions (O and C), in this respect applying an approach similar to observational principle A of Chapter 3. Alternatively, we could apply just one observational position (say in $x = 0$), and observe the rod as it passes along. This would correspond to observational principle B. We note that in order to apply this principle for lengths it is required that we already has measured the velocity, v between the two reference frames. (Both observers agree on this velocity, obtained by measuring the time it takes for a single point on the other system to pass a certain distance on his own reference frame.)

Now using this second principle, we will at time $t = 0$ again let O' be positioned at O; thus, $x = 0$, $x' = 0'$, also giving $t' = 0$. So we have the same starting position as above. Now, however, time t is given as the time when C' is positioned in O. So this t is defined by having $x = 0$ and $x' = - x_0$, resulting in $t = t'\sqrt{1-(\frac{v}{c})^2}$ and $x' = - vt'$. Utilizing both these results, the length of $O'C'$ observed from K will equal $x = vt = vt'\sqrt{1-(\frac{v}{c})^2} = -x'\sqrt{1-(\frac{v}{c})^2}$. Thus, we have the same result as obtained by the first observational principle.

So the Lorentz transformation gives that a rod, located parallel to the movement of the reference frames, will be observed to have a length contraction, irrespective of the observational principle applied.

Annex C. Examples of the occurrence of time dilation

When the assumptions of the STR are valid - *e.g.* no gravitational forces present - and we also assume that there is a complete symmetry between the two reference frames - one can hardly argue that time dilation represents a physical reality. It may be more problematic to specify situations of asymmetry allowing a 'true' time dilation to occur under the conditions of the STR. Thus, we now consider the standard examples of the 'travelling twin' and the μ-meson. Tentatively we argue that these do not necessarily support the idea that time dilation represents a physical reality within the framework of the STR; thus maintaining the views advocated in the present work.

C.1 The example of the μ-mesons

In particle physics we may obtain speeds close to c, and so this field is relevant for providing experimental evidence on the existence of an 'actual' time dilation. The example of the μ-meson is frequently referred, *e.g.* see [4], where we have extracted the following text. The μ-mesons are produced by cosmic rays in the upper atmosphere. When 'at rest' they have a lifetime of about 2 microseconds, so if their internal clocks ran at a rate independent of their speed, even if they travelled at the speed of light about half of them would be gone after they had travelled 2.000 feet. Yet about half of the μ-mesons produced in the upper atmosphere (about 100.000 feet up) manage to make it all the way down to the ground. This is because they travel at speed so close to the speed of light that the slowing down factor equals 1/50, and they can survive for 50 times as long as they can when being stationary.

The phenomenon is further explained as (again citing [4]): "The atomic particles can go much further because their internal clocks that govern when they decay are running much more slowly in the frame in which they rush along at speed close to c. This is a real effect, and it plays a crucial role in the operation of such particles accelerators."

I assume the experimental facts as such are unquestionable, and that gravitational forces do not essentially affect the phenomenon. Then the situation seems to be easily described in terms of the Lorentz transformation. First, consider just the reference frame of the μ-meson. The lifetime is approximately $t' = 2\cdot10^{-6}$ sec. So if it is moving at a speed close to c within this frame, then it will on the average go a distance, $x' = c\,t' = 10^9$ feet/sec $\cdot\, 2\cdot10^{-6}$ sec = 2000 feet during a time t'.

If, however, the reference frame of the μ-meson is moving relative to earth at a speed, v, close to c. then we apply the Lorentz transformation (6), (7), with the above value t' (and some x'). This means that we assume that the μ-meson is created at time 0 at the origin of this reference frame, and that we determine the x and t coordinate on earth when the μ-meson at time t' has traversed the distance x'. Since it is given that $\sqrt{1-\left(\frac{v}{c}\right)^2} \approx 1/50$, it follows that $v \approx 0{,}9998c \approx c$. Inserting these results in eq. (6) it follows that the result is virtually independent of x', and that $x \approx vt \approx ct$. Since it is given (observed) that $x = 10^5$ feet, we directly obtain $t \approx x/c \approx 10^{-4}$ sec. So in total we have $t = 50\,t'$, apparently confirming that the clock of the moving μ-meson goes slower by a factor 50.

In this argument we have applied observational principle A, (or rather principle C as an approximation, since $v \approx c$). This means that we imagine a clock following the μ-meson; this clock showing time t' when it has completed the (rather arbitrary) distance x'. Comparing this with two synchronized clocks on the reference frame of the earth, actually shows that in the earth frame a time, $t = 50t'$ has elapsed.

Of course, this seems rather natural to apply observational principle A in the present case, as we do not have any marks on the 'moving reference system', being required in principle B. Thus, observational principle B may seems unfeasible, and so it is perhaps really correct to say here that 'the moving clock goes slower'? However, I would add a few comments to this argument.

First, we could in principle also imagine a reference frame following the μ-meson, and from a fixed location on earth observe the time elapsing between the passings of marks on this frame, (*i.e.*

observational principle B). As we know, this will give a completely different relation between *t* and *t'*. One could object that such observations could hardly be carried out in practice. But the actual point is that if we really accepts the Lorentz transformation to give a 'true' description of the phenomenon, then this is also a valid way to describe it, and practical problems to actually achieve the observations are less relevant.

So the observations are (of course) in agreement the Lorentz transformation. But do they also confirm that the 'inner clock goes faster'? I consider this somehow to be a mystification of the observed phenomenon; hardly with any specific physical meaning. A major fact is that when the observed phenomenon ('lifetime' of μ-meson) is at rest with respect to the clock/observer, then the duration of this phenomenon on the average equals 2 microseconds. If anything, it is this duration that should be considered to express the 'inner clock' of the μ-meson. As the particle is essentially not affected by forces, it should 'see' itself as being at rest. And any 'inner clock' should hardly be affected by some observer passing by and making some observations, (apparently without affecting it).

When the relative speed between the two is high, (e.g. speed close to c), then the 'outside' observer will measure the distance that the particle goes during its lifetime to become correspondingly longer. However, such 'outside' observers are, as we have seen, quite 'unreliable', potentially obtaining a multitude of results. After all – utilizing the insight provided by the Lorentz equations – he will know that observers passing relative to the particle at various high speeds can observe essentially 'anything' regarding the particles lifetime. So we should hardly put too much weight into the fact that the chosen measurement principle of time duration found most appropriate 'from outside' gives a result which differs considerably from that of the 'inner clock' of the particle, (*i.e.* 2 microseconds). So we could rather suggest that the 'outside' observer should take this into account, and adjust his own 'direct measurement' by accounting for the relative speed between himself and the particle.

Thus, the question remains, whether time dilation should be considered a true physical phenomenon or an observational phenomenon. I see no need to refer to an 'inner clock going slower (or faster), and I do not find the experiment to contradict the view that the phenomenon itself is unaffected by any relative movement of constant velocity. What we observe might be seen is an observational peculiarity, rather than an inner property of the μ-meson being 'distorted'? And as the Lorentz transformation is fundamental, perhaps an observer being at rest with respect to the phenomenon should be given priority to a moving observer.

C.2 The travelling twin paradox

The so-called twin paradox provides an interesting example for discussing the interpretation of relative time, *e.g.* see [4]. As stated here this paradox illustrates that two identical clocks, initially in the same place and reading the same, can end up with different readings if they move apart from each other and then back together.

Reference [4] gives the following description, (Chapter 10): "If one twin goes to a star 3 light years away in a super rocket that travels at 3/5 the speed of light, the journeys out and back each takes 5 years in the frame of the earth. But since the slowing-down factor is $\sqrt{1-(\frac{3}{5})^2} = \frac{4}{5}$ (referring to the time dilation of the STR), the twin on the rocket will age only 4 years on the outward journey, and another 4 years on the return journey. When she gets back home, she will be 2 years younger than her stay-at-home sister, who has aged the full 10 years." Further, in the same Chapter: "In the frame of reference of the first clock [= clock on the earth] […], it is clear […] that when the trip is over, the second clock running slowly for the entire journey, will have advanced only by 8 (4 on the outward journey and 4 on the inward journey), while the first clock has advanced by 10."

So the authors in general consider the acceleration/deceleration period to be the actual cause of the slowing-down of the clock, (so rather than state ' moving clock goes slower', we should here say ' turning clock goes slower'. However, it definitely seems to be the claim that the slowing-down essentially occurs during the periods of the journey with a constant speed; (*i.e.* under the conditions of the STR)! The argument infers that the longer the period of constant relative speed, the more years are gained for the travelling twin. Thus, the gain is (essentially) not obtained during the acceleration period, even if this represents the root cause.

Now taking a closer look at this, we will in the present context focus on the periods of constant velocity. In order to make the argument clear, I suggest one should imagine that stationary space stations in advance are located at the borders of the distance of constant speed for the travelling twin. Both these space stations have a clock, synchronized with the clock on the earth. Thus, all these can be considered part of one and the same reference frame (the earth).

Now in the context of the STR, the really interesting topic is the clock comparisons at the start and end of this distance, both when travelling from the earth and on the return. It is easily verified, that as seen from earth, we both ways then apply observational principle A. That is, $t' = t\sqrt{1-(\frac{v}{c})^2}$, where t' is the time reading on the 'travelling clock', and t is the time on the clock stationary on the earth; or rather on all clocks belonging to this reference frame. Thus, we apply the Lorentz transformation with $x' = 0$. Now inserting $v = 0{,}6c$, this gives $t' = 0{,}8t$. So if we can ignore the acceleration/deceleration periods, we must simply agree with [4] when it states: "… the first clock reads 10 and the second reads only 8 when they are reunited ….".

So the argumentation might stop here, and we could simply accept that the 'moving clock goes slower'. However, we note that the clock on the rocket is involved in all comparisons, while the reference frame of the earth both ways applies two clocks (according to the above description). So before we definitely conclude regarding the importance of the above result, we should like to discuss observational principle, and will then consider a slight modification of the travelling twin experiment.

In this new thought experiment we start out by specifying two completely symmetric reference frames; each consisting of a series of rockets in a row. The equivalence of the systems could be verified when they are at rest with respect to each other. Then the reference frames are in a symmetric way brought into a situation of relative movement with speed, $v = 0{,}6c$. The twins are located in the center rockets O (O'). In addition there are rockets located ahead and behind the center, all have synchronized clocks. Figure 5 provides an illustration for the situation at time $t' = t = 0$ and O = O′. There should be some additional rockets (measure points), but for simplicity these are not included. This set-up provides the possibility of applying both principle A and B for time comparisons.

Distances are chosen so that when all clocks on K show time $t = 5$, then O has same location as C′, and when all clocks on K' show time $t' = 5$, then O′ is located at D.

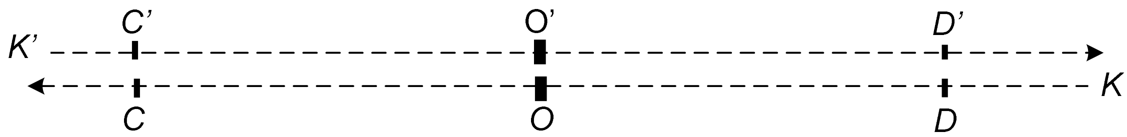

Figure 4 Two rocket moving relative to each other (time 0).

From the previous discussion of Chapter 3 we know that an observer at a fixed location in O will see time $t' = t/\sqrt{1-(\frac{v}{c})^2}$ on K', and so a fixed observer in O′ will see time $t = t'/\sqrt{1-(\frac{v}{c})^2}$ on K. Recall that in our case $v = 0{,}6c$, giving $\sqrt{1-(\frac{v}{c})^2} = 0.8$.

Thus, first taking the perspective of reference frame *K*: At the instant all clocks on *K* shows time $t = 5$ years, one will at positions O and D, respectively, observe times $t' = 5/0.8 = 6{,}25$ and $t' = 5 \cdot 0.8 = 4$ on *K*'. This corresponds to applying observational principle B and A, respectively.

Similarly, at time $t' = 5$ on *K'*, one will at positions C′ and O′, respectively, observe times $t = 4$ and $t = 6{,}25$ on *K*.

This should also tell us something about the standard travelling twin example. Again the crucial point is the perspective and observational set-up. As referred above, a standard statement is: On the outward journey the clock of the travelling twin has advanced 4 years while the clock of the earthbound twin has advanced 5 years. But this hardly the whole truth; this is essentially the narrative in the perspective of the earthbound twin. We could also say that at the moment when the deceleration starts for the travelling twin, then his clock shows 5 years, and at that time of the imagined clock he is just passing (giving earth time) shows 6.25 years.

So both reference frames will see their own clock showing time equal to 5 years when slow-down shall start. And, in principle we could – as described above - have arranged with clock comparisons to give the ‘contradictory’ result: By direct clock comparisons one would read that the clocks on the ‘other’ reference frame shows 4 years or 6,25 years, respectively, at this moment. Again practical problems to achieve these measurements are seen as rather irrelevant. If the conditions of Lorentz transformation are satisfied, the phenomenon will behave according to this. And this relation should imply that both answers are equally valid.

This is just an exemplification of the result discussed in Chapter 3. So the standard result of the travelling twin depends on applying the same perspective (earth) for the travel both ways. Considering the fully symmetric case of our new thought experiment, this can hardly be interpreted as 'actual time' going faster on any of the systems.

What does this tell us about the actual ‘travelling twin’ case? Accepting that we are entitled to ignore the acceleration period, the two cases (thought experiments) are up to this point (of deceleration) identical. And the turning, (that could be decided by the toss of a coin in our thought experiment), could hardly have any effect on the past 5 years, to decide which has aged more slowly so far. So one should believe that it is during the turning round (deceleration/acceleration) that the alleged slower ageing of the travelling twin occurs. However, it seems a paradox that the duration of the period of constant velocity (5 years) have such a significant effect on the ageing difference, (and it being rather strange that it is the time dilation formula for constant velocity that is applied).

Nevertheless, we will from the above discussion question whether the ‘travelling twin’ experiences an actual slowing down of the ageing during periods of constant velocity. The basic cause of the claimed slower ageing might be that one favours one observational principle; (as perhaps just one seems feasible). But in principle nothing should prevent us from applying another observational principle, giving another result. So, whatever the clocks say by the return, it is doubted that any difference should be ascribed to periods of constant velocity, when the clock readings are provided by the Lorentz transformation. Thus, we do not find this example sufficient to abandon the views advocated in the previous chapters.