A Static Cosmological Model, MOND and the Galactic Rotation Curve

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A static cosmological metric is derived that accounts for observed cosmic redshift without the requirement for an expanding universe. The metric is interpreted in such a way as to predict a universal potential that accounts the anomalous acceleration of outlying stars of spiral galaxies (the *galactic rotation curve*), obviating the need for dark matter or modifications to general relativity.

I. Introduction

The Big Bang (BB) theory has been questioned over its 85-year history for a number of fundamental reasons. That all the matter in the universe was created instantly at a single point some finite time in the past defies intuition as well as established conservation laws. That the dynamic general relativistic Friedmann-Lemaitre-Robertson-Walker (FLRW) metric, which forms the basis of the BB theory, is orthogonal in the space and time coordinates raises questions about its mathematical validity, a concern expressed by Friedmann himself in his original paper of 1922[1]. That Hubble deep-field images displaying thousands of distant galaxies reveal no apparent galactic structural evolution seems to suggest the universe is much older than the BB theory purports. Well-established patterns of stellar evolution indicate some stars predate by billions of years the event of the Big Bang, pointing to a flaw in the theory. Questions may also be raised about the superluminal expansion predicted by the FLRW metric, implying possible violations of locally observed special relativity. That Einstein himself believed singularities can't exist casts doubt on the singular origin of matter. These objections are foundational.

It is also unsatisfying that the BB theory fails to predict more recent observations, such as the apparent increase of expansion rate known as *cosmic acceleration*[2,3,4], currently explained by reintroducing Einstein's abandoned cosmological constant $\mathbf{\Lambda}$ and/or by postulating an ad hoc mysterious quantity called *dark energy*. The BB hypothesis also does not account for early-epoch *cosmic inflation*[5], nor does it accommodate the unexpectedly large rotation velocities of outlying stars in spiral galaxies, whose anomalous centripetal acceleration is often attributed to an unidentified substance called *dark matter*. A further defect in the theory is the premise of homogeneity. The universe in fact manifests large-scale inhomogeneities in the form of clusters, walls and voids. Averaging over these density variations has introduced difficulties of its own, a problem known as *back-reaction*[6].

Generally, astrophysicists as a community acknowledge these problems inherent in the BB hypothesis[7,8,9]. Scores of papers are published annually in peer-reviewed journals such as Physical Review D exploring possible solutions to these problems[10,11,12]. Nevertheless, the FLRW metric of the expanding universe remains the basis for modern cosmology, and is widely accepted among physicists as a valid model. This standard theory of cosmology has been dubbed the *Lambda-Cold-Dark-Matter* (ΛCDM) model due to its reliance on Einstein's cosmological constant to account for cosmic acceleration, and its appropriation of cold dark matter to explain the galactic rotation curve.

The ΛCDM model is the most comprehensive gravitational model of the universe currently known, and has a distinct appeal in that it preserves general relativity (GR) in its original form. Numerous theories of modified gravity, including bimetric, bigravity or massive gravity theories[13,14], scalar-tensor theories (eg. the Brans-Dicke theory[15]), tensor-vector-scalar (TeVeS) theories[16], vector-tensor theories[17], modified Newtonian gravity (MOND)[18], and f(R) gravity theories that modify Einstein's field equations (EFE)[19,20,21,22], have been investigated, but so far none have proven sufficiently compelling to replace GR as the prevailing theory of gravity. Many such variations suffer problems in the limit of solar system scales, where, to agree with observation, they must predict the same results as GR, giving rise to complicated schemes such as the chameleon mechanism[23,24,25] and Galileon fields[26]. Many of these theories fail the test of Occam's Razor.

The present article discusses the results of an investigation into an alternate description of the universe. This description obviates the need to modify general relativity, while also eliminating any requirement for dark matter. It accounts in a natural way for the redshift-distance relation, and postulates a simple metric explanation for the galactic rotation curve. This paper is organized as follows: Section II discusses the history of static theories of the cosmos. In Section III, the observational evidence for Einstein's field equations is briefly analyzed. A general equation for cosmic redshift and a static metric conforming to observed redshift are presented in Section IV. Section V offers a physical interpretation of the static metric. The consequences of the interpretation for MOND and the galactic rotation curve are discussed in Section VI, with a brief conclusion presented in Section VII.

II. Static Models

As a result of the current emphasis on ΛCDM and to some extent on modified gravity theories, other models, particularly static universe models, have received less attention in recent decades. Einstein's first cosmological model was that of a static universe, its heavenly bodies held in place by a form of background energy described by an ad hoc cosmological constant Λ. He abandoned the static universe model when the Hubble redshift-distance relation was discovered, as the latter seemed to indicate a dynamic expanding universe. This, it turns out, was a misconception, as will be discussed in section IV.

Fritz Zwicky reintroduced in 1929 a static model of the cosmos, postulating that photons lose energy during their transit across the universe due to an unknown phenomenon called *tired light*, resulting in the observed redshift. However, no verifiable physical mechanism for tired light was ever found, and his theory was subsequently discredited. Robert H. Dicke in the early 1950's briefly investigated a static universe theory, according to which redshift was induced by a slowing of the speed of light (also proposed by other theorists[27]) due to increasing visible matter along the past null cone, causing a vacuum refractive index (also proposed by others[28]) from Machian induction. Dicke soon abandoned this theory in favor of the Brans-Dicke theory, a scalar-tensor variation on general relativity. Recently, Dicke's original static theory has been re-investigated by Alexander Unzicker, who demonstrates its consistency with Dirac's large number hypothesis and, to some extent, with observations related to cosmic acceleration[29]. Other than Unzicker's treatise, little has been published about Dicke's static theory, and it is difficult to reproduce his variable-light-speed redshift mechanism. At any rate, in *metric theories* of gravity, variations in the speed of light can be transformed away, and any such redshift cancels. This paper, for reasons to be explained in Section IV, focuses exclusively on metric theories.

It is important to recall that the early steady state theories, such as that of Fred Hoyle, Hermann Bondi and Thomas Gold, are distinct from static models. Steady state theories predict a constant universal mass-energy density preserved by the continuous creation of matter in the vacuum between receding galaxies. They are thus dynamic models in which redshift arises from recession velocity.

III. Observational Evidence for Einstein's Field Equations

That the Friedmann-Lemaitre-Robertson-Walker metric is an exact solution to Einstein's field equations for uniform cosmic mass-energy density has been touted as a strong argument in favor of the Big Bang. To date, no observations have contradicted general relativity, and it remains our best descriptive theory of gravity (although as Einstein himself admitted, it does not address the *origin* of gravitational effects). However, one important consideration that is often overlooked in the literature is that all tests of GR up to the present time have dealt only with the Schwarzschild metric, which is a unique exact solution for the *vacuum*, ie. for $T_{\mu\nu} = 0$. There have so far been no observational tests of GR in the presence of a matter-energy distribution[30], ie. for $T_{\mu\nu} \neq 0$. Thus, non-vacuum solutions of EFE are without observational basis and remain speculative.

In this vein, the FLRW metric, a solution for a uniform mass-energy density, is also speculative. As an aside, this leads to the question of whether a complete theory of gravity could be constructed from Schwarzschild metrics alone. Some researchers have already suggested that elementary particles such as the electron are in fact tiny black holes, and the quantum properties of these particles are strikingly consistent with this hypothesis. If correct, it would mean the gravitational attraction of bulk matter derives from the superposition of numerous Schwarzschild metrics. The gravitational properties of the cosmos as a whole might then be approximated by an irregular grid of Schwarzschild metrics, one for each galaxy. The first problem with this model is that GR is nonlinear and metrics do not linearly superpose, making calculations difficult. Another deeper problem is that a Schwarzschild Grid Model (SGM) of the universe, while essentially sound from a physical standpoint, provides no obvious explanation for cosmic redshift. The most promising candidate redshift mechanism, the frame-dragging of photons as they traverse galactic gravitational fields, involves angular momentum and radial distance rather than the square of these quantities, causing a cancellation of energy changes for galaxies co-rotating and counter-rotating with the photon's velocity. Explaining cosmic redshift in SGM is equivalent to explaining tired light, which has proven elusive.

IV. Cosmic Redshift

That Einstein's field equations have never been verified for $T_{\mu\nu} \neq 0$ means the FLRW metric is purely theoretical. It is therefore valid to explore alternative metrics to describe cosmic space-time curvature. Such metrics, of course, must conform to the observed Hubble redshift-distance relation[31], as well as to approximate Minkowski space-time in local intergalactic neighborhoods

In the following analysis, only metric theories of gravity will be considered. Metric theories have the advantage that they represent gravitation as a purely geometrical phenomenon. They thus automatically obey the principle of equivalence. Metric theories (a category of tensor theory) are also simpler than bimetric, scalar-tensor, and tensor-vector-scalar theories, and are therefore more likely to pass the test of Occam's razor. For convenience, I will work in two dimensions t and r, since the transverse dimensions represented by θ and Φ are usually irrelevant to redshift.

Since any metric tensor $g_{\mu\nu}$ is an exact solution to EFE for *some* mass-energy distribution, we are free to postulate new metrics without violating GR. What this means is that any tensor $g_{\mu\nu}$ such that $ds^2 = g_{00}c^2dt^2 - g_{11}dr^2$ is consistent with the observed cosmic redshift and is approximately Minskowski for nearby intergalactic regions, is a valid candidate as a basis for cosmological theory. The question then becomes: what metrics other than FLRW approximate local Minkowski space and predict the observed redshift-distance relation?

Here, a brief digression is in order. In the original formulation of the BB theory, it was assumed cosmic redshift was a Doppler effect due to recession velocity. This tacit assumption led to the unavoidable conclusion that the universe was dynamic and galaxies were flying away from each other. As a result, most researchers rejected the possibility of a static universe. Later on, it was realized that cosmic redshift is not a Doppler effect but an intrinsic property of the expanding space of the FRW metric. Once it was understood that redshift is intrinsic to the metric, one could imagine some theorist might have gone back and revisited the Doppler shift assumption that dictated a dynamic description of the universe. Their effort might have led to the idea that redshift could be an intrinsic property of a *static* metric, obviating the need for adding a physical tired light mechanism. Apparently, this was not done, or if done, was not widely recognized. The present study is in part an attempt to rectify this omission.

Bearing this background in mind, we may ask if there exists a static metric that accounts for cosmic redshift. To answer this, we will need a general equation for redshift as a function of $g_{\mu\nu}$. Assuming the variables t and r of the metric are orthogonal and separable, the line element can be written:

$$ds^2 = g_{00}c^2dt^2 - g_{11}dr^2 = f_r^{\,2}(r)\,f_t^{\,2}(t)c^2dt^2 - h_r^{\,2}(r)h_t^{\,2}(t)dr^2 \tag{1}$$

For photons following null geodesics, we set ds=0, producing the differential equation:

$$\frac{f_t(t)}{h_t(t)}cdt = \frac{h_r(r)}{f_r(r)}dr,$$

which can be readily integrated. Using the resultant integral equation to calculate the proper time delay between wave crests at the points (t,r) of emission and observation, the general equation for redshift as a function of $g_{\mu\nu}$ turns out to be:

$$1+z = \frac{\lambda_o}{\lambda_e} = \frac{g_e\sqrt{(g_{00})_o}}{g_o\sqrt{(g_{00})_e}} = b_c(r_e) \tag{2}$$

where λ_o is the proper observed frequency, λ_e is the proper emitted frequency, subscripts o and e indicate evaluation at observation point (t_o, r_o) and emission point (t_e, r_e) respectively, and b_c is a monotonically increasing function of r_e. Here g_e and g_o are functions of time given by:

$$g_e \equiv \frac{f_t(t_e)}{h_t(t_e)},\, g_o \equiv \frac{f_t(t_o)}{h_t(t_o)} \tag{3}$$

Solving the equation 1+ z = b, for b a function of photon travel distance $\Delta r = r_e$, we obtain a family of static or time-independent metrics, ie metrics for which ft(t) = ht(t) = 1, as will be shown. Recall first that the Schwarzschild metric is also static and produces a distance-dependent redshift. One important difference between the Schwarzschild metric and cosmological metrics is that in the former case, the observer is assumed to be away from the origin, while in the latter, the observer is located at the origin. Stated another way, in the Schwarzschild case, a photon traveling toward the observer follows a path of increasing r, while in the cosmological case, it follows a path of decreasing r. Therefore, to formally compare the two metrics, the radial coordinate r needs to be inverted by some transformation $r \to r'$. This will be discussed in section V.

To solve $1+z=b(r_e)$ from the general redshift formula of Eq. (2), note that g_e and g_o are functions of time and are therefore equal to unity for a static metric. Thus we have

$$1+z=\frac{\sqrt{g_{oo\,o}}}{\sqrt{g_{oo\,e}}}$$

where g_{00} is a function of r alone. Hence from Eqs. (2) and (3):

$$1+z=\frac{f_r\ (r_o)}{f_r\ (r_e)}$$

The value 1 + *z* should conform, at least approximately for our present purpose, to current astronomically observed redshift as a function of distance. We can turn to standard cosmology to find out what this redshift relation is. For simplicity, we will use the standard model prior to postulation of cosmic acceleration. This model was based on the FLRW metric, which in the case of a flat (k=0) universe is given by

$$ds^2=c^2dt^2-a^2(t)dr^2$$

Here $a(t)=t^{2/3}$ is a scale function derived from thermodynamic equations for pressure and density. Redshift as a function of time is thus

$$1+z=\frac{a(t_o)}{a(t_e)}=\left(\frac{t_o}{t_e}\right)^{2/3}$$

as can be seen by substitution into Eq.(2). To express this as a function of r rather than t, we assume as an approximation that the speed dr/dt of light is constant over universal distances, so that $r_e-r_o=c(t_o-t_e)$ for incoming photons. Since $r_o=0$ for an observer at the origin, we have $r_e=c(t_o-t_e)$ and therefore

$$\frac{t_o}{t_e}=\frac{t_o}{t_o-r_e/c}=\frac{1}{1-r_e/ct_o}$$

Now, if we arbitrarily define a constant $R \equiv ct_o$, where t_o is the present time or the time elapsed since t=0, then R is the distance light has traveled since t=0. Substituting into the above gives:

$$\frac{t_o}{t_e} = \frac{1}{1 - r_e/R}$$

and thus

$$1 + z = \frac{f_r(r_o)}{f_r(r_e)} = \frac{1}{\left(1 - r_e/R\right)^{2/3}} \tag{4}$$

Hence f_r as a general function of r is given by

$$f_r(r) = \left(1 - \frac{r}{R}\right)^{2/3}$$

as can be seen by evaluating $f_r(r_o = 0) = 1$ for the numerator of Eq. (4). Inserting this into the static line element of Eq.(1), with functions of t set to unity, we obtain:

$$ds^2 = \left(1 - \frac{r}{R}\right)^{4/3} c^2 dt^2 - h_r^2(r) dr^2 \tag{5}$$

This represents a family of static metrics that predict the standard cosmic redshift. Note that according to Eq. (2), $h_r(r)$ does not affect redshift, and so can be any appropriate function.

The metric element $g_{00} = q(1 - r/R)$ where q(x) is some monotonic function of x, can be tailored to fit any redshift anomalies such as cosmic acceleration. While this may seem an arbitrary feature of the theory, it is irrelevant to the main point, which is that static metrics are capable of producing the observed cosmological redshift.

The constant R has several interpretations in the BB theory. For instance, in a closed FLRW metric (k=1), the cosmos forms a topological hypersphere for which R is often associated with the *radius of the universe*. R might also be interpreted as the distance light has traveled since moment of the BB. In addition, R is sometimes associated with the distance to the redshift horizon, meaning the distance from which approaching light is infinitely redshifted. In the static model described by the above line element, R is obviously a singularity of some sort, since at r=R, the time component of the metric vanishes. At the very least, R is an event horizon beyond which matter cannot be visually observed. R can therefore be called the *radius of the visible universe*. This does not mean that matter cannot exist beyond this radius, only that we cannot see it. It is still possible to detect such matter gravitationally, much as we might detect the matter inside a black hole. Thus the visible radius R is distinct from the *physical* radius of the universe R_u. There is nothing to prevent R_u from being located at infinity.

Before investigating the properties of the static cosmological metric, it is instructive first to evaluate the redshift equation for the static Schwarzschild metric. We have, from Eq. (2),

$$1+z=\frac{\sqrt{g_{00\ o}}}{\sqrt{g_{00\ e}}}=\frac{\sqrt{1-2M/r_o}}{\sqrt{1-2M/r_e}}=b_s(r_o) \tag{6}$$

where, for a fixed emission point r_e, b_s is a monotonically increasing function of observation point r_o due to an increase with r_o of the numerator. In the case of the static cosmological metric of Eqs.(1) and.(3), on the other hand, b_c is a monotonically increasing function of emission point r_e due to an *decrease* with r_e of the *denominator*. Thus, comparing the two cases, we have redshifts that seem, at first glance, to be counter-increasing with r:

$$(1-r/R)\text{ vs. }(1-2GM/c^2r)$$

but which in fact both increase with distance once we have inverted the r coordinate such that $r \to r'$, as previously mentioned. The inversion function is defined in the next section.

V. Interpretation of the Static Metric

How is a static cosmological metric as expressed by Eq.(3) to be interpreted physically, ie, what kind of material universe might give rise to such a metric? I will refrain from calculating until a later paper the plethora of Christoffel symbols $\Gamma^{\alpha\beta}_{\mu\nu}$ and curvatures $R_{\mu\nu}$

required for substitution into EFE with $T_{\mu\nu} \neq 0$, which might not even be physically correct, and claim that $g_{00} = q(1 - r/R)$ is the general form of the time component of the metric for an *inverted quasi-black hole*, to be defined. This result can be derived by first applying the radial inversion relation

$$r \to r = \frac{R^2}{r'} \qquad (7)$$

This relation is symmetric in r and r', and turns functions that decrease with r into functions that increase with r', as is needed to formally compare cosmological and Schwarzschild metrics. Note that when r is equal to R, r' is also equal to R, and when r approaches zero, r' approaches infinity and vice versa.

Applying Eq.(7) to the static time component $g_{00} = q(1 - r/R)$ yields $g_{00} = q(1 - R/r')$. Now, if we assume $R \sim 2GM_u/c^2$ (twice the "universal radius" $R_u \sim GM_u/c^2$ in standard terminology), where M_u is the estimated mass of the universe often assumed in the literature, the metric time component in terms of r' becomes

$$g_{00} = (1 - 2GM_u/c^2 r') \qquad (8)$$

This is the time component of the Schwarzschild metric for a mass M_u, and might be imagined to describe a black hole of Schwarzschild radius $r' = R_s \equiv 2GM_u/c^2 \sim R$. Why would this describe a black hole as opposed to some other extended mass distribution? To answer this question requires further interpretation. First, since $r' \to 0$ as $r \to \infty$, the metric component of Eq.(8) would be associated with a "black hole" centered on r at infinity. Loosely speaking, we, the observers at r=0, would thus be completely surrounded by this imagined black hole, whose center is an infinitely large sphere at $r = \infty$, and whose event horizon is an enclosed concentric sphere at $r = R$.

For this to be a physical black hole, its center must lie at a physical point r'=0. Thus, a sphere of infinite radius $r = \infty$ would coincide with a point r'=0. Geometrically, this requires that spheres centered on the observer at r=0 increase in size with increasing radius up to some maximum value, then decrease in size and "wrap back around" into a point at $r = \infty$, somewhat like latitude circles get bigger with increasing distance from the south pole, then "wrap back around" into a point at the north pole. The difference is that the earth's radius is finite, while the universe would be a closed topological hypersphere of infinite radius. These are attempts at visualizing the implications of the mathematics, which, despite any limitations in our imagination, is rigorous within the framework of assumptions.

Now we can discuss why Eq.(8) would be associated with a "black hole" rather than a more extended mass. The first reason is Olber's paradox. Were we completely surrounded by a sphere of visually observable matter, the night sky would not be dark. If instead we are surrounded by an event horizon at r=R, from which any approaching photons are infinitely redshifted, the night sky would be black as observed. The matter M_u in Eq.(8) must therefore lie on the other side of the event horizon, as it would for a black hole. The second reason is that postulating an event horizon at r=R is consistent with common cosmological thinking.

I have been placing "black hole" in quotes because the complete inverted cosmological metric is not precisely that of a black hole. So far, we have examined only the time component g_{00}. When we consider the space component g_{11}, which by analogy would be

$$g_{11} = h_r^2(r) = \frac{1}{1 - r/R} = \frac{1}{1 - 2GM_u / c^2 r'}$$

and substitute this into the cosmological metric, we must also transform the differential dr according to Eq.(7), with the result

$$dr = \frac{R^2}{r'^2} dr'.$$

This introduces a factor of R^4 / r'^4 into g_{11} Furthermore, the standard redshift-distance relation imposes a function $q(x) \sim x^{2/3}$ on both g_{00} and g_{11}, rendering them different from the $g_{\mu\nu}$ of the Schwarzschild metric.

Combining the above considerations, the line element for a static universe with observed cosmic redshift is given, in terms of orthogonal coordinates (t,r'), by:

$$ds^2 = q(1 - 2GM_u / c^2 r')c^2 dt^2 - \frac{1}{q(1 - 2GM_u / c^2 r')} \frac{R^4}{r'^4} dr'^2 \qquad (9)$$

where $q(x) = x^{k(t,r')}$ for some exponential function k ~ 2/3 to be determined by redshift observations. The mathematical form of this line element resembles that of a Schwarzschild metric in terms of inverted coordinate r', and so will be said to represent an *inverted quasi-black hole*.

VI. Consequences for MOND

It was recognized in the first half of the twentieth century, and famously noted by Fritz Zwicky, that the rotation rates of outlying stars in spiral galaxies were too high to be explained by the Newtonian gravitational attraction of visible matter alone. The fall-off rate of rotation velocity as a function of r is called the *galactic rotation curve*[32]. In particular, the rotation curve would be expected to fall off as 1/r in regions far from the nucleus where galactic matter can be approximated as a point mass. What is actually observed, however, is a rotation curve that increases with r as expected in the inner regions of galaxies, but which approaches a constant velocity independent of r in outlying regions. General relativity cannot account for the discrepancy, since the outer stars orbit at low velocity and acceleration, making relativistic corrections negligible. Lacking an alternative explanation, astronomers postulated that the nuclei of spiral galaxies were surrounded by a halo of unidentified invisible matter which would account for the unexplained centripetal acceleration. This came to be called *dark matter*. However, after decades of searching both theoretically and experimentally for massive particles that do not interact with light, no viable dark matter candidates have been verified to exist. This discrepancy led to the development of a number of theories of modified gravity intended to obviate the need for dark matter.

One compelling argument for modifying gravity itself arose from the fact that extensive statistical sampling of galactic rotation curves revealed a pattern common to nearly all spiral galaxies: Regardless of the galaxy's mass or size, its rotation curve begins to flatten at just that radius where the centripetal acceleration takes on the small value $a_0 \approx 1.2x10^{-8}\,\mathrm{cm/sec}^2 = c^2 / R'$, where R' is a cosmic-scale distance defined by this equation and approximately equal to 3.5 times the radius R of the visible universe . This value a_0 appears to be a universal constant applying to all galaxies. Dark matter distributions, on the other hand, would be expected to vary from galaxy to galaxy, and would not give rise to uniform behavior.

One theory of modified gravity first proposed by Milgromm, called Modified Newtonian Dynamics (MOND), is especially useful for analysis. MOND, a purely phenomenological descriptive formalism, applies curve fitting to a large statistical sample of galaxies to arrive at a mathematical equation for the average galactic rotation curve[18]. This equation is expressed as an interpolation formula joining the curve of the Newtonian inner region with that of the anomalous outer region. A central force law can be calculated from this formula and used to validate other causal theories, hence its usefulness. This MOND central force falls off as roughly $1/r^2$ for r << R_0 and $1/r$ for r>> R_0, where R_0 is a critical radius, called the *MOND radius*, usually located near the visible edge of the galaxy. The MOND radius R_0 is defined as that radius where the Newtonian acceleration $a_N = Gm(r)/r^2$ due to enclosed visible matter is equal to the observed anomalous acceleration a_0. This leads to the value $R_0 = \sqrt{r_s R'/2}$, where r_s is the Schwarzschild radius of the galaxy's visible matter, and R' ~ 3.5R is the cosmic-scale distance defined above.

The MOND acceleration a_0 can be expressed in terms of R' and R, with the result $a_0 = c^2/R' \sim c^2/3.5R$. That the universal constant a_0 is determined from observations independent of the mass M_u or radius R of the visible universe, yet involves a constant R' that is this close in value to R seems a remarkable coincidence. Indeed, it suggests that the rotation curve anomaly is not due to an unexpected feature of local gravity but rather to some unknown cosmic-scale phenomenon.

One candidate for such a cosmic-scale phenomenon is the inverted quasi-black hole derived in previous sections. As can be seen from the general relation $g_{00} = 1 - 2\phi/c^2$ for Schwarzschild metrics, the quasi-black hole line element of Eq. (9) suggests a potential $\phi = GM_u/r' = Rc^2/2r'$. Expressing this in terms of r, where $r = R^2/r'$, we have $\phi = rc^2/2R$. The acceleration a_ϕ associated with ϕ can be found by differentiating with respect to r, giving $a_\phi = c^2/2R$. This is remarkably close to the characteristic MOND acceleration $a_0 = c^2/3.5R$ derived above. The interpretation would be that outer galactic stars are accelerated not only by the gravitational potential of the galaxy but also by that of the surrounding cosmic inverted quasi-black hole. The latter field, being very small, is normally undetectable, but becomes observable in the particular case of very small accelerations at the outer edges of galaxies.

VII. Conclusion

Within the framework of a cosmic-scale inverted quasi-black hole, the anomalous galactic rotation curve can be approximately accounted for without resorting to modifications of general relativity, and without positing the existence of dark matter. The theory is derived from the single assumption that universal space-time is described by a static metric which approximates Minkowski space-time for local intergalactic regions and conforms to the observed redshift-distance relation. All specifics of the theory are deduced from this assumption. The inverse black hole cosmological theory is not necessarily a theory of modified gravity, since there is no reason to expect the inverted quasi-black hole metric is not a solution to Einstein's field equations for some physical distribution of matter. If the concept of a quasi-black hole centered at infinity seems paradoxical, one should bear in mind that the existence of the universe is itself paradoxical, and therefore its ultimate explanation should logically require a paradox of equal magnitude.

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