Game of Tradeoffs:

Beyond Imaginary Games, Bargaining in General, and Games with Games

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Abstract

The proposed approach generalizes between cooperative versus noncooperative games as well as across their otherwise disparate applications. *Inter alia*, both strategies and payoffs are shown to be entangled in a dual fashion, with the conventional solutions proving special reductions.

Rehashing on the Grand Game with Game: Pure Mixing

It has been proposed before that any decision making setup can be reduced to either a portfolio choice subject to preferences (with risk aversion as one salient parameter being embedded structurally) or to a game with a chance player (Shevenyonov, 2016k). Specifically, the opportunity frontier, possibly combining the endogenous production function or system alongside the exogenous state-setting environment, will act as either a chance player in the short run or as a more controllable as well as appropriable vehicle over the long haul. In the latter scenario, there is material as well as meaningful convergence between the payoffs (the chance player's lot acting as an add-on differential to be treated separably for ease of structural inference) and the strategies alike.

That said, what is it exactly that could be construed as the chance players "mixed strategy" if any? And does the social planner as the key player really come up with any such mixing for practical purposes? Incidentally, there is no alternative to mixing—whether conceptually or quantitatively so. In other words, it's a most natural underpinning behind the generalization attempted—even though further extension or relaxation, in particular of oversimplified exogeneity and orthogonality, will complete the demonstration as one missing bridge.

To begin with, the milieu does set states—which could pertain to any (not necessarily quantifiable) scenarios, modes or regimes, e.g. "bad" versus "good," "non-*kamban*" versus "*kamban*"[1] (referring to JIT-like or lean value management adding up as GDP maximization), and the like. Needless to say, rarely ever will there be any "corner" states or scenarios materializing, which lends every support and validity to mixes. On second thought, though

[1] The latter dichotomy, stemming from the Japanese approaches to logistics and working capital management, is what was deployed in my original research from day one, as in 1995-1997 presentations.

natural and inalterable conceptually, no *specific* value can possibly apply other than as posterior inference or backward induction. In other words, the chance player cannot react or "act upon" the key player's *ad-hoc move* (even though the long-term sensitivity analysis to cumulative strategy is more intricate)[2]. In effect, any mix or "strategy" can only be implied from the payoff distribution—based on the corner (or best attainable, optimal, potential) payoff as resulting from the key player choosing an optimum strategy (irrespective of the ad-hoc move that can at best materialize a corner case or a unity posterior probability with no mixing applicable on this player's part).

On the other hand, the key player (acting as an agent or social planner) will readily apply a particular mix as relative proximity to greater openness as opposed to protectionism. In fact, it should come as no surprise if the corner (and expected partial) payoff distribution proves far more *compressed* anywhere around the protected corner. Although ultimately chosen depending on one's risk preference map, it will rationally be bounded by uniform comparison metrics such as the Sharpe ratio adjusting the expected return differential to the risk or standard deviation attached. Largely the same would hold for investment project valuation, if one were to embark on the variability of earnings streams over and above the standardized criteria such as NPV or IRR which may in any event prove unreliable in either turning the blind eye to the varying initial outlay (as scale or entry barrier) or otherwise scrap some otherwise strategically crucial value contributors which may not fare as well on the strength of their standalone performances as opposed to contingent synergy.

To draw a tentative bottom line, the chance player's mix could be seen as one it *must have set* [implicitly or ex post] rather than *would rather opt for* [explicitly or ex ante]. Although similar logic could apply to the [aggregated] social planner as a disparate bureaucratic body, no such restrictions or inversions are required in simpler and direct-manned setups showing a minimalist hierarchy of agency, delegation, or voting stages.

Strategies: Non-Orthogonal, Non-Exogenous

To visualize a straightforward game-like setup, consider a payoff matrix (Appendix). It should be straightforward, again in line with portfolio and expected value mixing approaches, to reduce it to partial optimizations, with FOCs being as follows:

$$\frac{\partial EV}{\partial s} \equiv 0 = \frac{\partial}{\partial s}\{s * [\pi_{11} * \bar{s} + \pi_{12} * (1-\bar{s})] + (1-s) * [\pi_{21} * \bar{s} + \pi_{22} * (1-\bar{s})]\}$$

$$\frac{\partial \overline{EV}}{\partial \bar{s}} \equiv 0 = \frac{\partial}{\partial \bar{s}}\{\bar{s} * [\overline{\pi_{11}} * s + \overline{\pi_{12}} * (1-s)] + (1-\bar{s}) * [\overline{\pi_{21}} * s + \overline{\pi_{22}} * (1-s)]\}$$

$$\bar{s} = \frac{\pi_{22} - \pi_{12}}{\pi_{22} - \pi_{12} + \pi_{11} - \pi_{21}}$$

[2] The ultimate for diversification and price-taking?

$$(*)\ \ s = \frac{\overline{\pi_{22}} - \overline{\pi_{12}}}{\overline{\pi_{22}} - \overline{\pi_{12}} + \overline{\pi_{11}} - \overline{\pi_{21}}}$$

The above weights could be normalized or otherwise restrained to desolate, in particular, instances of pure (e.g. *{non-Kamban, Protection}={non-Kamban, Openness}*) versus explosive or less “well behaved” cases like zeros in the denominator (which may either second pure strategy or usher in structural inconclusiveness).

For now, suffice it to retort that, payoffs need not be exogenously imposed or assumed orthogonal as per the distributions or sharing between the players—and the same goes for the underlying strategies. Whereas the non-exogeneity cannot fully be captured in this *single*-stage setup, the intertwined nature of the payoffs and strategies alike (which is by and large overlooked in production functions other than CES, e.g. Cobb-Douglas assuming factor orthogonality) will now be treated succinctly yet adequately with an eye on arbitrarily chosen generality.

Entangled States as Complete Simplicity

One can, for starters, always conceive of and possibly construct a function linking the players’ payoffs as per each particular corner:

$$\exists\ F \equiv F(\pi, \bar{\pi})$$

The same, albeit based on a distinct and possibly dual rationale, would hold for the strategies supposedly being entangled for the short and long run alike:

$$\exists\ G \equiv G(s, \bar{s})$$

Just like the [complete] set of strategies being exercised maps into the resultant [complete] set of payoffs, this can be reversed by holding that an accommodating strategic set can be inferred from a particular one of distributed payoffs. This two-way-complete (yet not bijective in the one-to-one or per-element sense) rationale would suggest an identity for a particular *H* mapping:

$$\exists H: G(s, \bar{s}) \equiv H\{F(\pi, \bar{\pi})\} \equiv \hat{F}(\pi, \bar{\pi})$$

An expanded differential can be considered for ease of local tangency check, which would amount to comparison or match in the minor:

$$(\Delta)\quad \frac{\partial G}{\partial s}\Delta s + \frac{\partial G}{\partial \bar{s}}\Delta \bar{s} = \frac{\partial \hat{F}}{\partial \pi}\Delta \pi + \frac{\partial \hat{F}}{\partial \bar{\pi}}\Delta \bar{\pi}\ \ \exists \Delta$$

It should be straightforward to see that, anywhere around the optimum, both the LHS and RHS tend to zero, with the implicit function theorem suggesting that, locally,

$$(\Delta\Delta)\quad \left|\frac{ds}{d\bar{s}}\right| = \frac{\partial G/\partial \bar{s}}{\partial G/\partial s}, \qquad \left|\frac{d\pi}{d\bar{\pi}}\right| = \frac{\partial \hat{F}/\partial \bar{\pi}}{\partial \hat{F}/\partial \pi}$$

Among other things, the strategies as well as payoffs need not be orthogonal or Nash even if the maps have attained their optimal values. However, there is some independence coming in as invariance with respect to the chance player's share or weight. In place of linear orthogonality as in *(*)*, an alternative representation could be proposed:

$$0 = \Delta s * \{[\,]_1 - [\,]_2\}$$

$$(**)\ 0 = \Delta\bar{s} * \{\overline{[\,]_1} - \overline{[\,]_2}\}$$

With the indexed brackets referring to the exact same respective terms as in the original FOCs, the same linear orthogonality would obtain as before unless the strategies are shown to be optimal otherwise, i.e. with deltas being identically zero. It is this alternative concept of strategy that has yet to be recouped.

One starting point (in fact leading to further trivial linearization, albeit from a very different angle) would be to assume that the chance payoff is, again, the extension of the kernel—be it in corner or expected terms:

$$F^{OPT}(\pi, \bar{\pi}) = \pi^{OPT} + \bar{\pi}^{OPT}$$

$$EV^{OPT} = (EV + \overline{EV})^{OPT}$$

However, this could collapse the IFT expansion to a singular setting in case of local convergence between the strategies. Alternatively, anywhere near optimum, it could be posited that:

$$\frac{\partial G}{\partial s} = \frac{\Delta F}{\Delta s}$$

Now, once rendered in terms of expansion relative to the initial values, then reintegrated as a differential equation, it obtains that:

$$\frac{\partial G}{\partial s} = \frac{\Delta F}{\Delta s} \equiv \frac{F - F_0}{s - s_0}$$

$$G(s) = G(s_0) + (F - F_0) * \log(s - s_0)$$

$$e^{\Delta G} = (\Delta s)^{\Delta F}$$

Though far from trivial, this local-to-global transfer only captures a reduced case, with the conjugate strategy missing, taken as irrelevant (invariance), or exogenized.

The more stimulating as well as illuminating perspective would be to directly juxtapose (**) and (*Δ*) in a piecemeal fashion:

$$\frac{\partial G}{\partial s}\Delta s + \frac{\partial G}{\partial \bar{s}}\Delta\bar{s} \sim 0 \sim \{[\,]_1 - [\,]_2\}\Delta s + \{\overline{[\,]_1} - \overline{[\,]_2}\}\Delta\bar{s} \equiv [\quad]\Delta s + \overline{[\quad]}\Delta\bar{s}$$

By solving the implied differential equations pairwise, it obtains that:

$$G(s,\bar{s}) - G_0 = [\quad]s = \overline{[\,]}\,\bar{s} \equiv (\,)_1\bar{s}s + (\,)_2 s = \overline{(\,)_1}s\bar{s} + \overline{(\,)_2}\bar{s}$$

This simple implicit function, as a realization of that which was conjectured from the outset, ushers in a plethora of implications. For one thing, the *convergence* of strategies (as in the long-run appropriation case) does involve that of the payoffs as a matter of *contingency* rather than reduced-form attainments. On the other hand, this functional map may *not* have a *closed*-form solution, which is to suggest just how entangled the strategies really are. Better yet, depending on the signs of the particular ellipses or their differentials (which are inferred from within the *F* map as the dual implicit map), the actual CES-like relationships for strategies can be imputed:

$$\frac{(\,)_2}{\overline{(\,)_2}} \sim \frac{\bar{s}}{s} \;\leftrightarrow\; \rho \sim \frac{\rho}{\rho - 1} \sim 0$$

$$(\,)_1 \sim \overline{(\,)_1}\ AND\ \ (\,)_2 \sim -\overline{(\,)_2} \;\;\leftrightarrow\;\; \rho \sim 1$$

$$(\,)_1 \sim \overline{(\,)_1}\ AND\ \ (\,)_2 \sim +\overline{(\,)_2} \;\;\leftrightarrow\;\; \rho \sim -\infty$$

Whereas the former criterion refers to [perfect] neutrality or orthogonality, the other two point to [perfect] strategic substitutability and complementarity, respectively.

Implications on Extended Applications: More Contingencies Pinned Down

Apart from clear-cut implications for strategy being treated as an “input” along the lines of implied xHOS convergence (Shevenyonov, 2016m), similar modeling frameworks may apply to how the society opts for a particular level of democracy as a meta-choice depending on the quality or availability of human capital (with an eye on detecting and staying immune to manipulability and hype), and to integrative decisions for that matter, as when tossing up over China led versus US bred coalitions.

References

Shevenyonov, A. (2016k). Game of trade: A portfolio approach to theorizing on trade and investment as entangled domains. *viXra: 1611.0204*

Shevenyonov, A. (2016m). Implied intra-industry trade in information: Modified expositions. *viXra: 1611.0236*

Appendix: Game Matrix

	Non-Kamban	**Kamban**
Openness	$(\pi_{11}, \overline{\pi_{11}})$	$(\pi_{12}, \overline{\pi_{12}})$
Protectionism	$(\pi_{21}, \overline{\pi_{21}})$	$(\pi_{22}, \overline{\pi_{22}})$