Deng entropy in hyper power set and super power set

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Abstract

Deng entropy has been proposed to handle the uncertainty degree of belief function in Dempster-Shafer framework very recently. In this paper, two new belief entropies based on the frame of Deng entropy for hyper-power sets and super-power sets are respectively proposed to measure the uncertainty degree of more uncertain and more flexible information. Directly, the new entropies based on the frame of Deng entropy in hyper-power sets and super-power sets can be used in the application of DSmT.

Keywords: Uncertainty measure, Entropy, Belief entropy, Deng entropy, Hyper-power sets, Super-power sets, DSmT, Dempster-Shafer evidence theory

1. Introduction

Since firstly proposed by Clausius in 1865 for thermodynamics [1], various types of entropies are presented, such as information entropy [2], Tsallis

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entropy [3, 4], nonadditive entropy [5]. Information entropy [2], derived from the Boltzmann-Gibbs (BG) entropy [6] in thermodynamics and statistical mechanics, has been an indicator to measures uncertainty which is associated with the probability density function (PDF). Very recently, Deng entropy [7] was proposed to measure the uncertainty degree of of belief function in the frame of Dempster-Shafer evidence theory (DST). Some trying application of Deng entropy for managing the conflict of belief function has been presented [8].

Deng entropy roots in the classical DST with exclusive elements of the frame of discernment, which hasn't consider the situation in Dezert-Smarandache Theory (DSmT), which is based on hyper-power sets. Hyper power set and super power set consider more flexible relationship of the element, and there are no exclusive constraint for the elements in hyper power set and super power set, which can describe more uncertain information. In this paper, the frame of Deng entropy in hyper-power sets and super-power sets is proposed to measure the uncertainty degree of them, to be suitable for more uncertain and more flexible information.

The paper is organized as follows. The preliminaries Dempster-Shafer evidence theory and Deng entropy, DSmt, hyper power set and super power set are briefly introduced in Section 2. Section 3 proposed the new belief entropy base on the frame of Deng entropy for hyper power set and super power set, and four examples are used to illustrate the difference of new belief entropy among classical set, hyper power set and super powerset. Finally, this paper is concluded in Section 4.

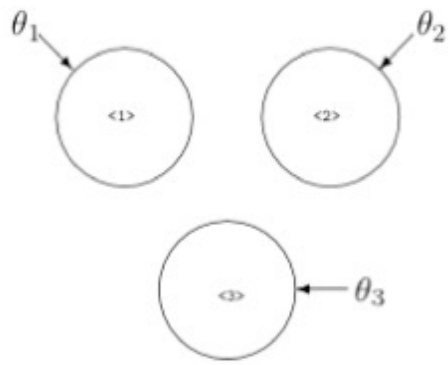

Figure 1: Power-sets

2. Preliminaries

In this section, some preliminaries are briefly introduced.

2.1. Dempster-Shafer evidence theory

Dempster-Shafer theory (short for DST) is presented by Dempster and Shafer [9, 10]. This theory is widely applied to uncertainty modeling [11, 12], decision making [13, 14], information fusion [15] and uncertain information processing [16]. Some basic concepts in D-S theory are introduced.

Let X be a set of mutually exclusive and collectively exhaustive events, indicated by

$$X = \{\theta_1, \theta_2, \cdots, \theta_i, \cdots, \theta_{|X|}\} \tag{1}$$

where set X is called a frame of discernment, and the three exclusive and exhaustive elements can be shown as Figure 1. The power set $2^X \triangleq (X, \cup)$ of X is indicated by 2^X, namely

$$2^X = \{\emptyset, \{\theta_1\}, \cdots, \{\theta_{|X|}\}, \{\theta_1, \theta_2\}, \cdots, \{\theta_1, \theta_2, \cdots, \theta_i\}, \cdots, X\} \tag{2}$$

For a frame of discernment $X = \{\theta_1, \theta_2, \cdots, \theta_{|X|}\}$, a mass function is a mapping m from 2^X to $[0, 1]$, formally defined by:

$$m: \quad 2^X \to [0, 1] \tag{3}$$

which satisfies the following condition:

$$m(\emptyset) = 0 \quad and \quad \sum_{A \in 2^X} m(A) = 1 \tag{4}$$

In DST, a mass function is also called a basic probability assignment (BPA). Assume there are two BPAs indicated by m_1 and m_2, the Dempster's rule of combination is used to combine them as follows:

$$m(A) = \begin{cases} \frac{1}{1-K} \sum\limits_{B \cap C = A} m_1(B) m_2(C) \ , & A \neq \emptyset; \\ 0 \ , & A = \emptyset. \end{cases} \tag{5}$$

with

$$K = \sum_{B \cap C = \emptyset} m_1(B) m_2(C) \tag{6}$$

Note that the Dempster's rule of combination is only applicable to such two BPAs which satisfy the condition $K < 1$.

2.2. Hyper power sets

The hyper-power set $D^\Theta \triangleq (\Theta, \cup, \cap)$ is defined as the set of all propositions from Θ with $\cap$ and $\cup$ operators, such that:

(i) $\emptyset, \theta_1, \theta_2, \cdots, \theta_n \in D^\Theta$.

(ii) If $A, B \in D^\Theta$, then $A \cap B \in D^\Theta$ and $A \cup B \in D^\Theta$.

(iii) No other elements belong to D^Θ, except those obtained by using rules (i) or (ii).

Example 1. *(1) In the degenerate case $(n = 0)$ where $\Theta = \{\}$, one has $D^\Theta = \left\{\alpha_0 \triangleq \emptyset\right\}$ and $|D^\Theta| = 1$.*

(2) When $\Theta = \{\theta_1, \theta_2\}$, one has $D^\Theta = \{\alpha_0, \alpha_1, \cdots, \alpha_4\}$ and $|D^\Theta| = 5$ with $\alpha_0 \triangleq \emptyset$, $\alpha_1 \triangleq \theta_1 \cap \theta_2$, $\alpha_2 \triangleq \theta_1$, $\alpha_3 \triangleq \theta_2$, and $\alpha_4 \triangleq \theta_1 \cup \theta_2$.

(3) When $\Theta = \{\theta_1, \theta_2, \theta_3\}$, the relations of the elements $\theta_1, \theta_2, \theta_3$ can be shown as Figure 2, the elements of $D^\Theta = \{\alpha_0, \alpha_1, \cdots, \alpha_{18}\}$ and $|D^\Theta| = 19$ are given by Table 1.

Table 1: Elements of $D^{\Theta=\{\theta_1,\theta_2,\theta_3\}}$

$\alpha_0 \triangleq \emptyset$	
$\alpha_1 \triangleq \theta_1 \cap \theta_2 \cap \theta_3$	$\alpha_{10} \triangleq \theta_2$
$\alpha_2 \triangleq \theta_1 \cap \theta_2$	$\alpha_{11} \triangleq \theta_3$
$\alpha_3 \triangleq \theta_1 \cap \theta_3$	$\alpha_{12} \triangleq (\theta_1 \cap \theta_2) \cup \theta_3$
$\alpha_4 \triangleq \theta_2 \cap \theta_3$	$\alpha_{13} \triangleq (\theta_1 \cap \theta_3) \cup \theta_2$
$\alpha_5 \triangleq (\theta_1 \cup \theta_2) \cap \theta_3$	$\alpha_{14} \triangleq (\theta_2 \cap \theta_3) \cup \theta_1$
$\alpha_6 \triangleq (\theta_1 \cup \theta_3) \cap \theta_2$	$\alpha_{15} \triangleq \theta_1 \cup \theta_2$
$\alpha_7 \triangleq (\theta_2 \cup \theta_3) \cap \theta_1$	$\alpha_{16} \triangleq \theta_1 \cup \theta_3$
$\alpha_8 \triangleq (\theta_1 \cap \theta_2) \cup (\theta_1 \cap \theta_3) \cup (\theta_2 \cap \theta_3)$	$\alpha_{17} = \theta_2 \cup \theta_3$
$\alpha_9 = \theta_1$	$\alpha_{18} = \theta_1 \cup \theta_2 \cup \theta_3$

2.3. Super power sets

Super-power sets $S^\Theta = (\Theta, \cup, \cap, c\,(.))$ is defined as the set of all composite propositions/subsets built from elements of Θ with $\cap$, $\cup$ and $c(.)$ operators such that:

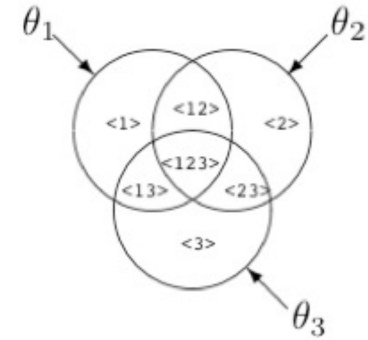

Figure 2: Hyper-power sets

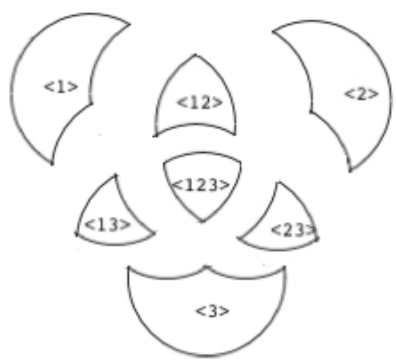

Figure 3: Super-power sets

(a) $\emptyset, \theta_1, \cdots, \theta_n \in S^\Theta$.

(b) If $A, B \in S^\Theta$, then $A \cap B \in S^\Theta$, $A \cup B \in S^\Theta$.

(c) If $A \in S^\Theta$, then $c(A) \in S^\Theta$.

(d) No other elements belongs to S^Θ, except those obtained by using rules (a), (b), and (c).

where $c(.)$ is the operator of complementation, and three elements in frame of discernment. When $\Theta = \{\theta_1, \theta_2, \theta_3\}$, the relations of the elements $\theta_1, \theta_2, \theta_3$ can be shown as Figure 3

Example 2. *Assume the frame* $\Theta = \{\theta_1, \theta_2\}$*, and* $\theta_1 \cap \theta_2 \neq \emptyset$*, then* $S^{\Theta=\{\theta_1,\theta_2\}}$ *is given by*

$$S^{\Theta} = \{\emptyset, \theta_1 \cap \theta_2, \theta_1, \theta_2, \theta_1 \cup \theta_2, c(\emptyset), c(\theta_1 \cap \theta_2), c(\theta_1), c(\theta_2), c(\theta_1 \cup \theta_2)\} \tag{7}$$

Since $c(\emptyset) = \theta_1 \cup \theta_2$ *and* $c(\theta_1 \cup \theta_2) = \emptyset$*, the super-power set is actually given by*

$$S^{\Theta} = \{\emptyset, \theta_1 \cap \theta_2, \theta_1, \theta_2, \theta_1 \cup \theta_2, c(\theta_1 \cap \theta_2), c(\theta_1), c(\theta_2)\} \tag{8}$$

2.4. Relation of power set, hyper-power set, and super-power set

In order to emphasize the differences of the power set 2^{Θ}, hyper-power set D^{Θ}, and super-power set S^{Θ}. In this part, we briefly introduce the relation of the power set 2^{Θ}, hyper-power set D^{Θ}, and super-power set S^{Θ} as follows:

$$2^{\Theta} \subseteq D^{\Theta} \subseteq S^{\Theta} \tag{9}$$

$$\left|2^{\Theta}\right| = 2^{|\Theta|} < \left|D^{\Theta}\right| < \left|S^{\Theta}\right| = 2^{2^{|\Theta|}-1} \tag{10}$$

Typically, Cardinalities of $2^{\Theta}, D^{\Theta}$ and S^{Θ} is shown as Table 2

2.5. Dezert-Smarandache Theory

Dezert-Smarandache Theory [17], short for DSmT, is an a natural extension of the classical Dempster-Shafer Theory (DST) but includes fundamental differences with the DST. DSmT allows to formally combine any types of independent sources of information represented in term of belief functions, but is mainly focused on the fusion of uncertain, highly conflicting and imprecise quantitative or qualitative sources of evidence [18, 19].

Table 2: Cardinalities of $2^\Theta, D^\Theta$ and S^Θ

$\lvert\Theta\rvert = n$	$\lvert 2^{\Theta}\rvert = 2^n$	D^Θ	$\lvert S^\Theta\rvert = \lvert 2^{\Theta_{ref}}\rvert = 2^{2^n-1}$
2	4	5	$2^3 = 8$
3	8	19	$2^7 = 128$
4	16	167	$2^{15} = 32768$
5	32	7580	$2^{31} = 2147483648$

Let $\Theta = \{\theta_1, \theta_2, \cdots, \theta_n\}$, here Θ is the frame of discernment, which contains n finite elements $\theta_i (i = 1, 2, \cdots, n)$. In the free DSm model, denoted $\mathcal{M}^f(\Theta)$, comparing with DST, the elements $\theta \in \Theta$ are not precisely defined and separated, so that no refinement of Θ in a new larger set Θ_{ref} of disjoint elementary hypotheses is possible [19].

General belief and plausibility functions are defined by a mapping $m(.) : D^\Theta \to [0, 1]$, when it satisfies

$$m(\emptyset) = 0 \;\; and \;\; \sum_{A \in D^\Theta} m(A) = 1 \tag{11}$$

The general belief function and plausibility function are define as the same manner within the DST respectively,

$$Bel(A) = \sum_{B \in D^\Theta, B \subseteq A} m(B) \tag{12}$$

$$Pl(A) = \sum_{B \in D^\Theta, B \cap A \neq \emptyset} m(B) \tag{13}$$

Let $\mathcal{M}^f(\Theta)$ be a free DSm model. The classical (free) DSm rule of combination (denoted (DSmC) for short) for $k \geq 2$ sources is given $\forall A \neq \emptyset$, and $A \in D^\Theta$ as follows:

$$m_{\mathcal{M}^f(\Theta)}(A) = [m_1 \oplus \cdots \oplus m_k](A) = \sum_{\substack{X_1,\cdots,X_k \in D^\Theta \\ X_1 \cap \cdots \cap X_k = A}} \prod_{i=1}^{k} m_i(X_i) \tag{14}$$

2.6. Deng entropy

With the range of uncertainty mentioned above, Deng entropy [7] can be presented as follows

$$E_d = -\sum_i m(F_i) \log \frac{m(F_i)}{2^{|F_i|} - 1} \tag{15}$$

where, F_i is a proposition in mass function m, and $|F_i|$ is the cardinality of F_i. As shown in the above definition, Deng entropy, formally, is similar with the classical Shannon entropy, but the belief for each proposition F_i is divided by a term $(2^{|F_i|} - 1)$ which represents the potential number of states in F_i (of course, the empty set is not included).

Specially, Deng entropy can definitely degenerate to the Shannon entropy if the belief is only assigned to single elements. Namely,

$$E_d = -\sum_i m(\theta_i) \log \frac{m(\theta_i)}{2^{|\theta_i|} - 1} = -\sum_i m(\theta_i) \log m(\theta_i)$$

In this section 3, the generalization of Deng entropy in hyper-power sets and super-power sets are denoted in definition 3.2 and definition 3.1.

3. Deng entropy in hyper-power sets and super-power sets

Firstly, the definition of the generalized entropy is described as follow

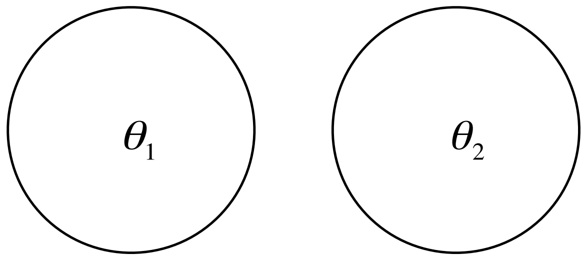

Figure 4: Two exclusively focal elements

3.1. Deng entropy in hyper-power set

Definition 3.1. *Assume F_i is the focal element and $m(F_i)$ is the basic probability assignment for F_i, then the possible generalization of Deng entropy in hyper-power set can be defined as H_h*

$$H_h = -\sum_i m\left(F_i\right) \log \left(\frac{m\left(F_i\right)}{\left|D^{F_i}\right| - 1} \right) \tag{16}$$

where $i = 1, 2, ..., \left|D^X\right| - 1$ (X is the scale of the frame of discernment), $D^X$is Dedekind's lattice.

Example 3. *Assume the frame $\Theta = \{\theta_1, \theta_2\}$, $\theta_1 \cap \theta_2 = \emptyset$, and $m_1\left(\theta_1\right) = m_1\left(\theta_2\right) = m_1\left(\theta_1 \cup \theta_2\right) = \frac{1}{3}$, just as Figure 4, then $D^{\Theta=\{\theta_1,\theta_2\}}$ is given by*

$$D^\Theta = \{\emptyset, \theta_1 \cap \theta_2, \theta_1, \theta_2, \theta_1 \cup \theta_2\} \tag{17}$$

For $\theta_1 \cap \theta_2 = \emptyset$, the super-power set is simplified as

$$D^\Theta = \{\emptyset, \theta_1, \theta_2, \theta_1 \cup \theta_2\} \tag{18}$$

$$m_1(\theta_1) = {}^1\!/_3 \tag{19}$$

$$m_1(\theta_2) = {}^1\!/_3 \tag{20}$$

$$m_1(\theta_1 \cup \theta_2) = {}^1\!/_3 \tag{21}$$

$$\begin{aligned}
&H_h(m_1) = -\sum_i m(F_i)\log\left(\frac{m(F_i)}{\left|D^{F_i}\right|-1}\right) \\
&= -\tfrac{1}{3}\cdot\log_2\left(\frac{{}^1\!/_3}{1}\right) - \tfrac{1}{3}\cdot\log_2\left(\frac{{}^1\!/_3}{1}\right) - \tfrac{1}{3}\cdot\log_2\left(\frac{{}^1\!/_3}{4}\right) \\
&= 2.2516
\end{aligned}$$

$$\begin{aligned}
&D(m_1) = -\sum_i m(F_i)\log\left(\frac{m(F_i)}{2^{|F_i|}-1}\right) \\
&= -\tfrac{1}{3}\cdot\log_2\left(\frac{{}^1\!/_3}{1}\right) - \tfrac{1}{3}\cdot\log_2\left(\frac{{}^1\!/_3}{1}\right) - \tfrac{1}{3}\cdot\log_2\left(\frac{{}^1\!/_3}{3}\right) \\
&= 1.5850
\end{aligned}$$

Hence,

$$\mathrm{H}_h(m_1) > D(m_1) \tag{22}$$

Example 4. *Assume the frame* $\Theta = \{\theta_1, \theta_2\}$*,* $\theta_1 \cap \theta_2 \neq \emptyset$*, and* $m_1(\theta_1) = m_1(\theta_2) = m_1(\theta_1 \cup \theta_2) = m_1(\theta_1 \cap \theta_2) = \frac{1}{4}$*, just as Figure 5, then* $D^{\Theta=\{\theta_1,\theta_2\}}$ *is given by*

$$D^{\Theta} = \{\emptyset, \theta_1 \cap \theta_2, \theta_1, \theta_2, \theta_1 \cup \theta_2\} \tag{23}$$

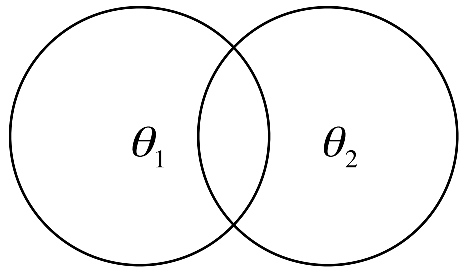

Figure 5: Two focal elements with intersection

$$m_2\left(\theta_1\right) = {}^1\!/_4 \tag{24}$$

$$m_2\left(\theta_2\right) = {}^1\!/_4 \tag{25}$$

$$m_2\left(\theta_1 \cup \theta_2\right) = {}^1\!/_4 \tag{26}$$

$$m_2\left(\theta_1 \cap \theta_2\right) = {}^1\!/_4 \tag{27}$$

$$\begin{aligned}
&H_h(m_2) = -\sum_i m\left(F_i\right) \log \left(\frac{m(F_i)}{\left|D^{F_i}\right| - 1} \right) \\
&= -\tfrac{1}{4} \cdot \log_2 \left(\frac{{}^1\!/_4}{1} \right) - \tfrac{1}{4} \cdot \log_2 \left(\frac{{}^1\!/_4}{1} \right) - \tfrac{1}{4} \cdot \log_2 \left(\frac{{}^1\!/_4}{4} \right) - \tfrac{1}{4} \cdot \log_2 \left(\frac{{}^1\!/_4}{4} \right) \\
&= 3
\end{aligned}$$

Here, we can not get the entropy of m_1 *with the Deng entropy by Eq. (15) for the elements are not exclusive.*

3.2. Deng entropy in super-power set

Definition 3.2. *Assume* F_i *is the focal element and* $m(F_i)$ *is the basic probability assignment for* F_i*, then the possible generalization of Deng entropy in* *super-power set can be defined as* H_s

$$H_s = -\sum_i m\left(F_i\right) \log\left(\frac{m\left(F_i\right)}{2^{2^{|F_i|-1}} - 1}\right) \tag{28}$$

where $i = 1, 2, ..., 2^{2^{|X|-1}} - 1$ *(X is the scale of the frame of discernment).*

Example 5. *Assume the frame* $\Theta = \{\theta_1, \theta_2\}$*,* $\theta_1 \cap \theta_2 = \emptyset$*, and* $m_3\left(\theta_1\right) = m_3\left(\theta_2\right) = m_3\left(\theta_1 \cup \theta_2\right) = \frac{1}{3}$*, just as Figure 4, then* $S^{\Theta = \{\theta_1, \theta_2\}}$ *is given by*

$$S^{\Theta} = \{\emptyset, \theta_1 \cap \theta_2, \theta_1, \theta_2, \theta_1 \cup \theta_2, c\left(\emptyset\right), c\left(\theta_1 \cap \theta_2\right), c\left(\theta_1\right), c\left(\theta_2\right), c\left(\theta_1 \cup \theta_2\right)\} \tag{29}$$

Since $c\left(\emptyset\right) = \theta_1 \cup \theta_2$ *and* $c\left(\theta_1 \cup \theta_2\right) = \emptyset$*, the super-power set is actually given by*

$$S^{\Theta} = \{\emptyset, \theta_1 \cap \theta_2, \theta_1, \theta_2, \theta_1 \cup \theta_2, c\left(\theta_1 \cap \theta_2\right), c\left(\theta_1\right), c\left(\theta_2\right)\} \tag{30}$$

For $\theta_1 \cap \theta_2 = \emptyset$*,* $c\left(\theta_1 \cap \theta_2\right) = \theta_1 \cup \theta_2$*,* $c\left(\theta_1\right) = \theta_2$*, and* $c\left(\theta_2\right) = \theta_1$*, the super-power set is simplified as*

$$S^{\Theta} = \{\emptyset, \theta_1, \theta_2, \theta_1 \cup \theta_2\} \tag{31}$$

$$m_3\left(\theta_1\right) = {}^1\!/_3 \tag{32}$$

$$m_3\left(\theta_2\right) = {}^1\!/_3 \tag{33}$$

$$m_3\left(\theta_1 \cup \theta_2\right) = {}^1\!/_3 \tag{34}$$

$$H_s(m_3) = -\sum_i m\left(F_i\right)\log\left(\frac{m(F_i)}{2^{2^{|F_i|-1}-1}}\right)$$
$$= -\tfrac{1}{3}\cdot\log_2\left(\frac{1/3}{1}\right) - \tfrac{1}{3}\cdot\log_2\left(\frac{1/3}{1}\right) - \tfrac{1}{3}\cdot\log_2\left(\frac{1/3}{7}\right)$$
$$= 2.5207$$
$$D(m_3) = -\sum_i m\left(F_i\right)\log\left(\frac{m(F_i)}{2^{|F_i|}-1}\right)$$
$$= -\tfrac{1}{3}\cdot\log_2\left(\frac{1/3}{1}\right) - \tfrac{1}{3}\cdot\log_2\left(\frac{1/3}{1}\right) - \tfrac{1}{3}\cdot\log_2\left(\frac{1/3}{3}\right)$$
$$= 1.5850$$

Hence

$$H_s\left(m_3\right) > D\left(m_3\right) \tag{35}$$

Example 6. *Assume the frame* $\Theta = \{\theta_1, \theta_2\}$, $\theta_1 \cap \theta_2 \neq \emptyset$, *and* $m_4\left(\theta_1\right) = m_4\left(\theta_2\right) = \frac{1}{7}$, $m_4\left(\theta_1 \cup \theta_2\right) = m_4\left(\theta_1 \cap \theta_2\right) = \frac{1}{7}$, $m_4\left(c(\theta_1)\right) = m_4\left(c(\theta_2)\right) = m_4\left(c(\theta_1 \cap \theta_2)\right) = \frac{1}{7}$ *just as Figure 5, then* $S^{\Theta=\{\theta_1,\theta_2\}}$ *is given by*

$$S^{\Theta} = \{\emptyset, \theta_1 \cap \theta_2, \theta_1, \theta_2, \theta_1 \cup \theta_2, c\left(\emptyset\right), c\left(\theta_1 \cap \theta_2\right), c\left(\theta_1\right), c\left(\theta_2\right), c\left(\theta_1 \cup \theta_2\right)\} \tag{36}$$

Since $c\left(\emptyset\right) = \theta_1 \cup \theta_2$ *and* $c\left(\theta_1 \cup \theta_2\right) = \emptyset$, *the super-power set is actually given by*

$$S^{\Theta} = \{\emptyset, \theta_1 \cap \theta_2, \theta_1, \theta_2, \theta_1 \cup \theta_2, c\left(\theta_1 \cap \theta_2\right), c\left(\theta_1\right), c\left(\theta_2\right)\} \tag{37}$$

$$m_4\left(\theta_1\right) = m_4\left(\theta_2\right) = \frac{1}{7} \tag{38}$$
$$m_4\left(\theta_1 \cup \theta_2\right) = m_4\left(\theta_1 \cap \theta_2\right) = \frac{1}{7} \tag{39}$$
$$m_4\left(c(\theta_1)\right) = m_4\left(c(\theta_2)\right) = m_4\left(c(\theta_1 \cap \theta_2)\right) = \frac{1}{7} \tag{40}$$

$$
\begin{aligned}
H_s &= -\sum_i m\left(F_i\right)\log\left(\frac{m(F_i)}{2^{2^{|F_i|}-1}-1}\right)\\
&= -\tfrac{1}{7}\cdot\log_2\left(\frac{1/7}{1}\right)-\tfrac{1}{7}\cdot\log_2\left(\frac{1/7}{1}\right)\\
&-\tfrac{1}{7}\cdot\log_2\left(\frac{1/7}{2^{2^2-1}-1}\right)-\tfrac{1}{7}\cdot\log_2\left(\frac{1/7}{2^{2^2-1}-1}\right)\\
&-\tfrac{1}{7}\cdot\log_2\left(\frac{1/7}{1}\right)-\tfrac{1}{7}\cdot\log_2\left(\frac{1/7}{1}\right)\\
&-\tfrac{1}{7}\cdot\log_2\left(\frac{1/7}{2^{2^2-1}-1}\right)=4.0105
\end{aligned}
$$

4. Conclusion

Hyper power set and super power set consider more flexible relationship of the element, and there are no exclusive constraint for the elements in hyper power set and super power set, which can describe more uncertain information. In this paper, the frame of Deng entropy in hyper-power sets and super-power sets is proposed to measure the uncertainty degree of them, to be suitable for more uncertain and more flexible information.

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References

[1] R. Clausius, The mechanical theory of heat: with its applications to the steam-engine and to the physical properties of bodies, J. van Voorst, 1867.

[2] C. E. Shannon, A mathematical theory of communication, ACM SIGMOBILE Mobile Computing and Communications Review 5 (1) (2001) 3–55.

[3] C. Tsallis, Possible generalization of Boltzmann-Gibbs statistics, Journal of Statistical Physics 52 (1-2) (1988) 479–487.

[4] C. Tsallis, et al., Introduction to nonextensive statistical mechanics, Springer, 2009.

[5] C. Tsallis, U. Tirnakli, Nonadditive entropy and nonextensive statistical mechanics–some central concepts and recent applications, in: Journal of Physics: Conference Series, Vol. 201, 2010, p. 012001.

[6] J. L. Lebowitz, Boltzmann's entropy and time's arrow, Physics today 46 (1993) 32–32.

[7] Y. Deng, Deng entropy, Chaos, Solitons & Fractals 91 (2016) 549–553.

[8] K. Yuan, F. Xiao, L. Fei, B. Kang, Y. Deng, Conflict management based on belief function entropy in sensor fusion, SpringerPlus 5 (1) (2016) 1–12.

[9] A. P. Dempster, Upper and lower probabilities induced by a multivalued mapping, Annals of Mathematics and Statistics 38 (2) (1967) 325–339.

[10] G. Shafer, A Mathematical Theory of Evidence, Princeton University Press, Princeton, 1976.

[11] P. Smets, R. Kennes, The transferable belief model, Artificial Intelligence 66 (2) (1994) 191–234.

[12] J. Abellán, Éloi Bossé, Drawbacks of uncertainty measures based on the pignistic transformation, IEEE Transactions on Systems, Man, and Cybernetics: Systems (2016) 1–7doi:10.1109/TSMC.2016.2597267.

[13] W. Jiang, B. Wei, C. Xie, D. Zhou, An evidential sensor fusion method in fault diagnosis, Advances in Mechanical Engineering 8 (3) (2016) 1–7.

[14] W. Jiang, C. Xie, B. Wei, D. Zhou, A modified method for risk evaluation in failure modes and effects analysis of aircraft turbine rotor blades, Advances in Mechanical Engineering 8 (4) (2016) 1–16. doi:10.1177/1687814016644579.

[15] J. Schubert, Conflict management in Dempster-Shafer theory using the degree of falsity, International Journal of Approximate Reasoning 52 (3) (2011) 449–460.

[16] B. Kang, Y. Deng, R. Sadiq, S. Mahadevan, Evidential cognitive maps, Knowledge-Based Systems 35 (2012) 77–86.

[17] J. Dezert, F. Smarandache, Presentation of dsmt, Advances and applications of DSmT for information fusion (2004) 3–35.

[18] J. Dezert, F. Smarandache, An introduction to dsmt for information fusion, New Mathematics and Natural Computation 8 (03) (2012) 343–359.

[19] F. Smarandache, J. Dezert, Advances and Applications of DSmT for Information Fusion (Collected works), second volume: Collected Works, Vol. 2, Infinite Study, 2006.