Sieve of Collatz

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Abstract

The sieve of Collatz is a new algorithm to trace back the non-linear Collatz problem to a linear cross out algorithm. Until now it is unproved.

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1 Introduction

The first view to collatz is simply to test the standard Collatz algorithm for integer numbers. To show that there is no other cycle and that every number gets to 1. To do that for infinite numbers it is impossible. So we have a look if there are numbers which can be crossed out. The simpliest consideration is to cross out the even numbers because they all will be united with the odd numbers. This will be the beginning of that special sieve of collatz. [**1**, **2**, **3**]

2 4k+3 unites with 4k+1

2.1 Sequences of 4k+3

First we have a look at the numbers of 4k+3. It can be shown that these numbers can be united with numbers of the form 4k+1. First we make a list. The first column consists of numbers of the form 4k+3. Now we starting the standard collatz algorithm horizontaly but only with the odd numbers. It stopps with the first number which is smaller than the predecessor. Lets have a look (Table 1):

2.2 Crossing out

Now it can be seen that the Sequences with more than 3 elements unites with the sequences with exactly 3 elements (unproved). It is a stringent pattern. So we can cross out the sequences with more than 3 elements and we get this following table 2 of numbers of the form 8k+3:

2.3 4k+3 unites with 8k+1

Now it can be seen that the second column consists of numbers of the form 8k+1. 8k+1 is element of 4k+1. So we now cross out the complete first column, and we have shown that all numbers of the form 4k+3 are united with numbers of the form 4k+1 (Table 3):

Table 1: Sequences of 4k+3

4k+3							
3	5	1					
7	11	17	13				
11	17	13					
15	23	35	53	5			
19	29	11					
23	35	53	11				
27	41	31					
31	47	71	107	161	121		
35	53	5					
39	59	89	67				
43	65	49					
47	71	107	161	121			
51	77	29					
55	83	125	47				
59	89	67					
63	95	143	215	323	485	91	
67	101	19					
71	107	161	121				
75	113	85					
79	119	179	269	101			
83	125	47					
87	131	197	37				
91	137	103					
95	143	215	323	485	91		
99	149	7					
103	155	233	175				
107	161	121					
111	167	251	377	283			
115	173	65					
119	179	269	101				
123	185	139					
127	191	287	431	647	971	1475	1093

Table 2: Sequences of 8k+3

8k+3		
3	5	1
11	17	13
19	29	11
27	41	31
35	53	5
43	65	49
51	77	29
59	89	67
67	101	19
75	113	85
83	125	47
91	137	103
99	149	7
107	161	121
115	173	65
123	185	139

Table 3: Sequences of 8k+1

8k+1	
5	1
17	13
29	11
41	31
53	5
65	49
77	29
89	67
101	19
113	85
125	47
137	103
149	7
161	121
173	65
185	139

Table 4: Sequences of 4k+1

4k+1	A067745
1	
5	1
9	7
13	5
17	13
21	1
25	19
29	11
33	25
37	7
41	31
45	17
49	37
53	5
57	43
61	23
65	49
69	13
73	55
77	29
81	61
85	1
89	67
93	35
97	73
101	19
105	79
109	41
113	85
117	11
121	91
125	47

3 Sieve of Collatz

3.1 Numbers of the form 4k+1

Lets have a look at the numbers of the form 4k+1. Here we can show that there is a way to unite these numbers with three different steps. These three steps are the sieve of collatz. In the first column we list the numbers of 4k+1 and the second column consists of the next odd number. It is interessting that the second column is build by the OEIS Sequence A067745 [**4**](Table 4).

Table 5:

A067745					
1					
7	11	17	13		
5	1				
13	5				
1					
19	29	11			
11	17	13			
25	19				
7	11	17	13		
31	47	71	107	161	121
17	13				
37	7				
5	1				
43	65	49			
23	35	53	5		
49	37				
13	5				
55	83	125	47		
29	11				
61	23				
1					
67	101	19			
35	53	5			
73	55				
19	29	11			
79	119	179	101		
41	31				
85	1				
11	17	13			
91	137	103			
47	71	107	161	121	

3.2 First step: Build new sequences

The first step of the sieve of collatz is now to cross out the complete first column and to build new sequences with the remaining numbers. Like before these sequences consists of only odd elements and are stopping with elements that are smaller than the predecessor (Table 5).

3.3 Second step: Crossing out same sequences

The second step is to cross out the sequences that appear more than one time. It shows a clear pattern that every fourth sequence will be crossed out (Table 6).

Table 6:

1					
7	11	17	13		
5	1				
13	5				
19	29	11			
11	17	13			
25	19				
31	47	71	107	161	121
17	13				
37	7				
43	65	49			
23	35	53	5		
49	37				
55	83	125	47		
29	11				
61	23				
67	101	19			
35	53	5			
73	55				
79	119	179	101		
41	31				
85	1				
91	137	103			
47	71	107	161	121	

Table 7:

1	
5	1
13	5
25	19
17	13
37	7
49	37
29	11
61	23
73	55
41	31
85	1

3.4 Third step: Crossing out sequences with more than 2 elements

The third step is to cross out the sequences with more than 2 elements. It can be shown that all these sequences with more than 2 elements unite with sequences of exactly two elements(Table 7)(unproved!). It is similiar to the step in subsection 2.1. Now we can repeat the three steps until the numbers of the intervall 1-125 disappear.

While the growth of the interval is exponentially, the number of steps of the sieve algorithm remains linear(Table 8).

Table 8: Expansion of steps

Interval	Steps
2^3	4
2^4	7
2^5	10
2^6	13
2^7	13
2^8	16
2^9	19
2^{10}	21

4 Summary

I have come to the conclusion, that this algorithm is helping us to solve the Collatz problem. This algorithm is only proved empirically, but i am convinced that it can be proved theoretically. It might be useful to build a code to check this algorithm empirically for more numbers than 2^{10}.

5 Acknowledgement

Thanks to the Matheplanet.

References

[1] *https://en.wikipedia.org/wiki/Collatz_conjecture*

[2] C. J. Everett Iteration of the number-theoretic function f(2n)=n, f(2n+1)=3n+2, Advances in Mathematics 25,42-45 (1977)

[3] Jeffrey C. Lagarias, The 3x+1 Problem and Its Generalizations, The American Mathematical Monthly,3-23, (1985)

[4] *https://oeis.org/*