Article

Equivalent Dirac Equation in 24 Irreducible Representations with Symmetry Violations Including CPT-Symmetry Violation

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Abstract

We demonstrate that if one adheres to a method akin to Dirac's method of arriving at the Dirac equation – then, the Dirac equation is not the only equation that one can generate but that there is a whole new twenty four equations that Dirac left out. Off these new equations – interesting is that; some of them violate C, P, T, CT, CP, PT and CPT-symmetry. If these equations are acceptable on the basis of them flowing from the widely – if not universally accepted Dirac prescription, then, the great riddle of why the preponderance of matter over antimatter might find a solution.

Keywords: Dirac equation, Matter Dominance, C Violation, CP Violation, CPT Violation.

"*The suppression of uncomfortable ideas may be common in religion and politics,*
but it is not the path to knowledge;
it has no place in the endeavor of science."

– **Carl Edward Sagan** (1934 – 1996)

1 Introduction

The Dirac (1928*a*,*b*) equation – discovered by the great British theoretical physicist, Paul Adrien Maurice Dirac (1904 – 1984); ranks amongst the greatest achievements of the human mind. Amongst its towering and touchstone achievements, Dirac (1930) used this equation to predict the existence of antimatter *i.e.*, the Position in this instance. In an unparalleled triumphant moment, the Positron was experimentally discovered in 1932 (by C. D. Anderson 1933). It has the same spin as the Electron but opposite electrical charge. For this monumental achievement – Carl David Anderson (1905 – 1991), was awarded the 1936 *Nobel Prize in Physics* "*…for his discovery of the Positron.*". After the direct detection of the Positron, the equally great German theoretical physicist – Werner Karl Heisenberg (1901 – 1976), who made major contributions to quantum theory; spoke glowingly of Dirac's theoretical discovery of antimatter as "*perhaps the biggest change of all the big changes in physics of our century … because it changed our whole picture of matter …*".

As a whole, the Dirac (1928*a*,*b*, 1930) theory predicts a perfect symmetry between matter and antimatter, that is to say – there must exist an equal amount of matter and antimatter in the Universe. Despite its esoteric grandeur and beauty, this prediction of the Dirac theory is in contempt of *Nature* as this prediction is completely at variance with physical and natural reality. The Universe has a clear preponderance of matter with minuscule quantities of antimatter which is produced in high energy particle interactions in the deep interiors of stars. This death of Dirac's theory has long been known and is considered a serious desideratum against observational evidence that the beautiful, esoteric and noble Dirac equation *via* its clear observance of C-symmetry, predicts a perfectly symmetric world with respect

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to matter and antimatter. Actually – if this where the case – that the Universe contained equal amounts of matter and antimatter – then, the Universe as we know it would be nothing other than a bath of radiation because matter and antimatter would annihilate to form radiation. Yes, the Universe does contain radiation but more than it contains radiation, it contains matter with little – *if any* – traces of antimatter. This evidence points to an asymmetrical world between matter and antimatter.

Since this problem was noticed, it has been considered a "*Holy Grail*" of particle physics. Current effort to solve it – are that CP-symmetry violation maybe the master-key to the resolution of these riddle and this thinking follows from the 1967 seminal work of the great Soviet nuclear physicist – Andrei Dimitriev Sakharov ($1921 - 1989$). Sakhorov (1967) described three minimum properties of *Nature* which are required for any *baryogenesis*[2] to occur, regardless of the exact mechanism leading to the excess of baryonic matter. In his seminal paper, Sakhorov (1967) did not list the conditions explicitly. Instead, he described the evolution of a Universe which goes from a Baryon-excess ($\mathcal{B}$-excess) while contracting in a *Big Crunch* to an anti-$\mathcal{B}$-excess after the resultant *Big Bang*. In summary, his three key assumptions are now known as they *Sakharov Conditions*, and these are:

1. At least one $\mathcal{B}$-*number violating* process.
2. C and CP-*violating* processes.
3. Interactions outside of thermal equilibrium.

These conditions must be met by any explanation in which ($\mathcal{B} = 0$) during the *Big Bang* but is very high in the present day. They are necessary but not sufficient – thus scientists seeking an explanation of the currently obtaining matter asymmetry on this basis (Sakhorov conditions) must describe the specific mechanism through which baryogenesis happens. Much theoretical work in cosmology and high-energy physics revolves around finding physical processes and mechanism which fit the three Sakhorov preconditions and correctly predicting the observed baryon density.

Therefore, as already said, the current thrust in research especially at *CERN*[3] is to search for physical processes in *Nature* that violate CP-*symmetry*. In 2011 during high-energy Proton collisions in the LHCb experiment (Aaij et al. 2013), scientists working at *CERN* created B_s^0 mesons – *i.e.* hadronic subatomic particles comprised of one quark and one antiquark – inside the LHCb experiment (Aaij et al. 2013) and this experiment seems to have yielded some very interesting results insofar as the Sakhorov conditions are concerned. By observing the rapid decay of the B_s^0, physicists of the *LHCb-Collaboration* (Aaij et al. 2013) were able to identify the neutral particle's decay products - *i.e.* the particles that it decayed into. After a significantly large number of Proton collisions and B_s^0 decay events, the *LHCb-Collaboration* (Aaij et al. 2013) concluded that more matter particles where generated than antimatter during neutral B_s^0 decays.

The first violations of CP-symmetry was first documented in Brookhaven Laboratory in the US in the 1960*s* in the decay of neutral Kaon particles. Since then, Japanese and US labs forty years later found similar behaviour in B^0-mesons systems where they detected similar CP-*symmetry violations*. *LHCb-Collaboration* (Aaij et al. 2013) results indicating that antimatter decays at a faster rate than antimatter only come in as further supporting evidence and from a Sakhorov (1967) standpoint, these observations certainly provide key insights into the problem of the preponderance of matter over antimatter.

Herein, we derive an irreducible Dirac equation that can be written in 24 representations and – with these new 24 Dirac equations – we have not C and CP violating processes, but C and CP violating *Physical Laws*, that is, equally legitimate *Laws of Physics* that predict an asymmetry in the Universe. Actually, a C-symmetry violating *Physical Law* is enough to explain the present imbalance of matter and antimatter. Ideally, what the 24 equations really mean is that, there must exist 24 types of Electrons.

[2] *Baryogenesis* is the generic term for the hypothetical physical processes that produced an asymmetry (imbalance) between baryons and antibaryons produced in the very early universe.

[3] European Organization for Nuclear Research (CERN) is located at the France-Swiss border near Geneva Swirtherland.

Our line of thinking in the resolution of the great riddle of why the imbalance between matter and antimatter, is that, we may need to search for `C`-violating *Physical Laws* as has been conducted herein and also in the reading Nyambuya (2015) – where the asymmetries (`C`, `P`, `T` and their combinations) exhibited by the proposed Curved Spacetime Dirac equations (Nyambuya 2008, 2013), have, as has been conducted here, been used to suggest a solution to this long standing riddle of *why the preponderance of matter over antimatter*.

More importantly we have the `CPT`-symmetry, where it is seen that twelve of the new twenty three Dirac equations violate this seemingly sacrosanct, inviolable and one of most basic precepts of particle physics – `CPT`-symmetry, which is considered (see *e.g.* Kostelecky 1998, Greenberg 2002, Villata 2011, Stadnik et al. 2014) to be an exact symmetry of *Nature*. To sanctify this symmetry, there is even the embellished `CPT` *Theorem* (Schwinger 1951, Lüders 1954, Pauli et al. 1955), which holds that `CPT`-symmetry holds for all physical phenomena, or more precisely, that: that any Lorentz invariant local Quantum Field Theory (QFT) with a Hermitian Hamiltonian must have `CPT`-symmetry. Spelt-out more clearly, the three prerequisite conditions for any physical theory to obey `CPT`-symmetry, are:

1. *Lorentz Invariance.*
2. *Locality.*
3. *Hermiticity of the Hamiltonian.*

All these three conditions the present twenty three new Dirac equations do meet them – *yet* – twelve of them go on to violate this important symmetry. We shall present and leave it to the reader to ponder on the meaning of these non-fundamental representations of the Dirac equation.

2 Original Dirac Equation

In this section – for instructive purposes, and for completeness, we formally present the original Dirac equation. That is, the Dirac equation is given by:

$$(i\hbar\gamma^\mu\partial_\mu - \mathcal{I}_4\mathrm{m}_0c)\,\psi = 0, \tag{2.1}$$

where $\mathcal{I}_4$ is the 4×4 identity matrix, ψ is the Dirac four component wavefunction and:

$$\gamma^0 = \left(\begin{array}{cc} \mathcal{I}_2 & 0 \\ 0 & -\mathcal{I}_2 \end{array}\right), \quad \gamma^i = \left(\begin{array}{cc} 0 & \sigma^i \\ -\sigma^i & 0 \end{array}\right), \tag{2.2}$$

are the 4×4 Dirac gamma matrices with $\mathcal{I}_2$ and 0 being the 2×2 identity and null matrices respectively; $\sigma^j : j = (1,2,3)$, are the usual 2×2 Pauli matrices. Throughout this reading, the Greek indices will be understood to mean $(\mu,\nu,... = 0,1,2,3)$ and lower case English alphabet indices $(i,j,k... = 1,2,3)$.

Dirac's γ-matrices satisfy the following equation:

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2\mathcal{I}_4\eta^{\mu\nu}, \tag{2.3}$$

where $\eta^{\mu\nu}$ is the Minkowski metric of flat spacetime *i.e.*:

$$\eta^{\mu\nu} := \left(\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right). \tag{2.4}$$

3 Dirac's Classic Derivation

As is now common knowledge – in deriving (arriving at) his equation, Dirac sought an equation that is first order in the spacetime derivatives (∂_μ), which, upon 'squaring' would give the Klein-Gordon equation. To that end, he first wrote down the future Dirac equation in the Schröndinger formalism as:

$$(\mathcal{H}-\mathcal{E})\,\psi=0, \tag{3.1}$$

where the Dirac Hamiltonian operator $\mathcal{H}$ and the energy operator $\mathcal{E}$ are defined such that:

$$\mathcal{H}=i\hbar\gamma^0\gamma^k\partial_k-\gamma^0\mathrm{m}_0c \text{ and } \mathcal{E}=-i\hbar\mathcal{I}_4\frac{\partial}{\partial t}. \tag{3.2}$$

He demanded of $\mathcal{H}$ and $\mathcal{E}$ to be such that if one multiplied (3.1) from the left by $(\mathcal{H}-\mathcal{E})$, the must obtained the usual Klein-Gordon equation, *i.e.*:

$$(\mathcal{H}+\mathcal{E})(\mathcal{H}-\mathcal{E})\psi=\left(\mathcal{H}^2+[\mathcal{H},\mathcal{E}]-\mathcal{E}^2\right)\psi=0, \tag{3.3}$$

where:

$$\mathcal{H}^2=\mathrm{m}_0^2c^2-\hbar^2\partial^k\partial_k, \tag{3.4}$$

$$\mathcal{E}^2=-\hbar^2\mathcal{I}_4\frac{\partial^2}{\partial t^2}, \tag{3.5}$$

$$[\mathcal{H},\mathcal{E}]=\mathcal{H}\mathcal{E}-\mathcal{E}\mathcal{H}=0. \tag{3.6}$$

In this way, the resulting equation $\left[\left(\mathcal{H}^2-\mathcal{E}^2\right)\psi=0\right]$, would give the desired Klein-Gordon equation, *i.e.*:

$$\left(\mathcal{H}^2-\mathcal{E}^2\right)\psi=0 \qquad \Rightarrow \qquad \hbar^2c^2\boldsymbol{\nabla}^2\psi-\hbar^2\frac{\partial^2\psi}{\partial t^2}-\mathrm{m}_0^2c^4\psi=0 \tag{3.7}$$

What we are now going to demonstrate below – is that, using the same prescription as Dirac, *albeit*, with a slight modification; the Dirac equation is just one in a set of 24 possible Dirac equations. Written in the usual form as given in equation (2.1), the Dirac equation is such that:

$$\left(i\hbar\gamma^\mu\partial_\mu+\mathcal{I}_4\mathrm{m}_0c\right)\left(i\hbar\gamma^\mu\partial_\mu-\mathcal{I}_4\mathrm{m}_0c\right)\psi=0 \implies \left[\hbar^2\Box-\mathrm{m}_0^2c^4\right]\psi=0, \tag{3.8}$$

where $\Box=\eta^{\mu\nu}\partial_\mu\partial_\nu$, is the usual *D'Ambelet operator*.

4 Representations of the Dirac Equation

In our new approach, the Hamiltonian has 24 representations and because of this, we shall add a new index-ℓ so that it is now represented as $\mathcal{H}_\ell:\ell=(1,2,\ldots,24)$. That said, we are to demanded of $\mathcal{H}_\ell$ and $\mathcal{E}$ to be such that a multiplication of (3.1) from the left by $(\mathcal{H}_\ell-\mathcal{E})^\dagger$, we must – as in the case of Dirac – obtain the Klein-Gordon equation, *i.e.*:

$$(\mathcal{H}_\ell-\mathcal{E})^\dagger(\mathcal{H}_\ell-\mathcal{E})\psi=\left(\mathcal{H}_\ell^\dagger\mathcal{H}_\ell-\left\{\mathcal{H}_\ell^\dagger,\mathcal{E}\right\}+\mathcal{E}^\dagger\mathcal{E}\right)\psi=0, \tag{4.1}$$

where now we will require $\mathcal{H}_\ell^\dagger\mathcal{H}_\ell$, $\mathcal{E}^\dagger\mathcal{E}$ and $\left\{\mathcal{H}_\ell^\dagger,\mathcal{E}\right\}$, to be such that:

$$\mathcal{H}_\ell^\dagger\mathcal{H}_\ell=|\mathcal{H}_\ell|^2=\mathrm{m}_0^2c^2-\hbar^2\partial^k\partial_k, \tag{4.2}$$

$$\mathcal{E}^\dagger\mathcal{E}=|\mathcal{E}|^2=\hbar^2\mathcal{I}_4\frac{\partial^2}{\partial t^2}, \tag{4.3}$$

Table 1: **List of the 24 $\mathcal{U}_\ell$-Matrices:**

List of the 24 $\mathcal{U}_\ell$-Matrices				
$\mathcal{U}_1 = \begin{pmatrix} \mathrm{I} & 0 \\ 0 & \mathrm{I} \end{pmatrix}$	$\mathcal{U}_2 = \begin{pmatrix} \sigma^1 & 0 \\ 0 & \sigma^1 \end{pmatrix}$	$\mathcal{U}_3 = \begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}$	$\mathcal{U}_4 = \begin{pmatrix} \sigma^3 & 0 \\ 0 & \sigma^3 \end{pmatrix}$	Group (I)
$\mathcal{U}_5 = \begin{pmatrix} \mathrm{I} & 0 \\ 0 & -\mathrm{I} \end{pmatrix}$	$\mathcal{U}_6 = \begin{pmatrix} \sigma^1 & 0 \\ 0 & -\sigma^1 \end{pmatrix}$	$\mathcal{U}_7 = \begin{pmatrix} \sigma^2 & 0 \\ 0 & -\sigma^2 \end{pmatrix}$	$\mathcal{U}_8 = \begin{pmatrix} \sigma^3 & 0 \\ 0 & -\sigma^3 \end{pmatrix}$	Group (II)
$\mathcal{U}_9 = \begin{pmatrix} 0 & \mathrm{I} \\ \mathrm{I} & 0 \end{pmatrix}$	$\mathcal{U}_{10} = \begin{pmatrix} 0 & \sigma^1 \\ \sigma^1 & 0 \end{pmatrix}$	$\mathcal{U}_{11} = \begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix}$	$\mathcal{U}_{12} = \begin{pmatrix} 0 & \sigma^3 \\ \sigma^3 & 0 \end{pmatrix}$	Group (III)
$\mathcal{U}_{13} = i\begin{pmatrix} 0 & \mathrm{I} \\ -\mathrm{I} & 0 \end{pmatrix}$	$\mathcal{U}_{14} = i\begin{pmatrix} 0 & \sigma^1 \\ -\sigma^1 & 0 \end{pmatrix}$	$\mathcal{U}_{15} = i\begin{pmatrix} 0 & \sigma^2 \\ -\sigma^2 & 0 \end{pmatrix}$	$\mathcal{U}^{16} = i\begin{pmatrix} 0 & \sigma^3 \\ -\sigma^3 & 0 \end{pmatrix}$	Group (IV)
$\mathcal{U}_{17} = \frac{1}{\sqrt{2}}\begin{pmatrix} -\mathrm{I} & \mathrm{I} \\ \mathrm{I} & \mathrm{I} \end{pmatrix}$	$\mathcal{U}_{18} = \frac{1}{\sqrt{2}}\begin{pmatrix} -\sigma^1 & \sigma^1 \\ \sigma^1 & \sigma^1 \end{pmatrix}$	$\mathcal{U}_{19} = \frac{1}{\sqrt{2}}\begin{pmatrix} -\sigma^2 & \sigma^2 \\ \sigma^2 & \sigma^2 \end{pmatrix}$	$\mathcal{U}_{20} = \frac{1}{\sqrt{2}}\begin{pmatrix} -\sigma^3 & \sigma^3 \\ \sigma^3 & \sigma^3 \end{pmatrix}$	Group (V)
$\mathcal{U}_{21} = \frac{1}{\sqrt{2}}\begin{pmatrix} \mathrm{I} & \mathrm{I} \\ \mathrm{I} & -\mathrm{I} \end{pmatrix}$	$\mathcal{U}_{22} = \frac{1}{\sqrt{2}}\begin{pmatrix} \sigma^1 & \sigma^1 \\ \sigma^1 & -\sigma^1 \end{pmatrix}$	$\mathcal{U}_{23} = \frac{1}{\sqrt{2}}\begin{pmatrix} \sigma^2 & \sigma^2 \\ \sigma^2 & -\sigma^2 \end{pmatrix}$	$\mathcal{U}_{24} = \frac{1}{\sqrt{2}}\begin{pmatrix} \sigma^3 & \sigma^3 \\ \sigma^3 & -\sigma^3 \end{pmatrix}$	Group (VI)

$$\left\{\mathcal{H}_\ell^\dagger, \mathcal{E}\right\} = \mathcal{H}_\ell^\dagger \mathcal{E} + \mathcal{E}^\dagger \mathcal{H}_\ell = 0. \tag{4.4}$$

In-order for (4.2), (4.3) and (4.4), to hold, where the new Hamiltonian $\mathcal{H}_\ell$ will have to be defined such that:

$$\mathcal{H} = i\hbar\tilde{\gamma}^0\tilde{\gamma}^k\partial_k - \tilde{\gamma}^0\mathcal{U}_\ell \mathrm{m}_0 c \text{ and } \mathcal{H} = i\hbar\frac{\partial}{\partial t}, \tag{4.5}$$

where the 24 unitary hermitian matrices $\mathcal{U}_\ell : \ell = (1, 2, \ldots, 24)$ are listed in Table (1), and $\tilde{\gamma}^0$ and $\tilde{\gamma}^k$ are such that:

$$\left\{\tilde{\gamma}^0 = \begin{pmatrix} 0 & \mathcal{I}_2 \\ -\mathcal{I}_2 & 0 \end{pmatrix} \text{ and } \tilde{\gamma}^k = \begin{pmatrix} \sigma^k & 0 \\ 0 & \sigma^k \end{pmatrix}\right\} \Rightarrow \tilde{\gamma}^\mu. \tag{4.6}$$

The matrices $\tilde{\gamma}^\mu$ satisfy the following *Dirac Algebra*:

$$\tilde{\gamma}^{\mu\dagger}\tilde{\gamma}^\nu + \tilde{\gamma}^{\nu\dagger}\tilde{\gamma}^\mu = -2\mathcal{I}_4\eta^{\mu\nu}, \tag{4.7}$$

The resulting set of *24* Dirac equations will be given by:

$$\left(i\hbar\mathcal{U}_\ell\tilde{\gamma}^\mu\partial_\mu - \mathrm{m}_0 c\right)\psi = 0. \tag{4.8}$$

This new set of *24* Dirac equations (4.8), is such that:

$$\left(i\hbar\mathcal{U}_\ell\tilde{\gamma}^\mu\partial_\mu - \mathrm{m}_0 c\right)^\dagger\left(i\hbar\mathcal{U}_\ell\tilde{\gamma}^\mu\partial_\mu - \mathrm{m}_0 c\right)\psi = 0 \implies \left[\hbar^2\Box - \mathrm{m}_0^2 c^4\right]\psi = 0. \tag{4.9}$$

Equation (4.8) is the same as the Dirac equation – albeit, with the Dirac γ^μ matrices now replaced with $\mathcal{U}_\ell\tilde{\gamma}^\mu$. If (4.8) is indeed equivalent to the Dirac equation, then, the matrices $\mathcal{U}_\ell\tilde{\gamma}^\mu$ must be such that:

$$\left(\mathcal{U}_\ell\tilde{\gamma}^\mu\right)^\dagger\left(\mathcal{U}_\ell\tilde{\gamma}^\nu\right) + \left(\mathcal{U}_\ell\tilde{\gamma}^\nu\right)^\dagger\left(\mathcal{U}_\ell\tilde{\gamma}^\mu\right) = \tilde{\gamma}^{\mu\dagger}\tilde{\gamma}^\nu + \tilde{\gamma}^{\nu\dagger}\tilde{\gamma}^\mu = -2\mathcal{I}_4\eta^{\mu\nu}. \tag{4.10}$$

As one can verify for themselves, equation (4.10) does indeed holds true. The meaning of this is that the new equation (4.8) is indeed equivalent to the Dirac equation.

5 Lorentz Invariance

Obviously, all the 24 representations (4.8) of the Dirac equation are Lorentz invariant. Perhaps, we must not just say this, but demonstrate this explicitly that equation (4.8), is indeed Lorentz invariant as claimed. For this purpose, we shall write equation (4.8) as follows:

$$i\hbar\tilde{\gamma}^\mu\partial_\mu\psi = \mathcal{U}_\ell \mathrm{m}_0 c\psi \tag{5.1}$$

To 'prove' (demonstrate) Lorentz invariance for equation (5.1), two *sine qua non* conditions must be satisfied and these are:

1. Given any two inertial observers O and O′ anywhere in spacetime, if in the frame O we have the state ψ described by the equation: $[i\hbar\tilde{\gamma}^\mu\partial_\mu\psi = \mathcal{U}_\ell \mathrm{m}_0 c\psi(x)]$, then: $[i\hbar\tilde{\gamma}^{\mu'}\partial_{\mu'}\psi'(x') = \mathcal{U}'_\ell \mathrm{m}_0 c\psi'(x')]$, is the corresponding equation describing the same state in the frame O′.

2. Given that $\psi(x)$ is the wavefunction as measured by observer O, there must exist a prescription for observer O′ to compute $\psi'(x')$ from $\psi(x)$ and this describes to O′ the same physical state as that measured by O.

In the above, the x in $\psi(x)$ represents the four spacetime coordinates x^μ.

Now – to proceed with our proof, we know that the Lorentz transformation is linear – and, because of this, it is to be required or expected of the transformations between $\psi(x)$ and $\psi'(x')$ to be linear too, that is:

$$\psi'(x') = \psi'(\Lambda x) = S(\Lambda)\psi(x) = S(\Lambda)\psi(\Lambda^{-1}x'), \tag{5.2}$$

where $S(\Lambda)$ is a non-singular 4×4 matrix which depends only on the relative velocities of O and O$'$ and Λ is the Lorentz transformation matrix $\Lambda_\mu^{\mu'} = \partial x^{\mu'}/\partial x^\mu$ or $\Lambda_{\mu'}^{\mu} = \partial x^\mu/\partial x^{\mu'}$. The matrix $S(\Lambda)$ has an inverse if O $\rightarrow$ O$'$ and also O$'\rightarrow$ O. This inverse is such that:

$$\psi(x) = S^{-1}(\Lambda)\psi'(x') = S^{-1}(\Lambda)\psi'(\Lambda x) \tag{5.3}$$

or we could write:

$$\psi(x) = S(\Lambda^{-1})\psi'(\Lambda x) \Longrightarrow S(\Lambda^{-1}) = S^{-1}(\Lambda). \tag{5.4}$$

By replacing the wavefunction $\psi(x)$ in (5.1), with $S^{-1}(\Lambda)\psi'(x')$, we can now write this equation as:

$$i\hbar\tilde{\gamma}^\mu S^{-1}(\Lambda)\partial_\mu\psi'(x') = \mathrm{m}_0 c\mathcal{U}_\ell S^{-1}(\Lambda)\psi'(x'). \tag{5.5}$$

Multiplying this equation (5.5) from the left by $S(\Lambda)$ and substituting $\partial_\mu = \Lambda_\mu^{\mu'}\partial_{\mu'}$ we will have:

$$i\hbar S(\Lambda)\tilde{\gamma}^\mu\Lambda_\mu^{\mu'} S^{-1}(\Lambda)\partial_{\mu'}\psi'(x') = \mathrm{m}_0 c S(\Lambda)\mathcal{U}_\ell S^{-1}(\Lambda)\psi'(x'). \tag{5.6}$$

As desired, the above equation (5.6) can be rewritten as:

$$i\hbar\tilde{\gamma}^{\mu'}\partial_{\mu'}\psi'(x') = \mathcal{U}'_\ell \mathrm{m}_0 c\psi'(x'), \tag{5.7}$$

where:

$$\gamma_\ell^{\mu'} = S(\Lambda)\gamma_\ell^\mu\Lambda_\mu^{\mu'} S^{-1}(\Lambda) \;\text{ and }\; \mathcal{U}'_\ell = S(\Lambda)\mathcal{U}_\ell S^{-1}(\Lambda). \tag{5.8}$$

In this way, we have shown that equation (5.1) – hence equation (5.1); is indeed Lorentz invariant as claimed since the two afore-stated *sine qua non* conditions for Lorentz invariance are satisfied. Before we close, we must make mention that – according to Pauli (1936)'s *Fundamental Theorem*, because $\tilde{\gamma}^\mu$ satisfies the *Dirac Algebra* (4.7), the matrix $S = S(\Lambda)$ does exist. In the next section, we shall proceed to investigate whether or not this equation obeys charge conjugation symmetry, parity symmetry, time reversal symmetry together with the different combinations of these discrete symmetries.

6 Symmetries of the New Dirac Equations

We here investigate the symmetries of the 24 new Dirac equations (5.1) *i.e.* their invariance (or lack thereof) under charge conjugation (C), parity (P), time reversal (T) together with the different combinations of these discrete symmetries. Before we can embark on this exercise, we need to consider first how the electric ($\boldsymbol{E}$) and magnetic fields ($\boldsymbol{B}$), currents ($\boldsymbol{J}$) and charges (ϱ) behave under C, P and T transformations. Under a P transformation, the positions of electrical charges will be interchanged and so the electric field will change sign as a consequence. Currents will flow in opposite direction so they also will change sign as a result. Since the magnetic field is proportional to $\boldsymbol{J}\times\boldsymbol{r}$, its sign will be preserved. All this can be summarised as:

$$\begin{array}{lcr} \mathtt{P}:\boldsymbol{E}(\boldsymbol{r},t) & \mapsto & -\boldsymbol{E}(-\boldsymbol{r},t) \\ \mathtt{P}:\boldsymbol{B}(\boldsymbol{r},t) & \mapsto & \boldsymbol{B}(-\boldsymbol{r},t) \\ \mathtt{P}:\boldsymbol{J}(\boldsymbol{r},t) & \mapsto & -\boldsymbol{J}(-\boldsymbol{r},t) \\ \mathtt{P}:\boldsymbol{\nabla} & \mapsto & -\boldsymbol{\nabla} \end{array}. \tag{6.1}$$

Under a T-transformation, the charges and positions will remain unchanged, whereas the currents will flow in opposite direction, in which case we will get:

$$\begin{array}{lcr} \mathtt{T}: \boldsymbol{E}(\boldsymbol{r},t) & \mapsto & \boldsymbol{E}(\boldsymbol{r},-t) \\ \mathtt{T}: \boldsymbol{B}(\boldsymbol{r},t) & \mapsto & -\boldsymbol{B}(\boldsymbol{r},-t) \\ \mathtt{T}: \boldsymbol{J}(\boldsymbol{r},t) & \mapsto & -\boldsymbol{J}(\boldsymbol{r},-t) \\ \mathtt{T}: \partial/\partial t & \mapsto & -\partial/\partial t \end{array}. \tag{6.2}$$

Using similar arguments as above, we will get for the C-transformation, the following:

$$\begin{array}{lcr} \mathtt{C}: \varrho(\boldsymbol{r},t) & \mapsto & -\varrho(\boldsymbol{r},t) \\ \mathtt{C}: \boldsymbol{E}(\boldsymbol{r},t) & \mapsto & -\boldsymbol{E}(\boldsymbol{r},t) \\ \mathtt{C}: \boldsymbol{B}(\boldsymbol{r},t) & \mapsto & -\boldsymbol{B}(\boldsymbol{r},t) \\ \mathtt{C}: \boldsymbol{J}(\boldsymbol{r},t) & \mapsto & -\boldsymbol{J}(\boldsymbol{r},t) \end{array}. \tag{6.3}$$

Finally under the combined CPT-transformation the charges and currents change sign and the electric and magnetic fields will retain their signs. These properties can be summarised in terms of the four vector potential $A_\mu^{\mathrm{ex}} = (A_0, A_k)$ of the ambient electromagnetic field as:

$$\begin{array}{lcl} \mathtt{C}: (A_0, A_k) & \rightarrow & (-A_0, -A_k) \\ \mathtt{P}: (A_0, A_k) & \rightarrow & (-A_0, A_k) \\ \mathtt{T}: (A_0, A_k) & \rightarrow & (A_0, -A_k) \end{array}. \tag{6.4}$$

Of particular importance here are the transformations (6.4) of the four vector potential $A_\mu = (A_0, A_k)$, this the reader will have to know as we will not remind them in the derivations that follow.

6.1 Commutation of $\mathcal{U}_\ell$ with γ^0, γ^2 and $\gamma^0\gamma^2$

In sections (6.2) to (6.8), we are going to investigate whether or not the new Lorentz invariant Dirac equation (5.1) does obey the discrete symmetries C, P, T and their combinations. In these investigations, the commutation (or lack thereof) the three matrices γ^0, γ^2 and $\gamma^0\gamma^2$ with $\mathcal{U}_\ell$ will be crucial. As such, we have worked these out and the results are presented in the self explanatory Table (2). Below we give an explanation of this table:

1. When the matrix γ^0, γ^2 or $\gamma^0\gamma^2$ commutes with $\mathcal{U}_\ell$, a value of " $+1$" appears in the table entry.
2. When the matrix γ^0, γ^2 or $\gamma^0\gamma^2$ anti-commutes with $\mathcal{U}_\ell$, a value of " -1" appears in the table entry.
3. When the matrix γ^0, γ^2 or $\gamma^0\gamma^2$ neither commutes nor anti-commutes with $\mathcal{U}_\ell$, a value of "0" appears in the table entry.

The reader must take note that the matrices γ^0, γ^2 & $\gamma^0\gamma^2$ are not the same as $\tilde{\gamma}^0$, $\tilde{\gamma}^2$ & $\tilde{\gamma}^0\gamma^2$, and that it is not a mistake that we have considered these matrices γ^0, γ^2 & $\gamma^0\gamma^2$ and not $\tilde{\gamma}^0$, $\tilde{\gamma}^2$ & $\tilde{\gamma}^0\tilde{\gamma}^2$.

6.2 C-Symmetry

To investigate whether or not equation (5.1) is invariant under charge conjugation, we proceed as usual, that is, we bring the Dirac particle ψ under the influence of an ambient electromagnetic magnetic field A_μ (which is a real function). The normal procedure of incorporating this ambient electromagnetic magnetic field into the Dirac equation is by making the canonical transformation of the derivatives, *i.e.*: $(\partial_\mu \longmapsto \mathcal{D}_\mu = \partial_\mu + iA_\mu)$, hence, under this transformation, equation (5.1) will now be given by:

$$\left[i\hbar\tilde{\gamma}^0\left(\partial_0 + iA_0\right) + i\hbar\tilde{\gamma}^k\left(\partial_k + iA_k\right) - \mathcal{U}_\ell \mathrm{m}_0 c\right]\psi = 0. \tag{6.5}$$

Table 2: **Commutation of $\mathcal{U}_\ell$ with γ^0, γ^2 and $\gamma^0\gamma^2$.**

ℓ	γ^0	γ^2	$\gamma^0\gamma^2$
1	$+1$	$+1$	$+1$
2	$+1$	-1	$+1$
3	$+1$	$+1$	$+1$
4	$+1$	-1	$+1$
5	$+1$	-1	-1
6	$+1$	$+1$	-1
7	$+1$	-1	-1
8	$+1$	$+1$	-1
9	-1	-1	$+1$
10	-1	$+1$	$+1$
11	-1	-1	$+1$
12	-1	$+1$	$+1$
13	-1	$+1$	-1
14	-1	-1	-1
15	-1	$+1$	-1
16	-1	-1	-1
17	0	-1	0
18	0	$+1$	0
19	0	-1	0
20	0	$+1$	0
21	0	-1	0
22	0	$+1$	0
23	0	-1	0
24	0	$+1$	0

Equation (6.5) represents the Dirac particle ψ which is immersed in an ambient electromagnetic magnetic represented by the four vector potential A_μ. If we are to reverse the ambient electromagnetic magnetic field *i.e.* ($A_\mu \longmapsto -A_\mu$), then, (6.5) becomes:

$$\left[i\hbar\tilde{\gamma}^0\left(\partial_0 - iA_0\right) + i\hbar\tilde{\gamma}^k\left(\partial_k - iA_k\right) - \mathcal{U}_\ell \mathrm{m}_0 c\right]\psi = 0. \tag{6.6}$$

Further, if (6.5; hence 5.1) is symmetric under charge conjugation, then, there must exist some mathematical transformation, which if applied to (6.6) would lead us back to an equation that is equivalent to (6.5).

Now, starting from (6.6), in-order to revert back to (6.5), the first mathematical operation to be applied to (6.6) is the complex conjugate operation on the entire equation – this operation allow us to get rid of the minus sign attached to the vector A_μ. So doing, we will have:

$$\left[-i\hbar\tilde{\gamma}^{0^*}\left(\partial_0 + iA_0\right) - i\hbar\tilde{\gamma}^{k*}\left(\partial_k + iA_k\right) - \mathcal{U}_\ell^* \mathrm{m}_0 c\right]\psi^* = 0. \tag{6.7}$$

Written more compactly, equation (6.7) is:

$$-i\hbar\tilde{\gamma}^{\mu^*}\left[\mathcal{D}_\mu + \mathcal{U}_\ell^* \mathrm{m}_0 c\right]\psi^* = 0. \tag{6.8}$$

Now, next, we need to get rid of the complex conjugate operator-$*$ attached to $\tilde{\gamma}^{\mu^*}$; to do this, we proceed by multiplying equation (6.8) throughout by $\gamma^0\gamma^2$ and having done this, knowing that:

$$\gamma^0\gamma^2\tilde{\gamma}^{\mu^*} = -\tilde{\gamma}^\mu\gamma^0\gamma^2, \tag{6.9}$$

we will require of $\mathcal{U}_\ell$ to be such that:

$$\gamma^0\gamma^2\mathcal{U}_\ell^* = \mathcal{U}_\ell\gamma^0\gamma^2. \tag{6.10}$$

If (6.10) holds true, it follows that, we will have:

$$\left[i\hbar\tilde{\gamma}^0\left(\partial_0 + iA_0\right) + i\hbar\tilde{\gamma}^k\left(\partial_k + iA_k\right) - \mathcal{U}_\ell \mathrm{m}_0 c\right]\gamma^0\gamma^2\psi^* = 0. \tag{6.11}$$

The above equation (6.11) can be written more conveniently as:

$$\left[i\hbar\tilde{\gamma}^0\left(\partial_0 + iA_0\right) + i\hbar\tilde{\gamma}^k\left(\partial_k + iA_k\right) - \mathcal{U}_\ell \mathrm{m}_0 c\right]\psi_c = 0, \tag{6.12}$$

where ($\psi_c = \gamma^0\gamma^2\psi^*$). This equation (6.12) is the same as equation (6.5), *albeit*, all we have done is to replace ψ with ψ_c. If the condition (6.10) holds for the case ℓ, then equation (5.1) is symmetric under charge conjugation for the case ℓ, else, it is not. So, of the *24* new Dirac equations, only those for which this condition (6.10) holds – will the corresponding equation be symmetric under change conjugation. Calculations with the *24* matrices $\mathcal{U}_\ell$ reveals that for the cases ($\ell = 1-4; 9-12$), C-symmetry is obeyed, while for the cases ($\ell = 5-8; 13-24$), it (C-symmetry) is not observed.

6.3 P-Symmetry

A parity transformation requires that we reverse the space coordinates *i.e.* ($x^k \longmapsto -x^k$). This transformation of the coordinates implies that the space derivatives will transform as: ($\partial_k \longmapsto -\partial_k$). Applying these transformations to (5.1), we will have this equation (5.1) now being given by:

$$\left[i\hbar\tilde{\gamma}^0\partial_0 - i\hbar\tilde{\gamma}^k\partial_k - \mathcal{U}_\ell \mathrm{m}_0 c\right]\psi = 0. \tag{6.13}$$

Now, we need to get rid of the minus sign that we have just introduced in the space derivatives. To do this, we multiply equation (6.13) throughout by γ^0 and then make use of the fact that:

$$\begin{array}{ccc}\gamma^0\tilde{\gamma}^0 & = & -\tilde{\gamma}^0\gamma^0 \\ \gamma^0\tilde{\gamma}^k & = & \tilde{\gamma}^k\gamma^0\end{array}. \tag{6.14}$$

Applying this fact (6.14) to equation (6.13) and then multiply the resulting equation by "-1", we will have:

$$\left[i\hbar\tilde{\gamma}^\mu\gamma^0\partial_\mu + \gamma^0\mathcal{U}_\ell \mathrm{m}_0 c\right]\psi = 0. \tag{6.15}$$

Now, if:

$$\gamma^0\mathcal{U}_\ell = -\mathcal{U}_\ell\gamma^0, \tag{6.16}$$

then, equation (6.15) will reduce to:

$$\left[i\hbar\tilde{\gamma}^0\partial_0 + i\hbar\tilde{\gamma}^k\partial_k - \mathcal{U}_\ell \mathrm{m}_0 c\right]\psi_p = 0, \tag{6.17}$$

where ($\psi_p = \gamma^0\psi$). This equation (6.17) is the same as equation (5.1), *albeit*, all we have done is to replace ψ with ψ_p. If (6.16) holds, then, equation (5.1) is symmetric under charge conjugation. So, of the 24 Dirac equations, only those for which this condition (6.16) holds – will the corresponding equation be symmetric under parity reversal. Calculations with the *24* matrices $\mathcal{U}_\ell$ reveals that for the cases ($\ell = 9-16$), P-symmetry is obeyed, while for the cases ($\ell = 1-8; 17-24$), it (P-symmetry) is not observed.

6.4 T-Symmetry

A time reversal transformation requires that we reverse the time coordinate *i.e.* $(t \longmapsto -t)$ and in-turn, this implies the time derivative will transform as: $(\partial_0 \longmapsto -\partial_0)$. Applying these transformations to (5.1), we will have equation (5.1) now being given by:

$$\left[-i\hbar\tilde{\gamma}^0\partial_0 + i\hbar\tilde{\gamma}^k\partial_k - \mathcal{U}_\ell \mathrm{m}_0 c\right]\psi = 0, \tag{6.18}$$

Now, in-order to revert back to equation (5.1), we apply commutation relations (6.14) where upon we will obtain:

$$\left[i\hbar\tilde{\gamma}^0\gamma^0\partial_0 + i\hbar\tilde{\gamma}^k\gamma^0\partial_k - \gamma^0\mathcal{U}_\ell \mathrm{m}_0 c\tilde{\gamma}^2\right]\psi = 0. \tag{6.19}$$

Now, if:

$$\gamma^0\mathcal{U}_\ell = \mathcal{U}_\ell\gamma^0, \tag{6.20}$$

then, equation (6.19) will reduce to:

$$\left[i\hbar\tilde{\gamma}^0\partial_0 + i\hbar\tilde{\gamma}^k\partial_k - \mathcal{U}_\ell \mathrm{m}_0 c\right]\psi_t = 0, \tag{6.21}$$

where $(\psi_t = \gamma^0\psi^*)$. This equation (6.21) is the same as equation (6.18), *albeit*, all we have done is to replace ψ with ψ_t. If (6.20) holds, then equation (5.1) is symmetric under charge conjugation. So, of the 24 Dirac equations, only those for which this condition (6.20) holds – will the corresponding equation be symmetric under parity reversal. Calculations with the *24* matrices $\mathcal{U}_\ell$ reveals that for the cases $(\ell = 1 - 8)$, T-symmetry is obeyed, while for the cases $(\ell = 9 - 24)$, it (T-symmetry) is not observed.

6.5 CP-Symmetry

A simultaneous reversal of the electronic charge and parity requires that we:

1. Introduce an ambient electromagnetic field $(\partial_\mu \longmapsto \mathcal{D}_\mu = \partial_\mu + iA_\mu)$.
2. Reverse the sign of the ambient electromagnetic field, that is to say $(A_\mu \longmapsto -A_\mu)$.
3. Reverse the sign in the space coordinates *i.e.* $(x^k \longmapsto -x^k)$ which implies $(\partial_k \longmapsto -\partial_k)$.
4. From the relation in equation (6.4), remember that $(x^k \longmapsto -x^k)$ which implies $(A_0 \longmapsto -A_0)$.

Effecting all these transformations into (5.1), we will have:

$$\left[i\hbar\tilde{\gamma}^0\left(\partial_0 + iA_0\right) - i\hbar\tilde{\gamma}^k\left(\partial_k + iA_k\right) - \mathcal{U}_\ell \mathrm{m}_0 c\right]\psi = 0. \tag{6.22}$$

Now, multiplying (6.22) by "$-\gamma^0$" from the left, thereafter applying the commutation relations (6.14), we will have:

$$\left[i\hbar\tilde{\gamma}^\mu\gamma^0\mathcal{D}_\mu + \gamma^0\mathcal{U}_\ell \mathrm{m}_0 c\right]\psi = 0. \tag{6.23}$$

If:

$$\gamma^0\mathcal{U}_\ell = -\mathcal{U}_\ell\gamma^0, \tag{6.24}$$

then, equation (6.23) will reduce to:

$$\left[i\hbar\tilde{\gamma}^\mu\gamma^0\mathcal{D}_\mu - \mathcal{U}_\ell \mathrm{m}_0 c\right]\psi_{cp} = 0. \tag{6.25}$$

where $(\psi_{cp} = \gamma^0\psi)$. This equation (6.25) is the same as equation (6.22), *albeit*, with ψ now replaced with ψ_{cp}. If the condition (6.24) holds, then, equation (5.1) is symmetric under simultaneous reversal of charge and parity. So, of the 24 Dirac equations, only those for which this condition (6.24) holds – will the corresponding equation be symmetric under change conjugation. Calculations with the *24* matrices $\mathcal{U}_\ell$ reveals that for the cases ($\ell = 9 - 16$), CP-symmetry is obeyed, while for the cases ($\ell = 1 - 8; 17 - 24$), it (CP-symmetry) is not observed.

6.6 CT-Symmetry

A simultaneous reversal of the electronic charge and time requires that we:

1. Introduce an ambient electromagnetic field ($\partial_\mu \longmapsto \mathcal{D}_\mu = \partial_\mu + iA_\mu$).
2. Reverse the sign of the ambient electromagnetic field, that is to say ($A_\mu \longmapsto -A_\mu$).
3. Reverse the sign in the time coordinate *i.e.* ($x^0 \longmapsto -x^0$) which implies ($\partial_0 \longmapsto -\partial_0$).
4. From the relation in equation (6.4), remember that ($x^0 \longmapsto -x^0$) which implies ($A_k \longmapsto -A_k$).

Effecting all these transformations into (5.1), we will have:

$$\left[-i\hbar\tilde{\gamma}^0\left(\partial_0 + iA_0\right) + i\hbar\tilde{\gamma}^k\left(\partial_k + iA_k\right) - \mathcal{U}_\ell \mathrm{m}_0 c\right]\psi = 0, \tag{6.26}$$

Now, multiplying (6.26) by "γ^0" from the left, thereafter applying the commutation relations (6.14), we will have:

$$\left[i\hbar\tilde{\gamma}^\mu\gamma^0\mathcal{D}_\mu - \gamma^0\mathcal{U}_\ell \mathrm{m}_0 c\right]\psi = 0. \tag{6.27}$$

If:

$$\gamma^0\mathcal{U}_\ell = \mathcal{U}_\ell\gamma^0, \tag{6.28}$$

then, equation (6.27) will reduce to:

$$\left[i\hbar\tilde{\gamma}^\mu\gamma^0\mathcal{D}_\mu - \mathcal{U}_\ell \mathrm{m}_0 c\right]\psi_{ct} = 0. \tag{6.29}$$

where ($\psi_{ct} = \gamma^0\psi$). This equation (6.29) is the same as equation (6.26), *albeit*, with ψ now replaced with ψ_{ct}. If the condition (6.28) holds, then, equation (5.1) is symmetric under charge conjugation. So, of the 24 Dirac equations, only those for which this condition (6.28) holds – will the corresponding equation be symmetric under change conjugation. Calculations with the *24* matrices $\mathcal{U}_\ell$ reveals that for the cases ($\ell = 1 - 8$), CT-symmetry is obeyed, while for the cases ($\ell = 9 - 24$), it (CT-symmetry) is not observed.

6.7 PT-Symmetry

If we are to reverse the space and time coordinates, that is ($x^\mu \mapsto -x^\mu$) $\Rightarrow$ ($\partial^\mu \mapsto -\partial^\mu$), and thereafter take the complex conjugate and then multiply the resulting equation by "$-\gamma^0\gamma^2$" from the left and then making use of the commutation relations (6.9), the resultant equation will be:

$$\left[i\hbar\tilde{\gamma}^{\mu^*}\gamma^0\gamma^2\partial_\mu - \gamma^0\gamma^2\mathcal{U}_\ell^* \mathrm{m}_0 c\right]\psi^* = 0, \tag{6.30}$$

If:

$$\gamma^0\gamma^2\mathcal{U}_\ell^* = \mathcal{U}_\ell\gamma^0\gamma^2, \tag{6.31}$$

then, equation (6.33) will reduce to:

$$\left[i\hbar\tilde{\gamma}^{\mu^*}\gamma^0\gamma^2\partial_\mu - \mathcal{U}_\ell \mathrm{m}_0 c\right]\psi_{pt} = 0. \tag{6.32}$$

where ($\psi_{pt} = \gamma^0\gamma^2\psi^*$). This equation (6.32) is the same as equation (6.30), *albeit*, with ψ now replaced with ψ_{pt}. If the condition (6.31) holds, then, equation (5.1) is symmetric under charge conjugation. So, of the 24 Dirac equations, only those for which this condition (6.31) holds – will the corresponding equation be symmetric under change conjugation. Calculations with the 24 matrices $\mathcal{U}_\ell$ reveals that for the cases ($\ell = 1, 2, 3$), PT-symmetry is observed while for the cases ($\ell = 1, 2, 3,$) this symmetry is not obeyed.

6.8 CPT-Symmetry

The seemingly sacrosanct CPT-symmetry involves the simultaneous reversal of electronic charge, parity and time. First, we place the particle in an ambient electromagnetic field – this implies: ($\partial_\mu \mapsto \partial_\mu + iA_\mu$), *i.e.*:

$$\left[i\hbar\tilde{\gamma}^0\left(\partial_0 + iA_0\right) + i\hbar\tilde{\gamma}^k\left(\partial_k + iA_k\right) - \mathcal{U}_\ell \mathrm{m}_0 c\right]\psi = 0. \tag{6.33}$$

Second, we reverse the spacetime coordinates, that is to say: ($x^\mu \longmapsto -x^\mu$) and this implies ($\partial^\mu \longmapsto -\partial^\mu$). Third, we reverse the ambient electromagnetic magnetic ($A_\mu \longmapsto -A_\mu$). Lastly, according to (6.4), the reversal of the spacetime coordinates requires that we reverse – once again – the ambient electromagnetic magnetic ($A_\mu \longmapsto -A_\mu$). Applying all these transformations to (6.33), we will have:

$$\left[-i\hbar\tilde{\gamma}^\mu\mathcal{D}^*_\mu - \mathcal{U}_\ell \mathrm{m}_0 c\right]\psi = 0. \tag{6.34}$$

Now, in-order to revert back to the original equation (6.33), we (1) take the complex conjugate, (2) multiply the resulting equation throughout by $\gamma^0\gamma^2$ and thereafter make use of the fact commutation relations (6.9). Applying the said operations to (6.34), this equation will now be given by:

$$\left[i\hbar\tilde{\gamma}^\mu\gamma^0\gamma^2\mathcal{D}_\mu + \gamma^0\gamma^2\mathcal{U}^*_\ell \mathrm{m}_0 c\right]\psi^* = 0. \tag{6.35}$$

If:

$$\gamma^0\gamma^2\mathcal{U}^*_\ell = -\mathcal{U}_\ell\gamma^0\gamma^2, \tag{6.36}$$

then, equation (6.35) will reduce to:

$$\left[i\hbar\tilde{\gamma}^\mu\mathcal{D}_\mu - \mathcal{U}_\ell \mathrm{m}_0 c\right]\psi_{cpt} = 0. \tag{6.37}$$

where ($\psi_{cpt} = \gamma^0\gamma^2\psi^*$). This equation (6.37) is the same as equation (6.33), *albeit*, with ψ now replaced with ψ_{cpt}. If the condition (6.36) holds, then, equation (5.1) is symmetric under charge conjugation. So, of the 24 Dirac equations, only those for which this condition (6.36) holds – will the corresponding equation be symmetric under change conjugation. Calculations with the *24* matrices $\mathcal{U}_\ell$ reveals that for the cases ($\ell = 5-8; 13-16$), CPT-symmetry is obeyed, while for the cases ($\ell = 1-4; 9-12; 17-24$), it (CPT-symmetry) is not observed.

6.9 Summary

Table (3) gives a summary of the symmetries of all the twenty four Dirac equations. For each ℓ-representation, if the a symmetry is obeyed, a value of "1" is entered and if it not obeyed, a value of "0" is entered. It is seen from this table that 33% of the equations obey C and CP-symmetry and that the CPT-symmetry is not spared as 66.6% of the equations obey violate this symmetry.

Table 3: **Symmetries:**

Case	**Symmetry**						
ℓ	C	P	T	CP	CT	PT	CPT
1	1	0	1	0	1	1	0
2	1	0	1	0	1	1	0
3	1	0	1	0	1	1	0
4	1	0	1	0	1	1	0
5	0	0	1	0	1	0	1
6	0	0	1	0	1	0	1
7	0	0	1	0	1	0	1
8	0	0	1	0	1	0	1
9	1	1	0	1	0	1	0
10	1	1	0	1	0	1	0
11	1	1	0	1	0	1	0
12	1	1	0	1	0	1	0
13	0	1	0	1	0	0	1
14	0	1	0	1	0	0	1
15	0	1	0	1	0	0	1
16	0	1	0	1	0	0	1
17	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0

7 General Discussion

We have shown that if one adheres to the prescription $[(\mathcal{H}+\mathcal{E})(\mathcal{H}-\mathcal{E})\psi = 0]$ handed down to us by Dirac on how to arrive at the Dirac equation – *albeit*, with a subtle change in prescription $[(\mathcal{H}-\mathcal{E})^{\dagger}(\mathcal{H}-\mathcal{E})\psi = 0]$; then, they would come to the realization that the Dirac equation is just one amongst a set of 25 possible equation. All the new 24 equations are Lorentz invariant. The basis (γ^{μ}) of Dirac equation is reducible while the basis $(\tilde{\gamma}^{\mu})$ of the present 24 new equations is irreducible. What is interesting about these new 24 Dirac-type equations is that unlike the Dirac equation which obeys all the seven discrete symmetries C, P, T, CP, CT, PT and CPT-symmetries, these equations violate at least one of these symmetries. Of particular interest to us here is the violation of C, CP and CPT-symmetries.

8 Conclusion

Assuming the acceptability of what has been presented herein, we hereby make the following conclusion:

1. As the original Dirac equation, the new twenty four Dirac equations have equal legitimacy to be considered as physical equations possibly describing a part of our Universe. If any of them are not equations describing the Universe (or a part thereof) that we live in, then, there must be some extra physical constraints presently

beyond us, that is, constraints that are a part of the Laws of *Nature*, constraints that ultimately rule out these equations.

2. The revered, seemingly sacrosanct and highly regarded `CPT`-*Theorem* that holds that all Lorentz invariant theories must uphold `CPT`-symmetry does not hold true for twelve of the Dirac equations.

9 Future Work

There is need to study the free particle solutions of the new twenty four equations. What one will have to check in these free particle solutions is whether or not they resulting equations give physically meaningful energy solutions. We have not done this in the present.

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