On the Secular Recession of the Earth-Moon System as an Azimuthal Gravitational Phenomenon (II)

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Abstract We here apply – *albeit*, with improved assumptions compared to our earlier work (Nyambuya et al., Astrophys. & S. Sci. $358(1) : \mathrm{pp}.1 - 12, 2015$); the ASTG-model to the observed secular trend in the mean Sun-(Earth-Moon) and Earth-Moon distances thereby providing an alternative explanation as to what the cause of this secular trend may be. For the semi-major axis rate of the Earth-Moon system, we now obtain a new value of about $+3.00\,\mathrm{cm/yr}$ while in the earlier work we obtained a value of about $+5.00\,\mathrm{cm/yr}$. This new value of $+3.00\,\mathrm{cm/yr}$ is closer to that of Standish (2005)'s measurement of $+(7.00 \pm 2.00)\,\mathrm{cm/yr}$. Our present value accounts for only 43% of Standish (2005)'s measurement. The other 57% can be accounted for by invoking the hypothesis that the θ-component of the angular momentum maybe non-zero. In the end, it can be said that the ASTG-model predicts orbital drift as being a result of the orbital inclination and the Solar mass loss rate. The Newtonian gravitational constant G is assumed to be an absolute time constant.

Keywords astrometry, celestial mechanics, ephemerides, planetary recession

1 Introduction

In our earlier reading (Nyambuya et al. 2015), we did demonstrate that the ASTG-model (Nyambuya 2010, 2015), is *in-principle* and to a reasonable extent capable of proffering an alternative explanation to the observed secular recession of the Earth-Moon system that has been measured by Krasinsky and Brumberg (2004) & Standish (2005). Krasinsky and Brumberg (2004) & Standish (2005) reported for the Earth-Moon system, an orbital recession from the Sun of about $+(15.00\pm4.00)\,\mathrm{cm/yr}$ and $+(7.00\pm2.00)\,\mathrm{cm/yr}$ respectively. In Nyambuya et al. (2015), we deduced from the ASTG-model a secular recession of $+(5.10 \pm 0.10)\,\mathrm{cm/yr}$. This result is in favor of the Standish (2005)'s measurement. An assumption made in our earlier work (Nyambuya et al. 2015), is that the tangential orbital speed of planets around the Sun stays the same. This assumption was made only as a first order approximation. In this short reading, we drop this assumption and conduct a new calculation.

It should be said that as argued (calculated) by *e.g.* Krasinsky and Brumberg (2004) & Noerdlinger (2008), a Newtonian gravitational calculation that takes into account the Solar mass loss rate is able to account for only $\sim 0.3\,\mathrm{cm}$ of the annual drift of the Earth-Moon system (this is about 3% of the measured value). In the ASTG-model which brings in a θ-dependence into the fold, one is able to account for much more of the drift. It is only interesting that this is the case; one wonders whether this model will stand the test to successfully account for the predicted drift of other planets.

2 New Calculation with Improved Assumptions

In conducting the new calculation, what we need is equation (24) of Nyambuya et al. (2015), *i.e.*:

$$\frac{\dot{J}_{\varphi}}{J_{\varphi}} + 2(\sin\theta - 1)\frac{\dot{r}}{r} = \frac{\dot{\mathcal{M}}_{\odot}}{\mathcal{M}_{\odot}} - \frac{\dot{r}}{r} + \frac{\dot{\gamma}}{\gamma}, \tag{1}$$

where J_{φ} is the orbital angular momentum of a planet around the Sun, θ is this planet's orbital inclination to the Solar equator, r is the planet's radial distance from the Sun, γ is the planet's gravitational to inertial mass ratio. For all practical purposes, the gravitational and inertial mass of a planet as it orbits the Sun can be assumed to be a constant, the meaning of which is that $(\dot{\gamma} = 0)$.

The assumption made in Nyambuya et al. (2015), namely that the tangential orbital speed of planets around the Sun stays the same implies that $(\dot{J}_{\varphi}/J_{\varphi} = \dot{a}/a)$ where a is the planet's semi-major axis. In-order to maintain Kepler's

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Table 1 Theoretical Predictions of Secular Solar Planetary Drifts

Planet	Tilt Angle (θ) (1.0°)	Mean Radius (AU)	New Value ($\dot{a}$) (cm/yr)	Old Value ($\dot{a}$) (cm/yr)	New Value ($10^{-13}\mathrm{yr}^{-1}$)	Old Value ($\dot{a}/a$) ($10^{-13}\mathrm{yr}^{-1}$)
Mercury	14.0	0.390 ± 0.080	$+16.00 \pm 3.00$	$+1.10 \pm 0.30$	$+28.30$	$+1.50 \pm 0.50$
Venus	10.4	0.726 ± 0.005	$+3.55 \pm 0.02$	$+2.74 \pm 0.02$	$+3.30$	$+2.50 \pm 0.03$
Earth	7.0	1.000 ± 0.020	$+2.70 \pm 0.04$	$+5.10 \pm 0.10$	$+1.80$	$+3.65 \pm 0.10$
Mars	8.9	1.500 ± 0.100	$+5.50 \pm 0.50$	$+6.50 \pm 0.50$	$+2.40$	$+2.50 \pm 0.50$
Jupiter	8.3	5.200 ± 0.300	$+16.90 \pm 0.90$	$+24.50 \pm 1.00$	$+2.20$	$+3.00 \pm 0.30$
Saturn	9.5	9.60 ± 0.500	$+39.00 \pm 2.00$	$+39.50 \pm 2.00$	$+2.70$	$+2.60 \pm 0.30$
Uranus	7.8	19.300 ± 0.900	$+57.500 \pm 3.00$	$+97.500 \pm 5.00$	$+2.00$	$+3.20 \pm 0.30$
Neptune	8.8	30.200 ± 0.300	$+106.00 \pm 1.00$	$+134.00 \pm 2.00$	$+2.40$	$+2.85 \pm 0.05$
Pluto	24.2	40.000 ± 10.000	-80.00 ± 20.00	-65.00 ± 20.00	-1.40	-0.90 ± 0.50

Note: The planetary data on the tilt angle θ of planetary orbits relative to the Solar spin equator, the perihelion and apehilion distances of planets used in the present table are adapted from the *NASA* website: `http://nssdc.gsfc.nasa.gov/planetary/factsheet/` on this day 15 Nov. 2014@16h07 GMT+2. The angle θ has been calculated as follows (1) we obtained the tilt of Solar planetary orbits relative to the ecliptic plane and these values are available on the *NASA* website; (2) we then add to this the tilt angle of the Solar spin equator relative to the ecliptic plane and this is known to be 7°. In this way, we obtained the tilt angles of the planes of these orbits relative to the Solar spin equator. This same method has been used in Table (1) of Nyambuya (2010).

Third Law ($\mathcal{T}^2_{\text{orb}} \propto \mathcal{R}^3_{\text{orb}}$) at all times as the planets undergo their secular drift, a more realistic assumption can be obtained from the first order Newtonian approximation relating the orbital angular momentum J_φ, $\mathcal{M}_\odot$, and a, namely $J^2_\varphi \simeq 2\gamma G\mathcal{M}_\odot a$ (see Eqn. [13] of Nyambuya and Simango 2014). Taking ($\dot{G} \equiv 0$) and setting ($r = a$), from this assumption (namely, $J^2_\varphi \simeq G\mathcal{M}_\odot a$), one obtains:

$$\frac{\dot{J}_\varphi}{J_\varphi} = \frac{1}{2}\frac{\dot{\mathcal{M}}_\odot}{\mathcal{M}_\odot} + \frac{1}{2}\frac{\dot{a}}{a}. \tag{2}$$

Inserting (2) into (1) and re-arranging, one obtains:

$$\frac{\dot{a}}{a} = -\frac{1}{1 - 4\sin\theta}\frac{\dot{\mathcal{M}}_\odot}{\mathcal{M}_\odot}. \tag{3}$$

Obviously, this result (3) will have a singularity when the angle ($\theta \sim 14.48^\circ$). What this implies is that in the regime where ($\theta \mapsto 14.48^\circ$), the assumption that we made, namely that ($J_\theta = 0$), this will have to be revisited. Therefore, this result (3) will apply for a θ that is significantly different from ($\theta = 14.48^\circ$).

If we do not assume ($J_\theta = 0$), then, equation (24) of Nyambuya et al. (2015) and the assumption (2), equation (3) will be given by:

$$(1 - 4\sin\theta)\frac{\dot{a}}{a} - 4\omega_\theta\cos\theta = -\frac{\dot{\mathcal{M}}_\odot}{\mathcal{M}_\odot}. \tag{4}$$

From this formula – for the case ($J_\theta \neq 0$), it is clear that at ($\theta \mapsto 14.48^\circ$), the singularity does not exist, thus is a result of the approximation ($J_\theta = 0$). Clearly, as ($\theta \mapsto 14.48^\circ$), we must have:

$$\omega_\theta \mapsto \frac{1}{4\cos\theta}\frac{\dot{\mathcal{M}}_\odot}{\mathcal{M}_\odot}. \tag{5}$$

We will now apply (3) to Solar planets.

3 Recession of Earth-Moon System

Just as in the reading Nyambuya et al. (2015), there are two parameters involved in the secular drift of the a planet around the Sun; these are – its tilt (θ) angle and the Solar mass loss rate ($\dot{\mathcal{M}}_\odot/\mathcal{M}_\odot$). The actual cause is the Solar mass loss rate – for if the Sun was non-luminous, there would be no recession according to (3). According (*e.g.*) to Noerdlinger (2008), the total Solar mass loss rate is ($\dot{\mathcal{M}}_\odot/\mathcal{M}_\odot = -9.13 \times 10^{-14}\ \mathrm{yr}^{-1}$). This Solar mass rate includes electromagnetic radiation, the Solar neutrino luminosity and Solar wind. Given that: the tilt of the Earth-Moon's orbit about the Solar equator is ($\theta \sim 7.0^\circ$), it follows from equation (3) that for the Earth-Moon system ($\dot{a}_{\text{em}} \sim +2.70\,\mathrm{cm/yr}$). This value is about 43% of the measurement by Standish (2005) and adds $\sim$ 40% to Krasinsky and Brumberg (2004) & Noerdlinger (2008)'s calculation of the recession due to the Solar mass loss rate.

We will discuss in the subsequent section, what the 57% discrepancy between the present theoretical and observational value really means.

Now, using this equation (3), we will – just as we did in the reading Nyambuya et al. (2015), make predictions about the the possible drift of other Solar planets. We are of the view that the improved assumption adopted here make these predictions superior to the our earlier prediction. These predictions are tabled in the self explanatory Table (1). As in Nyambuya et al. (2015), the case of Pluto still has a negative recession.

4 General Discussion

With improved assumptions, we herein have applied the ASTG-model to the observed secular drift in the mean Sun-(Earth-Moon). Our findings give a value that is about half the measurement of Standish (2005); *i.e.*, we obtain an annual recession of about $+2.70\,\mathrm{cm/yr}$. As before (Nyambuya et al. 2015), this prediction from the ASTG-model is seen as being a result of the orbital inclination, θ, and the Solar mass loss rate, $\dot{\mathcal{M}}_\odot/\mathcal{M}_\odot$.

If we take the Standish (2005) value for $\dot{a}_{\mathrm{em}}$, and the reason for this being that it is the closer value compared to our result $\sim +2.70\,\mathrm{cm/yr}$, then, there is an unaccounted for recession of $\sim +(4.00 \pm 2.00)\,\mathrm{cm/yr}$. According to (5), this can be attributed to the θ-component of the orbital angular momentum being non-zero ($J_\theta \neq 0$). If $(\dot{a}/a)_\Delta$ is a measure of this deficiency, then, according to (5):

$$\left(\frac{\dot{a}}{a}\right)_\Delta = \frac{4\omega_\theta \cos\theta}{1 - 4\sin\theta}. \tag{6}$$

From this, it follows that:

$$\omega_\theta = 1.30 \times 10^{-14}\,\mathrm{yr}^{-1} = 7.25 \times 10^{-22}\,\mathrm{s}^{-1}. \tag{7}$$

Hence:

$$J_\theta = 16.00\,\mathrm{m^2 s^{-1}}. \tag{8}$$

This value (8) is 14 orders of magnitude smaller than J_φ *i.e.*:

$$\frac{J_\theta}{J_\varphi} = 1.30 \times 10^{-14}. \tag{9}$$

Just as is the case with the Earth-Moon system's theoretical value for $\dot{a}$ exhibits a clear discrepancy between this value and the observational values of Standish (2005) & Krasinsky and Brumberg (2004), it is expected that – should observational measurements for the recession of other planets be made available, these will also exhibit a discrepancy between the ASTG-model's theoretical values and the observational values. This discrepancy can, as has been done for the Earth-Moon system; be attributed to a non-zero θ-component of the orbital angular momentum. So, the measurement of $\dot{a}$ can – according to the present ASTG-model; be taken as a way of also measuring J_θ for these planets.

It should be said that – at present, there does not exists any effort in the measurement of J_θ. The planetary orbital plane's inclination to the Solar equator is usually assumed to be fixed, *i.e.*, their inclination angles to the Solar equatorial plane has largely remained the same since these planets went into orbit around the Sun. The present ideas are suggesting this might not be the case.

Having assumed that a non-zero J_θ might account for the unaccounted for drift of the Earth-Moon system, at this point, it should be said, that the present model is a simplistic model that ignores processes such as tidal effects and Solar oblateness *etc*. These certainly have an effect on the Earth-Moon system drift. Thus, apart from the unaccounted drift being attributed to the a non-zero J_θ, it is possible that this may be accounted by these processes or a combination of these processes including a non-zero J_θ.

In-closing, allow us to say that the present letter should – perhaps – be taken more as an addendum to our earlier reading (Nyambuya et al. 2015). That is to say, despite the new addition of the a possible non-zero θ-component of the orbital angular momentum ($J_\theta \neq 0$), the present letter is not a fully-fledged research article.

5 Conclusion

Assuming the correctness (*i.e.*, acceptability) of the thesis posited herein, and its consequences thereof as applied herein, we hereby make the following conclusion that:

1. The observed secular recession of the Earth-Moon system may very well be a result of the Solar azimuthal gravitational field.

2. According to the present ideas, the unaccounted for recession of the Earth-Moon system suggests a non-zero θ-component of the orbital angular momentum being non-zero ($J_\theta \neq 0$).

Acknoweledgements: We are grateful to the National University of Science & Technology (NUST)'s Research & Innovation Department and Research Board for their unremitting support rendered toward our research endeavours; of particular mention, Prof. Dr. P. Mundy, Dr. P. Makoni, Dr. D. J. Hlatywayo and Prof. Dr. Y. S. Naik's unwavering support.

References

Krasinsky, G. A. and Brumberg, V. A. (2004), 'Secular Increase of Astronomical Unit from Analysis of the Major Planet Motions, and its Interpretation', *Celestial Mechanics and Dynamical Astronomy* **90**(3-4), 267–288.

Noerdlinger, P. D. (2008), 'Solar Mass Loss, the Astronomical Unit, and the Scale of the Solar System', *arXiv:0801.3807v1* pp. 1–31.

Nyambuya, G. G. (2010), 'Azimuthally Symmetric Theory of Gravitation (I) – On the Perihelion Precession of Planetary Orbits', *Monthly Notices of the Royal Astronomical Society* **403**(3), 1381–1391.

Nyambuya, G. G. (2015), 'Azimuthally Symmetric Theory of Gravitation II. On the Perihelion Precession of Solar Planetary Orbits', *Monthly Notices of the Royal Astronomical Society* **451**(3), 3034–3043.

Nyambuya, G. G., Makwanya, T., Tuturu, B. A. and Tsoka, W. (2015), 'On the Secular Recession of the Earth-Moon System as an Azimuthal Gravitational Phenomenon', *Astrophysics and Space Science* **358**(1).

Nyambuya, G. G. and Simango, W. (2014), 'On the Gravitational Bending of Light – Was Sir Arthur Stanley Eddington Right?', *International Journal for Astronomy and Astrophysics* **4**(2), 250–263.

Standish, E. M.: Kurtz, D. W. (2005), The Astronomical Unit Now, *in* 'Transits of Venus: New Views of the Solar System and Galaxy', number 196 *in* 'Proceedings IAU Colloquium', IAU, Cambridge University Press, UK, Cambridge. pp.163-179.

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