Dirac Equation for the Proton (I)

Why Three Quarks for Muster Mark?

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The present reading is the first in a series where we suggest a Dirac equation for the Proton. Despite its great success in explaining the physical world as we know it, in its bare form, not only is the Dirac equation at loss but fails to account *e.g.* for the following: (1) Why inside hadrons (Proton in this case) there are three, not four or five quarks; (2) Why quarks have fractional electronic charges; (3) Why the gyromagnetic ratio of the Proton is not equal to two as the Dirac equation requires. In the present reading, we make an attempt to answer the first question of why inside the proton, there are three, not four or five quarks.

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INTRODUCTION

In July of 2008, we presented [1] three equations that we proposed as the equivalent of the Dirac equation [2, 3] on a curved spacetime. These equations have thus far not generated the much anticipated interest in the scientific community. We hope this is set to change as we will demonstrate in the present reading that these equations [1] are capable of explaining the Proton as a fundamental particle comprising of three quarks with exactly the fractional electronic charges that quarks are found to have in *Nature*.

When the Dirac equation was serendipitously discovered in 1928 by the eminent British physicist, Professor Paul Adrien Maurice Dirac (1902 – 1984) in 1928 while staring at a fire [4], he christened it the "*Equation of the Electron*" because it did two remarkable things, *i.e.*:

(1).It explained the then mysterious origins of the Electron's spin as a relativistic phenomenon.

(2).It unprecedentedly gave the correct value of the gyromagnetic ratio of the Electron ($g_e \simeq 2$).

In our series of reading – of which the present is the first, we will write down an "*Equation for the Proton*" and our suggestion as to why this equation should be thought of as an equation for the Proton are threefold;

(1).We account for the existence of the three quarks believed to inhabit the Proton. This is accomplished in the present reading (Paper I).

(2).We deduce the the correct fractional electronic charges of these quarks *i.e.* $q_j = (\pm 1/3, \mp 2/3, \mp 2/3)$]. This is accomplished in the second reading (Paper II).

(3).We give an acceptable account of the gyromagnetic ratio of the Proton [$g_p = 5.585694710(50)$. We do not deduce the exact value of the gyromagnetic ratio of the Proton from our theory, but, an account why it must differ from the Dirac prediction of ($g = 2$). This is accomplished in the third reading (Paper III).

If these three outcomes are to be taken as ponderable achievements of the proposed theory, then, there might exist some very and credible strong grounds on which to call this proposed equation an "*Equation for the Proton*". However, before this can happen, there is much more that the proposed equation still has to do. At the moment, it must be taken as an interesting equation that may hold the potent seed that may one-day lead to it being christened the "*Equation for the Proton*"

The Proton is a Baryon[27] and deep inelastic scattering experiments [5, 6] have revealed it to be composed of two up quarks (u) and one down quark (d). Further, it is considered to be a stable particle, the meaning of which is that it does not decay (decompose/disintegrate) into smaller constituents. However, developments in the *Grand Unification Theories* (GUTs) have suggested that it might decay with a half-life of $\sim 10^{32}$ years. Despite this prediction, at present, there is currently no experimental evidence whatsoever that Proton decay actually occurs. Be that it may, at a 90% confidence level, recent experiments [7] at the Super-Kamiokande water Cherenkov radiation detector in Japan gave lower limits for Proton half-life of about 6.60×10^{33} years *via* anti-Muon decay ($p \mapsto \mu^+\pi^0$) and about 8.20×10^{33} years *via* positron decay ($p \mapsto e^+\pi^0$), while newer, preliminary results from the Super-Kamiokande seem to estimate a newer half-life of no less than $\sim 1.29 \times 10^{34}$ years *via* positron decay[28]. If anything, it looks like experiments are pushing this value further and further from the initial prediction of $\sim 10^{32}$ years.

The up-quark carries an electric charge of $+\frac{2}{3}e$ while the down-quark carries an electric charge of $-\frac{1}{3}e$. Why the Proton [together with the Neutron with $\mathrm{g}_n = -3.82608545(90)$] contains three quarks is not known and worse off, why these quarks contain fractional charges. Furthermore, it is presently a complete mystery as to why the Proton, together with the Neutron possess the gyromagnetic ratios that they possess. Their non-Dirac gyromagnetic ratios have been taken to mean they are not fundamental Dirac particles.

This work does not make the claim that it conclusively addresses these three issues – we merely lay down a proposal that seeks to address them. In so doing, we hope that this reading gives credence to the work present in Refs [1, 8]. Further, we hope this work that we present in series of papers is going to generate debate on the afore-cited works [1, 8]; on whose shoulders the present work stands.

Now, we give the synopsis of the present reading – it is organised as follows: in section "*Dirac Equation*", for instructive purpose, we formally present the Dirac equation. In section "*Curved Spacetime Dirac Equations*", we give an exposition of the proposed curved spacetime Dirac equations in the context of the proposed UFT [8]. In section "*Why Three Quarks*", we embark on the main task of the presenting where we make the endeavour to answer the question of why the Proton (or hadrons in general) has (have) three and not four quarks or any number of quarks. In section "*Derivation: Quark Equations*", in accordance with the present model, we present the equations governing the three quarks. Lastly, in section "*General Discussion*", we give a general discussion and the conclusion drawn from the present reading.

DIRAC EQUATION

For a particle whose rest-mass and wave-function are m_0 and $|\psi\rangle$ respectively, its Dirac equation is given by:

$$\left[\imath\hbar\gamma^{\mu}\partial_{\mu} - \mathrm{m}_0 c\right]|\psi\rangle = 0, \tag{1}$$

where:

$$\gamma^0 = \begin{pmatrix} \mathcal{I}_2 & 0 \\ 0 & -\mathcal{I}_2 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \tag{2}$$

are the 4×4 Dirac gamma matrices ($\mathcal{I}_2$ and 0 are the 2×2 identity and null matrices respectively) and $|\psi\rangle$ is the four component Dirac wave-function, $\hbar$ is the normalized Planck constant, c is the speed of light in vacuum, $\imath = \sqrt{-1}$ and:

$$|\psi\rangle = \left|\begin{matrix} \psi_0 \\ \psi_1 \\ \psi_2 \\ \psi_3 \end{matrix}\right\rangle = \left|\begin{matrix} \psi_L \\ \psi_R \end{matrix}\right\rangle, \tag{3}$$

is the Dirac 4×1 four component wavefunction and $|\psi_L\rangle$ and $|\psi_R\rangle$ are the Dirac bispinors that are defined such that:

$$|\psi_L\rangle = \left|\begin{matrix} \psi_0 \\ \psi_1 \end{matrix}\right\rangle \quad \text{and} \quad |\psi_R\rangle = \left|\begin{matrix} \psi_2 \\ \psi_3 \end{matrix}\right\rangle, \tag{4}$$

Throughout this reading – unless otherwise specified; the Greek indices will here-and-after be understood to mean $(\mu, \nu, \ldots = 0, 1, 2, 3)$ and the lower case English alphabet indices $(i, j, k \ldots = 1, 2, 3)$. Further - throughout this reading – in the presentation of the wavefunction, we shall use the Dirac Notion as has already been done.

CURVED SPACETIME DIRAC EQUATIONS

Since the time that we first set our mind on the Dirac equation [2, 3] and understood it at a common consensus level, we have had the general inexplicable inner feeling – call it intuition or what you will; that, despite its great and unparalleled success in explaining a vastness of phenomenon in the quantum world, there is dire need to revisit this equation at its most fundamental and elementary level if physics is to make its next great leap forward. This feeling is shared by other researchers as-well [9–11].

At the genesis of our quest to understand at a much deeper level the Dirac equation, our first port of call was to ask:

> *"If any, what is its [the Dirac equation] generalization in curved spacetime?"*

This question we asked despite our knowledge in the existence of alternative attempts [12–20] at a Curved Spacetime (CST) Dirac equation. Our general feeling which led us directly to search for yet another CST-Dirac equation is that we strongly felt the issue of the preponderance of matter over antimatter (which the bare Dirac equation fails to explain) should be explained by a more general Dirac equation such as a CST-Dirac equation.

As is well known, the Dirac equation exhibits a perfect symmetry, that is to say, it obeys the charge conjugation symmetry (C-symmetry), space conjugation symmetry (P-symmetry), time conjugation symmetry (T-symmetry) and any combination of these there symmetrices *i.e.* CP, CT, PT and CPT-symmetries. The afore-mentioned alternatives [12–20] also uphold these symmetries. We felt a CST-Dirac equation should violate some of these symmetries in which event one might be able to explain the preponderance of matter over antimatter. We have since demonstrated [21], that the CST-Dirac equations [1] do violate some of the above mentioned symmetries and this violation can – *in-principle* - be used to explain the preponderance of matter over antimatter. We will be clear here on the afore-cited work [21], that it is on-going work, the meaning of which is that its findings should be taken as preliminary and at the sametime seriously.

As is common knowledge, the Dirac equation is an equation applicable to a Minkowski spacetime and not to a curved spacetime. As pointed above, other researchers have proposed alternative versions of the curved spacetime Dirac equation [12–20]. However, this did not detour *nor* dent our searching spirit in seeking another version of the Dirac equation on a curved spacetime. Our quest culminated in the publication of the reading [1].

With all modesty – allow us to say that, what makes the effort [1] unique in comparison to these other efforts [12–20] on the same endeavour is that the approach, that has been used to arrive at the proposed curved spacetime Dirac equations therein [1], is the same as that used by Dirac in arriving at his equation. This approach [1] is the simplest imaginable when compared to these other efforts seeking a curved spacetime Dirac equation.

As is well known – *i.e.* – with regard to how Dirac arrived at his equation – is that - Dirac was motivative by his displeasure with the Klein-Gordon equation [22, 23]:

$$\Box\left|\psi\right\rangle \;=\; \left(\frac{\mathrm{m}_0 c}{\hbar}\right)^2\left|\psi\right\rangle\ldots \left(\text{where } \Box = \boldsymbol{\nabla}^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right), \tag{5}$$

because it gave negative probabilities which are obviously meaningless[29]. His suspicion was that this was as a result that the Klein-Gordon equation was second order in the space and time derivatives. So, he decided to write an equation which is first order in the space and time derivatives. He [Dirac] proposed that this equation be given by:

$$\left[\gamma^0\frac{\partial}{\partial ct} + \gamma^1\frac{\partial}{\partial x} + \gamma^2\frac{\partial}{\partial y} + \gamma^3\frac{\partial}{\partial z}\right]\left|\psi\right\rangle = -\imath\left(\frac{\mathrm{m}_0 c}{\hbar}\right)\left|\psi\right\rangle, \tag{6}$$

such that "upon squaring", this equation (6) should yield the usual Klein-Gordon equation [2]. What Dirac noted about this linearised equation is that the mathematical objects $\gamma^\mu : (\mu = 0,1,2,3)$ can not be ordinary numbers but that at the very least, these can only be 4×4 matrices. After the serendipitous milestone achievement of the discovery of the Dirac equation, our modest, feeble but very strong feeling is that the next logical step should have been to seek a curved spacetime version of this noble and beautiful equation. It is our strong feeling that this curved spacetime Dirac equation must have been sort alone the same lines as those championed by Dirac on his journey of discovering the Dirac equation. For some reason, researchers have had other ideas as this seems to have skipped them somehow.

On our part in the search for such an equation alone the Dirac's path, in [1], what we did was to take-off from the first step taken by Dirac (*i.e.* equation 6); were we judiciously added by the *sleight of hand*, a space and time varying four vector function $A^\mu = A^\mu(\boldsymbol{r}, t)$ as follows:

$$\left[A^0\gamma^0\frac{\partial}{\partial ct} + A^1\gamma^1\frac{\partial}{\partial x} + A^2\gamma^2\frac{\partial}{\partial y} + A^3\gamma^3\frac{\partial}{\partial z}\right]\left|\psi\right\rangle = -\imath\left(\frac{\mathrm{m}_0 c}{\hbar}\right)\left|\psi\right\rangle. \tag{7}$$

As Dirac did (provided $\gamma^\mu_{(1)}A^\mu\partial_\mu A^\nu\partial_\nu\left|\psi\right\rangle = 0$), if we are to demand that "upon squaring", this equation (7) should result in the curved spacetime Klein-Gordon equation *i.e.*:

$$g^{\mu\nu}_{(1)}\partial_\mu\partial_\nu\left|\psi\right\rangle = \left(\frac{\mathrm{m}_0 c}{\hbar}\right)^2\left|\psi\right\rangle, \tag{8}$$

then, inevitably, one comes to the interesting conclusion that the metric of spacetime $g^{\mu\nu}_{(1)}$ must now be such that it is given by $g^{\mu\nu}_{(1)} = \frac{1}{2}\{A^\mu\gamma^\mu, A^\nu\gamma^\nu\}$, where:

$$g^{\mu\nu}_{(1)} = \begin{pmatrix} +A^0A^0 & 0 & 0 & 0\\ 0 & -A^1A^1 & 0 & 0\\ 0 & 0 & -A^2A^2 & 0\\ 0 & 0 & 0 & -A^3A^3 \end{pmatrix}\mathcal{I}_4. \tag{9}$$

The object $\mathcal{I}_4$ in equation (9), is the 4×4 identity matrix. In this way, $g^{\mu\nu}_{(1)} = \frac{1}{2}\{A^\mu\gamma^\mu, A^\nu\gamma^\nu\}$ is now a function of a four vector. The bracket $\{,\}$ in $g^{\mu\nu}_{(1)} = \frac{1}{2}\{A^\mu\gamma^\mu, A^\nu\gamma^\nu\}$ is the usual anticommutator bracket. Additionally, we have written the metric $g^{\mu\nu}_{(1)}$ with a subscript "(1)"; this we have done with the anticipation of the two other metrics, namely $g^{\mu\nu}_{(2)}$ and $g^{\mu\nu}_{(3)}$. The metric $g^{\mu\nu}_{(1)}$ has all the off-diagonals equal to zero. We should in general be able to have a metric with non-zero off-diagonals.

In the new setting, the metric no-longer is a function of ten potentials as before but of four! This is an obvious reduction in complexity. Further, we noted that the proposal (7) resulted in three versions of the curved spacetime Dirac equation [1]. Thus far, not much exploration of these equations has been done by other researchers other than ourself. Because this reading unearths something that no other endeavours on the Dirac equation have unearthed, we are hoping that this reading might lead to a change of fortunes insofar as the appreciation of these CST-Dirac equations is concerned.

To obtain a second equation, we write:

$$\left[A^0\gamma^0_{(2)}\frac{\partial}{\partial ct} + A^1\gamma^1_{(2)}\frac{\partial}{\partial x} + A^2\gamma^2_{(2)}\frac{\partial}{\partial y} + A^3\gamma^3_{(2)}\frac{\partial}{\partial z}\right]\left|\psi\right\rangle = -\imath\left(\frac{\mathrm{m}_0 c}{\hbar}\right)\left|\psi\right\rangle, \tag{10}$$

where:

$$\gamma^0_{(2)} = \begin{pmatrix} \mathcal{I}_2 & 0 \\ 0 & -\mathcal{I}_2 \end{pmatrix} = \gamma^0,$$

$$\gamma^k_{(2)} = \frac{1}{2}\begin{pmatrix} 2\mathcal{I}_2 & \imath\sqrt{2}\sigma^k \\ -\imath\sqrt{2}\sigma^k & -2\mathcal{I}_2 \end{pmatrix}. \tag{11}$$

The object $\mathcal{I}_2$ is the 2×2 identity matrix.

Now, as Dirac did (provided $\gamma^\mu_{(2)}A^\mu\partial_\mu A^\nu\partial_\nu|\psi\rangle = 0$), if we are to demand that "upon squaring", this equation (10) should result in the curved spacetime Klein-Gordon equation *i.e.*:

$$g^{\mu\nu}_{(2)}\partial_\mu\partial_\nu|\psi\rangle = \left(\frac{\mathrm{m}_0 c}{\hbar}\right)^2|\psi\rangle, \tag{12}$$

then, inevitably, one comes to the interesting conclusion that the metric of spacetime $g^{\mu\nu}_{(2)}$ must now be such that it is given by $g^{\mu\nu}_{(2)} = \frac{1}{2}\left\{A^\mu\gamma^\mu_{(2)}, A^\nu\gamma^\nu_{(2)}\right\}$, where:

$$g^{\mu\nu}_{(2)} = \begin{pmatrix} +A^0A^0 & +A^0A^1 & +A^0A^2 & +A^0A^3 \\ +A^1A^0 & -A^1A^1 & +A^1A^2 & +A^1A^2 \\ +A^2A^0 & +A^2A & -A^2A^2 & +A^2A^2 \\ +A^3A^0 & +A^3A & +A^3A^2 & -A^3A^3 \end{pmatrix}\mathcal{I}_4. \tag{13}$$

As one would expect for a generally curved spacetime metric, the metric $g^{\mu\nu}_{(2)}$ has non-zero off-diagonals terms. We should in general be able to have a metric with non-zero off-diagonals.

For the third equation, we write:

$$\left[\gamma^0_{(3)}\frac{\partial}{\partial ct} + \gamma^1_{(3)}\frac{\partial}{\partial x} + \gamma^2_{(3)}\frac{\partial}{\partial y} + \gamma^3_{(3)}\frac{\partial}{\partial z}\right]|\psi\rangle = -i\left(\frac{\mathrm{m}_0 c}{\hbar}\right)|\psi\rangle, \tag{14}$$

where now:

$$\gamma^0_{(3)} = \begin{pmatrix} \mathcal{I}_2 & 0 \\ 0 & -\mathcal{I}_2 \end{pmatrix} = \gamma^0,$$

$$\gamma^k_{(3)} = -\frac{1}{2}\begin{pmatrix} 2\mathcal{I}_2 & \imath\sqrt{2}\sigma^k \\ -\imath\sqrt{2}\sigma^k & -2\mathcal{I}_2 \end{pmatrix}. \tag{15}$$

As before (provided $\gamma^\mu_{(3)}A^\mu\partial_\mu A^\nu\partial_\nu|\psi\rangle = 0$), if we are to demand that "upon squaring", this equation (14) should result in the curved spacetime Klein-Gordon equation *i.e.*:

$$g^{\mu\nu}_{(3)}\partial_\mu\partial_\nu|\psi\rangle = \left(\frac{\mathrm{m}_0 c}{\hbar}\right)^2|\psi\rangle, \tag{16}$$

then, inevitably, one comes to the interesting conclusion that the metric of spacetime $g^{\mu\nu}_{(3)}$ must now be such that it is given by $g^{\mu\nu}_{(3)} = \frac{1}{2}\left\{A^\mu\gamma^\mu_{(3)}, A^\nu\gamma^\nu_{(3)}\right\}$, where:

$$g^{\mu\nu}_{(3)} = \begin{pmatrix} +A^0A^0 & -A^0A^1 & -A^0A^2 & -A^0A^3 \\ -A^1A^0 & -A^1A^1 & +A^1A^2 & -A^1A^3 \\ -A^2A^0 & -A^2A^1 & -A^2A^2 & -A^2A^3 \\ -A^3A^0 & -A^3A^1 & +A^3A^2 & -A^3A^3 \end{pmatrix}\mathcal{I}_4. \tag{17}$$

Again, as one would expect for a generally curved spacetime metric, the metric $g^{\mu\nu}_{(3)}$ has non-zero off-diagonals terms.

In-general[30], equations (7), (10) and (14) can all be written in a condensed form as:

$$\imath\hbar A^\mu\gamma^\mu_{(a)}\partial_\mu|\psi\rangle = \mathrm{m}_0 c|\psi\rangle, \tag{18}$$

where the 4×4 matrices $\gamma^\mu_{(a)}$ are such that:

$$\gamma^0_{(a)} = \begin{pmatrix} \mathcal{I}_2 & 0 \\ 0 & -\mathcal{I}_2 \end{pmatrix} = \gamma^0,$$

$$\gamma^k_{(a)} = \frac{1}{2}\begin{pmatrix} 2\lambda_a\mathcal{I}_2 & \imath^{\lambda_a|\lambda_a|}\sqrt{2^{|\lambda_a|}}\sigma^k \\ -\imath^{\lambda_a|\lambda_a|}\sqrt{2^{|\lambda_a|}}\sigma^k & -2\lambda_a\mathcal{I}_2 \end{pmatrix}. \tag{19}$$

The λ_a's in (19) are defined such that when:

$$a = \begin{cases} 1, \text{ then } (\lambda_1 = 0): \text{ Quadratic Spacetime (QST).} \\ 2, \text{ then } (\lambda_2 = +1): \text{ Parabolic Spacetime (PST).} \\ 3, \text{ then } (\lambda_3 = -1): \text{ Hyperbolic Spacetime (HST).} \end{cases} \tag{20}$$

The index "a" is not an active index as are the Greek indices. This index labels a particular curvature of spacetime *i.e.* whether spacetime is flat[31], positive or negatively curved as defined by the resulting metric $g^{\mu\nu}_{(a)}$ which is given in equation (22).

In summary and in a condensed form – provided the condition holds ($\gamma^\mu_{(a)}A^\mu\partial_\mu A^\nu\partial_\nu|\psi\rangle = 0$); squaring (18) in the usual Dirac way of squaring, results in the Klein-Gordon equation:

$$g^{\mu\nu}_{(a)}\partial_\mu\partial_\nu|\psi\rangle = \left(\frac{\mathrm{m}_0 c}{\hbar}\right)^2|\psi\rangle, \tag{21}$$

where the general and condensed metric $g^{\mu\nu}_{(a)}$ is such that:

$$g^{\mu\nu}_{(a)} = \begin{pmatrix} +A^0A^0 & \lambda A^0A^1 & \lambda A^0A^2 & \lambda A^0A^3 \\ \lambda A^1A^0 & -A^1A^1 & \lambda A^1A^2 & \lambda A^1A^3 \\ \lambda A^2A^0 & \lambda A^2A^1 & -A^2A^2 & \lambda A^2A^3 \\ \lambda A^3A^0 & \lambda A^3A^1 & \lambda A^3A^2 & -A^3A^3 \end{pmatrix} \mathcal{I}_4. \quad (22)$$

The condition $(\gamma^{\mu}_{(a)}A^{\mu}\partial_{\mu}A^{\nu}\partial_{\nu}|\psi\rangle = 0)$, shall be taken as a gauge condition imposed upon the four vector potential A_{μ} and the wavefunction $|\psi\rangle$. Alternatively, the above metric can in-short, be written as:

$$\begin{aligned} g^{\mu\nu}_{(a)} &= \tfrac{1}{2}\left\{A^{\mu}\gamma^{\mu}_{(a)}, A^{\nu}\gamma^{\nu}_{(a)}\right\} \\ &= \tfrac{1}{2}\left\{\gamma^{\mu}_{(a)}, \gamma^{\nu}_{(a)}\right\} A^{\mu}A^{\nu} \; . \\ &= \sigma^{\mu\nu}_{(a)}A^{\mu}A^{\nu} \end{aligned} \quad (23)$$

where $\sigma^{\mu\nu}_{(a)} = \frac{1}{2}\left\{\gamma^{\mu}_{(a)}, \gamma^{\nu}_{(a)}\right\}$. What we have just done is to give a exposition of the seemingly banal CST-Dirac equations that we first presented in [1]. Equation (18) is the most general way of writing these three equations (7, 10, 14). Apart from the said exposition, what we have done is to demonstrate that the metric tensor is susceptible to decomposition into a tensor field describable by four unique fields that form a relativistic four vector. In [24]'s General Theory of Relativity (GTR), the metric tensor exists as a compound mathematical object comprising ten unique fields.

Now, after having written down this CST-Dirac equation (18), we wondered what interpretation to give to the four vector A^{μ}. It is then that we developed the all-encompassing *Unified Field Theory* (UFT) given in [8]. At first sight, it is tempting to identify A^{μ} with the electromagnetic four vector potential of the particle in question. Our investigation in [25] lead us on a different path. This path however still allows for the this four vector A^{μ} to be a function of the electromagnetic four vector potential of the particle in question. As such, in the present reading, we shall take A^{μ} to be the electromagnetic four vector potential of the particle in question.

WHY THREE QUARKS

We now come to the first part of what this reading is all about. Our first port of call is to answer the question as to why three quarks are found in a Proton. In-order for us to answer the question as to *why three quarks* and not four or any other number, we will have to do some little linear algebra. The fact that the Dirac wavefunction ψ can be written as a set of two by bispinors, this fact alone is enough to prove why there must exist three quarks in a Proton. This very fact that the wavefunction is a set of two by bispinors implies that it can be decomposed into a linear combination of three fundamental bispinors spanning a vector space.

In general, a basis of a vector space $\mathscr{V}$ is defined as a subset $(v_1, \ldots, v_n)$ of vectors in $\mathscr{V}$ that are linearly independent and this set of subset of vectors spans $\mathscr{V}$. Consequently, if $(v_1, \ldots, v_n)$ is a list of vectors in $\mathscr{V}$, then these vectors form a basis if and only if every $(v \in \mathscr{V})$ can be uniquely written as $(v = a_1v_1 + \cdots + a_nv_n)$ where a_k are elements of the base field. The base field is almost always a real number $(\in \mathbb{R})$ or a complex number $(\in \mathbb{C})$. A vector space $\mathscr{V}$ will have many different bases, but there are always the same number of basis vectors in each of them. The number of basis vectors in $\mathscr{V}$ is called the dimension of $\mathscr{V}$. Every spanning list in a vector space can be reduced to a basis of the vector space.

Given the above, it is not difficult to show that if the Dirac wavefunction is taken as an ordered pair of bispinors, then, this wavefunction can be decomposed into a set of three linearly independent set of bispinors $|\Psi_j\rangle$, that is to say:

$$|\psi\rangle = \sum_{j=1}^{3} q_j\tau_j |\Psi_j\rangle \, , \quad (24)$$

where $(q_j \in \mathbb{R})$ are the usual probability coefficients of a quantum mechanical wavefunction, and τ_j is such that:

$$\tau_j = \begin{pmatrix} \sigma_j & 0 \\ 0 & \sigma_j \end{pmatrix} \text{ for } (j = 1, 2, 3). \quad (25)$$

The matrix τ_j commutes with the $\gamma^{\mu}_{(a)}$, *i.e.*:

$$\left[\tau_j, \gamma^{\mu}_{(a)}\right] \equiv 0. \quad (26)$$

Like the Dirac wavefunction (2), the sub-system $|\Psi_j\rangle$, is defined:

$$|\Psi_j\rangle = \left| \begin{matrix} \Psi_{j0} \\ \Psi_{j1} \\ \Psi_{j2} \\ \Psi_{j3} \end{matrix} \right\rangle = \left| \begin{matrix} \Psi_{jL} \\ \Psi_{jR} \end{matrix} \right\rangle , \quad (27)$$

where, likewise:

$$|\Psi_{jL}\rangle = \left| \begin{matrix} \Psi_{j0} \\ \Psi_{j1} \end{matrix} \right\rangle \text{ and } |\Psi_{jR}\rangle = \left| \begin{matrix} \Psi_{j2} \\ \Psi_{j3} \end{matrix} \right\rangle . \quad (28)$$

From the above, it follows that:

$$\langle\psi| \, |\psi\rangle = \sum_{j=1}^{3} q_j^2 \langle\Psi_j| \, |\Psi_j\rangle \, , \quad (29)$$

The wavefunction $|\psi\rangle$ is normalised ($\langle\psi|\,|\psi\rangle = 1$), so are the sub-states $|\Psi_j\rangle$, that is to say: $\langle\Psi_j|\,|\Psi_j\rangle = 1$. It follows from this and from (29) as-well, that:

$$\sum_{j=1}^{3} q_j^2 = 1. \quad (30)$$

In general, the q's can be complex and time variable. However, in our model, these are fixed by the internal logic of the theory so that they (the q's) are real and eternally fixed. In the end, they (the q's) also determine the fractional charges of quarks.

DERIVATION: QUARK EQUATIONS

In-order to write down the equations that govern the three sub-systems $|\Psi_j\rangle$, we will need to decompose the matrix $\gamma^{\mu}_{(a)}$. Each of these sub-systems $|\Psi_j\rangle$ are going to have their own matrix $\gamma^{\mu}_{(a)}$ which we shall denote $\gamma^{\mu}_{(a_j)}$: a_j is defined as $(a_j = j)$. Now, in our decomposition of $\gamma^{\mu}_{(a)}$, we shall write $\gamma^{\mu}_{(a)}$ as a linear combination of the matrices $\gamma^{\mu}_{(a_j)}$, *i.e.*:

$$\gamma^{\mu}_{(a)} = \pm \sum_{j=1}^{3} q_j \gamma^{\mu}_{(a_j)}. \quad (31)$$

From the definitions of $|\Psi_j\rangle$ and $\gamma^{\mu}_{(a)}$ in (24) and (31) respectively, it follows that:

$$\gamma^{\mu}_{(a)} |\Psi_j\rangle = \sum_{i=1}^{3}\sum_{j=1}^{3} q_i q_j \gamma^{\mu}_{a_i} \tau_j |\Psi_j\rangle \quad (32)$$

For the equations that we seek, in-order for them to describe particles that behave independently with no clearly visible interference with one another – in their totality – the ij-cross-terms *i.e.*, the terms for which $(i \neq j)$, these must vanish identically. Without destroying the particle system into a senseless triviality, this can be achieved if we set:

$$\gamma^{\mu}_{(a_i)} \tau_j |\Psi_j\rangle + \gamma^{\mu}_{(a_j)} \tau_i |\Psi_i\rangle = 0, \ \text{ for } \ (i \neq j). \quad (33)$$

This attainable condition (33) can be summed-up as:

$$\langle\Psi_j| \left[\tau_j \left(\gamma^{\mu}_{(a_i)}\right)^{-1} \gamma^{\mu}_{(a_j)} \tau_i\right] |\Psi_i\rangle = -1, \ \text{ for } \ (i \neq j). \quad (34)$$

In (34), the matrix $\gamma^{\mu^{-1}}_{(a_i)}$ is the inverse of $\gamma^{\mu}_{(a_i)}$. With (33) as a given, it follows that:

$$\gamma^{\mu}_{(a)} |\Psi\rangle = \sum_{j=1}^{3} q_j^2 \gamma^{\mu}_{a_j} \tau_j |\Psi_j\rangle\,. \quad (35)$$

Now, inserting (24) and (35) into (18), we will have:

$$\sum_{j=1}^{3} \underbrace{\left[\imath\hbar q_j^2 \tau_j A^{\mu} \gamma^{\mu}_{(a_j)} \partial_{\mu} - q_j \tau_j \mathrm{m}_0 c\right] |\Psi_j\rangle}_{\text{Must Vanish Identically}} = 0. \quad (36)$$

Clearly, if the sub-systems $|\Psi_j\rangle$ are to act as independent non-interacting particles – as quarks do; then, the term in (36) in the under-brace (or, in the summation sign: $\sum_{j=1}^{3}$) must vanish identically. This implies:

$$\left[\imath\hbar q_j^2 \tau_j A^{\mu} \gamma^{\mu}_{(a_j)} \partial_{\mu} - q_j \tau_j \mathrm{m}_0 c\right] |\Psi_j\rangle = 0, \quad \text{for} \ \ (j = 1, 2, 3). \quad (37)$$

By dividing (37) throughout by q_j and multiplying by τ_j^{-1} from the left, this equation (37) reduces to:

$$\left[\imath\hbar q_j A^{\mu} \gamma^{\mu}_{(a_j)} \partial_{\mu} - \mathrm{m}_0 c\right] |\Psi_j\rangle = 0, \ \text{ for } \ \ (j = 1, 2, 3). \quad (38)$$

This equation (38), is our sought for equation which governs the behaviour and evolution of the three sub-systems $|\Psi_j\rangle$. We shall call these sub-systems $|\Psi_j\rangle$, quarks. From this equation, it is manifestly clear that if A^{μ} represents a quantum of the electromagnetic field, $q_j A^{\mu}$ represents a fraction of this quantum if $(q_j < 1)$. Thus, if the proposed equation (38) is that of the three quarks found inside the Proton, the q's must match with the q-values obtained from experimental philosophy.

Now, in order that our claim be believable or that this claim have some real substance to it – *i.e.*, the claim that the sub-systems $|\Psi_j\rangle$ be regarded as quarks; there obviously is need to at least demonstrate that the electronic charges q_j of these particles, are as they are found in *Nature* for quarks, that is: $(q_j = \pm 1/3, \mp 2/3, \mp 2/3)$. We shall indeed demonstrate in Paper (II) that $(q_j = \pm 1/3, \mp 2/3, \mp 2/3)$. In the present, all we have done is to demonstrate that the system of equations that we have proposed [1] allows for a composite particle to have or to comprise of three distinct "non-interacting" sub-systems (sub-particles). The three quark can each have distinct spacetime configurations: $g^{\mu\nu}_{(1)}$, $g^{\mu\nu}_{(2)}$ and $g^{\mu\nu}_{(3)}$.

GENERAL DISCUSSION

The all-beautiful Dirac equation has been in existence for 88 years now. After it was discovered and promptly accepted, it was taken-up [by Dirac; Richard Feynman (1918 – 1987); Julian Seymour Schwinger (1918 – 1994);

etc] and used to develop QED and QFT. Today, the Dirac equation is an integral part of the Standard Model of Particle Physics (SMPP). It is such an indispensable and integral part of the SMPP without which the SMPP can not be understood. Actually, it is unimaginable to imagine someone trying to understand the SMPP without the Dirac equation. Despite all this, there is still much more that we are to learn from this noble equation.

In our approach to trying to get a better understanding of the Dirac equation, we have had to go back in time to 1928 and seek a curved spacetime version of this equation directly from the curved spacetime version of the Klein-Gordon equation. This curved spacetime Dirac equation we have demanded that it be derived using the same prescription as that used by Dirac when he derived his beautiful equation from the Minkowski version of the Klein-Gordon equation. To a larger extent, we believe we did succeed in this endeavour. As we shall demonstrate in our series of readings – our seemingly banal approach [1] to a curved spacetime Dirac equation is not only different from the common approaches that have been used by other researchers [12–20], but it brings in very interesting insights into some of the intriguing questions in particle physics.

In the present, we have demonstrated that one can use these three equations to account for the three quarks found inside the Proton. This approach can also be employed to the original Dirac equation. However – as we shall demonstrate in the second reading is that, if one employs the Dirac equation alone, they will not be able to account for the fractional quark charges as in this Dirac theory with three independent subsystems, the electronic charges of these subsystems will have to be variable. This variability of the electronic charges does not entail a violation of the Law of Conservation of electronic charge. Despite this, we know that this variability of the electronic charge of quarks is not the case in *Nature*. In exact conformity with the reality of *Nature* as we have experience Her, we will demonstrate in Paper (II) that our proposed scheme of trying to explain the Proton yields the much desired "magic numbers" ($q_j = \pm 1/3, \mp 2/3, \mp 2/3$). Allow us with modesty to say that – this [theoratical prediction that ($q_j = \pm 1/3, \mp 2/3, \mp 2/3$)] is a significant noteworthy achievement of the theory.

In-closing – further allow us to say that, in all the existing literature – at least the vast literature that we have had the fortune of peruse; there seems to not exist a theory that explains why there should be three quarks (subsystems) inside a Proton and worse-off, any theory that has produced the exact electronic charges of these quarks. This achievement of the proposed CST-Dirac equations [1] to explain the fractional electronic charges as they are found *Nature*, this ought to be taken as the clearest evidence yet, that the proposed CST-Dirac equations [1] may contain in them a potent gem of truth that has a possible correspondence with physical and natural reality.

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[28] See http://www-sk.icrr.u-tokyo.ac.jp/whatsnew/new-20091125-e.html. Visited on this day 13 Oct 2014@10h58 GMT+2.
[29] We have provided very strong, plausible and credible arguments in [26] to the effect that one can literary avoid negative probabilities in QM by making an appropriate choice of the probability current density.
[30] In equation (18) above, the term $A^{\mu}\gamma^{\mu}_{(a)}$ must be treated as a single object with one index μ. This is what this object is. One can set $\Gamma^{\mu}_{(a)} = A^{\mu}\gamma^{\mu}_{(a)}$. The problem with this setting is that we need to have the objects, A^{μ} and $\gamma^{\mu}_{(a)}$, clearly visible in the equation.
[31] By *flat*, it here is not meant that the spacetime is Minkowski *flat*, but that the metric has no off diagonal terms. On the same footing, by *positively curved spacetime*, it meant that metric has positive off diagonal terms and likewise, a *negatively curved spacetime*, it meant that metric has negative off diagonal terms.