Dirac Equation for the Proton (III)

On the Gyromagnetic Ratio of the Proton

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(Dated: September 16, 2016)

The present reading is the third in series where we suggest a Dirac equation for the Proton. Despite its great success in explaining the physical world as we know it, in its bare form, not only is the Dirac equation at loss but fails to account *e.g.* for the following: (1) Why inside hadrons there are three, not four or five quarks; (2) Why quarks have fractional charges; (3) Why the gyromagnetic ratio of the Proton is not equal to two as the Dirac equation requires. In the present reading, we make an attempt to answer the third question of why the gyromagnetic ratio of the Proton is not equal to two as the Dirac equation requires. We show that from the internal logic of the proposed theory – when taken to first order approximation, we are able to account for $\sim 55.7\%$ [2.000000000] of the Proton's excess gyromagnetic ratio [3.585694710(50)]. The remaining $\sim 44.3\%$ [1.585694710(50)] can be accounted as a second order effect that has to do with the Proton having a finite size.

PACS numbers: 03.65.Pm, 11.30.j, 04.62.b, 04.62.+v, 98.80.Jk, 04.40.b

INTRODUCTION

The present reading is the third in series where we suggest a Dirac equation for the Proton (and in a latter reading, an equation for the Neutron as-well). In the first part [1] [hereafter Paper I], an attempt was made at answering the question of why the Proton has three quarks and not any other number like $4, 5, 6, \ldots$ *etc.* In the second part [2] [hereafter Paper II], further attempts where made at answering not only the question of why these quarks which are held '*eternal prisoners*' inside the Proton have fractional electrical charges, but it was demonstrated why these quarks have the very charges that they are found to have in *Nature*. In the present reading, we address the issue of the relatively large Proton gyromagnetic ratio (g-factor), *i.e.*, why the gyromagnetic ratio of the Proton [3–5] differs significantly from Dirac's bare and natural value of $(g = 2)$.

Historically, when the Dirac equation [6, 7] was discovered by the eminent British physicist, Paul Adrien Maurice Dirac $(1902 - 1984)$ in 1928 while staring at a fire [8], he christened it the "*Equation of the Electron*" because it did two remarkable things, *i.e.*:

(1). It explained the then mysterious origins of the Electron's spin as a relativistic phenomenon.

(2). Without precedent, it gave the correct value ($g_e \simeq 2$) of the gyromagnetic ratio of the Electron.

Given this, and what we have thus far done *i.e.*:

(1). In Paper (I), we have – on the legitimate mathematical grounds of the rich *Lie-Algebra* and the fact that the Dirac 4×1 wavefunction is a bispinor of two bispinors; accounted for the existence of the three quarks believed to make-up the Proton.

(2). In Paper (II), we have deduced from the internal logic of the proposed Proton model, the correct fractional electronic charges of these quarks *i.e.* $q_j = (\pm\frac{1}{3}, \mp\frac{2}{3}, \mp\frac{2}{3})$.

and that in the present reading we are going to give an acceptable account of the Proton g-factor, whose latest measured value is $g_p = 5.585694710(50)$ [9]; these facts all but point an "*Equation for the Proton*". Off cause, there is still a lot more to be done but these achievements should be considered sufficient to be considered worthwhile to explore the proposed three Curved Spacetime (CST)-Dirac equations of [10] as candidate equations for further understanding of not only the Proton, but the Neutron as-well.

The Proton is a Baryon and is considered to be composed of two up quarks (u) and one down quark (d). Further, it is considered to be a stable particle, the meaning of which is that it does not decay into smaller constituents. However, developments in the *Grand Unification Theories* (GUTs) have suggested that it might decay with a half-life of $\sim 10^{32}$ years. Despite this prediction, at present, there is currently no experimental evidence that Proton decay occurs. Be that it may, at a 90% confidence level, recent experiments at the Super-Kamiokande water Cherenkov radiation detector in Japan gave lower limits for Proton half-life of about 6.60×10^{33} years *via* anti-Muon decay ($p \mapsto \mu^+\pi^0$) and about 8.20×10^{33} years *via* positron decay ($p \mapsto e^+\pi^0$) [11], while newer, preliminary results from the Super-Kamiokande seem to estimate a newer half-life of no less than $\sim 1.29 \times 10^{34}$ years *via* positron decay[26]. If anything, it looks like experiments are pushing this value further and further from the initial prediction of $\sim 10^{32}$ years.

The up-quark carries an electric charge of $+\frac{2}{3}e$ while the down-quark carries an electric charge of $-\frac{1}{3}e$. Why the Proton [together with the Neutron with $\mathrm{g}_n = -3.82608545(90)$] contains three quarks is not known and worse off, why these quarks contain fractional charges. Furthermore, at present, it is a complete mystery as to why the Proton, together with the Neutron possess the gyromagnetic ratios well in excess of the Dirac prediction of $(g = 2)$.

This reading (and the entire work) does not make the claim that it conclusively addresses these three issues – but, we merely lay down a proposal that seeks to address them. This proposal is meant to generate debate. We make use of the recently Unified Field Theory (UFT) proposed in the reading [12] and as-well the proposed Curved Spacetime (CST) Dirac Equations proposed in [10]. In so doing, we hope that this reading gives credence to these works: [10, 12]. Further, we hope this reading will generate debate on these said works – [10, 12], on whose shoulders the present work stands.

CURVED SPACETIME DIRAC EQUATIONS

For instructive and completeness purposes, we here give a re-cap of the CST-Dirac equations. For a particle of rest mass m_0 whose Dirac 4×1 wavefunction is ψ, the proposed general curved spacetime Dirac equation [10] for such a particle is such that:

$$\left[\imath\hbar A^{\mu}\gamma^{\mu}_{(a)}\partial_{\mu} - \mathrm{m}_0 c\right]\psi = 0, \tag{1}$$

where we shall take A^{μ} is the four electromagnetic vector potential of the particle in question and 4×4 matrices $\gamma^{\mu}_{(a)}$ are such that:

$$\begin{aligned} \gamma^{0}_{(a)} &= \begin{pmatrix} \mathcal{I}_2 & 0 \\ 0 & -\mathcal{I}_2 \end{pmatrix} = \gamma^0, \\ \gamma^{k}_{(a)} &= \frac{1}{2}\begin{pmatrix} 2\lambda_a\mathcal{I}_2 & \imath^{\lambda_a|\lambda_a|}\sqrt{2^{|\lambda_a|}}\sigma^k \\ -\imath^{\lambda_a|\lambda_a|}\sqrt{2^{|\lambda_a|}}\sigma^k & -2\lambda_a\mathcal{I}_2 \end{pmatrix}. \end{aligned} \tag{2}$$

In (2), σ^k and $\mathcal{I}_2$ are the usual 2×2 Pauli matrices and the identity matrix respectively. The λ_a's in (2) are defined such that when:

$$a = \begin{cases} 1, \text{ then } (\lambda_1 = 0): \text{ Quadratic Spacetime (QST).} \\ 2, \text{ then } (\lambda_2 = +1): \text{ Parabolic Spacetime (PST).} \\ 3, \text{ then } (\lambda_3 = -1): \text{ Hyperbolic Spacetime (HST).} \end{cases} \tag{3}$$

The index "a" is not an active index as are the Greek indices. This index labels a particular curvature of spacetime *i.e.* whether spacetime is flat[27], positive or negatively curved as defined by the resulting metric $g^{\mu\nu}_{(a)}$ which is given in equation (6). In a condensed form – provided the 'gauge' condition:

$$\gamma^{\mu}_{(a)}A^{\mu}\partial_{\mu}A^{\nu}\partial_{\nu}\psi = 0, \tag{4}$$

holds – then, squaring (1) in the usual Dirac way of squaring[28], results in the Klein-Gordon equation:

$$g^{\mu\nu}_{(a)}\partial_{\mu}\partial_{\nu}\psi = \left(\frac{\mathrm{m}_0 c}{\hbar}\right)^2\psi, \tag{5}$$

where the general and condensed metric $g^{\mu\nu}_{(a)}$ is such that:

$$g^{\mu\nu}_{(a)} = \begin{pmatrix} +A^0A^0 & \lambda_a A^0A^1 & \lambda_a A^0A^2 & \lambda_a A^0A^3 \\ \lambda_a A^1A^0 & -A^1A^1 & \lambda_a A^1A^2 & \lambda_a A^1A^3 \\ \lambda_a A^2A^0 & \lambda_a A^2A^1 & -A^2A^2 & \lambda_a A^2A^3 \\ \lambda_a A^3A^0 & \lambda_a A^3A^1 & \lambda_a A^3A^2 & -A^3A^3 \end{pmatrix}\mathcal{I}_4. \tag{6}$$

The condition (4), shall be taken as a gauge condition imposed upon the four vector potential A_{μ} and the wavefunction ψ.

GENERAL SPIN CURVED SPACETIME DIRAC EQUATIONS

As we did in the reading [13], for instructive purposes – we are now going to transform the CST-Dirac equation (1), into a General Spin Curved Spacetime (GSCST) Dirac equation. As it stands, equation (1) would be a horribly complicated equation insofar as its solutions are concerned because the vector A^{μ} is expected to be a function of space and time *i.e.* $A^{\mu} = A^{\mu}(\boldsymbol{r}, t)$. Other than a numerical solution, there is no foreseeable way to obtain an exact solution is if that is the case. However, while pondering on possible solutions of (1), we found a way round the problem of taming this otherwise horribly complicated equation; we realised that this vector can actually be used to arrive at a GSCST-Dirac equation thereby drastically simplifying the equation. This simplification scheme requires that:

(1).For A^{μ}, we set:

$$A^{\mu} = \phi s^{\mu}, \tag{7}$$

where s^{μ} is vector that has no space nor time dependence (we shall define this vector shortly) and

$\phi = \phi(\boldsymbol{r}, t)$ is a scalar field. It has been shown in the UFT presented in [12], that, indeed, one can take A^μ as the electromagnetic four vector potential of the particle in question as one can show [12] that this vector represents not only the electromagnetic field of the particle in question, but its *Weak* and *Strong Force Fields* as-well.

(2).For the rest-mass, m_0 of the particle, we replace this with $\mathrm{m}_0\phi$, that is to say:

$$\mathrm{m}_0 \longrightarrow \mathrm{m}_0\phi. \tag{8}$$

Inserting (7) and (8) into (1), one obtains:

$$\left[i\hbar\gamma^\mu_{(as)}\partial_\mu - \mathrm{m}_0 c\right]\psi = 0. \tag{9}$$

where the new 4×4 γ-matrices ($\gamma^\mu_{(as)} = s^\mu \gamma^\mu_{(a)}$) are now defined such that:

$$\gamma^0_{(as)} = s^0 \begin{pmatrix} \mathcal{I}_2 & 0 \\ 0 & -\mathcal{I}_2 \end{pmatrix},$$

$$\gamma^k_{(as)} = \tfrac{1}{2}s^k \begin{pmatrix} 2\lambda_a\mathcal{I}_2 & \imath^{\lambda_a|\lambda_a|}\sqrt{2^{|\lambda_a|}}\sigma^k \\ -\imath^{\lambda_a|\lambda_a|}\sqrt{2^{|\lambda_a|}}\sigma^k & -2\lambda_a\mathcal{I}_2 \end{pmatrix}, \tag{10}$$

and $[(s^0 = 1); (s^k = \pm1, \pm2, \pm3, \dots\, etc)]$ (for justification of this, see[13–15]). Since s^μ is four vector, $\gamma^k_{(as)}$ is a four vector as-well. Further, the space vector s^k determines the spin of the particle. Furthermore, multiplication of (9) from the left by $\left[i\hbar\gamma^\nu_{(as)}\partial_\nu + \mathrm{m}_0 c\right]$, results in the Klein-Gordon equation:

$$g^{\mu\nu}_{(as)}\partial_\mu\partial_\nu\psi = \left(\frac{\mathrm{m}_0 c}{\hbar}\right)^2\psi, \tag{11}$$

where:

$$g^{\mu\nu}_{(as)} = \frac{1}{2}\left\{\gamma^\mu_{(as)}, \gamma^\nu_{(as)}\right\} = \sigma^{\mu\nu}_{(as)}. \tag{12}$$

The space and time variable four vector A^μ has been removed from our midst and has been replaced by the non-space and non-time variable from vector s^μ. From here-on, we shall use equation (9) as our GSCST-Dirac equation.

PROTON

As pointed out in the two readings [10, 13], each of the three configuration $g^{\mu\nu}_{(as)}$ (as represented by the a-index) represent a particle of different energy. Consequently, what this means is that one is able to explain the existence of the three generations that are seen in Leptons and Quarks. We do not intend to go deep into these matters here as there is a reading – which is part of the present series of readings – where this issue is tackled. What we want to point out here is that – for a given spin setting, the present theory predicts three possible energy states and this applies to all fundamental particles including the Proton.

In the case of the Proton, in-order that they contain quarks as we known them, these three possible configurations must yield:

$$q_j := \left[\underbrace{\pm\frac{1}{3}}_{q_1}, \underbrace{\mp\frac{2}{3}}_{q_2}, \underbrace{\mp\frac{2}{3}}_{q_3}\right]. \tag{13}$$

As one can verify for themselves, the following three configurations do just that, *i.e.*:

$$\begin{aligned} \gamma^\mu_{(1s)} &= q_1\gamma^\mu_{(1s)} + q_2\gamma^\mu_{(2s)} + q_3\gamma^\mu_{(3s)} \quad :\mapsto \quad (123) \ \text{Configuration.} \\ \gamma^\mu_{(2s)} &= q_1\gamma^\mu_{(2s)} + q_2\gamma^\mu_{(2s)} + q_3\gamma^\mu_{(3s)} \quad :\mapsto \quad (223) \ \text{Configuration.} \\ \gamma^\mu_{(3s)} &= q_1\gamma^\mu_{(3s)} + q_2\gamma^\mu_{(2s)} + q_3\gamma^\mu_{(3s)} \quad :\mapsto \quad (323) \ \text{Configuration.} \end{aligned} \tag{14}$$

The configuration $\gamma^\mu_{(1s)}$ represents the spin $s/2$ quadratic Proton ($a = 1$), $\gamma^\mu_{(2s)}$ represents the spin $s/2$ parabolic Proton ($a = 2$) while $\gamma^\mu_{(3s)}$ represents the spin $s/2$ hyperbolic Proton ($a = 3$). As afore-stated, we reiterate that each configuration ($\gamma^\mu_{(1s)}, \gamma^\mu_{(2s)}$ & $\gamma^\mu_{(3s)}$) has a different mass and represents a generation of the Proton (or the fundamental particle in question).

The (123)-configuration should contain three different quarks, while the (223) and (323)-configuration contains two quarks of the same kind with the third being of the different kind. Given that the Proton contains two u-quarks and one d-quark, it follows that the Proton must be described by one of the two configurations $[(223), (323)]$. At the moment, we can not discern which of the two represents the Proton as we know it. This is something for latter readings. For the purposes of calculating the g-factor, it will not affect our outcome as these two configurations have the same gyromagnetic ratio. In the next section, we present the calculation for the gyromagnetic ratio for the three configurations.

GYROMAGNETIC RATIO

Now, we are going to calculate the gyromagnetic ratio for the hyperbolic and parabolic particle respectively. As before, we proceed by taking the particle [*i.e.* the hyperbolic and parabolic particle: where $a = (2,3)$] and then placing it in an ambient magnetic field $A_\mu^{\rm ex}$. To incooperate this ambient magnetic field $A_\mu^{\rm ex}$ into the Dirac equation (9), we have to replace the partial derivatives ∂_μ with $(D_\mu = \partial_\mu - \imath q A_\mu^{\rm ex}/\hbar)$ where q is the electrical charge of the Proton (or the particle in question), *i.e.* $(\partial_\mu \mapsto D_\mu)$. So doing will result in (9) reducing to:

$$\left[\imath\hbar\gamma^\mu_{(as)}D_\mu - \mathrm{m}_0 c\right]\psi = 0. \tag{15}$$

Acting on this equation (15) from the left handside using the operator $\left(\imath\hbar\gamma^\mu_{(as)}D_\mu + \mathrm{m}_0 c\right)$, one obtains:

$$\left[\gamma^\mu_{(as)}\gamma^\nu_{(as)}D_\mu D_\nu + \frac{\mathrm{m}_0^2c^2}{\hbar^2}\right]\psi = 0. \tag{16}$$

We know that:

$$\begin{aligned}\gamma^\mu_{(as)}\gamma^\nu_{(as)}D_\mu D_\nu &= \tfrac{1}{2}\left(\left\{\gamma^\mu_{(as)},\gamma^\nu_{(as)}\right\} + \left[\gamma^\mu_{(as)},\gamma^\nu_{(as)}\right]\right)D_\mu D_\nu\,, \\ &= \eta^{\mu\nu}_{(as)}D_\mu D_\nu + \sigma^{\mu\nu}_{(as)}D_\mu D_\nu\end{aligned} \tag{17}$$

where $(\eta^{\mu\nu}_{(as)} = \left\{\gamma^\mu_{(as)},\gamma^\nu_{(as)}\right\})$ & $(\sigma^{\mu\nu}_{(as)} = \left[\gamma^\mu_{(as)},\gamma^\nu_{(as)}\right])$; and:

$$\sigma^{\mu\nu}_{(as)}D_\mu D_\nu = \frac{1}{2}\sigma^{\mu\nu}_{(as)}\left[D_\mu, D_\nu\right] = \frac{\imath q}{2\hbar}\sigma^{\mu\nu}_{(as)}F_{\mu\nu} = \frac{\imath n e}{2\hbar}\sigma^{\mu\nu}_{(as)}F_{\mu\nu}. \tag{18}$$

In the above equation (18), we have written $(q = ne)$, where $[e = 1.6021766208(98)\times 10^{-19}\,\mathrm{C}]$ is the elementary electrical charge and $(n = 0, \pm 1)$ where $(n = 0)$ for an electrically neutral particle, $(n = -1)$ for negatively change particle like an Electron and $(n = +1)$ for negatively changed particle like a Proton.

The components of $\sigma^{\mu\nu}_{(as)}$ are such that $(\sigma^{ii}_{(as)} = 0)$, with $(\sigma^{0k}_{(as)}$ & $\sigma^{k0}_{(as)})$, defined such that:

$$\sigma^{0k}_{(as)} = \sigma^{k0}_{(as)} = \begin{pmatrix} 0 & \imath^{\lambda_a|\lambda_a|}\sqrt{2^{|\lambda_a|}}s^k\sigma^k \\ \imath^{\lambda_a|\lambda_a|}\sqrt{2^{|\lambda_a|}}s^k\sigma^k & 0\end{pmatrix}, \tag{19}$$

and:

$$\begin{aligned}\sigma^{ij}_{(as)} &= -\begin{pmatrix} \frac{1}{2}\imath^{2\lambda_a|\lambda_a|+1}2^{|\lambda_a|}s^is^j[\sigma^i,\sigma^j] & \lambda_a\imath^{\lambda_a|\lambda_a|}\sqrt{2^{|\lambda_a|}}(s^i\sigma^i - s^j\sigma^j) \\ \lambda_a\imath^{\lambda_a|\lambda_a|}\sqrt{2^{|\lambda_a|}}(s^i\sigma^i - s^j\sigma^j) & \frac{1}{2}\imath^{2\lambda_a|\lambda_a|+1}2^{|\lambda_a|}s^is^j[\sigma^i,\sigma^j]\end{pmatrix} \\ &= -\begin{pmatrix} \imath^{2\lambda_a|\lambda_a|+1}2^{|\lambda_a|}s^is^j\sigma^k & \lambda_a\imath^{\lambda_a|\lambda_a|}\sqrt{2^{|\lambda_a|}}(s^i\sigma^i - s^j\sigma^j) \\ \lambda_a\imath^{\lambda_a|\lambda_a|}\sqrt{2^{|\lambda_a|}}(s^i\sigma^i - s^j\sigma^j) & \imath^{2\lambda_a|\lambda_a|+1}2^{|\lambda_a|}s^is^j\sigma^k\end{pmatrix}\end{aligned}. \tag{20}$$

In the above calculation (equation 20), we must remember the *Lie-Algebra*: $(\sigma^i\sigma^j = +\imath\sigma^k;\ \sigma^j\sigma^i = -\imath\sigma^k)$. Further, let us set $(s^is^j = s^k)$ where the s^k's do not satisfy the usual *Lie-Algebra*; so doing, we will have:

$$\sigma^{ij}_{(as)} = -\begin{pmatrix} \imath^{2\lambda_a|\lambda_a|}2^{|\lambda_a|}s^k\sigma^k & \lambda_a\imath^{\lambda_a|\lambda_a|}\sqrt{2^{|\lambda_a|}}(s^i\sigma^i - s^j\sigma^j) \\ \lambda_a\imath^{\lambda_a|\lambda_a|}\sqrt{2^{|\lambda_a|}}(s^i\sigma^i - s^j\sigma^j) & \imath^{2\lambda_a|\lambda_a|}2^{|\lambda_a|}s^k\sigma^k\end{pmatrix}. \tag{21}$$

Furthermore, all the above calculations when put into effect, they will reduce equation (16) to:

$$\left[D_i^2 + D_0^2 + 2D_jD_0 + 2(D_iD_j)_{i\neq j} + \frac{\imath n e}{2\hbar}\sigma^{\mu\nu}_{(as)}F_{\mu\nu} + \frac{\mathrm{m}_0^2c^2}{\hbar^2}\right]\psi = 0. \tag{22}$$

The terms $[2D_jD_0 + 2(D_iD_j)_{i\neq j};\ D_jD_0]$ will be considered negligible [8] so that the resulting equation is:

$$\left[D_i^2 + \frac{\imath n e}{2\hbar}\sigma^{\mu\nu}_{(as)}F_{\mu\nu} + \frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \frac{\mathrm{m}_0^2c^2}{\hbar^2}\right]\psi = 0, \tag{23}$$

where in equation (23) above, we have set $(D_0^2 = \partial_0^2)$ because the electric field is zero (or because the $\boldsymbol{B}$-field is a static field).

Now, for the ambient magnetic field, we consider a weak constant magnetic field in the z-axis and from the Lorenz [16] and Coulomb gauge, we (can and) shall choose that the magnetic vector potential of this magnetic field be such that $\boldsymbol{A} = \frac{1}{2}\boldsymbol{r} \times \boldsymbol{B}$ where the magnetic field $\boldsymbol{B}$ is such that $\boldsymbol{B} = (0, 0, B)$ so that:

$$\boldsymbol{A} = \frac{1}{2}x^2 B\hat{\boldsymbol{\imath}} - \frac{1}{2}x^1 B\hat{\boldsymbol{\jmath}}, \tag{24}$$

where $(x = x^1; y = x^2)$, and:

$$F_{\mu\nu} = \begin{pmatrix} 0 & -v_y B/2 & +v_x B/2 & 0 \\ +v_y B/2 & 0 & -B & 0 \\ -v_x B/2 & +B & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{25}$$

Now, to proceed with our calculation – on the assumption that these terms are small, we will neglect second order and higher terms. So doing, we obtain:

$$\begin{aligned} D_i^2 &= \partial_i^2 - \imath ne(\partial_i A_i^{ex} + A_i^{ex}\partial_i) - n^2e^2\mathrm{O}(A_{ex,i}^2) \\ &= \partial_i^2 - \imath neB(x^2\partial_1 - x^1\partial_2) - n^2e^2\mathrm{O}(A_{ex,i}^2) \ , \\ &= \quad \boldsymbol{\nabla}^2 + ne\boldsymbol{B}\cdot\boldsymbol{L}/\hbar - n^2e^2\mathrm{O}(A_{ex,i}^2) \end{aligned} \tag{26}$$

where $(\mathbf{L} = \mathbf{r} \times \mathbf{p} = -\imath\hbar\boldsymbol{r} \times \boldsymbol{\nabla})$ is the orbital angular momentum operator which means that the orbital angular momentum generates orbital magnetic moment that interacts with the magnetic field. In this instance, the angular momentum comprises only the z-component – that is to say: $[\mathbf{L} = -\imath\hbar(x^1\partial_2 - x^2\partial_1)\hat{\boldsymbol{k}}]$. As a starting point in the calculation of the gyromagnetic ratio, the second order term $n^2e^2\mathrm{O}(A_{ex,i}^2)$ will be neglected. We will take up this term in the next section when we try to fully account for the Proton's gyromagnetic ratio. As will be seen shortly, neglecting this term results in $\sim 22.8\%$ of the Proton's excess g-ratio not begin accounted for.

Now, if we write the Dirac four component wavefunction as:

$$\psi = \begin{pmatrix} \Phi \\ \chi \end{pmatrix}, \tag{27}$$

we find that in the non-relativistic limit – the component Φ dominates, thus we will consider this component as describing the wavefunction of the our particle.

To proceed, we need to compute $\imath ne\sigma_{(as)}^{\mu\nu}F_{\mu\nu}/2\hbar$ and we must remember that we have to consider the terms that affect Φ and also take not that the terms with σ^1 and σ^2 do not contribute to the gyromagnetic ratio. First – as one can verify for themselves the term $(\sigma^{0j}F_{0j} + \sigma^{j0}F_{j0})$ is such that $(\sigma^{0j}F_{0j} + \sigma^{j0}F_{j0} = 0)$, and $\sigma^{ij}F_{ij}$ is effectively equal to $(\sigma^{12}F_{12} + \sigma^{21}F_{21})$ which in-turn equals to $(\sigma^{12} - \sigma^{21})F_{12}$, hence:

$$\sigma^{ij}F_{ij} = 4\imath^{2\lambda_a|\lambda_a|+1}2^{|\lambda_a|}\epsilon\boldsymbol{S}\cdot\boldsymbol{B}/\hbar, \tag{28}$$

where ϵ is a directional term which is such that $(\epsilon = +1)$ if $(\boldsymbol{S}\cdot\boldsymbol{B} > 0)$ and $(\epsilon = +1)$ if $(\boldsymbol{S}\cdot\boldsymbol{B} < 0)$. To justify this, let us consider an Electron in a magnetic field. It is known that in presence of external magnetic fields, the spin of the Electrons would be aligned along the direction of the field, or at least a component of the external magnetic field along the direction of the Electron's spin – this, would point in the same direction as the Electron's spin. If we flipped the Electron's charge, we would expect the spin of the new positively charged particle to point in the exact opposite direction as before. In this way, a positive g-factor would mean alignment of the component of the unit vector of the magnetic field along the direction of the spinning particle and likewise, a negative g-factor would mean alignment in the opposite direction of the component of the unit vector of the magnetic field along the direction of the spinning particle – hence $(\epsilon = \pm 1)$.

Now, in writing $\sigma^{ij}F_{ij}$ as given above, we have neglected terms with σ^1 and σ^2 as these do not contribute to the gyromagnetic ratio[29]. From (28), it follows that:

$$\frac{\imath ne}{2\hbar}\sigma_{(as)}^{\mu\nu}F_{\mu\nu} = -4ne\imath^{2\lambda_a|\lambda_a|}2^{|\lambda_a|}\epsilon\boldsymbol{S}\cdot\boldsymbol{B}/\hbar. \tag{29}$$

where $(\boldsymbol{S} = \frac{1}{2}s^3\hbar\sigma^3\hat{\boldsymbol{z}})$ is the spin along the z-axis. In our calculation, we have set $(s^1 = s^2 = 1)$ because the spin is along the z-axis: remember that we have set $(s^is^i = s^i)$, which in this case implies that $(s^1s^2 = s^3)$. Further, since the Proton is a spin 1/2 particle, $(s^3 = 1)$.

Now writing $(\Phi = e^{-im_0c^2t/\hbar}\Psi)$ where Ψ oscillates much more slowly with time than $e^{-im_0c^2t/\hbar}$, then, to first order approximation where the terms in $\ddot{\Psi}$ are considered to be small hence negligible, we will have:

$$\left[\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \frac{\mathrm{m}_0^2c^2}{\hbar^2}\right]e^{-im_0c^2t/\hbar}\Psi \simeq -\frac{2i\mathrm{m}_0c}{\hbar}e^{-im_0c^2t/\hbar}\frac{\partial\Psi}{\partial t}. \tag{30}$$

Putting all the bits and pieces together, we will have:

$$\left[-\frac{\hbar^2}{2\mathrm{m}_0}\boldsymbol{\nabla}^2 + n\mu_{\mathrm{B}}\boldsymbol{B}\cdot(\boldsymbol{L} + \mathrm{g}_a\boldsymbol{S})\right]\Psi = i\hbar\frac{\partial\Psi}{\partial t}, \tag{31}$$

where $(\mu_{\mathrm{B}} = e\hbar/2\mathrm{m}_0c)$ is the Bohr magneton (in Gaussian CGS units) and $[\mathrm{g}_a = 2^{|\lambda_a|+1}\imath^{2\lambda_a|\lambda_a|}\epsilon = (-2)^{|\lambda_a|+1}\epsilon]$ so that:

$$\frac{\mathrm{g}_a - 2}{2} = (-2)^{|\lambda_a|}\epsilon - 1, \tag{32}$$

hence, we have:

$$\frac{g_a - 2}{2} = \begin{cases} 0, & \text{for } (a = 1) : If \; (\boldsymbol{S}\cdot\boldsymbol{B} > 0 : i.e., \; \epsilon = +1). \\ -2, & \text{for } (a = 1) : If \; (\boldsymbol{S}\cdot\boldsymbol{B} < 0 : i.e., \; \epsilon = -1). \\ +1, & \text{for } (a = 2,3) : If \; (\boldsymbol{S}\cdot\boldsymbol{B} > 0 : i.e., \; \epsilon = +1). \\ -3, & \text{for } (a = 2,3) : If \; (\boldsymbol{S}\cdot\boldsymbol{B} < 0 : i.e., \; \epsilon = -1). \end{cases} \tag{33}$$

This result (35), is the sought for gyromagnetic ratio for all the three types of particle configuration as represented by the a-index $[a = (1,2,3)]$. For the case $(a = 0)$, we obtain as expected, the Dirac gyromagnetic ratio $(g_0 = \pm 2)$ for the Electron $(g_0 = -2)$ and the Positron $(g_0 = +2)$. For the case $a = (1,2)$, we obtain $(g_{2,3} = \pm 4)$ with $(g_{2,3} = +4)$ being the g-factor for the Proton and $(g_{2,3} = -4)$ expected to the the g-factor for the anti-Proton. In the case of the Proton, a g-factor of $(g_{2,3} = +4)$ accounts for $\sim 55.7\%$ of the Proton's extra-anomalous g-ratio $(g_p - 2 = 3.585...)$. As suggested in the reading [17], we propose to account for this $\sim 44.3\%$ unaccounted extra-anomalous g-ratio of the Proton as a second effect arising from the neglected second order term in equation (26) *i.e.*, the term $n^2e^2\mathrm{O}(A^2_{\mathrm{ex},i})$. We present this proposal in §() below.

EXTRA-ANOMALOUS GYROMAGNETIC RATIO

The fact that the gyromagnetic ratio of the Electron differs slightly from the predicted Dirac value of $(\mathrm{g_D} = 2)$, this implies that there is an extra unaccounted for interaction of the spin with the ambient magnetic field or an aspect of it. To take this into account, naturally, we would modify equation (31) so that it reads:

$$\left[-\frac{\hbar^2}{2\mathrm{m}_0}\boldsymbol{\nabla}^2 + n\mu_{\mathrm{B}}\boldsymbol{B}\cdot[\boldsymbol{L} + (\mathrm{g}_a + 2\Delta_{\mathrm{g}})\boldsymbol{S}]\right]\Psi = i\hbar\frac{\partial\Psi}{\partial t}, \tag{34}$$

where Δ_{g} is the extra-anomalous gyromagnetic ratio. This modification will lead to $(\mathrm{g}^{\mathrm{eff}}_a = 2[(-2)^{|\lambda_a|}\epsilon - 1 + \Delta_{\mathrm{g}}])$, so that:

$$\frac{\mathrm{g}^{\mathrm{eff}}_a - 2}{2} = (-2)^{|\lambda_a|}\epsilon - 1 + \Delta_{\mathrm{g}}, \tag{35}$$

where $\mathrm{g}^{\mathrm{eff}}_a$ is the *Effective g-ratio*. In-order to calculate Δ_{g}, we will make now in-cooperate the second order term the we neglected earlier in (26) *i.e.*, the term $n^2e^2\mathrm{O}(A^2_{\mathrm{ex},i})$. We must remember that at the instance of equation (26), we left out this term on the pretext that it was small and negligible. This second order term is such that:

$$n^2e^2\mathrm{O}(A^2_{\mathrm{ex},i}) = \left[\frac{ne\mathrm{O}(A^2_{\mathrm{ex},i})}{\boldsymbol{B}\cdot\boldsymbol{S}}\right] ne\boldsymbol{B}\cdot\boldsymbol{S}. \tag{36}$$

Inserting this into our system equations at the instance of equation (26) as has been done in the reading [17], one will find out that:

$$\Delta_{\mathrm{g}} = \frac{ne\mathrm{O}(A^2_{\mathrm{ex},i})}{2\boldsymbol{B}\cdot\boldsymbol{S}} = \left(\frac{neR^2}{4\hbar}\right) B\cos\vartheta, \tag{37}$$

where R is the radius of the particle in question which in this case is the Proton. If the there alignment of the component of the unit vector of the magnetic field along the direction of the spinning particle then $(\cos\vartheta > 0)$, and likewise, if there is alignment in the opposite direction of the component of the unit vector of the magnetic field along the direction of the spinning particle, then $(\cos\vartheta < 0)$; from this, we write $(\cos\vartheta = \epsilon|\cos\vartheta|)$, therefore, from all this, it follows that:

$$\frac{\mathrm{g}^{\mathrm{eff}}_a - 2}{2} = (-2)^{|\lambda_a|}\epsilon - 1 - \epsilon\left(\frac{qR^2}{4\hbar}\right)|\boldsymbol{B}\cos\vartheta|. \tag{38}$$

As pointed out in the reading [17], the constancy of Δ_{g} as revealed by experiments (for the Electron, Proton, Neutron and other papers), this is indication that the term "$|\boldsymbol{B}\cos\vartheta|$" is also a constant. This term "$|\boldsymbol{B}\cos\vartheta|$" is the magnitude of the component of the $\boldsymbol{B}$-field along the spin axis of the Electron. Why this is so, we have no answer to this question as the theory does not say why.

GENERAL DISCUSSION

In the present work, we have demonstrated that the CST-Dirac equations [10] can in general account for the g-factor of the Proton. At a *prima face* level, our motivation for an alternative explanation of the Proton's gyromagnetic ratio may appear ill-founded because the conventional explanation for these anomalies relies on Quantum Field Theory (QFT) and is quite adept at describing them. As already said in Paper (I), Quantum Electrodynamics (QED) can predict the g-factor of the Electron to about a part-per-trillion. Quantum Chromodynamics (QCD) does a lower-precision, but an admirable job with the Proton and Neutron [18–20], which are composite particles while the Electron is assumed to a point particle. Given the aforesaid, our contribution may appear unnecessary if not useless altogether. For example, one may ask the following question:

> *By putting forward a new proposal, are we discarding present QFT, QED and QCD efforts in explaining these the g-values of the Proton, Neutron and Electron?*

To this important question, we have the following to say. When the large magnetic moments of the Proton (and the Neutron) where first discovered, many puzzling questions regarding the nature of the Proton (and the Neutron) where raised [5] and these where not answered until the advent of the quark model [21–23]. Resident quarks found inside the Proton (and the Neutron) are what is believed to be the reason for the large g-factor for the Proton (and Neutron). In this quark model, the magnetic moment of the Proton (and Neutron) is (or can) be modelled as a sum of the magnetic moments of the constituent quarks [24]. From the present model, this is not the reason. The reason is that the Proton (and any other particle for that matter) has (is here predicted to have) three configurations with two of these configurations $[a = (2, 3)]$ having these large g-factors. The other configuration $(a = 1)$ has a g-factor that should be close to that of the Electron. So, a prediction of the present model is that one of the three states (or configurations) of the Proton must have a g-factor comparable to that of the Electron.

In-closing, allow us to that, it is our modest view, the present result is surely a profound result for it – *in-principle*, now allows us to say that the Dirac equation does – on a qualitative level, explain not only the Proton, but the Neutron and any general spin-1/2 particle. The very fact that one would not account qualitatively for the Proton and Neutron's gyromagnetic ratio from the bare Dirac equation, this led physicists to think that in its bare form, the Dirac equation is not an equation for the Electron and because of this, some physicists have sought for a Dirac equation for the Proton and the Neutron [25]. Certainly, there is need to ponder on this result further than has been conducted herein.

CONCLUSION

Assuming the acceptability of what has been presented herein, we hereby make the following conclusion:

(1). The Dirac equation can be considered to account for the extra-anomalous gyromagnetic ratio of all spin-1/2 particles and this extra-anomalous gyromagnetic ratio arise as second order effect so to the spin's alignment with the ambient magnetic field.

(2). The present finding gives us insight into the nature of how the Electron (and other particles in general), interacts with the ambient magnetic field. The constancy of $\Delta_{\rm g}$ implies that the Electron (and other particles in general)'s spin does not randomly align itself with the ambient magnetic field but does this in a systematic and well ordered manner.

Acknowledgments: We are grateful to the National University *of* Science *&* Technology (NUST)'s Research *&* Innovation Department and Research Board for their unremitting support rendered toward our research endeavours; of particular mention, Prof. Dr. P. Mundy, Dr. P. Makoni, Dr. D. J. Hlatywayo and Prof. Dr. Y. S. Naik's unwavering support. This reading is dedicated to my mother *Setmore Nyambuya* and to the memory of departed father *Nicholas Nyambuya* (*27.10.1947 – 23.09.1999*).

[1] G. G. Nyambuya, Submitted to: *Physical Review D.* pp. 1–8: Manuscript No. *es*2016*sep*16_543. (2016).
[2] G. G. Nyambuya, Submitted to: *Physical Review D.* pp. 1–8: Manuscript No. *es*2016*sep*16_608. (2016).
[3] I. Estermann and O. Stern, Zeitschrift für Physik **85**, 17 (1933), ISSN 0044-3328".
[4] R. Frisch and O. Stern, Zeitschrift für Physik **85**, 4 (1933), ISSN 0044-3328.
[5] G. Breit and I. I. Rabi, Phys. Rev. **46**, 230 (1934).
[6] P. A. M. Dirac, Proc. Roy. Soc. (London) **A117**, 610 (1928).
[7] P. A. M. Dirac, Proc. Roy. Soc. (London) **A118**, 351 (1928).
[8] A. Zee, *Quantum Field Theory in a Nutshell* (Princeton University Press, Princeton, USA, 2003), pp. 177–179, 1st ed.
[9] A. Mooser, S. Ulmer, K. Blaum, K. Franke, H. Kracke, C. Leiteritz, W. Quint, C. C. Rodegheri, C. Smorra, and J. Walz, Nature **509**, 596 (2014).
[10] G. G. Nyambuya, Foundations of Physics **37**, 665 (2008).
[11] H. Nishino, S. Clark, K. Abe, Y. Hayato, T. Iida, M. Ikeda, J. Kameda, K. Kobayashi, Y. Koshio, M. Miura, et al. (Super-Kamiokande Collaboration), Phys. Rev. Lett. **102**, 141801 (2009).
[12] G. G. Nyambuya, J. Mod. Phys. **5**, 1733 (2014).
[13] G. G. Nyambuya, J. Mod. Phys. **4**, 1050 (2013).
[14] G. G. Nyambuya, Apeiron **16**, 516 (2009).
[15] G. G. Nyambuya, Prespacetime J. **7**, 1733 (2016), ISSN 2153-8301, (ISSN: 2153-8301; ISBN: 1530161681).
[16] H. A. Lorentz, Arch. Nèerl. Sci. **25**, 287 (1892).
[17] G. G. Nyambuya, In Press: J. Modern Phys. pp. 1–5 (2016).
[18] J. Engel, M. J. Ramsey-Musolf, and U. van Kolck, Progress in Particle and Nuclear Physics **71**, 21 (2013), ISSN 0146-6410, fundamental Symmetries in the Era of the LHC.
[19] G. Martinelli, G. Parisi, R. Petronzio, and F. Rapuano,

Physics Letters B **116**, 434 (1982), ISSN 0370-2693.
[20] I. Balitsky and A. Yung, Physics Letters B **129**, 328 (1983), ISSN 0370-2693.
[21] M. Gell-Mann, Physics Letters **8**, 214 (1964), ISSN 0031-9163.
[22] G. Zweig, Tech. Rep. 8182/TH.401, CERN, Geneva - Switzerland (1964), pp.1-24.
[23] G. Zweig, Tech. Rep. 8182/TH.412, CERN, Geneva - Switzerland (1964), pp.1-77.
[24] Y. Gell and D. B. Lichtenberg, Il Nuovo Cimento A (1971-1996) **61**, 27 (2016), ISSN 1826-9869.
[25] B. H. Chirgwin and H. T. Flint, Nature **155**, 724 (1945).
[26] See http://www-sk.icrr.u-tokyo.ac.jp/whatsnew/new-20091125-e.html. Visited on this day 13 Oct 2014@10h58 GMT+2.
[27] By *flat*, it here is not meant that the spacetime is Minkowski *flat*, but that the metric has no off diagonal terms. On the same footing, by *positively curved spacetime*, it meant that metric has positive off diagonal terms and likewise, a *negatively curved spacetime*, it meant that metric has negative off diagonal terms.
[28] The usual Dirac way of squaring is multiply this equation (1) from the left by $\left[\imath\hbar A^{\mu}\gamma^{\mu}_{(a)}\partial_{\mu} + \mathrm{m}_0 c\right]$.
[29] For the record, this term is: $2\lambda_a \imath^{\lambda_a|\lambda_a|}\sqrt{2^{|\lambda_a|}}(s^1\sigma^1 - s^2\sigma^2)B$. The particle is assumed to be confined along the xy-axis – as such, $s^1\sigma^1$ and $s^2\sigma^2$ are not the spin along the x and y-axis, but linear momentum of the particle along the x and y-axis on the xy-plane. Hence this term does not contribute to the g-factor, hence unnecessary for the present purposes and investigations.