Conjecture that there exist an infinity of Poulet numbers which are also Harshad-Coman numbers

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Abstract. OEIS defines the notion of Harshad numbers as the numbers n with the property that n/s(n), where s(n) is the sum of the digits of n, is integer (see the sequence A005349). In this paper I define the notion of Harshad-Coman numbers as the numbers n with the property that (n – 1)/(s(n) – 1), where s(n) is the sum of the digits of n, is integer and I make the conjecture that there exist an infinity of Poulet numbers which are also Harshad-Coman numbers.

Definition:

The Harshad-Coman numbers are the numbers n with the property that (n – 1)/(s(n) – 1), where s(n) is the sum of the digits of n, is integer.

The sequence of Harshad-Coman numbers:

: 2, 3, 4, 5, 6, 7, 8, 9, 11, 13, 19, 20, 21, 22, 25, 28, 31, 37, 40, 41, 43, 46, 49, 51, 55, 61, 64, 71, 73, 81, 82, 85, 91, 101, 103, 109, 110, 111, 112, 113, 115, 118, 121 (...)

Conjecture:

There exist an infinity of Poulet numbers P which are also Harshad-Coman numbers, i.e. that have the property that (P – 1)/(s(P) – 1), where s(P) is the sum of the digits of P, is an integer.

The first n Poulet numbers which are also Harshad-Coman numbers:
(From the first 12 Poulet numbers, 9 are also Harshad-Coman numbers)

: 645 (indeed, (645 – 1)/(15 – 1) = 46, integer);
: 1105 (indeed, (1105 – 1)/(7 – 1) = 184, integer);
: 1387 (indeed, (1387 – 1)/(19 – 1) = 77, integer);
: 1729 (indeed, (1729 – 1)/(19 – 1) = 96, integer);
: 1905 (indeed, (1905 – 1)/(15 – 1) = 136, integer);
: 2465 (indeed, (2465 – 1)/(17 – 1) = 154, integer);
: 2701 (indeed, (2701 – 1)/(10 – 1) = 300, integer);
: 2821 (indeed, (2821 – 1)/(13 – 1) = 235, integer);
: 3277 (indeed, (3277 – 1)/(19 – 1) = 182, integer).

Few larger Poulet numbers which are also Harshad-Coman numbers:

: 999710032321 (indeed, (999710032321 – 1)/(46 – 1) = 22215778496, integer);
: 999746703869 (indeed, (999746703869 – 1)/(77 – 1) = 13154561893, integer);
: 999986341201 (indeed, (999986341201 – 1)/(73 – 1) = 16666439020, integer).

Notes:

: For some Poulet numbers the number obtained is rational (example: for Poulet number 999828475651 is obtained 13886506606.25).
: For some Poulet numbers the number obtained is irrational (example: for Poulet number 999666754801 is obtained 14487923982.60869565217391304347826086956521739130434782...).

Definition:

OEIS also defines the notion of Moran numbers as the numbers n with the property that n/s(n), where s(n) is the sum of the digits of n, is prime (see the sequence A001101). I also define the notion of Moran-Coman numbers as the numbers n with the property that p = (n – 1)/(s(n) – 1), where s(n) is the sum of the digits of n, is prime and I make the conjecture that there exist an infinity of Moran-Coman numbers.

The sequence of Moran-Coman numbers:

: 3, 4, 6, 8, 19, 20, 22, 28, 40, 43, 46, 64, 85, 110, 112, 115, 118 (...) for which were obtained the primes p = 2, 3, 5, 7, 2, 19, 7, 3, 13, 7, 5, 7, 7, 109, 37, 19, 13 (...)