Fractional Matrix : A new Eigenvalue method on Spectral Analysis .

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A new member in Matrix representation has been introduced under the name **Fractional − Matrix** and defined properly. Further we show how one can address spectral analysis using this **Fractional − Matrix**. Interesting examples have been considerd .

PACS 02.10.Sp,03.65.Ge

Key words - fractional matrix , inverse matrix,eigenvalue,matrix diagonalisation method , differential equation.

I.Introduction.

Mathematics has been a beautiful subject for some one , who understands it in a fruitful way . Mathematics has been a tool to understand physical systems in a more secialised way [1]. In the context of differential equation

$$\ddot{x} + f(x)\dot{x}^2 + g(x) = 0 \tag{1}$$

or in eigenvalue relation

$$A|\Psi >= \lambda|\Psi_n > \tag{2}$$

its role is out standing .In the above $f(x), g(x)$ represent function of co-ordinate 'x' and the derivative is w.r.t time. In the context eigenvalue relation A stands for a standard matrix preferably symmetric and non-singular in its behaviour i.e $Det(A) \neq 0$. In this context we would like to state that for physical application matrix 'A' must be symmetric.Further we would like to state that all the literature on matrix analysis appears to be incomplete as no where it has been discussed "**Fractional – Matrix** hence forward (FM) . Hence aim of this communication is to introduce the FM and suggest how one can utilise it understanding the eigenvalues of a physical system.

II.Fractional -Matrix:Definition,Calculation and Eigenvalue relation.

Let us consider a matrix' A ' defined by the symbol

$$A = a_{i,j} \tag{3}$$

where $i, j = 1.....n$. In other worgd we condider (nxn) matrix .Let the inverse of matrix A is defined by the standard relation[1]

$$A^{-1} = Inv(A) = \frac{adj(A)}{det(A)} \tag{4}$$

Here the "adj(A)" i.e adjoint matrix is written interms of co-factors.

Now we denine the 'Fractional-Matrix' as

$$B = \frac{\alpha + A}{A} = I + \alpha A^{-1} \tag{5}$$

where I stands for an unit matrix of same dimension as that of A . Let the eigenvalue relation of B is

$$B|\Psi >= \beta|\Psi_n > \tag{6}$$

then the eigenvalue of original matrix A can be written as

$$\lambda = \frac{\alpha}{(\beta - 1)} \tag{7}$$

In derieving the above relation we have taken standard relation (for symmetric matrix) as [2]9 for detail proof see [2])

$$A\Psi = \lambda\Psi \to A^{-1}\psi = \frac{1}{\lambda}\Psi \tag{8}$$

III.Example for Mathematical Sciences :Simple example of (2x2) matrix.

Let us consider a simple (2x2) matrix as

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \tag{9}$$

having inverse as

$$A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix} \tag{10}$$

Now we fine fractional matrix $B_{\alpha=1,2,3,4}$ as for $\alpha = 1$

$$B_1 = \begin{bmatrix} 2 & 0 \\ 0 & 1.5 \end{bmatrix} \tag{11}$$

forv $\alpha = 2$ as

$$B_2 = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \tag{12}$$

for $\alpha = 3$ as

$$B_3 = \begin{bmatrix} 4 & 0 \\ 0 & 2.5 \end{bmatrix} \tag{13}$$

for $\alpha = 4$ as

$$B_4 = \begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix} \tag{14}$$

for $\alpha = 5$ as

$$B_5 = \begin{bmatrix} 6 & 0 \\ 0 & 3.5 \end{bmatrix} \tag{15}$$

Now we calculate the eigenvalues of original matrix A through the formula

$$\lambda_n = \frac{\alpha}{(\beta_n - 1)} \tag{16}$$

and tabulate in table-1 .

Table -I : Eigenvalues of Fractional- Matrix B_α and comparison with A

. A	α	B_α	A from B_α	remarks
1	1	2	1	same
2		1.5	2	same
1	2	3	1	same
2		2	2	same
1	3	4	1	same
2		2.5	2	same
1	4	5	1	same
2		3	2	same
1	5	6	1	same
2		3.5	2	same

IV.Example for Physical Sciences

Let us consider the differential equation as stated above [3] with $f(x) = \frac{x}{(1+x^2)}$ and $g(x) = \frac{x}{(1+x^2)}$. The corresponding Hamiltonian can be written as

$$H = \frac{1}{2}[x^2 + \frac{p^2}{1+x^2}] \tag{17}$$

The second term is not not invariant under exchane of momentum [4] ,so we consider the Hamiltonian as

$$H = \frac{1}{2}[p\frac{1}{1+x^2}p + x^2] \tag{18}$$

,which is invariant under exchange of momentum. Using matrix diagonalisation method [5] we compute the eigenvalues of H and B (for $\alpha = 5$ and tabulate them in table-II as follows.

Table -II : Eigenvalues of H , B for $\alpha = 5$ and Comparison

n	H	$B \to \alpha = 5$	H from B	Remarks
0	0.355 026 280 726	15.083 464 440 362	0.355 026 28	same
1	1.226 397 537 460	5.076 981 441 394	1.226 397 53	same
2	1.846 999 994 984	3.707 092 589 916	1.846 999 99	same
3	2.445 481 397 035	3.044 587 215 450	2.445 481 39	same
4	2.977 016 520 362	2.679 533 843 967	2.977 016 52	same

V. Discussion

We have introduced a new member in the theory of Linear Algebra under the section " Matrix - Analysis" as " Fractional - Matrix" . The illustrated examples will motivate many to add new information to this new member . Last but not the least , the present method is a self - checking method on eigen-

value relation .

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TheAfrican.Rev.Phys.9:0027,201(2014)

; B.Rath:TheAfrican.Rev.Phys.(2016)(Accepted for publication).