The generalized Goldbach's conjecture: symmetry of prime number

Jian Ye

Department of Mathematics Sichuan University

Chengdu 610207,China

yoyoo20082005@sina.com

July 29,2014

Keywords: the generalized Goldbach's conjecture , symmetry of prime number

Abstract:

Goldbach's conjecture: symmetrical primes exists in natural numbers.

the generalized Goldbach's conjecture: symmetry of prime number in the former and tolerance coprime to arithmetic progression still exists.

MSC: 11B25;11N13.

Notation:

$G(x)$: the number of representatives a large even integer x as a sum of two primes.

Let $G(x,q,L)$: the number of representatives a large even integer x as a sum of two primes in the former L and tolerance q coprime to arithmetic progression.

p: a prime number.

a: a constant number, $a>0$.

$\mathcal{O}$: mean big O notation describes the limiting behavior of a function when the argument tends towards a particular value or infinity, usually in terms of simpler functions.

$p|k$: p divides k.

$\phi(q)$: $\phi(q)$ is the Euler phi-function.

$Li_2(x)$: express the logarithmic integral function or integral logarithm. $Li_2(x)$ is a special function such as $Li_2(x) = \int_2^x \frac{dt}{\ln^2 t}$.

Symmetry of prime number:

In the former L and tolerance q ($L<q, q\geq 2$) coprime to arithmetic progression, for any one item of this progression such as $\frac{x}{2}$ where $\frac{x}{2} > \phi(q)\cdot q^2$, There are at least a pair of symmetrical primes on $\frac{x}{2}$, namely even integer x can be expressed for a sum of two primes in its series.

Let $G(x,q,L)$ is the number of representatives a large even integer x as a sum of two primes in the former L and tolerance q coprime to arithmetic progression.

1.) If $q = 2^m$,

$$G(x,q,L) = \frac{1}{\phi(q)} \cdot 2C \cdot \prod_{p \geq 3} \frac{(p-1)}{(p-2)} \cdot Li_2(x) + \mathcal{O}(x \cdot e^{-a\sqrt{\ln x}}) \quad (1)$$

where ($p > 2$, $p|x$).

2.) If q is an odd prime number,

$$G(x,q,L) = \frac{1}{q-2} \cdot 2C \cdot \prod_{p \geq 3} \frac{(p-1)}{(p-2)} \cdot Li_2(x) + \mathcal{O}(x \cdot e^{-a\sqrt{\ln x}}) \quad (2)$$

where ($p > 2$, $p|x$).

3.) If q is an odd number, let $\Upsilon(q) = q \cdot \prod (1 - \frac{2}{p})$, ($p > 2, p|q$),

$$G(x,q,L) = \frac{1}{\Upsilon(q)} \cdot 2C \cdot \prod_{p \geq 3} \frac{(p-1)}{(p-2)} \cdot Li_2(x) + \mathcal{O}(x \cdot e^{-a\sqrt{\ln x}}), \quad (3)$$

where ($p > 2$, $p|x$).

4.) If $q = 2^m \cdot j$ (j is an odd number), let $\Upsilon(j) = j \cdot \prod (1 - \frac{2}{p})$, ($p > 2, p|j$),

$$G(x,q,L) = \frac{1}{\phi(2^m)} \cdot \frac{1}{\Upsilon(j)} \cdot 2C \cdot \prod_{p \geq 3} \frac{(p-1)}{(p-2)} \cdot Li_2(x) + \mathcal{O}(x \cdot e^{-a\sqrt{\ln x}}), \quad (4)$$

where ($p > 2$, $p|x$).

It can be seen, classify two situations.

While $q = 2$; $q = 3$ or $q = 6$ alternative,

$$G(x) = G(x,q,L) = 2C \cdot \prod_{p \geq 3} \frac{(p-1)}{(p-2)} \cdot Li_2(x) + \mathcal{O}(x \cdot e^{-a\sqrt{\ln x}}), \quad (5)$$

where ($p > 2$, $p|x$, $a > 0$), this formula prompts the expression of Goldbach conjecture.

The Goldbach's conjecture expression is same with the twin prime conjecture. Since

$$C = \prod_{p \geq 3} (1 - \frac{1}{(p-1)^2}), \quad (6)$$

where $p > 2$, p is a prime number.

Conclusion:

In the former L and tolerance q $(L < q, q \geq 2)$ coprime to arithmetic progression,
Let $G(x, q, L)$ is the number of representatives a large even integer x as a sum of two primes in its arithmetic progression,

$$G(x,q,L) = \frac{1}{\phi(2^m)} \cdot \frac{1}{\Upsilon(j)} \cdot 2C \cdot \prod_{p \geq 3} \frac{(p-1)}{(p-2)} \cdot Li_2(x) + \mathcal{O}(x \cdot e^{-a\sqrt{\ln x}}).$$

where $p > 2$, $p|x$, $q = 2^m \cdot j$, $\Upsilon(j) = j \cdot \prod (1 - \frac{2}{p})$ $(p > 2, p|j)$,
p is a prime number, j is an odd number.