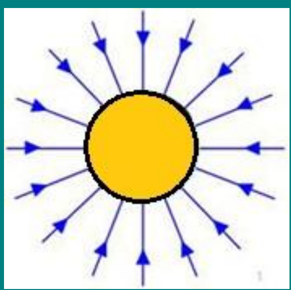
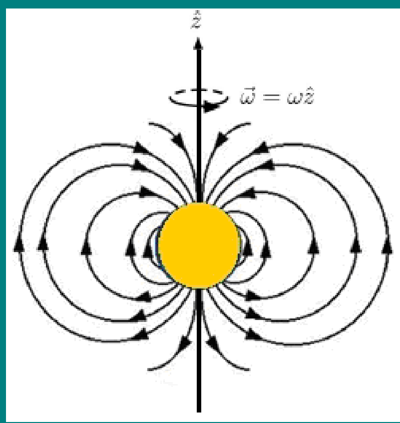
Alexander G. Kyriakos

Lorentz-invariant Gravitation Theory.

$$\frac{1}{c^2}\cdot\frac{\partial^2\varphi}{\partial t^2}-\nabla^2\varphi=4\pi\rho \qquad\qquad \frac{1}{c^2}\cdot\frac{\partial^2\vec{A}}{\partial t^2}-\nabla^2\vec{A}=\frac{4\pi}{c}\,\rho\vec{\upsilon}$$





$$ds^2=\left(1-\frac{r_s}{r}\right)c^2dt^2-\frac{dr^2}{\left(1-\frac{r_s}{r}\right)}-r^2\left(\sin^2\theta\, d\varphi^2+d\theta^2\right)+\frac{4\gamma_N M_s a}{c^2 r}\sin^2\theta\, d\varphi dt$$

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Annotation

The modern theory of gravity, which is conditionally called General Theory of Relativity (GR), was verified with sufficient accuracy and adopted as the basis for studying gravitational phenomena in modern physics. However, it has certain features that make it impossible to connect with other theories, on which almost all the techniques and technology of modern civilization is built. Another formal disadvantage of general relativity is that the study and the use of its mathematical apparatus require much more time than the study of any of the branches of modern physics. This book is an attempt to build a version of the theory of gravitation, which is in the framework of the modern field theory and would not cause difficulties when teaching students. A characteristic feature of the proposed theory is that it is built on the basis of the nonlinear quantum field theory.

From author

The Lorentz-invariant theory of gravitation (LIGT) is the conditional name of the proposed theory of gravity, since Lorentz-invariance is a very important, although not the only feature of this theory.

Note that our approach was used in the past in relation to the gravitational theories that have some similarities with our theory. Therefore the results obtained by well-known scientists are widely cited in the book. However, for posing the problem and for some of the basic elements of the theory which are obtained by the author of the book, the only person responsible is the author.

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NOTATIONS.

(In almost all instances, meanings will be clear from the context. The following is a list of the usual meanings of some frequently used symbols and conventions).

Mathematical signs

- $\alpha, \beta, \mu, \nu ...$ - Greek indices range over 0,1,2,3 and represent space-time coordinates, components, etc.
- $i, j, k...$ - Latin indices range over 1,2,3 and represent coordinates etc. in 3- dimensional space
- $\hat{\alpha}_\mu, \hat{\beta}$ - Dirac matrices
- $\vec{A}$ - 3-dimensional vector
- A^μ - 4-dimensional vector
- $A^{\mu\nu}$ - Tensor components
- ∇ - Covariant derivative operator
- $\nabla^2 \equiv \Delta$ - Laplacian
- $\Box$ - d'Alembertian, operator $\equiv \nabla^2 - \partial^2 / \partial t^2$
- $g_{\mu\nu}$ - metric tensor of curvilinear space-time
- $g_{\mu\nu}^{GR}$ - metric tensor of GR space-time
- $R_{\alpha\beta\gamma\delta}$ - Riemann tensor
- $R_{\alpha\beta}$ - Ricci tensor $R^\gamma{}_{\alpha\beta\delta}$
- R - Ricci scalar $\equiv R^\alpha{}_\alpha$
- $G_{\alpha\beta}$ - Einstein tensor
- $\eta_{\mu\nu}$ - Minkowski metric
- $h_{\mu\nu}$ - Metric perturbations
- $\Lambda_{\mu\nu}$ - Lorentz transformation matrix

Physical values

- u_μ - velocity
- a_μ - 4-acceleration $\equiv du_\mu / d\tau$
- p_μ - 4-momentum
- $T^{\mu\nu}$ - Stress-energy tensor
- $F^{\mu\nu}$ - Electromagnetic field tensor
- j_μ - Current density
- $J^{\mu\nu}$ - Angular momentum tensor
- γ_N - Newton's constant of gravitation
- γ_L - Lorentz factor (L-factor)
- m - mass of particle
- M_S - mass of the star (Sun)
- $\vec{J}$, $\vec{L}$ - angular momentum

Abbreviations:

- LIGT - Lorentz-invariant gravitation theory;
- EM - electromagnetic;
- EMTM - electromagnetic theory of matter;
- EMTG - electromagnetic theory of gravitation;
- SM - Standard Model;
- NQFT – nonlinear quantum field theory
- NTEP - nonlinear theory of elementary particles;
- QED - quantum electrodynamics.
- HJE - Hamilton-Jacobi equation
- SR or STR - Special Ttheory of Relativity
- GR or GTR - General Theory of Relativity
- L-transformation - Lorentz transformation
- L-invariant - Lorentz-invariant

Indexes

- e - electrical
- m - magnetic,
- em - electromagnetic,
- g - gravitational, within the framework of EMTG
- ge - gravito-electric,
- gm - gravito-magnetic
- N - Newtonian

Table of contents

Chapter 1. Introduction. Statement of problem

1.0. The place of gravitation theory in a number of other physical theories

The consequence of observation and experiments is a series of statements that allow us to make important conclusions about the nature of gravity.

First of all, let us define and explore some important terms and conceptions.

Material objects whose behaviour, in practice, does not reveal their wave nature, are referred to as macro-objects; otherwise the objects are called micro-objects. Sciences regarding macro-objects are called classical. Sciences regarding micro-objects are called quantum. Objects of the macrocosm consist of the objects of the microcosm (although, due to the wave nature of elementary particles, the boundary between macro-objects and micro-objects is not clear, but in practice it does not cause too much difficulty). The laws of the microworld are Lorentz-invariant. The properties of matter of microcosm do not differ from the properties of matter of the macrocosm

The first classical mechanics theory was created by Newton. Two types of laws of mechanics exist: laws of motion of material bodies under the action of forces, and laws that define these forces (which are often called equations of sources). In frames of Newton's theory, his second law is the primary law of motion, while Newton's gravitational law defines the force of gravity.

It should also be mentioned that during the further development of mechanics, numerous mathematical formulations of the original laws of Newtonian mechanics were found, which are physically almost equivalent, including the ones that use energy characteristics of the motion of bodies, rather than force.

As was revealed later, Newtonian mechanics is valid for speeds, well below the speed of light $c \cong 300000$ km/sec. Mechanics, which is valid for speeds υ from zero to the speed of light was conditionally named relativistic mechanics. The deviation of relativistic mechanics from non-relativistic is usually of second order to υ/c (Pauli, 1981)). Under the condition $\upsilon << c$, Newton's laws have very high accuracy.

2.0. Relativistic theories

The definition "relativistic" is equivalent to the requirement "to be invariant under Lorentz transformations". Therefore we will use the definition of "relativistic" equally with the definition of "Lorentz-invariant (briefly "L-invariant"). Let us emphasize that these transformations were obtained by Lorentz on the base of research of invariance of the laws of the EM field - Maxwell's equations.

In relativistic mechanics, there are also several forms of equations of motion and equations of sources. As the relativistic law of motion (including the theory of gravity) the relativistic Hamilton-Jacobi equation is often used.

2.1. How non-relativistic mechanics is related to the relativistic mechanics

Let us note one important feature of any Lorentz-invariant (relativistic) theories in comparison to non-Lorentz-invariant (non-relativistic) theories.

Non-relativistic theories give correct predictions at speeds much less than the speed of light. The relativistic theories give exact values in the entire range of speed from R until the speed of light c = 300 000 km/sec. The inaccuracy of non-relativistic theories compared to the relativistic can be attributed to the Lorentz factor $\gamma_L = 1/\sqrt{1-\upsilon^2/c^2}$, a factor of the Lorentz transformation (see in reference book the diagram of Lorentz-factor as a function of speed). Most amendments to the non-relativistic theory are determined not by the Lorentz factor of the first degree, but of the second degree, which makes the corrections even less.

As seen from the graph, factor is not very different from the unit, up until the velocity of the particle reaches the 1/10 of the velocity of light. The maximum speeds of the planets and the massive bodies on the Earth and in the solar system are: projectile - 1.5 km/s, the rocket - 10-12 km/s, meteorites - 18-25 km/s, the Earth around the Sun - 30 km/s, the Sun in the direction to the galactic center - 200 km/s, our galaxy - up to 400 km/s. Higher speed is achieved only by elementary particles in cosmic space or in accelerators, but they do not play any role in the theory of gravity. If we talk about non-mechanical applications, the velocity of the electrons in the home appliances (e.g., in wires) is comparable to the velocity of thermal motion of atoms and molecules (fraction of km/s).

Thus, the value υ^2/c^2 in real problems of mechanics is very small. This means that the Lorentz factor is not very different from unit. Perhaps the only case we need to use the relativistic equations in real life is in spectroscopy in the study of the emission of very fine lines.

This means that Newtonian mechanics is valid in practical applications with great accuracy. Any corrections thereto, regarding relativism, are insignificant and mainly caused by curiosity of scientists than the practical needs of society. Given that the technical calculations are made with an accuracy of no more than three decimal places, apparently, there is no meaning in this life to enter the so-called relativistic corrections.

This was already understood by one of the founders of the Lorentz-invariant physics – A. Poincare, who had warned (Poincaré, 1908):

"*I tried in a few words to give the fullest possible understanding of new ideas and explain how they were born ... In conclusion, if I may, I express a wish. Suppose that in a few years, this new theory will be tested and come out victorious from this test. Then, our school education is in serious danger: some teachers will undoubtedly want to find a place to new theories .... And then [the students] will not grasp the usual mechanics*.

Is it right to warn students that it gives only approximate results? Yes! But later! When they will be permeated by it, so to speak, to the bone, when they will be accustomed to think only with its help, when there will not be a risk that they forget how to do this, then we can show them its borders. They will have to live with the ordinary mechanics, the only mechanic that they will apply. Whatever the success of automobilism would be, our machines will never reach those speeds where ordinary mechanics is not valid. Other mechanics is a luxury, but one can think about a luxury only when it is unable to cause harm to the necessary."

3.0. The general theory of relativity

The modern theory of gravitation, conventionally called the general theory of relativity (GTR or GR), refers to classical mechanics. As the equation of source is considered to be the Einstein-Hilbert equation (EHE) of general theory of relativity (GTR), which was found by these researchers almost independently and almost simultaneously (Pauli, 1981; Vizgin, 1981). As the basis for theory building, Hilbert used a variational principle. The approach of Einstein was heuristic, emanating from the experimental fact of equality of gravitational and inertial masses (note that this equivalence is also valid in nonrelativistic theories).

A very difficult question, is whether the GTR and its equation are relativistic in terms of the Lorentz invariance. Strictly speaking, it is not. Einstein assumed that the general covariance of the equations of general relativity includes special relativity. Indeed, (Katanaev, 2013, p. 742):

«*Lorentz metric satisfies the Einstein's vacuum equations* . [But] "*in GTR is postulated that space-time metric is not a Lorentz metric, and is found as a solution of Einstein's equations. Thus, the space-time is a pseudo-Riemanian manifold with metric of a special type that satisfies the Einstein equations*."

As is known, EHE is very different from other equations of mechanics, since it is based on the Riemann geometry in general covariant system of coordinates. For this reason, it is not compatible with the laws of quantum mechanics.

Besides, EHE has several disadvantages, which have not been overcome to date (see critics in the works of V. Fock, A. Logunov, etc (Fock, 1964; Logunov, 2002; Rashevskyi, 1967)). These disadvantages have been for many years the cause of searching the new relativistic source equation of gravity. The L-invariant theory of gravitation is regarded as one of the basic, because it could completely eliminate the disadvantages of the GTR. (see, e.g., the Lorentz-invariant theory of A. Logunov with scholars (Logunov, 2002)).)

It should be noted that GTR showed that the cause of gravity is matter in the form of concentrated and unconcentrated material field. The first type of matter is usually called a massive body or an 'island matter' (e.g., massive elementary particles and bodies, composed of them). The second type is, properly, called a field. According to modern concepts, the fields also consist of elementary particles - quanta of this field (as, for example, the electromagnetic field, which consists of massless elementary particles - photons).

Both kind of matter are characterized by the energy, momentum, mass, current of mass and other dynamic characteristics (Fock, 1964). Moreover, the majority of these characteristics may be defined either as integral features - namely, energy, momentum, mass, and current of mass, etc., or in form of differential characteristics, such as densities of these parameters of matter.

Obviously, within the framework of LIGT, the reason of gravity is also matter in the above sense.

As we noted, the question about why the search for other theories of gravity still continues, is due to the fact that general relativity has a number of peculiarities and drawbacks (see Fock, 1964; Logunov, 2002; Rashevskyi, 1967, etc.), that impede its relationships with the rest of physics. Let us enumerate them.

3.1. The peculiarities and drawbacks of GTR

1) The left side of the nonlinear equations of general relativity (in fact it is a short record of 10 equations) has no physical meaning, but only a geometric meaning.

The right side includes a pseudo-tensor of energy-momentum (Landau and Lifshitz,1975), which has a conditional physical sense (Logunov et al.). the choice of this tensor is quite arbitrary. For this reason, in general relativity the law of conservation of energy is absent.

Recall that Einstein spoke about his gravitational field equations in the book "Physics and Reality» (Einstein, 1936):

"[GTR] *is similar to a building, one wing of which is made of fine marble (left part of the equation), but the other wing of which is built of low grade wood (right side of equation). The phenomenological representation of matter is, in fact, only a crude substitute for a representation which would correspond to all known properties of matter*."

2) In 1918, Schrodinger (Schroedinger, 1918) first showed that by the appropriate choice of coordinate system all components, characterizing the energy-momentum of the gravitational field in the interpretation of Einstein, can be turned to zero.

This was confirmed by D. Hilbert and other scientists. For example, in the book (Landau and Lifshitz, 1975, page 283), we can read the following:

"*On the other hand, from the vanishing of a pseudo-tensor at some point in one reference system it does not at all follow that this is so for another reference system, so that it is meaningless to talk of whether or not there is gravitational energy at a given place. This corresponds completely to the fact that by a suitable choice of coordinates, we can "annihilate" the gravitational field in a given volume element, in which case, from what has been said, the pseudotensor ttk also vanishes in this volume element*."

3) Due to the strong nonlinearity of the equation, the exact analytical solution of GR is obtained and experimentally verified only for a small number of tasks, and, by ideally setting them - in a vacuum, out of the source of gravity.

4) GTR has no connection with quantum field theory (i.e., with the theory of elementary particles - the smallest particles of matter, capable to produce the gravitational field). Some prominent scientists even argue that gravity is some independent object of nature, which has no connection with the rest of physics.

4.0. The scientific goals

"*The nature of time, space* and *reality* are to large extent dependent on our interpretation of - *Special (SRT) and General Theory of Relativity (GTR). In STR essentially two distinct interpretations exist; the "geometrical" interpretation by Einstein based on the Principle of Relativity and the Invariance of the velocity of light and, the "physical" Lorentz-Poincare* interpretation with underpinning by *rod contractions*, *clock slowing* and *light synchronization*, see e.g. (Bohm, 1965; Bell, 1987). *It can be questioned whether the Lorentz-Poincare-interpretation of STR can be continued into GTR*" (Broekaert, 2005).

It can be said that the purpose of creation of Lorentz-invariant theory of gravitation (LIGT) is to show that the Lorentz-Poincare-interpretation of STR can be continued into gravitation theory. Such a theory could allow to overcome all the shortcomings of general relativity.

Since the Hilbert-Einstein equations give proven results, obviously, we have to show that such a LIGT gives equivalent results.

Our additional goal will be to explain the features of general relativity within the framework of nongeometric physics.

For the purity of the theoretical conclusions of LIGT we will not use anywhere of ideas of GTR or of similar metric theory as the basis of our theory (this does not apply to those cases, in which we will compare the results of these theories).

In the book we shall use the CGS system of units, in particular, the system of units of Gauss, since here all units are a unified system of mechanical units.

Chapter 2. Origin of the gravitation field source

1.0. The source of gravitation in the theories of gravitation and conservation laws

1.1. The source of gravity in general relativity

Initially Einstein assumed that the source of gravity in the Hilbert-Einstein equations is symmetric energy-momentum tensor $T_{\mu\nu}$ of the Lorentz-invariant mechanics since (Fock, 1964) in a theory working with Euclidean space-time the classical conservation laws can be stated in differential form, namely in the form of relations:

$$\sum_{k=0}^{3} \frac{\partial T^{ik}}{\partial x_k} = 0, \tag{1.1}$$

which corresponds to ten integrals of motion of Lorentz-invariant mechanics. As the generalization of $T_{\mu\nu}$ in GR should be the general covariant derivative and instead (1.1) we have:

$$\nabla_\nu T^{\mu\nu} \equiv \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x_\nu}\left(\sqrt{-g}T^{\mu\nu}\right) + \Gamma^{\mu}_{\alpha\beta} T^{\alpha\beta} = 0 \ , \tag{1.2}$$

But, it appears that (Landau and Lifshitz, 1971) “in this form, however, this equation does not generally express any conservation law whatever”.

As a way out of this situation Einstein's formulation of energy-momentum conservation laws in the form of a divergence involved the introduction of a pseudo-tensor quantity t^{ik} which is not a true tensor (although covariant under linear transformations).

To determine the conserved total four-momentum for a gravitational field plus the matter located in it, Einstein choose a system of coordinates of such form that at some particular point in space-time all the first derivatives of the g_{ik} vanish.

Then we can enter the value t^{ik} by the following expression:

$$(-g)\left(T^{ik} + t^{ik}\right) = \frac{\partial h^{ikl}}{\partial x^l}, \tag{1.3}$$

where $\partial h^{ikl} = \dfrac{c^4}{16\pi\gamma_N} \dfrac{\partial}{\partial x^m}\left[(-g)\left(g^{ik}g^{lm} - g^{il}g^{km}\right)\right]$.

From the definition (96.4) it follows that for the sum $T^{ik} + t^{ik}$ the equation

$$\frac{\partial}{\partial x^k}(-g)\left(T^{ik} + t^{ik}\right) = 0, \tag{1.4}$$

is identically satisfied. This means that there is a conservation law for the quantities

$$P^i = \frac{1}{c}\int (-g)\left(T^{ik} + t^{ik}\right) dS_k, \tag{1.5}$$

In the absence of a gravitational field, in galilean coordinates, $t^{ik} = 0$, and the integral goes over into into the four-momentum of the matter. Therefore the quantity (1.5) must be identified with the total four-momentum of matter plus gravitational field. But it is obvious that this result depends on the choice of coordinates and is ambiguous.

Unfortunately, there is still no generally accepted definition of energy and momentum in GR. Attempts aimed at finding a quantity for describing distribution of energy-momentum due to matter, non-gravitational and gravitational fields only resulted in various energy-momentum complexes, which are non-tensorial under general coordinate transformations.

1.2. The source of gravity in LIGT and conservation laws

In the Lorentz-invariant mechanics, in general, the values that make up the energy-momentum tensor (see above), are used in the theory, without being recorded in the form of the tensor (Fock, 1964) (it is noteworthy that W. Fock called this tensor the mass tensor (Fock, 1964, §31)).

Note, that after being divided by the square of the speed of light, these values are identical to the mass and mass flow (in general case, densities of mass and mass flow). Therefore in framework of LIGT, for the sake of brevity, we will call the source of gravity "mass/energy" (meaning by this term any element of the energy-momentum tensor of given task).

Mass as a source of gravitation is called gravitational mass or gravitational charge. Currently, the origin of the gravitational mass is unknown. But we know that it is equal with great precision to inertial mass, which appears in the laws of motion in mechanics.

Thus, if we find out the origin of inertial mass/energy, we can conclude that gravitational mass/energy and gravitation field have the same origin.

The question now is what do we know about the origin of inertial mass, particularly, of the elementary particles as initial source of gravitation?

2.0. The mass theories (classical and modern views)

To state the existing views on the considered issues, we will use the works of contemporary scientists (Quigg, 2007; Dawson, 1999; etc):

"Mass remained an essence - part of the nature of things - for more than two centuries, until J.J. Thomson (1881), Abraham (1903) and Lorentz (1904) sought to interpret the electron mass as electromagnetic self-energy".

2.1. Classical views

Theory, created by J.J. Thomson and H. Lorentz (1881 - 1926), lies entirely in the field of classical electromagnetic theory. According to this theory, the inertial mass has electromagnetic origin. Unfortunately, attempts to apply this theory to quantum theory has not been undertaken. Nevertheless, there is still no evidence that the inertial mass is not fully electromagnetic (Feynman et al, 1964):

"We only wish to emphasize here the following points:

1) the electromagnetic theory predicts the existence of an electromagnetic mass, but it also falls on its face in doing so, because it does not produce a consistent theory – and the same is true with the quantum modifications;

2) there is experimental evidence for the existence of electromagnetic mass; and

3) all these masses are roughly the same as the mass of an electron.

So we come back again to the original idea of Lorentz - may be all the mass of an electron is purely electromagnetic, maybe the whole 0.511 MeV is due to electrodynamics. Is it or isn't it? We haven't got a theory, so we cannot say."

As we will be convinced later, the results of modern theory of elementary particles do not contradict to the original idea of Lorentz that all the mass of an electron may be purely electromagnetic.

2.2. Modern views

The modern mass theory is the, so-called, Higgs mechanism of the Standard Model theory (SM) (Quigg, 2007; Dawson, 1999; etc).

"Our modern conception of mass has its roots in known Einstein's conclusion: "The mass of a body is a measure of its energy content. Among the virtues of identifying mass as $m_0 = \varepsilon_0 / c^2$, where ε_0 designates the body's rest energy, is that mass, so understood, is a Lorentz-invariant quantity, given in any frame as $m = \left(1/c^2\right)\sqrt{\varepsilon^2 - p^2c^2}$. But not only is Einstein's a precise definition of mass, it invites us to consider the origins of mass by coming to terms with a body's rest energy.

We understand the mass of an atom or molecule in terms of the masses of the atomic nuclei, the mass of the electron, and small corrections for binding energy that are given by quantum electrodynamics.

Nucleon mass is an entirely different story, the very exemplar of $m_0 = \varepsilon_0 / c^2$. Quantum chromodynamics (QCD), the gauge theory of the strong interactions, teaches that the dominant contribution to the nucleon mass is not the masses of the quarks that make up the nucleon, but the energy stored up in confining the quarks in a tiny volume. The masses m_u and m_d of the up and down quarks are only a few MeV each. The quarks contribute no more than 2% to the 939MeV mass of an isoscalar nucleon (averaging proton and neutron properties).

Hadrons such as the proton and neutron thus represent matter of a novel kind. In contrast to macroscopic matter and beyond what we observe in atoms, molecules and nuclei, the mass of a nucleon is not equal to the sum of its constituent masses - quarks; it is, basically, a confinement energy of gluons!"

Thus, the consequences of Higgs mechanism can not be used in the gravitation theory. The Higgs mechanism, under certain assumptions, allows us to describe the generation of masses of fundamental elementary particles: intermediate bosons, leptons and quarks. But as it is mentioned above (Quigg, 2007), more than 98% of the visible mass in the Universe is composed by the non-fundamental (composite) particles: protons, neutrons and other hadrons.

Although there are some parallels in the interpretation of results between the Higgs mechanism and the theory of mass of Thomson-Lorentz, mathematically they have nothing in common. A much closer relationship exists between the Higgs mechanism and the quantum wave theory of mass.

3.0. Quantum wave theory of mass

In the history of physics a theory under this name is absent. But a number of meaningful results, obtained even in the 19^{th} century, taking into account the results of quantum field theory, makes it possible to choose this approach. We will name it the "quantum wave theory of mass".

In the Maxwell-Lorentz electromagnetic theory the existing waves are the electromagnetic waves only. At the beginning of the 20th century it was revealed that these waves are quantized and consist of particles: of the quanta of EM field, photons. Moreover, photons are unique massless particles in the nature. Thus, the only possibility of the generation of massive particles in the electromagnetic theory is some transformation of photons, as a result of which special massive electromagnetic waves-particles must appear.

As is known, QCD is a nonlinear analog of linear quantum theory of electromagnetic field – quantum electrodynamics (QED). According to that, gluons are analogous to photons. Thus, the energy of gluons is the energy of nonlinear electromagnetic waves, which constitutes the main part of the energy of atoms and molecules.

Here we will recall some results of the 19^{th} century and estimate them from the results of contemporary theory point of view of the. Because of the dualism wave-particle, we will further examine photon as electromagnetic wave and particle simultaneously.

3.1. Energy and momentum of electromagnetic wave

The fact that EM wave has an energy and a momentum, it was discovered already into the 19^{th} century. The EM wave presses the metallic wall, and also it can revolve a light rotator (Lebedev, and otthers). By this we can assume that EM wave (photon) has a mass.

For the time average of the pressure of the train of EM waves with area s and length l, the following expression (Becker, 1982) is obtained: $\mathrm{P} = \frac{1}{8\pi}\left(\vec{\mathrm{E}}^2 + \vec{\mathrm{H}}^2\right) = u$, where u is the energy density of EM wave. The important dependence between energy and momentum of wave is already included in this equation. The total momentum, transmitted from EM train to wall will be equal to: $p = u \cdot s \cdot t$, where $t = l/c$ is the time of action of train. Thus, the transmitted momentum is equal to: $p = u \cdot s \cdot l/c$. Since the numerator $u \cdot s \cdot l = \varepsilon$ is the energy of train, we obtain $p = \varepsilon/c$. If we assign to EM wave a mass m', then it is possible to consider that $p = m'c$. In that case we obtain $m' = \varepsilon/c^2$ - the known relationship of Einstein.

Nevertheless, later it was proven that photon is a mass-free particle in the sense that its rest mass is equal to zero. But if we interpret the collision of EM wave with the wall as the stoppage of EM wave, then it is possible to say that the "stopped" photon acquires mass m'.

This result led, evidently, to a study of other methods of the "stoppage" of EM waves for the purpose of understanding the origin of mechanical mass of the material bodies.

3.2. The Mass of a Box Full of Light

According with (The authors, 2005) *"the experimental confirmation of the pressure of light in* 1901 *led to new theoretical work. In* 1904, *Max Abraham computed the pressure produced by radiation upon a moving surface, when the beam of light reaches the surface in a mirror in any angle. Starting from Abraham's results, Friedrich Hasenoehrl* (1874-1916) *studied the dynamics of a box full of radiation.*

Imagine a cubic box with perfectly reflecting internal surfaces, full of light. When the box is at rest, the radiation produces equal forces upon all those surfaces. Now, suppose that the box is accelerated, in such a way that one of its surfaces moves in the x direction. It is possible to prove that, when the radiation inside the box strikes this surface, the pressure will be smaller, and when it strikes the opposite surface, the pressure will be greater, than in the case when the box is at rest (or in uniform motion). Therefore, the radiation inside the box will produce a resultant force against the motion of the box. So, to accelerate a box full of light requires a greater force than to accelerate the same box without light. In other words, the radiation increases the inertia of the box. In the case when the radiation inside the box is isotropic, there is a very simple relation between its total energy E and its contribution m to the inertia of the box (Hasenoehrl, 1904; 1905): $m = \frac{4\varepsilon}{3c^2}$.

Note that here, as in the theory of the electron, there appears a numerical factor 4/3. *This is not a mistake. The relation between those equations and the famous* $\varepsilon = mc^2$ *will be made clear later* (Fadner, 1988).

Hasenoehrl also computed the change of the radiation energy as the box was accelerated. He proved that the total radiation energy would be a function of the speed of the box. Therefore, when the box is accelerated, part of the work done by the external forces is transformed into the extra radiation energy. Since the inertia of the radiation is proportional to its energy, and since this energy increases with the speed of the box,, the inertia of the box will increase with its speed. Of course, if the internal temperature of the box were increased, the radiation energy would augment, and the inertia of the box would also increase. Therefore, Hasenoehrl stated that the mass of a body depends on its kinetic energy and temperature".

As Pauli notes in his review (Pauli, 1958) "black-body radiation in a moving cavity… case is of historical interest, since it can be treated entirely on the basis of electrodynamics, without relativity. When this is done, one comes to the inevitable conclusion that a momentum, and thus also an inertial mass, must be ascribed to the moving radiation energy. It is of interest that this result should have been found by Hasenoehrl already before the theory of relativity had been formulated".

3.3. A “Box Full of Light” as massive particle

As we can see, the radiation inside the box behaves like a massive body or particle. Let us conditionally name the totality of EM waves in a box as ‘EM-particle’.

From the foresaid above it is obvious that the mass of ‘EM-particle’, calculated according to Lorentz's theory, will also have a coefficient of 4/3 like the mass of classical electron. Obviously, upon consideration of the stresses of Poincare we will obtain the coefficient one. The stresses of Poincare were introduced for the stabilization of the electrostatic field of classical electron. In the case in question the stability exists due to interaction of EM wave with the walls of the box. These interactions play in this case the role of the stresses of Poincare, which ensure the stability of ‘EM particle’. Naturally, if we take into account the presence of these stresses, we will also obtain the coefficient one (of course this result will also appear, if we use Einstein's approach).

With the perpendicular fall of EM waves on the walls of the box the stress is pressure. With inclined fall the components of stress will formally consist both of pressures and tangent stresses (as a result of the resolution of momentum on perpendicular and tangential components). The stress tensor of Maxwell (and generally, continuous medium tensor consists precisely of such components. In this example the stresses are not mechanical: EM waves interact with the electrons of the wall atoms by means of EM Lorentz's forces. Nevertheless, these stresses are external with respect to EM waves in the box, i.e., they are not organized by the EM waves themselves.

We can improve our model for the purpose to do approach the quantum field theory. Let us select a box with mirror walls of the size of the order of a wavelength λ. If we consider resonance conditions, the box itself will select the appropriate wavelength. This corresponds to the case when we placed into this box one photon. In the case of quantum theory we can speak about the photon in a cell of phase size. If we ignore the presence of walls, it is possible to consider photon in the box as particle. This particle possesses spin one and mass, determined by its energy $m' = \varepsilon / c^2 = \hbar / \lambda c$. In other words, we have a model of the neutral massive boson, similar to intermediate boson.

3.4. The mathematical description of EM-particle in the classical case

The mathematical description of this model in the classical case can be given on the basis of the theory of waveguides and resonators (Crawford Jr., 1968; Broglie, 1941). As is known, the motion of waves is determined by the dispersion equation (or by another dispersion relationship).

Dispersion equation is the relationship, which connects angular frequencies ω and wave vectors k of natural harmonic waves (normal waves) in linear uniform systems: continuous media, waveguides, transmission lines and others. Dispersion equation is written in the form:

$$\omega = \omega(k), \qquad (3.1)$$

Dispersion equations are the consequence of the dynamic (in the general case integrodifferential) equations of motion and of boundary conditions. And also, vice versa, on the base of the form of dispersion equation the dynamic equations of processes can be restored with the replacement:

$$i\omega \to \frac{\partial}{\partial t}, \; ik_x \to -\frac{\partial}{\partial x}, \; \frac{1}{i\omega} \to \int (\ldots) dt, \; \frac{1}{ik_x} \to \int (\ldots) dx, \qquad (3.2)$$

It is easy to obtain the dispersion equation for the infinite wave without any limiting conditions, $\vec{\Phi} = \vec{\Phi}_o e^{-i(\omega t - ky)}$, using the homogeneous wave equation:

$$\left(\frac{\partial^2}{\partial t^2} - c^2 \vec{\nabla}^2 \right) \vec{\Phi} = 0, \qquad (3.3)$$

where $\vec{\Phi}$ are in our case any vector components of electrical and magnetic field. Putting this solution, we obtain $\omega^2 - \upsilon^2 k^2 = 0$ or $\omega = \upsilon \cdot k$.

In the case of the presence of limitations, superimposed on the wave by medium or by it self, the equation becomes heterogeneous:

$$\left(\frac{\partial^2}{\partial t^2} - c^2 \vec{\nabla}^2\right)\vec{\Phi} = \vec{\Phi}_0, \tag{3.4}$$

where $\vec{\Phi}_0$ is certain function of the electromagnetic fields. In this case dispersion relationship becomes more complex: new terms are introduced and its linearity is disrupted.

The same relationship dispersion equation:

$$\omega^2 = \omega_0^2 + \upsilon^2 k^2, \tag{3.5}$$

can correspond to: 1) EM waves in the isotropic plasma; 2) plasma waves; 3) waves in the waveguides; 4) waves in the acoustic waveguides; 5) elementary particle in relativistic wave mechanics ($\upsilon = c$, $\omega_0 = m_0 c^2 / \hbar$, m_0 is rest mass).

In the latter case the discussion deals with de Broglie wave dispersion relation. Energy, momentum, and mass of particles are connected through the relativistic relation

$$\varepsilon^2 = \left(m_0 c^2\right)^2 + (pc)^2, \tag{3.6}$$

Elementary particles, atomic nuclei, atoms, and even molecules behave in some context as matter waves. According to the de Broglie (Broglie, 1941) relations, their kinetic energy ε can be expressed as a frequency ω : $\varepsilon = \hbar\omega$, and their momentum p as a wave number k : $p = \hbar k$.

The relationships (Broglie, 1941), *obtained for EM wave in a waveguides or in a box, are completely analogous to those, which exist in wave mechanics, in which the rectilinear and uniform particle motion with the rest mass* m_0 *depicts in the form of propagation of plane simple harmonic wave* $\psi = \psi_0 e^{i(\omega t - kr)}$.

As we noted, $\omega = ck$ *corresponds to the propagation of EM wave in the vacuum. But if EM wave is in the waveguide, then between* ω *and* k *we have the relationship* (3.6), *where* ω_0 *is different from zero and it is equal to one of its eigenvalues, which correspond to the form of the waveguide in question. From the point of view of wave mechanics everything happens as if the photon had its own mass, determined by the form of waveguide and by the eigenvalue* $\omega_{0i} = m_{0i}/\hbar$ *. Thus, it is possible to say that in this waveguide the photon can possess a series of possible own masses.*

From a contemporary point of view we can interpret the appearance of photon mass as follows. A photon, until its entry into a waveguide or resonator, obeys to the linear equation

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{\Phi} \equiv \sum_\nu \frac{\partial^2}{\partial x_\nu^2}\vec{\Phi} \equiv \partial_\nu \partial^\nu \vec{\Phi} = 0, \tag{3.7}$$

Lagrangian of which

$$L = \frac{1}{2}\left\{\left(\frac{\partial\vec{\Phi}}{\partial t}\right)^2 - c^2\left(\vec{\nabla}\psi\right)^2\right\} \equiv \frac{1}{2}c^2 \sum_\nu \left(\frac{\partial\vec{\Phi}}{\partial x_\nu}\right)^2 \equiv \partial_\nu \vec{\Phi} \partial^\nu \vec{\Phi}, \tag{3.8}$$

describes the mass-free field. After entry to a box the photon experiences a certain spontaneous transformation and becomes massive particle. Each component of the field of this massive particle obeys to Klein-Gordon wave equation (Wentzel, 2003):

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - m^2\right)\vec{\Phi} \equiv \left(\sum_\nu \frac{\partial^2}{\partial x_\nu^2} - m^2\right)\vec{\Phi} \equiv \left(\partial_\nu \partial^\nu - m^2\right)\vec{\Phi} = 0, \tag{3.9}$$

This is achieved by choosing the following, evidently Lorentz-invariant Lagrangian:

$$L = \frac{1}{2}\left\{\left(\frac{\partial\vec{\Phi}}{\partial t}\right)^2 - c^2\left(\vec{\nabla}\vec{\Phi}\right)^2 - c^2 m^2 \vec{\Phi}^2\right\} \equiv -\frac{1}{2}c^2\left\{\sum_{\nu}\left(\frac{\partial\vec{\Phi}}{\partial x_\nu}\right)^2 + m^2\vec{\Phi}\vec{\Phi}^+\right\} \equiv \partial_\nu\vec{\Phi}\partial^\nu\vec{\Phi} - c^2 m^2\vec{\Phi}^2 \quad (3.10)$$

The question arises: whether can EM wave ensure themselves the stability as 'EM-particle', but without the presence of external actions (vessel walls)?

In this case we will actually have a massive "particle", generated from EM wave. Obviously this case can be realized only as a result of the self-interaction of fields of EM waves. This means that the equation of EM wave-particle must be nonlinear.

Namely this mechanism of particle masses production – from free wave to the self-interaction wave - is realized in nonlinear quantum field theory (NQFT). We pass on to a brief consideration of this mechanism.

4.0. Quantum wave mass production mechanism

4.1. Parallels and differences between mass production mechnisms in CM and in NQFT

The description of generation of mass in NQFT has close parallels with this procedure in SM. But there are also differences, which allow us to obtain the same results in a more simple way. Here are the main ones.

As we know (Quigg, 2007), the CM is constructed as a heuristic theory: by the trial and error method. At the basis of finding suitable mathematical description forms the various conservation laws (symmetry) and their violation are widely used here.

NQFT is constructed as a strictly axiomatic theory. All its results follow logically from the selected axioms without additional hypotheses. Symmetries themselves and their violation are consequence of the theory, but not its foundation.

The CM is based on the hypothesis that all the particles at the time of generation of the Universe were massless.

The mechanism of mass production in NQFT is based on the conversion of the unique in nature massless boson - photon into the massive vector boson.

Another major difference between the SM and NQFT is closely related to the interpretation of the wave functions. The existence of two possible choices of wave functions is known for a long time (Fermi, 1952). ; Akhiezer and Berestetskii 1965; Levich et al, 1973)

In the SM, the wave functions of the gauge bosons (e.g., photons or intermediate bosons) are 4- potentials; in the same time the wave functions of the spinor particles do not a physical meaning.

In NQFT the wave functions of all particles are the vectors of field strengths of the corresponding particles.

The most important assumption in the SM is local gauge (or phase) symmetry: symmetry with respect to local rotation of the wave functions in the interior space of the particles. At the same time, to achieve this symmetry it is necessary to enter (pick manually) the additional (compensating) terms, related to the characteristics of the particles. These terms define the interaction between the particles.

Instead, in NQFT postulate of rotation of the particle field (wave function) is introduced directly. This postulate is the source of particle self-action. The additional terms appear automatically by means of transport of the field vectors along a curved path. It is essential that these terms disclose a simple physical meaning.

4.2. The postulate of generation of massive elementary particles

In the proposed theory of massive elementary particles we accept (see in detail (Kyriakos, 2009)) the following basic postulate, which ensure the generation of massive particles:

The fields of an electromagnetic wave quantum (photon) can under specified conditions undergo a rotation transformation and initial symmetry breaking, which generate massive elementary particles.

It follows from these hypothesis that the equations of elementary particles must be nonlinear modifications of the equations of quantized electromagnetic (EM) waves.

4.3. Photon as a gauge field

As one of the simplest examples of the generation of massive fields (particles) we can consider the photoproduction of the electron-positron pair:

$$\gamma + p \rightarrow e^{+} + e^{-} + p \ , \qquad \text{(A)}$$

Actually, the photon γ is a mass-free gauge vector boson. The EM field of proton p (or some nucleous of atom) initiates its transformation into two massive particles: electron and positron**.** The fields of the electron and positron e^{+}, e^{-} are spinors, which are not transformed like vector fields. *Thus, we can say that the reaction* (A) *describes the process of symmetry breaking of the initial mass-free vector field in order to generate the massive spinor particles.*

Let us examine the Feynman diagram of the above reaction of a pair production (Fig. 4.1):

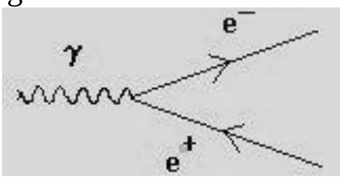


Fig. 4.1

It is known that using Feynman's diagrams within the framework of SM, we can precisely calculate all characteristics of particles with the exception of charge and mass. Nevertheless, the reaction (A) remains mysterious: we do not know, for example, how the process of transformation of the mass-free boson field into the massive fields happens, and how the electrical charge appears.

Based on this evidence, let us assume that in the vertex of the Feynman diagram, the rotation transformation of the "linear" photon (in the sense that it obeys to a linear wave equation) into the "nonlinear" photon (which obeys a nonlinear wave equation) is achieved. This "nonlinear" photon acquires rest mass and can be conditionally named "intermediate photon"..

We show that these ideas can be translated into mathematical language.

4.4. Wave equation of a photon in matrix form

Let us consider the general case of a circularly polarized electromagnetic (EM) wave that is moving, for instance, along the y- axis. This wave is the superposition of two plane-polarized waves with mutually perpendicular vectors of the EM fields: $\vec{\mathrm{E}}_x, \vec{\mathrm{H}}_z$ and $\vec{\mathrm{E}}_z, \vec{\mathrm{H}}_x$. The electric and magnetic wave fields can be written in a complex form as follows:

$$\begin{cases} \vec{\mathrm{E}} = \vec{\mathrm{E}}_o e^{-i(\omega t - ky)} + \vec{\mathrm{E}}_o^{*} e^{i(\omega t - ky)} \\ \vec{\mathrm{H}} = \vec{\mathrm{H}}_o e^{-i(\omega t - ky)} + \vec{\mathrm{H}}_o^{*} e^{i(\omega t - ky)} \end{cases}, \qquad (4.1)$$

An electromagnetic wave propagating in any direction can have two plane polarizations; it contains only four field vectors. For example, in the case of y-direction, we have:

$$\vec{\Phi}(y) = \{\mathrm{E}_x, \mathrm{E}_z, \mathrm{H}_x, \mathrm{H}_z\}, \qquad (4.2)$$

and $E_y = H_y = 0$ for all transformations. Here, note that the Dirac bispinor also has only four components.

The EM wave equation has the following known form (Jackson, 1999):

$$\left(\frac{\partial^2}{\partial t^2} - c^2\vec{\nabla}^2\right)\vec{\Phi}(y) = 0, \qquad (4.3)$$

where $\vec{\Phi}(y)$ is any of the above electromagnetic wave field vectors (4.2). In other words, this equation represents four equations: one for each vector of the electromagnetic field.

We can also write this equation in the following operator form:

$$\left(\hat{\varepsilon}^{\,2} - c^2\hat{\vec{p}}^{\,2}\right)\Phi(y) = 0, \qquad (4.4)$$

where $\hat{\varepsilon} = i\hbar\frac{\partial}{\partial t}$, $\hat{\vec{p}} = -i\hbar\vec{\nabla}$ are correspondingly the operators of energy and momentum; Φ is a matrix which consists of the four components $\vec{\Phi}(y)$.

Taking into account that $\left(\hat{\alpha}_o\hat{\varepsilon}\right)^2 = \hat{\varepsilon}^{\,2}$, $\left(\hat{\vec{\alpha}}\,\hat{\vec{p}}\right)^2 = \hat{\vec{p}}^{\,2}$, where $\hat{\alpha}_0 = \begin{pmatrix} \hat{\sigma}_0 & 0 \\ 0 & \hat{\sigma}_0 \end{pmatrix}$; $\hat{\vec{\alpha}} = \begin{pmatrix} 0 & \hat{\vec{\sigma}} \\ \hat{\vec{\sigma}} & 0 \end{pmatrix}$; $\hat{\beta} \equiv \hat{\alpha}_4 = \begin{pmatrix} \hat{\sigma}_0 & 0 \\ 0 & -\hat{\sigma}_0 \end{pmatrix}$ are Dirac's matrices and $\hat{\sigma}_0, \hat{\vec{\sigma}}$ are Pauli's matrices, equation (4.4) can also be represented in a matrix form:

$$\left[\left(\hat{\alpha}_o\hat{\varepsilon}\right)^2 - c^2\left(\hat{\vec{\alpha}}\,\hat{\vec{p}}\right)^2\right]\Phi = 0, \qquad (4.5)$$

Recall that in case of a photon $\omega = \varepsilon/\hbar$ and $k = p/\hbar$. From equation (4.5), using (4.1), we obtain $\varepsilon = cp$, which is the same as for a photon. Therefore, we can consider the wave function Φ of the equation (4.5) both as that of an EM wave and (taking into account its quantization) of a photon.

Factoring (4.5) and multiplying it on the left by the Hermitian-conjugate function Φ^+, we get:

$$\Phi^+\left(\hat{\alpha}_o\hat{\varepsilon} - c\hat{\vec{\alpha}}\,\hat{\vec{p}}\right)\left(\hat{\alpha}_o\hat{\varepsilon} + c\hat{\vec{\alpha}}\,\hat{\vec{p}}\right)\Phi = 0, \qquad (4.6)$$

Equation (4.6) may be broken down into two Dirac-like equations without mass:

$$\begin{cases} \Phi^+\left(\hat{\alpha}_o\hat{\varepsilon} - c\hat{\vec{\alpha}}\,\hat{\vec{p}}\right) = 0 \\ \left(\hat{\alpha}_o\hat{\varepsilon} + c\hat{\vec{\alpha}}\,\hat{\vec{p}}\right)\Phi = 0 \end{cases}, \qquad (4.7)$$

Note that the system of equations (4.7) is identical to the equation (4.5), and can be represented (Akhiezer and Berestetskiy, 1969; Levich et al, 1973) as a system of quantum equations for a photon in Hamilton's form. At the same time in the electromagnetic interpretation they are the equations of EM waves.

Actually, it is not difficult to show that only in the case when the Φ-matrix has the form:

$$\Phi = \begin{pmatrix} E_x \\ E_z \\ iH_x \\ iH_z \end{pmatrix}, \quad \Phi^+ = \left(E_x \quad E_z \quad -iH_x \quad -iH_z\right), \qquad (4.8)$$

the equations (4.7) are the right Maxwell-like equations of the retarded and advanced electromagnetic waves. Using (4.8), and substituting it into (4.7), we obtain:

$$\begin{cases} \frac{1}{c}\frac{\partial \mathrm{E}_x}{\partial t} - \frac{\partial \mathrm{H}_z}{\partial y} = 0 \\ \frac{1}{c}\frac{\partial \mathrm{H}_z}{\partial t} - \frac{\partial \mathrm{E}_x}{\partial y} = 0 \\ \frac{1}{c}\frac{\partial \mathrm{E}_z}{\partial t} + \frac{\partial \mathrm{H}_x}{\partial y} = 0 \\ \frac{1}{c}\frac{\partial \mathrm{H}_x}{\partial t} + \frac{\partial \mathrm{E}_z}{\partial y} = 0 \end{cases}, \qquad (4.9) \qquad \begin{cases} \frac{1}{c}\frac{\partial \mathrm{E}_x}{\partial t} + \frac{\partial \mathrm{H}_z}{\partial y} = 0 \\ \frac{1}{c}\frac{\partial \mathrm{H}_z}{\partial t} + \frac{\partial \mathrm{E}_x}{\partial y} = 0 \\ \frac{1}{c}\frac{\partial \mathrm{E}_z}{\partial t} - \frac{\partial \mathrm{H}_x}{\partial y} = 0 \\ \frac{1}{c}\frac{\partial \mathrm{H}_x}{\partial t} - \frac{\partial \mathrm{E}_z}{\partial y} = 0 \end{cases}, \qquad (4.9')$$

For waves of any other direction the same results can be obtained by cyclic transposition of indices, or by a canonical transformation of matrices and wave functions.

We will further conditionally name each of (4.7) equations the linear semi-photon equations, remembering that it was obtained by division of one wave equation of a photon into two equations of the electromagnetic waves: retarded and advanced.

4.5. The rotation transformation of photon fields

The rotation transformation of the “linear” photon wave to a “curvilinear” one can be conditionally written in the following form:

$$\hat{R}\Phi \to \Psi \,, \qquad (4.10)$$

where $\hat{R}$ is the rotation operator (see in detail (Kyriakos, 2009)) for the transformation of a photon wave from linear state to curvilinear state, and Φ' is some final wave function:

$$\Phi' = \begin{pmatrix} \Psi_1 \\ \Psi_2 \\ \Psi_3 \\ \Psi_4 \end{pmatrix} = \begin{pmatrix} \mathrm{E}'_x \\ \mathrm{E}'_z \\ i\mathrm{H}'_x \\ i\mathrm{H}'_z \end{pmatrix}, \qquad (4.11)$$

which appears after the nonlinear transformation (4.10); here, $\left(\mathrm{E}'_x \quad \mathrm{E}'_z \quad -i\mathrm{H}'_x \quad -i\mathrm{H}'_z\right)$ are electromagnetic field vectors after the rotation transformation, which correspond to the wave functions Ψ.

It is known that the transition of vector motion from linear to curvilinear state is described by differential geometry (Eisenhart, 1909). Note also that this transition is mathematically equivalent to a vector transition from flat space to curvilinear space, which is described by Riemann geometry (Rashevski, 1956). In relation to this, let us remind ourselves that the Pauli matrices, as well as the photon matrices, are the space rotation operators in 2-D and 3-D space, correspondingly (Ryder, 1985).

4.5.1. *The rotation transformation description in differential geometry*

Recalling that the Pauli matrices are generators of rotation transformation in 2D space, we can assume that this curved path lies in a plane.

Let us consider a plane-polarized EM wave, which has the field vectors $(\mathrm{E}_x, \mathrm{H}_z)$. Let this wave is rotated in the plane (X', O', Y') of a fixed co-ordinate system (X', Y', Z', O') around the axis Z' at some radius r_p, so that E_x is parallel to the plane (X', O', Y'), and H_z is perpendicular to this plane (fig. 4.2).

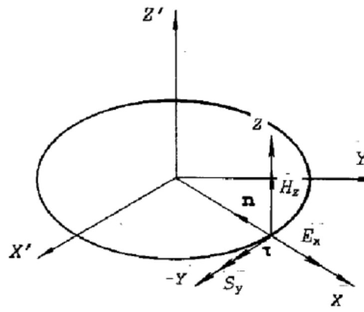


Fig. 4.2

According to Maxwell, the following term of equations (4.7)

$$\hat{\alpha}_0 \hat{\varepsilon} \Phi = i\hbar \frac{\partial \Phi}{\partial t}$$

contains the Maxwell's displacement current, which is defined by the expression:

$$j_{dis} = \frac{1}{4\pi} \frac{\partial \vec{\mathrm{E}}}{\partial t}, \qquad (4.12)$$

The electrical field vector $\vec{\mathrm{E}}$ above, which moves along the curvilinear trajectory (assume its direction is from the center), can be written in the form:

$$\vec{\mathrm{E}} = -\mathrm{E} \cdot \vec{n}, \qquad (4.13)$$

where $\mathrm{E} = |\vec{\mathrm{E}}|$, and $\vec{n}$ is the normal unit-vector of the curve, directed to the center. Then, the derivative of $\vec{\mathrm{E}}$ can be represented as follows:

$$\frac{\partial \vec{\mathrm{E}}}{\partial t} = -\frac{\partial \vec{\mathrm{E}}}{\partial t}\vec{n} - \mathrm{E}\frac{\partial \vec{n}}{\partial t}, \qquad (4.14)$$

Here, the first term has the same direction as $\vec{\mathrm{E}}$. The existence of the second term shows that at the rotation transformation of the wave an additional displacement current appears. It is not difficult to show that it has a direction tangential to the ring:

$$\frac{\partial \vec{n}}{\partial t} = -\upsilon_p \mathrm{K} \vec{\tau}, \qquad (4.15)$$

where $\vec{\tau}$ is the tangential unit-vector, $\upsilon_p \equiv c$ is the electromagnetic wave velocity, $\mathrm{K} = \frac{1}{r_p}$ is the curvature of the trajectory, and r_p is the curvature radius. Thus, the displacement current of the plane wave moving along the ring can be written in the following form:

$$\vec{j}_{dis} = -\frac{1}{4\pi}\frac{\partial \mathrm{E}}{\partial t}\vec{n} + \frac{1}{4\pi}\omega_p \mathrm{E} \cdot \vec{\tau}, \qquad (4.16)$$

where $\omega_p = \frac{m_p c^2}{\hbar} = \frac{\upsilon_p}{r_p} \equiv c\mathrm{K}$ is an angular velocity. Furthermore, here, $m_p c^2 = \varepsilon_p$ is photon energy, where m_p is some mass, corresponding to the energy ε_p.

Obviously, the terms $\vec{j}_n = \frac{1}{4\pi}\frac{\partial \mathrm{E}}{\partial t}\vec{n}$ and $\vec{j}_\tau = \frac{\omega_p}{4\pi}\mathrm{E} \cdot \vec{\tau}$ are the normal and tangent components of the displacement current of the rotated electromagnetic wave accordingly. Thus:

$$\vec{j}_{dis} = \vec{j}_n + \vec{j}_\tau, \qquad (4.17)$$

This is a remarkable fact that the currents $\vec{j}_n$ and $\vec{j}_\tau$ are always mutually perpendicular, so that we can write (4.17) in complex form as follows:

$$j_{dis} = j_n + ij_\tau \,, \qquad (4.17')$$

where $j_n = \frac{1}{4\pi}\frac{\partial \mathrm{E}}{\partial t}$ is the absolute value of the normal component of the displacement current, and

$$j_\tau = \omega_p \frac{1}{4\pi}\mathrm{E} \equiv \frac{m_p c^2}{\hbar}\frac{1}{4\pi}\mathrm{E} \equiv \frac{\upsilon_p}{r_p}\frac{1}{4\pi}\mathrm{E} \equiv \mathrm{K}\frac{c}{4\pi}\mathrm{E}\,, \qquad (4.18)$$

is the absolute value of the tangential component of the displacement current.

Thus, the appearance of the tangent current leads to origination of the imaginary unit in a complex form of particles' equation. So, we can assume that the appearance of the imaginary unit in the quantum mechanics is tied to the appearance of tangent currents .

4.6. An equation of the massive intermediate photon

As it follows from the previous sections, some additional terms $K = \hat{\beta}\, m_p c^2$, corresponding to tangent components of the displacement current, must appear in equation (4.6) due to a curvilinear motion of the electromagnetic wave:

$$\Psi'\left(\hat{\alpha}_o\hat{\varepsilon} - c\hat{\vec{\alpha}}\cdot\hat{\vec{p}} - K\right)\left(\hat{\alpha}_o\hat{\varepsilon} + c\hat{\vec{\alpha}}\cdot\hat{\vec{p}} + K\right)\Psi = 0\,, \qquad (4.19)$$

Thus, in the case of the curvilinear transformation of the electromagnetic fields of a photon, we obtain the wave equation with mass (Schiff, 1955), instead of equation (4.5):

$$\left(\hat{\varepsilon}^2 - c^2\hat{\vec{p}}^2 - m_p^2 c^4\right)\Psi = 0, \qquad (4.20)$$

Note that the free term is proportional to the square of the curvature of the trajectory of the particle wave field: $m_p^2 c^4 = c^2\hbar^2\mathrm{K}^2$.

Equation (4.4.2) is similar to the Klein-Gordon equation. However, the latter describes the **scalar** field, i.e. the massive boson with zero spin of the type of the Higgs boson. It is not difficult to prove, using an electromagnetic form that (4.4.2) is an equation of a massive **vector** particle, which we will conditionally name the "*massive photon*".

5.0. Electron and positron generation

Using the results obtained above, it is easy to show that the electron and positron have electromagnetic origin.

5.1. The massive semi-photon equation

Our analysis of an initial stage of photoproduction of electron-positron pair, made above, shows that an intermediate photon can be divided into two parts, in order to produce an electron and a positron. Let us describe this process mathematically in order to find equations for these particles.

Introducing $K = \hat{\beta}\, mc^2$ in equation (4.20), we obtain:

$$\Psi^+\left(\hat{\alpha}_o\hat{\varepsilon} - c\hat{\vec{\alpha}}\cdot\hat{\vec{p}} - \hat{\beta}m_p c^2\right)\left(\hat{\alpha}_o\hat{\varepsilon} + c\hat{\vec{\alpha}}\cdot\hat{\vec{p}} + \hat{\beta}m_p c^2\right)\Psi = 0\,, \qquad (5.1)$$

Now, we can separate the intermediate photon equation (5.1) into two transformed waves, *advanced* and *retarded*, in order to obtain two new equations for the massive particles:

$$\left[\left(\hat{\alpha}_o\hat{\varepsilon} + c\hat{\vec{\alpha}}\,\hat{\vec{p}}\right) + \hat{\beta}\,m_p c^2\right]\psi = 0, \tag{5.2}$$

$$\psi^+\left[\left(\hat{\alpha}_o\hat{\varepsilon} - c\hat{\vec{\alpha}}\,\hat{\vec{p}}\right) - \hat{\beta}\,m_p c^2\right] = 0, \tag{5.2'}$$

where

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} = \begin{pmatrix} E_x \\ E_z \\ iH_x \\ iH_z \end{pmatrix}, \quad \psi^+ = \begin{pmatrix} E_x & E_z & iH_x & iH_z \end{pmatrix}, \tag{5.3}$$

is some new transformed EM wave function which appears after the intermediate photon breaking. Further, in this connection, we will conditionally name the equations (5.2) as *semi-photon equations*, and the passage from (4.20) to (5.2) as the *symmetry breaking* of an intermediate photon).

Now, we will analyze the peculiarities of equations (5.2). We can see that the latter are similar to the Dirac electron and positron equations. However, instead of electron mass m_e, equations (5.2) contain the intermediate photon mass m_p. The question is, what type of particles do equations (5.2) describe?

In the case of an electron-positron pair production, it must be $m_p = 2m_e$. So, we have from (5.2):

$$\left[\left(\hat{\alpha}_o\hat{\varepsilon} + c\hat{\vec{\alpha}}\,\hat{\vec{p}}\right) + 2\hat{\beta}\,m_e c^2\right]\psi = 0, \tag{5.4}$$

$$\psi^+\left[\left(\hat{\alpha}_o\hat{\varepsilon} - c\hat{\vec{\alpha}}\,\hat{\vec{p}}\right) - 2\hat{\beta}\,m_e c^2\right] = 0, \tag{5.4'}$$

and after the breaking of the intermediate photon, the non-charged massive particle must be divided into two charged massive semi-photons, the positively and negatively charged particles acquire electric fields. At the same moment each particle begins to move in the field of the other. In order to become independent (i.e. free) particles, the electron and positron must be drawn sufficiently far away from each other .

The external field of particles defines the amount of work, so that the release energy is the field's production energy, and at the same time this is annihilation energy. Therefore, due to the law of energy conservation, this value of energy for each particle must be equal to $\varepsilon_{rel} = m_e c^2$.

So, equations (5.2) can be written in the following form:

$$\left[\left(\hat{\alpha}_0\hat{\varepsilon} + c\hat{\vec{\alpha}}\,\hat{\vec{p}}\right) + \hat{\beta}\,m_e c^2 + \hat{\beta}\,m_e c^2\right]\psi = 0, \tag{5.8}$$

$$\psi^+\left[\left(\hat{\alpha}_o\hat{\varepsilon} - c\hat{\vec{\alpha}}\,\hat{\vec{p}}\right) - \hat{\beta}\,m_e c^2 - \hat{\beta}\,m_e c^2\right] = 0, \tag{5.8'}$$

Using a linear equation for the description of the law of energy conservation, we can write:

$$\pm\hat{\beta}\,m_e c^2 = -\varepsilon_{ex} - c\hat{\vec{\alpha}}\,\vec{p}_{ex} = -e\varphi_{ex} - e\hat{\vec{\alpha}}\,\vec{A}_{ex}, \tag{5.9}$$

where "*ex*" means "external". Substituting (5.9) into (5.8), we obtain the Dirac equations with an external field:

$$\left[\hat{\alpha}_0\left(\hat{\varepsilon} \mp \varepsilon_{ex}\right) + c\hat{\vec{\alpha}}\cdot\left(\hat{\vec{p}} \mp \vec{p}_{ex}\right) + \hat{\beta}\,m_e c^2\right]\psi = 0, \tag{5.10}$$

which at $d \to \infty$ gives the Dirac free - plus and minus - particle equations:

$$\left[\left(\hat{\alpha}_o\hat{\varepsilon} + c\hat{\vec{\alpha}}\,\hat{\vec{p}}\right) + \hat{\beta}\,m_e c^2\right]\psi = 0, \tag{5.11}$$

$$\psi^+\left[\left(\hat{\alpha}_o\hat{\varepsilon} - c\hat{\vec{\alpha}}\,\hat{\vec{p}}\right) - \hat{\beta}\,m_e c^2\right] = 0, \tag{5.11'}$$

In other words, the equations of mssive semi-photons are Dirac equations. Consequently, the particles corresponding to massive semi-photons are leptons, including electron and a positron.

5.2. Electromagnetic representation of Dirac's equations

Using the electromagnetic representation (5.3) of the semi-photon wave function ψ and the displacement electric tangential currents (4.16) from previous chapter $\vec{j}^{e}_{dis} = \frac{1}{4\pi}\omega_p \mathrm{E}\cdot\vec{\tau}$, we obtain an electromagnetic form of equations (5.11).

$$\begin{cases} \frac{1}{c}\frac{\partial E_x}{\partial t} - \frac{\partial H_z}{\partial y} = -i\frac{4\pi}{c} j^e_x \\ \frac{1}{c}\frac{\partial H_z}{\partial t} - \frac{\partial E_x}{\partial y} = i\frac{4\pi}{c} j^m_z \\ \frac{1}{c}\frac{\partial E_z}{\partial t} + \frac{\partial H_x}{\partial y} = -i\frac{4\pi}{c} j^e_z \\ \frac{1}{c}\frac{\partial H_x}{\partial t} + \frac{\partial E_z}{\partial y} = i\frac{4\pi}{c} j^m_x \end{cases}, \quad (5.12') \qquad \begin{cases} \frac{1}{c}\frac{\partial E_x}{\partial t} + \frac{\partial H_z}{\partial y} = -i\frac{4\pi}{c} j^e_x \\ \frac{1}{c}\frac{\partial H_z}{\partial t} + \frac{\partial E_x}{\partial y} = i\frac{4\pi}{c} j^m_z \\ \frac{1}{c}\frac{\partial E_z}{\partial t} - \frac{\partial H_x}{\partial y} = -i\frac{4\pi}{c} j^e_z \\ \frac{1}{c}\frac{\partial H_x}{\partial t} - \frac{\partial E_z}{\partial y} = i\frac{4\pi}{c} j^m_x \end{cases}, \quad (5.12'')$$

Let us note that for the symmetry we included in the equations the displacement magnetic tangential currents (4.16) $\vec{j}^{m}_{dis} = \frac{1}{4\pi}\omega_p \mathrm{H}\cdot\vec{\tau}$. It is known that the existence of the magnetic current $\vec{j}^{m}$ does not contradict to quantum theory (see Dirac's theory of a magnetic monopole (Dirac, 1931)). In case of the plane polarized wave (see previous chapters), the magnetic currents are equal to zero (but not for other polarizations).
As it is easily see, the currents' forms (5.12) of the electron equation is similar to Maxwell-Lorentz's equations with complex fields, which is frequently used in the classical theory of electromagnetic waves.

6.0. An equation of a massive neutrino of NEPT

In the previous chapters, we showed that the electron is a half of the period of the intermediate ***plane-polarized*** *photon*, or the ***plane-polarized*** (this is how we called it for brevity) semi-photon.

According to contemporary experimental data, a neutrino, similar to an electron, is lepton, and must have a mass. However, in contrast to electron, neutrino must have a zero charge and preserved helicity. Moreover, the neutrino and antineutrino must have mirror asymmetry.

In order to satisfy these requirements, we propose the following hypothesis about the structure of EM neutrino: *neutrino is a semi -photon with a* ***circular*** *polarization*.

On the basis of this hypothesis, we will show below that within the framework of NEPT the massive neutrino is fully described by Dirac's lepton equation, and it has a preserved inner poloidal helicity (p-helicity) which produces the above-mentioned electromagnetic features.

7.0. Electromagnetic origin of weak and strong interactions

Thus, we have shown that certainly all fields in the framework of quantum electrodynamics (photons and leptons) have electromagnetic origin.

What do we have with respect to all other particles?

To avoid overloading the book with details that are set out in a variety of books, let us cite words of the active developer of the Standard Model, the Nobel laureate Mary Gell-Mann (Gell-Mann, 1985) :

We have the working renormalizable theory of strong, electromagnetic and weak interactions... This is of course the Yang-Mills theory… Essentially, all that we managed to do is *just to generalize quantum electrodynamics (QED). QED was invented around 1929 and since then has never changed... Now QED is generalized and includes strong and weak interactions along with electromagnetic, quarks and neutrinos, along with electrons.*

Thus, one could argue that QED and theories of weak and strong interactions also have electromagnetic origin. As we know, these theories cover all types of elementary particles: massless photons and massive leptons, bosons and hadrons. Therefore, it can be argued that the mass of elementary particles and hence of the whole matter has electromagnetic origin.

Conclusion

Thus we have shown that if as electromagnetic theory we imply quantum nonlinear electromagnetic field theory, the inertial mass has a fully electromagnetic origin. This answers the Feynman question in the above passage.

From this follows that gravitational mass and gravitational field also have an electromagnetic origin. Obviously, then the theory of gravity should be some variant of the nonlinear theory of the electromagnetic field. We will present an attempt to build such a theory in the following articles.

Chapter 3. The axiomatics of LIGT and its consequences

1.0. Lemma of electromagnetism

In the previous chapter of LITG, we presented evidence of the electromagnetic origin of inertial mass. Feynman noted (Feynman et al, 1964), that this statement does not contradict the experimental data.

On this basis, we state here the following lemma, which will serve as a foundation for building LIGT (let us call it "*Lemma of electromagnetism*").

Lemma of electromagnetism: ***The electromagnetic field is the basis for the origin of matter in the Universe***

From here follow a number of conclusions that are important for the theory of gravitation.

1) The equivalence of gravitational and inertial masses leads to the conclusion that gravity has an electromagnetic origin.

This conclusion is of fundamental importance for the construction of the Lorentz-invariant theory of gravitation.

2) The Lorentz-invariance of the laws of electromagnetism, determines Lorentz-invariance of the laws of gravity.

3) Elementary particles are the primary carriers of matter and its characteristics. Hence, the equation of gravitation should follow from the equations of elementary particles.

4) Matter is involved in the creation of the gravitational field as its source, without quantization of this source. Thus, the gravitational field can be regarded as a classical field, which does not require quantization. The assumed origin of this equation from quantum equations of elementary particles, is not a limitation here, because a transition exists from quantum to classical equations.

5) In the elementary particles' theory, inertial mass is associated with energy and momentum of particle by the equation:

$$\varepsilon^2 - c^2 p^2 = m_0^2 c^4 ,$$

where m_0 is the rest mass (invariant quantity). From this follows, what in general is the equivalence of mass and energy-momentum

$$m_0 = \frac{1}{c^2}\sqrt{\varepsilon^2 - c^2 p^2} ,$$

According to the above mentioned cause we can consider mass, energy and momentum as the gravitation sources.

6) Since in general case, the original equations of microcosm are nonlinear, we should assume that the gravitational equations are non-linear (it is easy to show that the same should follow from the principle of the equivalence of mass and energy-momentum).

Based on formulated above Lemma of electromagnetism, we can choose the following axioms for LIGT, which do not contradict to the experimental data.

2.0. Axiomatics of LIGT

As the first and second postulates we will take the experimental facts:

1. Postulate of source: ***the source of the gravitational field is matter in the form of an island matter or a field mass.***

2. Postulate of the masses' equivalence: ***the gravitational charge (mass) is proportional to the inertial mass.***

The electromagnetic origin of the mass of all elementary particles, as well as the weakness of the gravitational field compared to the electromagnetic field, allow us to take the following postulate.

3. Postulate of Mossotti -Lorentz: ***the gravitational field is a residual electromagnetic field.***

(*Note: we do not associate this axiom with the Mossotti model which explains how this residue is formed, but have in mind the general idea that the gravitational field is a small part of the electromagnetic field, which acts attractively*).

4. **The locality postulate:** ***gravitational field is locally Lorentz-invariant, that is Lorentz-invariant on any infinitely small time interval and on any infinitely small distance.***

(*Note: since the EM field is itself Lorentz- invariant, this axiom can be seen as a consequence of the axiom of Mossotti-Lorentz. But classical mechanics is globally Lorentz- invariant. With the introduction of postulate 4 we actually emphasize that gravitation, in the general case, is not globally Lorentz-invariant*).

From these axioms the next consequences follow, proof of which may serve as a confirmation of the axioms.

Corollary 1: since the gravitational field is residual, it is much weaker than the electromagnetic field, but in the case of a neutral matter (in the electromagnetic sense), the gravitational field is decisive.

Corollary 2: the gravitational constant is determined as a portion of full electromagnetic interaction.

Corollary 3: as in the theory of electromagnetism the interaction is described by the Lorentz force, the same (or its modification) describes the theory of gravitation.

Corollary 4: the equations of massive elementary particles can be regarded as the source equations of the gravitational field.

Corollary 5: all the features of motion of matter in the gravitational field come from the electromagnetic theory, in particular, from the effects associated with the Lorentz transformations.

Corollary 6: all the characteristics of gravitational field (its energy, momentum, angular momentum, etc) have an electromagnetic origin and obey the laws of electromagnetism.

The foregoing allows us to give a new interpretation of the equivalence of inertial and gravitational masses, different from that of Einstein.

Using our interpretation of the masses’ equivalence and the abovementioned axiomatics we shall try to build a Lorentz-invariant theory of gravitation.

Chapter 4. Electromagnetic base of relativistic mechanics

1.0. Lorentz-invariant amendments to Newton's mechanics

Under the moving masses (gravitational charges) we will understand the two interacting bodies, one of which we call the source of the gravitational field, and the other - the test particle.

Our approach to the theory of gravitation is based on a modern version of the electromagnetic theory of matter (EMTM) (Lorentz, 1916; Richardson, 1914; Becker, 1933). In framework of EMTM the mass is of electromagnetic (EM) origin. Therefore in framework of our axiomatics, the main results of EM theory are equivalent to results of the theory of gravitation.

The Maxwell EM theory was the first theory, whose properties were found to depend on the speed of the charge (in this case, electric). This theory is called the Lorentz-invariant (L-invariant) or relativistic theory.

The microscopic non-quantum EM theory, cover the microscopic level of matter and is called the Maxwell-Lorentz theory (M-L-theory) (see some details below). Before the creation of the EM theory, all other physical theories, including Newton's theory of gravity, were non-L-invariant (i.e., "non-relativistic").

A peculiarity of the L-invariant theory is a nonlinear dependence of the parameters of physical quantities from the speed of the charges (electric or gravitational). At speeds of up to one-tenth of the speed of light, these parameters are hardly different from the parameters of static objects.

This suggests that the basis of mechanics is still the classical Newtonian mechanics, and relativistic mechanics is Newton's mechanics plus minor amendments thereto.

Moreover, in the framework of EMTM it is easy to show that the calculation of corrections to the non-L-invariant theory is determined by the non-L-invariant theory. The amendments are calculated on the basis of non-L-invariant laws that take into account the changes in the parameters at high speeds. The calculation procedure is equivalent to the method of calculation which is based on perturbation theory, when the zero approximation is the non-relativistic theory. Below we show this, based on the known results presented in textbooks.

2.0. The Lorentz transformation as transition from rest to motion

2.1.General principles of electromagnetic theory of matter

The general equations of the electromagnetic theory of matter (EMTM) are formulated on the basis of Maxwell's equations, taking into account the Lorentz hypothesis. Under this hypothesis, all elementary particles (and, consequently, atoms, molecules and bodies) are composed of an electromagnetic field, which is in a concentrated ("condensed" according to Einstein) state. Since among these particles are the free EM fields (photons), they are also included in this list. At the same time, charges and currents are also determined by the electromagnetic fields. Consequently, there is only one kind of vectors, describing the field, namely the electromagnetic (EM) field strengths in vacuo $\vec{E}$ and $\vec{H}$ or equivalent quantities.

The self-consistent Maxwell-Lorentz microscopic equations are the independent fundamental field equations. The Maxwell-Lorentz equations are following four differential (or, equivalent, integral) equations for any electromagnetic medium (Jackson, 1965; Tonnelat, 1966):

$$rot\vec{B} - \frac{1}{c}\frac{\partial \vec{E}}{\partial t} = \frac{4\pi}{c}\vec{j}\,, \tag{2.1}$$

$$rot\vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0\,, \tag{2.2}$$

$$di\upsilon\vec{E} = 4\pi\rho\,, \tag{2.3}$$

$$di\upsilon\vec{B} = 0\,, \tag{2.4}$$

where $\vec{E}, \vec{H}, \vec{D}, \vec{B}$ are electric field vector, magnetic field vector, electric induction vector, magnetic induction vector, correspondingly; in vacuum $\vec{D} = \vec{E}$ and $\vec{B} = \vec{H}$; $\vec{E} = -grad\varphi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}$, $\vec{B} = rot\vec{A}$, where $\varphi, \vec{A}$ are scalar and vector potentials, correspondingly; ρ is the charge density; $\vec{j}$ is the current density: c is the speed of light.

The difference between these equations and Maxwell's equations is that $\vec{E}$ and $\vec{H}$, as well as all other quantities needed to describe a matter, refer to an arbitrarily small volume of space. In this case the equations (2.1-2.4) are called the Maxwell-Lorentz (ML) equations.

The Maxwell's macroscopic quantities $\bar{\vec{E}}$ and $\bar{\vec{H}}$ can be deduced from the microscopic quantities $\vec{E}$ and $\vec{H}$ only by averaging over space and time. This averaging and deducation of the actual Maxwell equations from (2.1-2.4) is considered in many courses on electromagnetism (Becker, 1933).

In the equations (2.1-2.4) nothing is said about how the velocity $\vec{\upsilon}$ of the charges changes over time. For this purpose the Lorentz law is used. According to Lorentz the density of force has the form:

$$\vec{f} = \rho\left(\vec{E} + \frac{1}{c}\vec{\upsilon} \times \vec{B}\right), \qquad (2.5)$$

hence $fd\tau$ is a force, acting on the volume $d\tau$.

2.2. The field of the uniformly and arbitrarily fast moving charge.

For the analysis of the field equations (2.1-2.4), it is advisable to go from fields themselves to the electromagnetic field potentials (Becker, 1933). This is done as follows: first of all, we satisfy the equation $di\upsilon\vec{H} = 0$ (2.4) by substituting:

$$\vec{H} = rot\vec{A}, \qquad (2.6)$$

where vector $\vec{A}$ is named the vector potential.

Then from the equation of (2.2) it follows that $\left(\vec{E} + \partial\vec{A}/c\,\partial t\right)$ should be zero. Therefore, we demand that the value $rot\left(\vec{E} + \partial\vec{A}/c\,\partial t\right)$ is equal to the gradient of a scalar φ:

$$\vec{E} = -\frac{1}{c}\frac{\partial \vec{A}}{\partial t} - grad\varphi, \qquad (2.7)$$

where vector φ is named the scalar potential.

The vector field is uniquely determined by divergence and vorticity of this field. Until now, we determined only $rot\vec{A}$. Now we can in addition freely dispose by the divergence of the vector $\vec{A}$. We will use this in order to put

$$di\upsilon\vec{A} + \frac{1}{c}\frac{\partial\varphi}{\partial t} = 0, \qquad (2.8)$$

If we now substitute (2.6) and (2.7) in the two remaining equations (2.1-2.4), then with the help of (2.8), we obtain two equations for the potentials:

$$\begin{aligned} &\frac{\partial^2 \vec{A}}{\partial x^2} + \frac{\partial^2 \vec{A}}{\partial y^2} + \frac{\partial^2 \vec{A}}{\partial z^2} - \frac{\partial^2 \vec{A}}{c^2 \partial t^2} = -\frac{4\pi}{c}\rho\vec{\upsilon}, \\ &\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} - \frac{\partial^2 \varphi}{c^2 \partial t^2} = -4\pi\rho, \end{aligned} \qquad (2.9)$$

For their integration we use the well-known fact that the field at time t at in any point is equal to the field at time $(t - dt)$ at the point, shifted back to the segment $\vec{\upsilon}dt$. This means that for all the quantities, characterizing the field, we will again have the relation:

$$\frac{\partial \chi}{\partial t} = -(\vec{\upsilon} \cdot grad)\chi$$

where $\chi = \chi(x, y, z)$ is a function of the field.

For example, for change in time of the electric vector $\vec{E}$ we will obtain:

$$\frac{\partial \vec{E}}{\partial t} = -(\vec{\upsilon} \cdot grad)\vec{e}$$

Thus, if the velocity is parallel to the positive x-axis, in our equations for the potentials (2.9), second time derivatives are replaced by derivatives with respect to the coordinate x according to the formula

$$\frac{\partial^2}{\partial t^2} = \upsilon^2 \frac{\partial^2}{\partial x^2}$$

Therefore, for the potentials $\vec{A}$ and φ we get the equation:

$$\left(1 - \frac{\upsilon^2}{c^2}\right)\frac{\partial^2 \vec{A}}{\partial x^2} + \frac{\partial^2 \vec{A}}{\partial y^2} + \frac{\partial^2 \vec{A}}{\partial z^2} = -\frac{4\pi}{c}\rho\vec{\upsilon},$$
$$\left(1 - \frac{\upsilon^2}{c^2}\right)\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} = -4\pi\rho, \qquad (2.10)$$

Note that the equations for the components of the vector potential differ from the equation for the scalar potential only by constant factor $\vec{\upsilon}/c$. Therefore, if we resolve the equation for φ, then a solution for the vector potential follows directly from it:

$$\vec{A} = \frac{\vec{\upsilon}}{c}\varphi, \qquad (2.11)$$

From this we obtain two equations:

$$rot\vec{A} = \frac{1}{c}rot(\vec{\upsilon}\varphi) = -\frac{1}{c}\vec{\upsilon} \times grad\varphi$$

$$\frac{\partial \vec{A}}{\partial t} = \frac{\vec{\upsilon}}{c}\frac{\partial \varphi}{\partial t} = -\frac{1}{c}\vec{\upsilon}(\vec{\upsilon} grad\varphi)$$

If we introduce them to the definitions (2.6) and (2.7), we get:

$$\vec{E} = -grad\varphi + \frac{1}{c}\vec{\upsilon}(\upsilon grad\varphi) \quad \text{and} \quad \vec{H} = \frac{1}{c}\vec{\upsilon} \times \vec{E}, \qquad (2.12)$$

It turns out that the relation (2.11) between the vector and scalar potentials leads to known dependence $\vec{H} = \frac{1}{2}\left[\vec{\upsilon} \times \vec{E}\right]$ between $\vec{H}$ and $\vec{E}$.

Therefore, to solve our problem, we can confine ourselves to integrating the equation (2.10) for φ. Note that this equation differs from the equation for the ordinary electrostatic potential only by constant coefficients $\left(1 - \upsilon^2/c^2\right)$ at $\partial^2\varphi/\partial x^2$. So technically we can reduce our problem to a simple electrostatic problem, if instead of coordinates x, y, z, t we introduce the new coordinates x', y', z', t' using the transformation:

$$x = x'\sqrt{1 - \beta^2}, \; y = y', \; z = z', \; t = t', \qquad (2.13)$$

where for brevity we put $\upsilon/c = \beta$. Due to this change, the functions $\varphi(x, y, z, t)$ and $\rho(x, y, z, t)$ pass to functions φ' and ρ' from x', y', z', t', so that we have the identities:

$$\rho'(x',y',z',t') \equiv \rho\left(x'\sqrt{1-\beta^2}, y', z', t'\right),$$
$$\varphi'(x',y',z',t') \equiv \varphi\left(x'\sqrt{1-\beta^2}, y', z', t'\right), \tag{2.14}$$

Therefore, our equation for the potential in the primed coordinates is

$$\frac{\partial^2\varphi'}{\partial x'^2} + \frac{\partial^2\varphi'}{\partial y'^2} + \frac{\partial^2\varphi'}{\partial z'^2} = -4\pi\rho, \tag{2.15}$$

As such, this equation is completely identical to the equation that determines the potential of the *fixed charge system*. Therefore, its integration can produced according to the well-known theory of the electrostatic potential. We get:

$$\varphi'(x',y',z',t') = \iiint \frac{\rho'(\xi',\eta',\varsigma',t')d\xi' d\eta' d\varsigma'}{\sqrt{(x'-\xi')^2 + (y'-\eta')^2 + (z'-\xi')^2}}$$

If we again turn to the unprimed coordinates with the help of (2.13) and (2.9), we will obtain the solution of equation (2.10) for the scalar potential in the form

$$\varphi(x,y,z,t) = \iiint \frac{\rho(\xi,\eta,\varsigma,t)d\xi d\eta d\varsigma}{\sqrt{(x-\xi)^2 + (1-\beta^2)(y-\eta)^2 + (z-\xi)^2}}, \tag{2.16}$$

Now let us find a particular solution for the time $t = t_0$ when the electron is in the beginning of the coordinate system, and restrict ourselves to the case of the point electron, i.e., assume that the charge density is different from zero only in the immediate vicinity of the origin of coordinates $(\xi = \eta = \varsigma = 0)$. Then the integration can be done, and we get the solution:

$$\varphi(x,y,z,t_0) = \frac{\vec{E}}{\sqrt{x^2 + (1-\beta^2)(y^2+z^2)}}, \tag{2.17}$$

For the purposes of brevity, we introduce for the expression that appears in the denominator instead of the distance r, the designation:

$$s = \sqrt{x^2 + (1-\beta^2)(y^2+z^2)}, \tag{2.18}$$

Then we will be able to present the solution to our problem in the form

$$\varphi = \frac{e}{s}, A_x = \frac{e\upsilon}{sc}, A_y = A_z = 0, \tag{2.19}$$

Using these potentials we can calculate the field $\vec{E}$ and $\vec{H}$ by the formulas (2.6), (2.7) or (2.12), and taking into account that differentiation by time is always replaced by $-\upsilon\frac{\partial}{\partial x}$, for the electrical strength in vector form we will get:

$$\vec{E} = (1-\beta^2)\frac{e}{s^3}\vec{r}, \tag{2.20}$$

Further, the magnetic strength $\vec{H} = \frac{1}{c}\vec{\upsilon}\times\vec{E}$; hence:

$$H_x = 0, H_y = -\frac{e\upsilon}{c}\frac{1-\beta^2}{s^3}z, H_z = \frac{e\upsilon}{c}\frac{1-\beta^2}{s^3}y, \tag{2.21}$$

2.3. Lorentz transformations and their consequences

Our aim (Lorentz, 1904; Lorentz, 1916; Poincaré, 1905) must again be to reduce the equations for a moving system to the form of the ordinary formulae that hold for a system at rest. It is found that the transformations needed for this purpose may be left indeterminate to a certain extent; our formulae will contain a numerical coefficient l, of which we shall provisionally assume only that it is a function of the velocity of translation υ, whose value is equal to unity for $\upsilon = 0$, and differs from 1 by an amount of the order of magnitude υ^2/c^2 for small values of the ratio υ/c.

If *x*, *y*, *z* are the coordinates of a point with respect to axes fixed in the vacuum, or, as we shall say, the "absolute" coordinates, and if the translation takes place in the direction of *OX*, the coordinates with respect to axes moving with the system, and coinciding with the fixed axes at the instant $t = 0$, will be

$$x_r = x - \upsilon t\,,\quad y_r = y\,,\quad z_r = z\,, \qquad (2.22)$$

Now, instead of x_r, y_r, z_r we shall introduce new independent variables differing from these "relative" coordinates by certain factors that are constant throughout the system. Putting

$$\frac{c^2}{c^2 - \upsilon^2} = \frac{1}{1-\beta^2} = \gamma_L^2, \qquad (2.23)$$

we define the new variables by the equations

$$x' = \gamma_L l x_r\,,\quad y' = l y_r\,,\quad z' = l z_r \qquad (2.24)$$

or

$$x' = \gamma_L l (x - \upsilon t)_r\,,\quad y' = l y_r\,,\quad z' = l z_r\,, \qquad (2.25)$$

and to these we introduce as our fourth independent variable

$$t' = \frac{l}{\gamma_L} t - \gamma_L l \frac{\upsilon}{c^2}(x - \upsilon t) = \gamma_L l \frac{\upsilon}{c^2}\left(t - \frac{\upsilon}{c^2} x\right), \qquad (2.26)$$

It was Poincaré (Poincaré, 1905) who first introduced that the real meaning of the substitution (2.25), (2.26) lies in the relation

$$x'^2 + y'^2 + z'^2 - c^2 t'^2 = l^2\left(x^2 + y^2 + z^2 - c^2 t^2\right), \qquad (2.27)$$

that can easily be verified, and from which we may infer that we shall have

$$x'^2 + y'^2 + z'^2 = c^2 t'^2\,, \qquad (2.28)$$

when

$$x^2 + y^2 + z^2 = c^2 t^2, \qquad (2.29)$$

This may be interpreted as follows. Let a disturbance, which is produced at the time $t = 0$ at the point $x = 0$, $y = 0$, $z = 0$ be propagated in all directions with the speed of light *c*, so that at the time t it reaches the spherical surface determined by(2.29). Then, in the system *x′*, *y′*, *z′*, *t′*, this same disturbance may be said to start from the point $x' = 0$, $y' = 0$, $z' = 0$, at the time $t' = 0$ and to reach the spherical surface (2.28) at the time t'. Since the radius of this sphere is *ct′*, the disturbance is propagated in the system *x′*, *y′*, *z′*, *t′* as it was in the system *x*, *y*, *z*, *t*, with the speed c. Hence, the velocity of light is not altered by the transformation.

3.0. The static fields of Coulomb and Newton as fundamental fields with respect to fields of moving sources.

The moving source of the gravitational field is a gravitational current, i.e., the movement of gravitational charge (mass). As we have seen (see above), the transition from the fixed charge and their fields to the mobile charge and fields, and vice versa, is described by Lorentz transformations.

In the theory of gravity the transition from the non-L-invariant theory to the L-invariant theory requires, first and foremost, to find the L-invariant expression for the static Newton's law of gravity $\vec{F}_N = \gamma_N mM\vec{r}^{\,0}/r^2$, or $div\vec{G} = 4\pi\gamma_N \rho_m$, where $\vec{G} \equiv \vec{F}/m$ (or by introducing potential φ_N through $\vec{F}_N = grad\varphi$, in the form $\vec{\nabla}^2\varphi_N = 4\pi\gamma_N \rho_m$).

According to LIGT the Newton law of gravity is a consequence of the law of the static interaction between charges of Coulomb (or of more general assertion - of Gauss theorem). This gives us an opportunity to consider the gravity problem on the basis of the electromagnetic problem that has been solved.

At first glance, here lies the contradiction. Static (non-L-invariant) Coulomb's law $\vec{F}_C = \gamma_e qQ\vec{r}^{\,0}/r^2$ (where in the CGS $\gamma_e = 1$) in the form $div\vec{E} = 4\pi\rho_e$ or $\vec{\nabla}^2\varphi_e = 4\pi\rho_e$, is included as part in the M-L equation, which, in its totality, is of course, L-invariant.

The exit from this contradiction is somewhat unexpected. We will show below that in the transition from source шт a stationary reference frame to the same source in moving frame, new additional fields are generated, which together with the same static field, meet the requirements of the L-invariance.

Farther we assume that all the statements that we can make with respect to EM theory, are valid for the theory of gravity, taking into account the established terminology (for example, the charge in EM theory is called mass in the theory of gravity, etc).

(Farther to confirm our ideas, we will use the quotes from the book of E. Purcell (Purcell, 1985)).

3.1. Gauss's law

The flux of the electric field $\vec{E}$ through any closed surface, that is, the integral $\int \vec{E}\cdot d\vec{s}$ over the surface, equals 4π times the total charge enclosed by the surface:

$$\int \vec{E}\cdot d\vec{s} = 4\pi\sum_i q_i = 4\pi\int \rho d\tau, \tag{3.1}$$

We call the statenlent in the box a law because it is equivalent to Coulomb's law and it could serve equally well as the basic law of electrostatic interactions, after charge and field have been defined. Gauss's law and Coulomb's law are not two independent physical laws, but the same law expressed in different ways.

This suggests that Gauss's law, rather than Coulomb's law, offers the natural way to define quantity of charge for a moving charged particle, or for a collection of moving charges.

It would be embarrassing if the value of $Q = \frac{1}{4\pi}\int_{S(t)} \vec{E}\cdot d\vec{s}$ so determined depended on the size and shape of the surface S. For a stationary charge it doesn't-that is Gauss's law.

But how do we know that Gauss's law holds when charges are moving? We can take that as an experimental fact.

3.2. Invariance of charge

There is conclusive experimental evidence that the total charge in a system is not changed by the motion of the charge carriers.

This invariance of charge lends a special significance to the fact of charge quantization. It is known the fact that every elementary charged particle has a charge equal in magnitude to that of every other such particle. And this precise equality holds not only for two particles at rest with respect to one another, but for any state of relative motion.

3.3 Electric field measured in different frames of reference

If charge is to be invariant under a Lorentz transformation, the electric field $\vec{E}$ has to transform in a particular way. “Transforming $\vec{E}$” means answering a question like this: if an observer in a certain inertial frame F measures an electric field $\vec{E}$ as X volts/cm, at a given point in space and time, what field will be measured at the same space-time point by an observer in a different inertial frame F'? For a certain class of fields, we can answer this question by applying Gauss's law to some simple systems.

Gauss's law tells us that the magnitude of E' must be

$$E' = \frac{E}{\sqrt{1-\beta^2}} = \gamma_L E$$

But this conclusion holds only for fields that arise from charges stationary in F. As we shall see below, if charges in F are moving, the prediction of the electric field in F' involves knowledge of two fields in F, the electric and the magnetic.

3.4 Force on a moving charge

At some place and time in the lab frame we observe a particle carrying charge q which is moving, at that instant, with velocity $\vec{\upsilon}$ through the electrostatic field. What force appears to act on q?

Force means rate of change of momentum, so we are really asking, What is the rate of change of momentum of the particle, $d\vec{p}/dt$, at this place and time, as measured in our lab frame of reference? That is all we mean by the force on a moving particle.

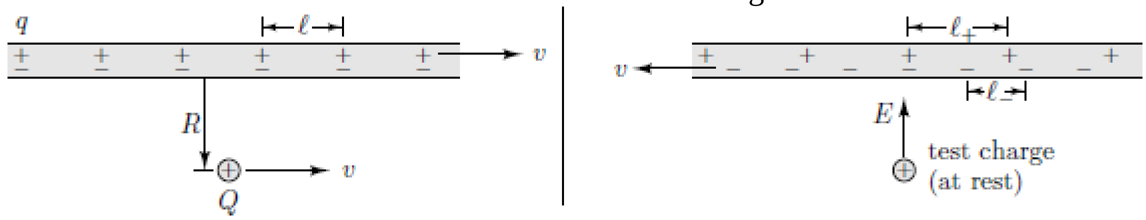
3.5 Interaction between a moving charge and other moving charges

We know that there can be a velocity-dependent force on a moving charge. That force is associated with a magnetic field, the sources of which are electric currents, that is, other charges in motion.

3.5.1 *Magnetism as a Consequence of Length Contraction*

Model a current-carrying wire (Schroeder, 1999) as a line of negative charges ($-q$) at rest and a line of positive charges ($+q$) moving to the right at speed $\vec{\upsilon} = \upsilon\vec{x}^0$, where $\vec{x}^0$ is unit vector of x-axis,. The average linear separation between charges is l. Consider a "test charge" Q moving parallel to the wire, at the same speed υ (for simplicity). In the frame of the test charge it is at rest and so are the (+)-charges in the wire, but the − charges are moving to the left. According to relativity, the distance between the (−)-charges is length-contracted to $l_- = l\sqrt{1-(\upsilon/c)^2}$, while the distance between the (+)-charges is un-length-contracted to $l_+ = l\sqrt{1-(\upsilon/c)^2}$. Therefore the wire carries a net negative charge and exerts an attractive electrostatic force on the test charge. Back in the lab frame, we call this a magnetic force.

Lab Frame: Test Charge Frame:



test charge (at rest)

To calculate the strength of the force, first we find the linear charge density of the wire in the test charge frame (assuming $\upsilon << c$ for simplicity):

$$\lambda = \frac{q}{l_+} - \frac{q}{l_-} = \frac{q}{l}\left(\sqrt{1-(\upsilon/c)^2} - \frac{1}{\sqrt{1-(\upsilon/c)^2}}\right) \approx \frac{q}{l}\left[1 - \frac{1}{2}\left(\frac{\upsilon}{c}\right)^2 - 1 - \frac{1}{2}\left(\frac{\upsilon}{c}\right)^2\right] = -\frac{q}{l}\left(\frac{\upsilon}{c}\right)^2, \quad (3.2)$$

In a typical household wire $\upsilon/c \sim 10^{-13}$, so the Lorentz factor differs from 1 by only about one part in 10^{26}. This tiny amount of length contraction is still observable, because the total charge of all the moving electrons is enough to exert enormous electrostatic forces.

The same derivation can be adapted to more complicated cases where the test charge has an arbitrary velocity, in either direction. To understand the case where the test charge is moving toward or away from the wire, you need to digress to show how the electric field of a point charge in motion is weaker in front of and behind the charge but stronger in the transverse directions. (This can be derived using length contraction and some simple gedanken experiments.)

From our present vantage point (Purcell, 1985), the magnetic interaction of electric currents can be recognized as an inevitable corollary to Coulomb's law. If the postulates of relativity are valid, if electric charge is invariant, and if Coulomb's law holds, then, as we shall now show, the effects we commonly call "magnetic" are bound to occur. They will emerge as soon as we examine the electric interaction between a moving charge and other moving charges.

Two charge distributions experience Lorentz contraction of various values - this is the solution of the problem.

A more general and detailed analysis of the problem is described, for example, in the book Let us use the results of book (Purcell, 1985) to get the mathematical expression of the arising force and magnetic field (for brevity we use the notation introduced earlier $\beta = \upsilon/c$, $\gamma_L = 1/\sqrt{1-\beta^2}$)

In general case the total linear density of charge in the wire in the test charge frame, λ, can be calculated:

$$\lambda = \lambda_+ - \lambda_- = -\frac{2\lambda\gamma_L\upsilon\upsilon_0}{c^2}, \tag{3.2'}$$

(the meaning of the unknown variables in (3.2') is explained below

The wire is positively charged. The use of Gauss's law (applied to the cylinder which surrounds the line) guarantees the existence of a radial electric field E'_r given by the formula for the field of any infinite line charge:

$$E'_r = \frac{2\lambda}{r} = \frac{4\lambda\gamma_L\upsilon\upsilon_0}{rc^2}, \tag{3.3}$$

Hence, the test charge q will experience a force, which is directed inwardly radially

$$F_r' = qE_r' = \frac{2q\lambda}{r} = \frac{4q\lambda\gamma_L\upsilon\upsilon_0}{rc^2}, \tag{3.4}$$

Now let's return to the lab frame. What is the magnitude of the force on the charge q as measured there? If its value is qE_r' in the rest frame of the test charge, observers in the lab frame will report a force smaller by the factor $(1/\gamma_L)$. Since $r = r'$, the force on our moving test charge, measured in the lab frame, is:

$$F_r = \frac{F_r'}{\gamma_N} = \frac{4q\lambda\upsilon\upsilon_0}{rc^2}, \tag{3.5}$$

Now $2\lambda\upsilon_0$ is just the total current I in the wire, in the lab frame, for it is the amount of charge flowing past a given point per second. We'll call current positive if it is equivalent to positive charge flowing in the positive x direction. Our current in this example is negative. Our result can be written this way:

$$F = \frac{2q\upsilon I}{rc^2}, \tag{3.6}$$

We have found that in the lab frame the moving test charge experiences a force in the y direction which is proportional to the current in the wire, and to the velocity of the test charge in the x direction.

If we had to analyze every system of moving charges by transforming back and forth among various coordinate systems, our task would grow both tedious and confusing. There is a better way. The overall effect of one current on another, or of a current on a moving charge, can be described completely and concisely by introducing a new field, the magnetic field.

3.6 Introduction of the magnetic field

Thus, a charge which is moving parallel to a current of other charges experiences a force perpendicular to its own velocity. We can see it happening in the deflection of the electron beam.

Let us state it again more carefully. At some instant t a particle of charge q passes the point (x, y, z) in our frame, moving with velocity υ. At that moment the force on the particle (its rate of change of momentum) is $\vec{F}$. The electric field at that time and place is known to be $\vec{E}$. Then the magnetic field at that time and place is defined as the vector $\vec{B}$ which satisfies the vector equation

$$\vec{F} = q\vec{E} + \frac{q}{c}\vec{\upsilon} \times \vec{B}, \qquad (3.7)$$

What kind of vector should be $\vec{B}$, in order to make the equation (3.6) compatible with the equation (3.7).

For fields that vary in time and space equation (3.7) is to be understood as a local relation among the instantaneous values of $\vec{F}$, $\vec{E}$, $\vec{\upsilon}$ and $\vec{B}$. Of course, all four of these quantities must be measured in the same inertial frame.

In the case of our "test charge" in the lab frame, the electric field $\vec{E}$ was zero. With the charge q moving in the positive x direction, $\vec{\upsilon} = \upsilon\vec{x}^0$, we found that the force on it was in the negative y direction, with magnitude $2q\upsilon I/rc^2$

$$\vec{F} = -\vec{y}^0 \frac{2q\upsilon I}{rc^2}, \qquad (3.8)$$

In this case the magnetic field must be

$$\vec{B} = \vec{z}^0 \frac{2I}{rc}, \qquad (3.9)$$

for then equation (3.7) becomes

$$\vec{F} = \frac{q}{c}\vec{\upsilon} \times \vec{B} = \left(\vec{x}^0 \times \vec{z}\right)\frac{q\upsilon}{c}\frac{2I}{rc} = -\vec{y}^0 \frac{2q\upsilon I}{rc^2}, \qquad (3.10)$$

in agreement with equation (3.8).

3.7 Vector potential

We found that the scalar potential function $\varphi(x, y, z)$ gave us a simple way to calculate the electrostatic field of a charge distribution. If there is some charge distribution $\rho(x, y, z)$, the potential at any point (x_1, y_1, z_1) is given by the volume integral

$$\varphi(x_1, y_1, z_1) = \int \frac{\rho(x_2, y_2, z_2)}{r_{12}} d\upsilon_2, \qquad (3.11)$$

The integration is extended over the whole charge distribution, and r_{12} is the magnitude of the distance from (x_2, y_2, z_2) to (x_1, y_1, z_1). The electric field $\vec{E}$ is obtained as the negative of the gradient of cp:

$$\vec{E} = -grad\varphi, \qquad (3.12)$$

The same trick won't work here, because of the essentially different character of $\vec{B}$. The curl of $\vec{B}$ is not necessarily zero, so $\vec{B}$ can't, in general, be the gradient of a scalar potential. However, we know another kind of vector derivative, the curl. It turns out that we can usefully represent $\vec{B}$, not as the gradient of a scalar function but as the curl of a vector function, like this:

$$\vec{B} = rot\vec{A}, \qquad (3.13)$$

By obvious analogy, we call $\vec{A}$ the vector potential. It is not obvious, at this point, why this tactic is helpful. That will have to emerge as we proceed. It is encouraging that equation (2.4) ($div\vec{H} = 0$) is automatically satisfied, since $div\, rot\vec{A} = 0$, for any $\vec{A}$.

In view of equation (2.1), the relation between $\vec{J}$ and $\vec{A}$ is

$$rot\left(rot\vec{A}\right) = \frac{4\pi\vec{J}}{c}. \tag{3.14}$$

Equation (3.14) , being a vector equation, is really three equations. We shall work out one of them, say the x-component equation. Among the various functions which might satisfy our requirement (3.13), let us consider as candidates only those which also have zero divergence $div\vec{A} = 0$. Then, after a series of transformations we get from (3.14):

$$\frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_x}{\partial y^2} + \frac{\partial^2 A_x}{\partial z^2} = \frac{4\pi J_x}{c}, \tag{3.15}$$

Conclusion

Thus, we shown that the calculation of corrections to the non-L-invariant theory is determined by the non-L-invariant theory.

Chapter 5. The connection of electromagnetic theory and gravitation

Introduction

Here, we will show that the adopted by us the Mossotti-Lorentz postulate does not contradict the existing results of physics, in particular, general relativity. To do this, we will show that the basic solutions of general relativity equations can be expressed in electromagnetic form.

1.0. Introduction

"*The nature of time, space* and *reality* (Broekaert, 2005) are to large extent dependent on our interpretation of Special (SRT) and General Theory of Relativity (GTR). In STR essentially two distinct interpretations exist; the "geometrical" interpretation by Einstein based on the *Principle of Relativity* and the *Invariance of the velocity of light* and, the "physical" Lorentz-Poincar´e interpretation with underpinning by *rod contractions, clock slowing* and *light synchronization,* see e.g. (Bohm, 1965; Bell, 1987). *It can be questioned whether the "Lorentz-Poincar´e"-interpretation of STR can be continued into GTR*" (Broekaert, 2005).

It can be said that the purpose of our Lorentz-invariant theory of gravitation (LIGT) lie namely in distributing the "physical" interpretation of the Lorentz-Poincaré to gravitation.

Recall that we use the absolute system of units of Gauss, in which all physical units are expressed in terms of mechanical units: centimeter (cm), gram (g), second (s). In our case, it is essential that in this system six vectors of EM theory: $\vec{E}$, $\vec{D}$, $\vec{P}$, $\vec{B}$, $\vec{H}$, $\vec{M}$ (where $\vec{E}$ is electric field strength, $\vec{D}$ is electric displacement, $\vec{P}$ is electric polarization, $\vec{B}$ is magnetic induction, $\vec{H}$ is magnetic field strength, $\vec{M}$ is magnetic polarization), have the same dimension ($g^{1/2}\,cm^{-1/2}\,s^{-1}$). It is believed that this is a major cause why the CGS-Gaussian units have more common use among theoretical physicists. Let also M_s is mass of a big body ("source"), in the gravitational field of which we will explore the movement of another body ("particle") with small mass which is equal to m, and $m << M_s$.

Note that the variety of tasks is determined here not only by the usual parameters of the Newtonian theory of gravitation, but also by whether the source and the particle rotate or not.

2.0. The existing solutions of the Einstein-Hilbert equation

Shortly (Berman, 2007) after the appearance of Einstein's General Relativistic field equations, the first static and spherically symmetric solution became available: it was Schwarzschild's-Droste's metric (Schwarzschild, 1916; Droste, 1917; Hilbert, 1917). It described the gravitational field around a point like mass *M*.

Afterwards, the first rotational metric was developed: Lense-Thirring solution (Thirring and Lense, 1918). It described the field around a rotating sphere at the origin. Nevertheless, it was only an approximate solution, that only represented a weak field, in the slow rotation case.

Reissner and Nordstrom's metric (Reissner, 1916), generalized Schwarzschild's by charging the mass source in the origin.

It was only in the sixties of last century, that a rigorous solution representing the rotation of a central mass, was discovered by R. Kerr, in what is now called Kerr's metric (Kerr, 1963). Immediately afterwards, the generalization to a charged rotating central mass was supplied, which is now called Kerr-Newman's metric (Newman, Couch et al. 1965).

Note that the effects associated with enumerated solutions have been tested with different accuracy. Three effects described by the static Schwarzschild-Droste solution, were checked with an accuracy higher than 1%. Some effects associated with the rotation of the source and the particles were checked with less precision. Most of the other solutions can not be verified by the current state of the art, or because there are no corresponding objects of observation.

3.0. GTR, EMG and EMGT

A lot of the solutions of general relativity are obtained in linear approximation, using the method of perturbation. It was found that the results of this linear theory may be presented in the form of Maxwell's equations. Such a representation has been called gravitoelectromagnetism, or, briefly, GEM.

3.1. Gravito-electromagnetism (EMG)

In general relativity (GR) (Overduin, 2008), “space and time are inextricably bound together. In special cases, however, it becomes feasible to perform a "3+1 split" and decompose the metric of four-dimensional spacetime into a scalar time-time component, a vector time-space component and a tensor space-space component.

When gravitational fields are weak and velocities are low compared to *c*, then this decomposition takes on a particularly compelling physical interpretation: if we call the scalar component a "gravito-electric **(ge-)** potential" and the vector one a "gravito-magnetic **(gm-)** potential", then these quantities are found to obey almost exactly the same laws as their counterparts in ordinary electromagnetism.

In other words, one can construct a "gravito-electric field" $\vec{E}_{ge}$ and a "gravito-magnetic field $\vec{H}_{gm}$, and these fields are obeyed equations that are identical to Maxwell's equations and the Lorentz force law of ordinary electrodynamics.

From symmetry considerations we can infer that the earth's **gravito-electric field** must be radial, and its **gravito-magnetic** one dipolar, as shown in the diagrams 3.1 and 3.2. below:

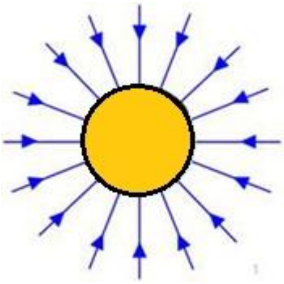


Fig.3.1. Radial gravitation field lines of Earth

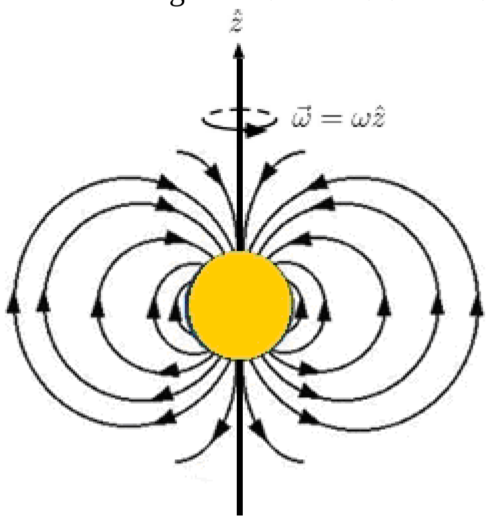


Fig. 3.2. Dipole gravitation field lines of Earth

These facts allow one to derive the main predictions of general relativity, simply by replacing the electric and magnetic fields of ordinary electrodynamics $\vec{E}$ and $\vec{H}$ by $\vec{E}_{ge}$ and $\vec{H}_{gm}$ respectively. However any such identification must be treated with care because the distinction between gravito-magnetism and gravito-electricity is frame-dependent, just like its counterpart in Maxwell's theory”.

The mathematical aspect of GEM theory is described in many papers (see, for example, (Mashhoon, 2008))

To avoid misunderstanding, it should be noted that the electromagnetic theory of gravitation (EMGT) and gravitoelectromagnetism (GEM) - are not the same (Mashhoon, 2008). GEM is an auxiliary discipline of general relativity, which allows to physically imagine the phenomena generated by the metric (i.e., geometric) theory: general relativity. In turn, EMGT is an independent theory of gravitation, which arose on the basis of the hypothesis Mossotti and then was developed by number of scientists, including O. Heaviside, H.Lorentz and others (Heaviside, 1912; Lorentz, 1900; Webster, 1912; Wilson, 1921; etc).

4.0. Problem of motion of two interacting bodies in electrodynamics

4.1. Statement of the Problem

For simplicity, we will use the motion law of Newton. In the relativistic case it is of the form:

$$\frac{d\vec{p}}{dt} = \vec{F}\,, \tag{4.1}$$

where $\vec{p} = m\vec{\upsilon}$ is the momentum of the particle, $\vec{F}$ is the force, acting on the particle. In the case of EM field this $\vec{F}$ is Lorentz force:

$$\vec{F} = \vec{F}_e + \vec{F}_m\,, \tag{4.2}$$

where $\vec{F}_e = q\vec{E}$ is the electric part of the Lorentz force, $\vec{F}_m = \frac{q}{c}\vec{\upsilon}\times\vec{H}$ is the magnetic part of the Lorentz force. If we express the vectors of the EM field in terms of potentials $\vec{E} = -grad\varphi - \frac{1}{c}\frac{\partial\vec{A}}{\partial t}$, $\vec{H} = rot\vec{A}$, for the Lorentz force we obtain the expression:

$$\vec{F} = q\vec{\nabla}\varphi - \frac{q}{c}\frac{\partial\vec{A}}{\partial t} + \frac{q}{c}\vec{\upsilon}\times\left(\vec{\nabla}\times\vec{A}\right) = q\vec{\nabla}\varphi - \frac{q}{c}\frac{\partial\vec{A}}{\partial t} + \frac{q}{c}\left[\vec{\nabla}\left(\vec{\upsilon}\cdot\vec{A}\right) - \left(\vec{\upsilon}\cdot\vec{\nabla}\right)\vec{A}\right], \tag{4.3}$$

Thus, the law (4.1) in the case of motion of a particle in an electromagnetic field takes the form:

$$\frac{d\left(m\vec{\upsilon}\right)}{dt} = q\vec{E} + \frac{q}{c}\vec{\upsilon}\times\vec{H}\,, \tag{4.4}$$

As we have noted equations of EM fields can be written in the force form (via field strength, i.e., through the force per unit charge) or in the energy form (via potentials, i.e., through the energy and momentum per unit charge). The connection between them is easy to to be found by multiplying the vector fields at the charge q:

$q\vec{E} = -grad\;q\varphi - \frac{q}{c}\frac{\partial\vec{A}}{\partial t}$, $q\vec{H} = c\cdot rot\frac{q}{c}\vec{A}$. Since the force, energy and momentum are expressed in terms of EM values as $\vec{F}_e = q\vec{E}$, $\varepsilon = q\varphi$, $\vec{p} = \frac{q}{c}\vec{A}$, respectively, and $q\vec{H} = rot\;c\vec{p}$, so for the electric and magnetic forces we obtain correspondingly: $\vec{F}_e = q\vec{E} = -\,grad\,\varepsilon - \frac{\partial\vec{p}}{\partial t}$, $\vec{F}_m = q\left[\vec{\upsilon}\times\vec{H}\right] = \vec{\upsilon}\times rot\;c\vec{p} = \vec{\upsilon}\times\vec{\nabla}\times c\vec{p}$.

Thus, within the framework of EMGT the Lorentz-invariant theory of gravitation must contain not only energy, but also momentum. In other words, it must be characterized not only by gravi-electric field or by corresponding gravi-electric potential, but also by a gravi-magnetic field or appropriate vector potential.

Since we consider the relativistic effects as corrections to the Newtonian mechanics, we will begin from the non-relativistic Newton's law, when the mass of the particle $m = const$. In this case, equation (4.4) can be rewritten as:

$$m\frac{d\vec{\upsilon}}{dt} = \vec{F}_e + \vec{F}_m, \tag{4.5}$$

Depending on the experimental conditions a few problems for the equation (4.5) can be set, solutions of which are of practical interest and the results of which can be tested in an experiment with sufficient accuracy:

1st task comes from the equation $m\frac{d\vec{\upsilon}}{dt} = \vec{F}_e$ for a spinless or spinning particle, rotating around the stationary source. This task corresponds to the fall of the particle on the source along the radius (the, so-called, radial infall), including the case when the particle rotates in a circular orbit, which is defined by the equilibrium of the force of gravity and centrifugal force.

2nd task comes from the equation $m\frac{d\vec{\upsilon}}{dt} = \vec{F}_m$ for the field, created by the rotation of the source. Since for the usual range of characteristics of the source the field $\vec{H}$ is much smaller than the field $\vec{E}$, this solution can be considered as a small perturbation of the particle motion of the first task. This task is a more difficult one, since $\vec{F}_m$ contains the speed. In addition, as compared with the first problem, the solution is dependent on the particle trajectories around the source; there may be tasks for latitude, azimuth and intermediate, between them, motions of the spinning and spinless particles.

3rd task comes from the equation $m\frac{d\vec{\upsilon}}{dt} = \vec{F}_e + \vec{F}_m$ is a more common type of motion and its solution, of course, is much more difficult relatively to the first two. However, under certain limitations on the parameters of the problem, it is possible to obtain interesting solutions.

Some of these problems in the case of EM theory have already been solved. In this case, it is sufficient only to transfer correctly the solution for the case of the gravitational field. In other cases, there is a need to solve problems from a clean slate.

4.2. Stationary electric field (Coulomb field)

Consider the interaction of two charges q and Q belonging to the bodies with masses m and M, respectively, where $m << M$.

The expression for the interaction of two point (or spherical) charges q and Q is given by the Coulomb force:

$$\vec{F}_C = k_e \frac{qQ}{r^2}\vec{r}^{\,0}, \tag{4.6}$$

wherein $k_e = 1$ in Gauss system of units, r is distance to particles, $\vec{r}^{\,0} = \vec{r}/r$ is a unit vector.

Expression for the field strength of the point source of the electric field is by definition equal:

$$\vec{E} = k_e \frac{\vec{F}_C}{q} = k_e \frac{Q}{r^2}\vec{r}^{\,0}, \tag{4.7}$$

Using the expression of the electric field through the potential $\vec{E} = \vec{\nabla}\varphi \equiv grad\varphi \equiv \partial\varphi/\partial\vec{r}$, it is easy to find:

$$\varphi = k_e \frac{Q}{r}, \tag{4.8}$$

4.3 Magnetic field of a charged particle

A charged particle, as some body, produces a magnetic field, firstly, due to its movement along a certain trajectory, $\vec{H}_l$, and, secondly, due to its own rotation, $\vec{H}_s$.

If the particle is involved in both movements simultaneously, the total field is a superposition of both fields: $\vec{H} = \vec{H}_l + \vec{H}_s$.

Consider how these fields are defined in electrodynamics.

1) If the particle moves along a path with a speed $\vec{\upsilon}$, in the laboratory frame a magnetic field appears, which is equal to:

$$\vec{H}_l = \frac{1}{c}\left[\vec{\upsilon}\times\vec{E}_l\right]. \tag{4.9}$$

The expression for the magnetic field (4.9) can be obtained from the Biot-Savart law when it is applied to a single particle. This magnetic field has the form of circular rings around the axis of the particle motion. It has maximum in a plane, which is perpendicular to the particle motion, and in this cross-section is equal to: $H_l = \frac{\upsilon}{c}\frac{q}{r^2}$. From (4.9) it also follows that this field is υ/c times smaller than the electric.

2) If a charged particle has a rotation (spin), it has a magnetic dipole moment $\vec{\mu}$. The magnetic field of a magnetic dipole is:

$$\vec{H}_s = \frac{3(\vec{r}^{\,0}\vec{\mu})\vec{r}^{\,0} - \vec{\mu}}{r^3}, \tag{4.10}$$

The force lines of the magnetic field are shown in Fig. 3.2. Along the axis of the dipole the field strength is $H_{sII} = \frac{2\mu}{r^3}$, where μ is the absolute value of the magnetic moment of the particle. In the direction, perpendicular to the dipole $H_{s\perp} = \frac{\mu}{r^3}$.

In our case, the task is analogous with the motion of the electron around the proton in a hydrogen atom. In this case, (Davydov, 1965) the atomic nucleus can be considered as a point magnetic dipole with moment $\vec{\mu}$. This dipole creates the potentials:

$$\varphi = 0,\ \vec{A} = \frac{\left[\vec{\mu}\times\vec{r}\right]}{4\pi\, r^3} = \left[\vec{\nabla}\times\frac{\vec{\mu}}{4\pi\, r}\right], \tag{4.11}$$

which correspond to the magnetic field:

$$\vec{H} = rot\vec{A} = \left[\vec{\nabla}\times\vec{A}\right] = \vec{\nabla}\left(\vec{\nabla}\frac{\vec{\mu}}{4\pi\, r}\right) - \vec{\nabla}^2\left(\frac{\vec{\mu}}{4\pi\, r}\right), \tag{4.12}$$

Operator

$$W = -\frac{e\hbar}{2mc}\vec{\sigma}\vec{H} = -\frac{e\hbar}{2mc}\sigma_z H, \tag{4.13}$$

characterizes the interaction of the magnetic moment of the electron with the magnetic field of the nucleus (here $\vec{\sigma}$ is particle spin).

The expressions for the torque $\vec{M}$ exerted by means of the magnetic field on the magnetic dipole, and the potential energy U of a constant magnetic dipole in a magnetic field, are similar to the corresponding formulas for the electric dipole interaction with the electric field:

$$\vec{M} = \vec{\mu}\times\vec{H},\quad U = -\vec{\mu}\cdot\vec{H}, \tag{4.14}$$

It is important to note that the relationship between q and m EMGT is not the same as in classical electrodynamics. In electrodynamics, these are independent quantities. In EMGT there is a close relationship between them due to the electromagnetic origin of matter and interactions, which means that any massive body consists of concentrated electromagnetic field and charges.

Within the framework of the nonlinear theory of elementary particles (NTEP) the concentrated EM field is a combination of different types of the self-interacting electromagnetic waves, which are elementary particles. The mass of these particles is equal to the energy of the EM waves, divided by the square of the speed of light. Since the mass m is determined via the EM field, it appears that mass may be defined through gravitational charge q_g ($m = q_g/\sqrt{\gamma_N}$) so that the dimension of q_g coincides with the dimension of electric charge q. This does not mean that q_g is q. The g-charge q_g is much smaller than the electric charge q and its action is manifested only when the bulk of the charge q is neutralized, as in the case for neutral bodies.

Hypothesis of Mossotti – Lorentz suggests (chapter 3) that for the passage to the gravitational field equations it is sufficient to move from electrical quantities to gravity quantities according to certain rules that allow the transition from the electromagnetic field to the residual electromagnetic field of the body. These rules we will try to elucidate below.

5.0. Transition from EM field theory to gravitational field theory

In the works of Lorentz (see, e.g., (Lorentz, 1900)) it was shown in sufficient detail that in the theory of electromagnetic fields the residual electromagnetic field can actually arise. But for the specific purpose of its introduction it is easier and more convenient to use the methods of similarity theory and dimensional analysis (Sedov, 1993).

We will compare the expressions of EM theory with the parallel expressions of gravitational theory and select the correspondences between them. For the control of the conclusions we use dimensional analysis.

The main characteristic of the source field in the one and in the other theory is the expression of the interaction force or the corresponding interaction energy between the two bodies.

5.1. Gravity electrostatic (ge-) field. The transition from the Coulomb's field to the Newton's field

If we assume that gravity is generated by electric field, but quantitatively, by very small part of it (see Appendix A1), then Newton's gravitation law:

$$\vec{F}_N = \gamma_N \frac{m \cdot M}{r^2} \vec{r}^{\,0}, \tag{5.1}$$

should take the form of Coulomb's law:

$$\vec{F}_C = k_0 \frac{q \cdot Q}{r^2} \vec{r}^{\,0}, \tag{5.2}$$

where m and q are the mass and electric charge of the particle, M and Q are the mass and electric charge of the source, γ_N is Newton's gravitational constant, and the coefficient k_0 in Gauss's units is $k_0 = 1$. In this case, the definitions of gravitational field strengths of Newton and Coulomb electric field have the form $\vec{E}_N = \dfrac{\vec{F}_N}{m} = \gamma_N \dfrac{M}{r^2} \vec{r}^{\,0}$ and $\vec{E} = \dfrac{\vec{F}_C}{q} = k_0 \dfrac{Q}{r^2} \vec{r}^{\,0}$, respectively.

We introduce the gravitational charge q_g, corresponding to mass m (Ivanenko and Sokolov, 1949) , by means of the relation:

$$q \to q_g = \sqrt{\gamma_N}\, m, \tag{5.3}$$

In this case, Newton's law can be rewritten in the form of Coulomb's law:

$$\vec{F}_g = \frac{q_g \cdot Q_g}{r^2} \vec{r}^{\,0} = \gamma_N \frac{m \cdot M}{r^2} \vec{r}^{\,0} = \vec{F}_N, \tag{5.4}$$

where $Q_g = \sqrt{\gamma_N}\, M$ is the gravitational charge of source, corresponding to the mass M of the source.

From the comparison of equations (5.2) and (5.4) it follows that the dimensions of the electromagnetic and gravitational charges coincide. At the same time, a gravitational charge (5.3) has electromagnetic origin, and, hence, the corresponding mass is the inertial mass. On the other hand, the law (5.4) comprises the gravitational masses. This implies the equivalence of inertial and gravitational masses.

We introduce the g-field strength within framework of EMGT as:

$$\vec{E} \to \frac{\vec{E}_g}{\sqrt{\gamma_N}}, \tag{5.5}$$

where the tension of the Coulomb field is equal to: $\vec{E} = \frac{Q}{r^2}\vec{r}^{\,0}$. Substituting the values of gravitation theory here, we get:

$$\vec{E}_g = \sqrt{\gamma_N}\frac{Q_g}{r^2}\vec{r}^{\,0} = \gamma_N \frac{M}{r^2}\vec{r}^{\,0} = \vec{E}_N, \tag{5.6}$$

where $\vec{E}_N$ is the strength of the Newton gravitational field.

Let us introduce the scalar gravitational potential within the framework of EMTG as:

$$\varphi \to \frac{\varphi_g}{\sqrt{\gamma_N}}, \tag{5.7}$$

where the potential of the Coulomb field is: $\varphi = \frac{Q}{r}$. Substituting the values of gravitation theory here, we get:

$$\varphi_g = \sqrt{\gamma_N}\frac{Q_g}{r} = \gamma_N \frac{M}{r} = \varphi_N, \tag{5.8}$$

where φ_N is the potential of the Newton gravitational field.

The Poisson equation for the g-field can serve as test for (5.7). Indeed, for the EM field the Poisson equation can be written as:

$$\Delta\varphi = 4\pi\,\rho_e, \tag{5.9}$$

where $\rho_e = \frac{dq}{d\tau}$ is the electric charge density, $d\tau$ is the volume element. We introduce the density of gravitational charge ρ_g similarly to the electric density:

$$\rho_e \to \rho_g = \frac{dq_g}{d\tau} = \sqrt{\gamma_N}\rho_m, \tag{5.10}$$

where $\frac{dm}{d\tau} = \rho_m$ is mass density. Then, replacing the potential and the charge density in (5.9) according to (5.7) and (5.10), we obtain the Poisson equation for the gravitational potential:

$$\Delta\varphi_g = 4\pi\sqrt{\gamma_N}\rho_g = 4\pi\,\gamma_N\rho_m, \tag{5.11}$$

which corresponds to the Poisson equation for the Newton gravitational field.

5.2. Gravi-magnetic field (gm-field)

In this case, by analogy with electrodynamics, the existence of the variable ge-field and associated with them alternating or direct gm-fields is assumed. The existence of a similar field is confirmed by general relativity and experiments. Unfortunately, since the Newton theory does not contain an analog of magnetic field, the verification of existence of the g-magnetic field within framework of EMGT, can presently be done only by dimensional analysis. Serious confirmation should be obtained by the solution of the corresponding equations of gravitation, which will give equivalent results to the general theory of relativity.

As is known, the magnetic field is generated by the motion of electric charges or movement of an electric field. In this case, we need to obtain an expression for the magnetic field, similar to Coulomb's law for the electric field. This is the Biot-Savart–Laplace law.

For simplicity, we will consider the special case of uniform motion of a source charge Q, which create a current I (current from motion of charge q will be denoted by i). In real tasks, of course, charges and masses are divided into point (differential) values, and field calculated by integrating over a set of point charges.

Magnetic vector $\vec{H}$ that occurs when the charge Q moves at a speed $\vec{\upsilon}$ in circuit element dl, will be:

$$\vec{H} = \frac{Q[\vec{\upsilon} \times \vec{r}]}{|r|^3} = \frac{I\left[d\vec{l} \times \vec{r}\right]}{|r|^3}, \tag{5.12}$$

Using the gravitational charge density ρ_g according to (5.10), similarly to the electric current $i = dq/dt$ and the current density $\vec{j}$, we will define respective g-current (or current of mass) as:

$$i \to i_g = \frac{dq_g}{dt} = \rho_g \upsilon_n dS = \sqrt{\gamma_N}\, \rho_m \upsilon_n dS, \tag{5.13}$$

and the density of g-current of mass, as:

$$\vec{j} \to \vec{j}_g = \left(i_g/dS\right)\vec{n} = \sqrt{\gamma_N}\, \rho_m \upsilon_n, \tag{5.14}$$

where $\vec{\upsilon}$ is the velocity of the charge in a conductor with a cross section dS, and υ_n the projection of the velocity on the normal to dS.

If the e-charge q moves close to the e-current (or permanent magnet field $\vec{H}$), this current (or field $\vec{H}$) acts on the charge via the magnetic part of the Lorentz force F_{Lm}:

$$\vec{F}_{Lm} = q\left[\vec{\upsilon} \times \vec{H}\right] = \frac{q \cdot Q}{|r|^3}\left[\vec{\upsilon} \times (\vec{\upsilon}' \times \vec{r})\right] = \frac{i \cdot I}{|r|^3}\left[d\vec{l} \times \left(d\vec{l}' \times \vec{r}\right)\right], \tag{5.15}$$

Let us introduce the strength of gm-field within framework of EMGT as:

$$\vec{H} \to \frac{\vec{H}_g}{\sqrt{\gamma_N}}, \tag{5.16}$$

where the magnetic field $\vec{H}$ is given by (5.12).

Substituting the corresponding physical quantities according to (5.13) and (5.16) in (5.12), we will obtain the gravi-magnetic (gm-) vector that arises when the charge $Q_g = M\sqrt{\gamma}$ moves at a speed $\vec{\upsilon}$ in an element of a circuit dl:

$$\vec{H}_g = \sqrt{\gamma_N}\, \frac{Q_g\left[\vec{\upsilon} \times \vec{r}\right]}{|r|^3} = \gamma_N \frac{M\left[d\vec{l} \times \vec{r}\right]}{|r|^3}, \tag{5.17}$$

or

$$\vec{H}_g = \sqrt{\gamma_N}\, \frac{I_g\left[d\vec{l} \times \vec{r}\right]}{|r|^3} = \gamma_N \frac{\left[d\vec{l} \times \vec{r}\right]}{|r|^3} \rho_m \upsilon_n dS, \tag{5.18}$$

where $\vec{r}$ is the distance between the test particle and the moving charged source or element of current I_g, which generate the gm-vector.

Using (5.15), for the gravi-magnetic Lorentz force we obtain:

$$\vec{F}_{Lgm} = \frac{q_g Q_g}{|r|^3}\left[\vec{\upsilon}\times\left(\vec{\upsilon}'\times\vec{r}\right)\right] = \gamma_N \frac{mM}{|r|^3}\left[\vec{\upsilon}\times\left(\vec{\upsilon}'\times\vec{r}\right)\right], \qquad (5.19)$$

Since the magnetic field $\vec{H}$ in the electrodynamics can be expressed via a vector potential $\vec{A}$ by the expression:

$$\vec{H} = rot\vec{A}, \qquad (5.20)$$

it is useful to define the transition from the EM vector potential $\vec{A}$ to the gravitational $\vec{A}_g$. We assume that:

$$\vec{A} \to \frac{\vec{A}_g}{\sqrt{\gamma_N}}, \qquad (5.21)$$

Then, using (5.16), we can rewrite (5.20) in the form:

$$\vec{H}_g = rot\vec{A}_g, \qquad (5.22)$$

Expression (5.21) also satisfies the full EM expression for the electric strength vector:

$$\vec{E} = -grad\varphi - \frac{1}{c}\frac{\partial\vec{A}}{\partial t}, \qquad (5.21)$$

Using (5.5) , (5.7) and (5.21) , we obtain for g-field:

$$\vec{E}_g = -grad\varphi_g - \frac{1}{c}\frac{\partial\vec{A}_g}{\partial t}, \qquad (5.21')$$

Thus, we have shown that the basic EM quantities and equations can be associated with similar quantities and equations for the g-field.

Recall that in framework of EMGT the Hamilton-Jacobi equation serves as the Lorentz-invariant law of motion. In the external source field, characterized by energy ε_{ex} and momentum $\vec{p}_{ex}$, this equation has the form:

$$\frac{1}{c^2}\left(\frac{\partial S}{\partial t} + \varepsilon_{ex}\right)^2 - \left(\frac{\partial S}{\partial x} - p_{x\,ex}\right)^2 - \left(\frac{\partial S}{\partial y} - p_{y\,ex}\right)^2 - \left(\frac{\partial S}{\partial z} - p_{z\,ex}\right)^2 = m^2c^2, \qquad (5.4)$$

Then, the Hamilton-Jacobi equation of motion of a particle in an EM field of the source:

$$\frac{1}{c^2}\left(\frac{\partial S}{\partial t} + q\varphi\right)^2 - \left(\frac{\partial S}{\partial x} - \frac{q}{c}A_x\right)^2 - \left(\frac{\partial S}{\partial y} - \frac{q}{c}A_y\right)^2 - \left(\frac{\partial S}{\partial z} - \frac{q}{c}A_z\right)^2 = m^2c^2, \qquad (5.22)$$

can be rewritten for the g-field in the same form:

$$\frac{1}{c^2}\left(\frac{\partial S}{\partial t} + m\varphi_g\right)^2 - \left(\frac{\partial S}{\partial x} - \frac{m}{c}A_{xg}\right)^2 - \left(\frac{\partial S}{\partial y} - \frac{m}{c}A_{yg}\right)^2 - \left(\frac{\partial S}{\partial z} - \frac{m}{c}A_{zg}\right)^2 = m^2c^2, \qquad (5.23)$$

As an illustration of the correctness of the relationship of electromagnetic and gravitoelectromagnetic quantities, we put in Appendix A2. to this article the table of dimensions of physical quantities, used in the article.

Appendixes:

A1. Relationship between electric and gravitational charges

It is easy to show that the gravitational field is a small fraction of the electromagnetic field.

To this corresponds the fact that the gravitational charge of the electron is less than its electric charge $e >> q_g$, where $q_g = m_e\sqrt{\gamma}$ (here $e = 4{,}8\cdot10^{-10}$ unit. SGSEq is electron charge (1 unit CGSEq = $g^{1/2}sm^{3/2}s^{-1}$), $m_e = 0{,}91\cdot10^{-27}$ g is electron mass, $\gamma = 6{,}67\cdot10^{-8}$ cm^3/g sec^2 is the gravitational constant. It is easy to see that the dimension of the gravitational charge of the electron coincides with the dimension of electric charge and its magnitude in 10^{21} times less. Indeed, $e/m_e\sqrt{\gamma} \approx 2\cdot10^{21}$.

For a proton (the only stable heavy particle), this value is of the order $e/m_p\sqrt{\gamma} \approx 2\cdot10^{18}$. The heaviest known elementary particles are the highly unstable bosons $W^{\pm}$ (mass ≈80 GeV). This is about 100 times more than the mass of the proton, giving a ratio of no less than 10^{16}.

A2. Dimensions of electromagnetic and gravi-electromagnetic quantities

For the verification of the correctness of correlations in the transition from the EM physical quantities to the gravitation quantities, the accordance of their dimensions plays an important role. The worded below (far from exhaustive) list confirms that electrodynamics can be considered as the basis of mechanics.

Electromagnetic theory

e-charge	$[q] = g^{1/2}\,cm^{3/2}\,s^{-1}$
e-charge density	$[\rho_e] = g^{1/2}\,cm^{-3/2}\,s^{-1}$
e-current	$[\,i\,] = g^{1/2}\,cm^{3/2}\,s^{-2}$
e-current density	$[\,j\,] = g^{1/2}\,cm^{-1/2}\,s^{-2}$
Coulomb force	$[F_C] = g\ cm\ s^{-2}$
Strength of e-fields	$[E] = g^{1/2}\,cm^{-1/2}\,s^{-1}$
Strength of m-field	$[H] = g^{1/2}\,cm^{-1/2}\,s^{-1}$
Scalar potential	$[\varphi] = g^{1/2}\,cm^{1/2}\,s^{-1}$
Vector potential	$[A] = g^{1/2}\,cm^{1/2}\,s^{-1}$
Field energy	$[\varepsilon_e] = g\ cm^2\,s^{-2} = [q\varphi]$
Field energy density	$[\rho_\varepsilon] = g\ cm^{-1}\,s^{-2} = [E^2] = [H^2]$

Gravitation theory of Newton

Newton's force	$[F_N] = g\ cm\ s^{-2}$
Newton's gravitational constant	$[\gamma_N] = g^{-1}\,cm^3\,s^{-2}$ $([\sqrt{\gamma_N}] = g^{-1/2}\,cm^{3/2}\,s^{-1})$
Field strength	$[E_N]$ = cm/s^2 (acceleration)
Scalar potential	$[\varphi_N]$ = cm^2/s^2 ≡ $(cm/s)^2$ (velocity square)
Scalar potential	$[\varepsilon_N] = [m\varphi_N]$ = g cm^2/s^2

Electromagnetic gravitation theory (EMGT)

g-charge	$[q_g] = g^{1/2}cm^{3/2}s^{-1} = [q_e] = [m\sqrt{\gamma_N}]$
g-charge density	$[\rho_g] = g^{1/2}\,cm^{-3/2}\,s^{-1}$
g-current	$[i_g] = g^{1/2}\,cm^{3/2}\,s^{-2}$

g-current density	$[\, j_g \,] = \text{g}^{1/2}\,\text{cm}^{-1/2}\,\text{s}^{-2}$
g-force	$[\, F_g \,] = \text{g cm s}^{-2} = [\, F_N \,]$
Strength of ge-fields	$[\, E_g \,] = \text{cm/s}^2 = [\, E_N \,] = [\, E \sqrt{\gamma_N} \,]$
Strength of gm-field	$[\, H_g \,] = \text{cm/s}^2 = [\, H \sqrt{\gamma_N} \,]$
Scalar potential of g-fields	$[\varphi_g] = \text{cm}^2/\text{s}^2 = [\, \varphi \sqrt{\gamma_N} \,]$
Vector potential of g-fields	$[\, A_g \,] = \text{cm}^2/\text{s}^2 = [\, A \sqrt{\gamma_N} \,]$
g-field energy	$[\, \varepsilon_g \,] = \text{g cm}^2/\text{s}^2 = [m \varphi_g]$

Chapter 6. Electromagneticly-like solutions of the GR

1.0. Introduction. Stating the problem

As it is known, all testable predictions of general relativity (GR) in respect of test particle movement in the field of moving sourse made in the framework linearized GR, which in the form of electromagnetic (EM) theory is called gravitoelectromagnetic theory (GEM).

In the present chapter, the proposed by us EM nonlinear theory of gravity (called conditionally "Lorentz-invariant theory of gravitation" - LIGT), is compared with GEM.

The bases are pointed that the existence of quantum theory of gravitation is possible, but practically is useless with such mass-energy, at which gravity can be measured. In addition, the claim is substantiated that under these conditions the quantum equation coincides with the classical equation.

Note that, in general case, under "source motion "we mean both the translational and rotational motion of the source.

Within the framework of general relativity (GR) the gravitational field around a massive body (the source) is described as the curvature of space-time, which is characterized by the metric tensor.

In framework of LIGT we consider this field as some kind of matter, which is described by the same characteristics as any matter: energy, momentum, angular momentum, etc.

Driving the motion of source, in general relativity the metric tensor changes, while in LIGT a deformation of field takes place. Both the metric tensor and the field in this case acquire additional features. Due to the fact that the field has a finite speed of propagation, the transmission of motion from the source to the field is delayed and is not set by the same values that are valid in Newtonian gravitational theory.

The source may have translational or rotational motion, as well as these two cases combined. As is shown by R. Forward (see later) it is difficult to experimentally examine the linear field motion of the source. But it was shown that the effect of rotation is experimentally verifiable. Therefore, in this article we focus on this occasion.

In a previous chapters we pointed out that when the gravitational field moves, as a gravitoelectric field of source, it causes the gravitomagnetic field, which will additionally influence the motion of a massive particle in the field of a rotating source.

We also noted there that the linear approximation of general relativity allows the calculation of this influence, which leads to expressions, which almost exactly coincide with the expression of EM theory. On this basis, this approach is in general relativity called gravitolectromagnetism (GEM).

It should be noted as before, that GEM should not be confused with electromagnetic theory of gravity, which is not connected with GR but arised much earlier; see (Mossotti, 1836; Zollner, 1982; Heaviside, 1894; Lorentz, 1900, etc). To the use of the GEM approximation of general relativity in the calculation of the amendments to the motion the moving source, are dedicated a lot of articles lately (Moeller, 1952; Sciama, 1953; Davidson, 1957; etc., see further). Therefore, we will very briefly indicate the basis of this approach, but for details we refer the reader to the relevant articles.

Further, as a basis for comparison with LIGT, we present a summary of the formulation and solution within the framework of the GEM, of the problems of particle motion in the gravitational field of a moving source in various cases (see paragraphs 2.0 to 4.0 inclusively)

(Note that we will continue to consider the terms "relativistic" and "Lorentz-invariant" as equivalent).

2.0. From GR to GEM

2.1. From Einstein field equations to Gravitoelectromagnetism

2.1.1. *The Linear Field Approximation*

Einstein's theory of general relativity (Ruggiero and Tartaglia, 2002; Grøn and Hervik, 2007) leads to Newtonian gravity in the linear limit when the gravitational field is weak and static and the particles in the gravitational field moves slowly compared to the velocity of light. But in the linear field approximation the field need not be static, and particles are allowed to move with relativistic velocities.

For example (Ruggiero and Tartaglia, 2002), we can use this kind of linear approximation, if the source is rotating and its rotation is not relativistic.In this case Einstein's equations may be written in a very simple way, which leads straight to the analogy with Maxwell's equations.

Even if we are more interested in the rotation effects, that is the gravitomagnetic effects, in the general case we need to talk about gravitoelectromagnetic fields, which include also the gravitoelectric, or newtonian, part.

Note, that with the partial exception of the binary pulsar PSR 1913+16, the weak field is the normal condition for all the tests of General Relativity up to this moment.

(We will follow the standard treatment and use the standard notations: Latin indices run from 1 to 3, while Greek indices run from 0 to 3; the flat space time metric tensor is $\eta_{\mu\nu}$ = diag(1,−1,−1,−1), in Cartesian coordinates $x^0 = ct$, $x^1 = x$, $x^2 = y$, $x^3 = z$, and, as usual, summation over repeated indexes is assumed).

So, let us start from the full non linear equations.

$$G_{\mu\nu} = 8\pi \frac{\gamma_N}{c^4} T_{\mu\nu}, \tag{2.1}$$

where γ_N - Newton's constant of gravitation

We put

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \tag{2.2}$$

where $\eta_{\mu\nu}$ is the Minkowski metric tensor, and $|h_{\mu\nu}| << 1$ is a "small" deviation from it. Then we define:

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h; \;\; h = h^{\alpha}_{\alpha} \;, \tag{2.3}$$

Thus if we expand the field equations (2.1) in powers of $\bar{h}_{\mu\nu}$ keeping only the linear terms, we obtain:

$$\Box \bar{h}_{\mu\nu} = 16\pi \frac{\gamma_N}{c^4} T_{\mu\nu}, \tag{2.4}$$

where we have also imposed the so called Lorentz gauge condition $h^{\mu\alpha}_{,\alpha} = 0$.

Equations (2.4) constitute the "linearized»Einstein's field equations. The analogy with the corresponding Maxwell equations

$$\Box A^{\nu} = 4\pi \, j^{\nu}, \tag{2.5}$$

is evident.

The weak field approximation (Grøn and Hervik, 2007) of Einstein's equations is valid to great accuracy in, for example, the Solar system. The resemblance between the electromagnetic wave equation, eq. (2.5), and eq. (2.4) is evident. The similarity between electromagnetism and the linearised Einstein equations goes even further.

The solution of (2.4) may be written exactly in terms of retarded potentials:

$$\bar{h}_{\mu\nu} = -4\frac{\gamma_N}{c^4}\int \frac{T_{\mu\nu}\left(t-\left|\vec{x}-x'\right|/c, \vec{x}'\right)}{\left|\vec{x}-x'\right|} d^3x', \tag{2.6}$$

The role of the electromagnetic vector potential A^ν is played here by the tensor potential $\bar{h}_{\mu\nu}$, while the role of the four-current j^ν is played by the stress-energy tensor $T^{\mu\nu}$.

We look for solutions such that $\left|\bar{h}_{00}\right| >> \left|\bar{h}_{ij}\right|$, $\left|\bar{h}_{0i}\right| >> \left|\bar{h}_{ij}\right|$, and neglect the other terms, which are smaller.

The explicit expression for the tensor potential $\bar{h}_{\mu\nu}$ is then:

$$\bar{h}^{00} = \frac{4\varphi_g}{c^2} , \tag{2.7}$$

$$\bar{h}^{0l} = -2\frac{A_g{}^l}{c^2} , \tag{2.8}$$

where φ_g is the Newtonian or ”gravitoelectric” potential

$$\varphi_g = -\frac{\gamma_N M_S}{r} , \tag{2.9}$$

while $\vec{A}_g$ is the ”gravitomagnetic” vector potential in terms of the total angular momentum of the system $\vec{L}$:

$$A_g{}^l = \frac{\gamma_N}{c}\frac{L^n x^k}{r^3}\varepsilon^l_{nk} , \tag{2.10}$$

(here ε^l_{nk} is the three-dimensional completely antisymmetric tensor of Levi-Civita).

It follows that $T^{00}/c^2 = \rho_m$ is the ”mass-charge” density. Hence the total mass M_S of the system is:

$$\int \rho_m \cdot d^3x = M_S , \tag{2.11}$$

while $T^{i0}/c = j_m{}^i$ represents the mass-current density, and the total angular momentum of the system is:

$$L^i = 2\int \varepsilon^i_{jk} x'^j \frac{T^{k0}}{c} d^3x' , \tag{2.12}$$

In terms of the potentials $\varphi_g, \vec{A}_g$ the Lorentz gauge condition becomes:

$$\frac{1}{c}\frac{\partial \varphi_g}{\partial t} + \frac{1}{2}\vec{\nabla}\cdot\vec{A}_g = 0 , \tag{2.13}$$

which, apart from a factor ½ , is the Lorentz condition of electromagnetism.

In this equation there is a factor ½ which does not appear in standard electrodynamics: *the effective gravitomagnetic charge is twice the gravitoelectric one*. It is supposed that this is a consequence of the fact that the linear approximation of GR involves a spin-2 field, while ”classical” electrodynamics involves a spin-1 field.

It is then straightforward to define the gravitoelectric and gravitomagnetic fields:

$$\vec{E}_g = -\vec{\nabla}\varphi_g - \frac{1}{2c}\frac{\partial \vec{A}_g}{\partial t} , \tag{2.14}$$

$$\vec{B}_g = \vec{\nabla}\times\vec{A}_g , \tag{2.15}$$

Using equations (2.4), (2.13), (2.14), (2.15), and the definitions of mass density and current, we finally get the complete set of Maxwell's equations for the so called gravitoelectromagnetic (GEM) fields:

$$div\vec{E}_g = 4\pi\gamma_N \rho_m , \tag{2.16}$$

$$div\vec{B}_g = 0, \tag{2.17}$$

$$rot\vec{E}_g + \frac{1}{c}\frac{\partial \vec{B}_g}{\partial t} = 0 , \tag{2.18}$$

$$rot\vec{B}_g - \frac{1}{c}\frac{\partial \vec{E}_g}{\partial t} = -\frac{4\pi\gamma_N}{c}\vec{j}_m , \tag{2.19}$$

Einstein's field equations in this form correspond to a solution that describes the field around a rotating object in terms of gravitoelectric and gravitomagnetic potentials. The metric tensor can be read from the corresponding space-time invariant:

$$ds^2 = \left(1 + \frac{2\varphi}{c^2}\right)c^2 dt^2 + 4\left(d\vec{r} \cdot \frac{\vec{A}}{c}\right)dt - \left(1 - \frac{2\varphi}{c^2}\right)\delta_{ij} dx^i dx^j , \tag{2.20}$$

Hence the gravitational field is understood in analogy with electromagnetism. For instance, the gravitomagnetic field of the Earth as well as of any other weakly gravitating and rotating spherical mass may be written as a dipolar field:

$$\vec{B}_g = -4\frac{\gamma_N}{c}\frac{3\vec{r}\left(\vec{r} \cdot \vec{L}_E\right) - \vec{L}_E r^2}{2r^5} , \tag{2.21}$$

where $\vec{L}_E$ is the angular momentum of the Earth.

2.2. The wave equation

As we noted above, the relationship (Wald, 1984, section 4.4):

$\partial^\nu \bar{h}_{\mu\nu} = 0$, (2.22)

is the analog of the Lorentz gauge condition. In this gauge, the linearized Einstein equation simplifies to become

$\partial^\alpha \partial_\alpha \bar{h}_{\mu\nu} = -16\pi k T_{\mu\nu}$, (2.4)

and is closely analogous to Maxwell's equation (2.5).

In vacuum ($T_{\mu\nu} = 0$) equations (2.22) and (2.4) are precisely the equations written down by Fierz and Pauli (Fierz and Pauli; 1939) to describe a massless spin-2 field propagating in flat spacetime. Thus, in the linear approximation, general relativity reduces to the theory of a massless spin-2 field. The full theory of general relativity thus may be viewed as that of a massless spin-2 field which undergoes a nonlinear self-interaction

It should be noted, however, that the notion of the mass and spin of a field require the presence of a flat background metric $h_{\mu\nu}$ which one has in the linear approximation but not in the full theory, so the statement that, in general relativity, gravity is treated as a massless spin-2 field is not one that can be given precise meaning outside the context of the linear approximation.

In EM equation (2.5) the constant c is the speed of light in vacuum while in GEM equation (2.4), corresponding constant (let's denote it by c) is the speed of gravitational wave in vacuum. We can not a priory assert that these speeds are equal to each other.

In electromagnetic theory this constant c (according to Maxwell) is equal to $c = 1\big/\sqrt{\mu_0 \varepsilon_0}$ in SI system units and is the speed of light (here ε_0 is permittivity or electric constant and μ_0 permeability or magnetic constant of vacuum).

The electromagnetic field may be a potential (electrostatic) field and/or vortex (magnetic) field. The Newtonian gravitational field is only potential (gravitostatic) field. Obviously, the gravitomagnetic field is a vortex field.

The electrostatic field is described by Coulomb's law and is characterized by a electric constant ε_0. The Newtonian gravitational field is characterized by the corresponding constant γ_N. R. Forward (Forward, 1961) termed this constant for GEM theory "capacitivity of space" and suggested that it is equal to: $\varepsilon_0 \to \varepsilon_g = 1/4\pi\gamma_N = 1{,}19 \cdot 10^9 \left(kg \cdot s^2/m^3\right)$

The magnetic field is always a vortex field and is characterized by the magnetic constant μ_0, which is contained in the Biot - Savart and Ampere laws . Obviously, we have to assume that in GEM there is a gravitomagnetic constant. R. Forward termed this quantity «permeability of space" and suggested that it is equal to: $\mu_0 \to \mu_g = 16\pi\gamma_N/c^2 = 3{,}73 \cdot 10^{-26}(m/kg)$

Using the relationship $c = 1/\sqrt{\mu_0\varepsilon_0}$, R. Forward obtained the speed of gravitational wave $c_g = 1/\sqrt{\gamma_0\eta_0} = c/2$

However R. Forward noted that "we are reasonable sure , however, that the velocity of propagation of gravitational energy will be the same as the speed of light since the value obtained by Einstein for the rotation of the perihelion of Mercury depends upon this value".

On the other hand, F. Forrester (Forrester, 2010) adopted a complete analogy to the electromagnetic theory and received for value $\varepsilon_g = 1/4\pi\gamma_N$ that gives $c_g = c$. In this regard, he noted that: "we may have to do some constant shuffling to get a more ideal and analogous combination. It may be that the SI definition of $\vec{B}$ or $\vec{A}$ is not ideal since we have $\vec{E} = -\vec{\nabla}\varphi - \partial_t\vec{A}$ rather than $\vec{E} = -\vec{\nabla}\varphi - (1/c)\partial_t\vec{A}$".

2.3. The equation of motion

To complete the picture (Ruggiero and Tartaglia, 2002), a further analogy can be mentioned. In fact, using the present formalism, the geodesic equation

$$\frac{d^2x^\alpha}{ds^2} + \Gamma^\alpha_{\mu\beta}\frac{dx^\mu}{ds}\frac{dx^\beta}{ds} = 0, \tag{2.23}$$

for a particle in the field of a weakly gravitating, rotating object, can be cast in the form of an equation of motion under the action of a Lorentz force.

The linearized Einstein equation (Wald, 1984, section 4.4) predicts that the space-space components of $\bar{h}_{\mu\nu}$, satisfy the source free wave equation, but the space-time and time-time components now satisfy

$$\partial^\mu\partial_\mu\bar{h}_{0\nu} = 16\pi j_\nu \tag{2.4'}$$

where j_ν is the mass-energy current density 4-vector; the tensor $T_{\mu\nu}$ has only a 'time-time' component and the neglect of the 'time-space' components is essentially the statement that velocities (and thus, momentum densities) are small while the neglect of the 'space-space' components is the statement that the stresses are small.

The potentials (2.7)-(2.8) satisfy precisely Maxwell's equations in the Lorentz gauge with source j_μ. If again we assume that the time derivatives of $\bar{h}_{\mu\nu}$ are negligible, then the space-space components of $\bar{h}_{\mu\nu}$ vanish, and we find that to linear order in the velocity of the test body, the geodesic equation now yields:

$$\vec{a} = -\vec{E}_g - 4\frac{\vec{\upsilon}}{c} \times \vec{B}_g , \tag{2.24}$$

where $\vec{a} = d^2\vec{x}/dt^2 = d\vec{\upsilon}/dt$ is the acceleration of the body relative to global inertial coordinates. $\vec{E}_g$ and $\vec{B}_g$ are defined in terms of A_μ by the same formulas as in electromagnetism. This is identical to the Lorentz force equation of electromagnetism (with $q \to m$) except for an overall minus sign and a factor of 4 in the "magnetic force" term. Thus, linearized gravity predicts that the motion of masses produces magnetic gravitational effects very similar to those of electromagnetism.

This is the basic background in which all tests of GEM take place.

2.4. The momentum flow vector

Poynting vector is a quantity describing the magnitude and direction of the flow of energy in electromagnetic field. In CGS unit system, the Poynting vector is defined as

$$\vec{S}_P = \frac{c}{4\pi}\vec{E}\times\vec{B}, \tag{2.25}$$

where c is the speed of light, $\vec{B}$ is the magnetic field, and $\vec{E}$ is the electric field.

(Ruggiero and Tartaglia, 2002) Still in 1893 Oliver Heaviside investigated the analogy between gravitation and electromagnetism; in particular, he explained the propagation of energy in a gravitational field, in terms of a gravitoelectromagnetic Poynting vector.

In particular (Mashhoon, 2008), the GEM Poynting vector is given by

$$\vec{S}_{Pg} = \frac{c}{2\pi\gamma_N}\vec{E}_g\times\vec{B}_g, \tag{2.26}$$

For instance, gravitational energy circulates around a stationary source of mass m and angular momentum $\vec{L} = L\vec{z}^0$ with a flow velocity

$$\vec{\upsilon}_g = k\frac{L}{M_S r}\sin\theta\vec{\varphi}^0 , \tag{2.27}$$

in the same sense as the rotation of the mass. Here we employed spherical polar coordinates and k = 4/7. The flow given by (2.27) is divergence-free and the corresponding circulation is independent of the radial distance r and is given by $2\pi k(L/M_s)\sin^2\theta$.

2.5. The conservation equation

General relativity, which is a field theory of gravitation, contains a gravitomagnetic field due to mass current (Mashhoon, 2008).

The GEM field equations (2.14)-(2.19) contain, obviousy, the continuity equation:

$$\vec{\nabla}\cdot\vec{j}_m + \partial\rho_m/\partial t = 0, \tag{2.28}$$

This is the basic background in which all tests of GEM take place.

It is useful to exploit the analogy with electromagnetism, because it simplifies the solutions of some problems.

3.0. The GEM solutions for the rotating sourses

3.1 The Lense-Thirring solution

As a special case of a stationary gravitational field (Landau and Lifshitz, 1975, p. 254), let us consider a uniformly rotating reference system. To calculate the interval ds we carry out the transformation from a system at rest (inertial system) to the uniformly rotating one. In the coordinates r,φ,z,t of the system at rest (we use cylindrical coordinates r,φ,z, the interval has the form

$$ds^2 = c^2dt^2 - dz^2 - r^2d\varphi^2 - dr^2, \tag{3.1}$$

Let the cylindrical coordinates in the rotating system be r', φ', z'. If the axis of rotation coincides with the axes Z and Z', then we have $r = r'$, $\varphi = \varphi' + \omega t$, $z = z'$, where ω is the angular velocity of rotation. Substituting in (3.1), we find the required expression for ds^2 in the rotating system of reference:

$$ds^2 = c^2\left(1 - \omega^2 r^2/c^2\right)dt^2 + 2\omega r^2 d\varphi' dt - dz^2 - r^2 d\varphi'^2 - dr^2, \qquad (3.2)$$

It is necessary to note that the rotating system of reference can be used only out to distances equal to c/ω. In fact, from (3.2) we see that for $r > c/\omega$, g_{00} becomes negative, which is not admissible. The inapplicability of the rotating reference system at large distances is related to the fact that there the velocity would become greater than the velocity of light, and therefore such a system cannot be made up from real bodies.

The most famous (Ruggiero and Tartaglia, 2002) rotational or gravitomagnetic effect is the Lense and Thirring effect (Lense and Thirring, 1918).

The expected effect is small, but, using artificial satellites with appropriately chosen orbits, it could appear as a precession of the orbital plane around the rotation axis of the Earth.

In the framework of GR the metric tensor in the vicinity of a spinning mass in weak field approximation, may be read off from the space-time invariant

$$ds^2 = \left(1 - \frac{r_s}{r}\right)c^2 dt^2 - \frac{dr^2}{\left(1 - \frac{r_s}{r}\right)} - r^2\left(\sin^2\theta\, d\varphi^2 + d\theta^2\right) + \frac{4\gamma_N M_s a}{c^2 r}\sin^2\theta\, d\varphi dt, \qquad (3.3)$$

where $r_s = \dfrac{2\gamma_N M_s}{c^2}$ is gravitational radius, M_s is a mass of central body, $a = L/M_s$ represents the angular momentum of the source per unit mass; more precisely stated, it is the projection of the angular momentum three-vector on the direction of the rotation axis, divided by the mass.

To calculate the entity of the looked for precession rate, we have to solve the equations of motion of a test body in the gravitational field corresponding to the metric tensor (3.3): it is what Lense and Thirring did in their papers. It is however simpler and clearer to use the GEM equations.

The Lense-Thirring precession is analogous to the precession of the angular momentum of a charged particle, orbiting about a magnetic dipole; the orbital momentum of the particle divided by 2 and by c will then be the equivalent of the magnetic dipole moment of the particle (Here the particle is assumed not to be itself a gyroscope, i.e. it is spinless). So, if (2.21) is the gravitomagnetic field of the Earth, the torque on the angular momentum $\vec{s}$ of the orbiting particle is:

$$\vec{m} = \frac{\vec{s}}{2c} \times \vec{B}_g, \qquad (3.4)$$

The time derivative of the orbital angular momentum can then be written as:

$$\frac{d\vec{s}}{dt} = -\gamma_N \frac{\vec{s} \times \left(3\vec{r}\vec{r} \cdot \vec{L} - \vec{L}r^2\right)}{c^2 r^5}, \qquad (3.5)$$

From (3.5) we find the angular velocity of the precession ($d\vec{s}/dt = \vec{\Omega} \times \vec{s}$):

$$\vec{\Omega} = \gamma_N \frac{3\vec{r}\vec{r} \cdot \vec{L} - \vec{L}r^2}{c^2 r^5}, \qquad (3.6)$$

If we take the average of $\vec{\Omega}$ along the orbit, the effective angular velocity of precession is

$$\left\langle \vec{\Omega} \right\rangle = \gamma_N \frac{3\left\langle \vec{r}\vec{r} \cdot \vec{L} \right\rangle - \vec{L}r^2}{c^2 r^5}, \qquad (3.7)$$

For an orbit with $r \cong R_{Earth}$ the magnitude of the precession rate is about 0.05 arcsec per year.

3.2. The Kerr solution by means of electrodynamics analogy

In general, stars and planets rotate and the Schwarzschild metric does not describe the correct associated space-time. Instead one should consider the metric associated to a rotating black-hole: Kerr metric.

The gravitational field of the rotating black hole is given by the following axially symmetric stationary Kerr metric (Landau and Lifshitz, 1975) :

$$ds^2 = \left(1 - \frac{r_s r}{\rho^2}\right)dt^2 - \frac{\rho^2}{\Delta}dr^2 - \rho^2 d\theta^2 - \\ - \left(r^2 + a^2 + \frac{r_s r a^2}{\rho^2}\sin^2\theta\right)\sin^2\theta\, d\varphi^2 + \frac{2 r_s r a}{\rho^2}\sin^2\theta\, d\varphi dt \quad , \qquad (3.8)$$

where we have introduced the notation

$$\Delta = r^2 - r_s r + a^2\,,\ \ \rho^2 = r^2 + a^2\cos^2\theta\,,\ \ a = L/M_S\ , \qquad (3.9)$$

This solution of the Einstein equations was discovered by R. Kerr, (Kerr, 1963), in a different form, and reduced to (3.8) by R. H. Boyer and R. W. Lindquist, 1967. There is no constructive analytic derivation of the metric (3.8) in the literature that is adequate in its physical ideas, and even a direct check of this solution of the Einstein equations involves cumbersome calculations.

3.2.1. An electrodynamics analogy of the Kerr result

In connection with the purposes of our article, we note the article “Linearized Kerr and spinning massive bodies: An electrodynamics analogy” by authors (Franklin and Baker, 2007), which examine the correspondence between spinning of charged spherical sources in electrodynamics and spinning, massive spherical sources in linearized general relativity and show that the form of the potentials and equations of motion are similar in the two cases in the slow motion limit. This similarity allows us to interpret the Kerr metric in analogy with a spinning sphere in electrodynamics and aids in understanding linearized general relativity, where the “forces” are effective and come from the intrinsic curvature of space-time. In particular they show that the Kerr metric lends itself to a Newtonian potential plus a “gravito-magnetic” vector potential interpretation.

Authors also conclude that the linearized Kerr metric represents the space-time generated by a rotating, massive, spherically symmetric body. The two parameters that appear in the derivation of the full metric take on physical significance in the linearized, slow-moving test body regime they have studied.

By analogy with the EM theory, the authors also considered the case of the presence of the spin for test particles, and based on this correspondence, they showed that the analogous spin-orbit coupling should be observable in general relativity.

3.3. The Kerr solution limits

3.3.1. *Lense-Thirring metric*

For slowly rotating spherical mass, Kerr metric becomes the Lense-Thirring metric:

$$ds^2 = ds^2{}_{Schw} - \frac{4\gamma_N L}{c^3 r^2}\sin^2\theta(rd\varphi)(cdt), \qquad (3.10)$$

or this metric in Cartesian coordinates:

$$ds^2 = ds^2{}_{Schw} - \frac{4\gamma_N L}{c^3 r^2}(cdt)\left(\frac{xdy - ydx}{r}\right), \qquad (3.11)$$

3.3.2. *Schwarzschild metric*

If the body does not rotate as a whole (L = 0), then it the external gravitational field of the is the centrally symmetric Schwarzschild field. In this case the Kerr metric is asymptotically flat and the Schwarzschild metric when a = 0.

3.4. A spinning test mass

This problem is analogous to calculation of the motion of an electron with spin around the proton in a hydrogen atom.

Suppose (Franklin and Baker, 2007) the test charge is a charged, spinning sphere. Because it is a test mass, we are not concerned with the electric or magnetic fields it generates, but the spin will introduce charge motion in addition to the orbital motion. Spinning test particles are usually introduced in EM in the context of Larmor precession to provide a classical model for quantum spin. In the usual examples, there is a uniform magnetic field, a circular orbit for the center of mass, and the familiar precession comes directly from the angular equations of motion.

Because most astrophysically relevant bodies carry very little charge, from the EM point of view, the problem of macroscopic spin-orbit and spin-spin coupling is just a toy problem, although in the microscopic setting, the model leads to fine and hyperfine splitting. But in the context of gravity for a relatively dense, fast spinning macroscopic source (supermassive black holes), the orbital motion of spinning test particles is more relevant.

It is easy to see that the form of the linearized Kerr metric with the nonrelativistic Lagrangian will also pick up the dipole-field interaction term, and the physical trajectories are similar to the ones that come from Eq. (3.6). In the general relativity setting, the EM Larmor precession is present and is known as Lense-Thirring precession; the Gravity Probe B experiment was sent up in 2004 to measure it.

4.0. The practical use of gravitoelectromagnetic analogy

Instead (Forrester, 2010) of imagining space-time as being warped by mass and energy, one can speak of a classical spin-2 graviton field in flat, Minkowski space-time that generates gravitation. Although we don't know yet how to quantize this field, we can think of it in a way similar to how we think of electromagnetism being mediated by photons. And just as a $1/r^2$ Coulomb force generates magnetism when the finite speed of the mediating photon is taken into account, a $1/r^2$ Newtonian gravitational force generates "gravito-magnetism" when the finite speed of the mediating graviton is taken into account. Magnetism is fundamentally an electric-force effect, and gravity must have some analogous "magnetic" force, meaning a gravitational force proportional and perpendicular to the velocity of a test mass. Einstein showed that gravity should be non-linear, so we know that the graviton should self-interact. General relativity also implies that the graviton should be spin 2. The self-interaction and spin-2 bring us all the way to the EM equivalent of general relativity. But it may be that in most of the Universe (barring black holes, supernovae, et cetera), all you really need to know about gravitation is the electromagnetic-analogue part (gravito-electromagnetism or GEM) to have an accurate description.

Perhaps, first who used the analogy between EM and GEM theories for practical calculations, was Robert Forward in an interesting paper (Forward, 1961), that is perhaps insufficiently well-known due in part to its place of publication (McDonald, 1996). As a variant of our approach, it is interesting for us that R. Forward was not looking for the gravitoelectromagnetic (GEM) equivalents on the basis of general relativity. He, in fact, postulated this equivalence and then, based on the results, obtained in the electromagnetic theory, he found the consequences which were received in the linearized GR.

Following to the work of Moeller, R.Forward presented an analogy between electromagnetism and gravitation, which allows calculation of various gravitational forces by considering the equivalent electromagnetic problem, including a number of examples. The tensor formulation was not used.

Robert Forward notest “that all gravitational effects are correctly described by Einstein’s GR… But because of the difficulty of the description process, there exist only a few solutions of Einstein’s equation which are of experimental interest in that they describe some physically observable effect of GR. The process is so specialized and so difficult that it is practically impossible to attempt solution of a problem unless months of study on the specialized terminology, procedures, and conventions of the general relativity theorist have been completed.

The most disappointing aspect is that in most cases, after struggling through the calculations, it will only be found that the effect calculated is too small to be observed. Fortunately, it is possible to bypass this complicated procedure…if Einstein's equations will be linearized.

If we have a certain mass distribution and flow, all that is necessary is to find a similar charge and current distribution in EM text, such as that of Smythe (Smythe, 1989). We then use the formulas derived for the electric and magnetic fields and make the substitutions in the EM formulas to obtain the gravitational formulas.

With these analogies, it is possible for anyone, who has had electromagnetic theory to study a situation of experimental interest, to calculate the effects, to be expected with sufficient accuracy to determine whether they warrant further study."

When the analogy is carried out and all the constants are evaluated, we obtain an isomorphism between the gravitational and the electromagnetic quantities.

To show how this analogy can be used, R. Forward calculated a few simple examples:

1) Newton's gravitational force between two stationary masses in analogy to the Coulomb force;

2) Interaction of two linear mass currents;

3) Motion of satellite of spinning body;

4) The gravitation analogy to electromagnetic radiation (which we mentioned earlier); etc.

The first problem of R. Forward – the Newton problem of the gravitational attraction between two masses by analogy with the law of attraction between two opposite charges – exhibits no difficulty and was solved along time ago. However, it should be noted that while Newton's law of gravity in general relativity is required to determine the constant of gravitation, in the electromagnetic theory of gravitation is possible to obtain a gravitation constant based on the principle of residual electromagnetic field.

Much more interesting is the second task: the attractive effect between two parallel pipes, into which molten metal (iridium) is flowing, is much more interesting. This example is included primarily to show why the gravitational equivalent of the magnetic field has never bee observed.

To find their interaction due to the mass currents j_{m1} and j_{m2}, he looked at the equivalent magnetic case of two wires with electric currents j_{q1} and j_{q2}.

The force between the two mass-current pipes will be about 2×10^{-13} newtons per meter of pipe. If we use Newton's gravitation law, we get a force of about 3×10^{-4} newtons per meter of pipe, so that the forces due to attractive effect of the mass currents are hidden by the gravitational effect, which also is not usually observable.

The purpose of the third task is similar to that of the Lense-Thirring, but its solution is quite different.

R. Forward "estimates the effect of the earth's rotation on an artificial satellite. First we need to know the gravitomagnetical field of the earth. From Smythe book (Smythe, 1950) we find an expression for the external magnetic field produced by a ring current I at a latitude on a spherical shell of radius *R*. By transforming the magnetic quantities in gravitational quantities, we obtain an expression for the gravitomagnetical field of a rotating massive ring with mass current j_m:

$$P_\theta = \frac{-\eta\, j_m \sin\alpha}{2R}\sum_{n=1}^{\infty}\frac{1}{(n+1)}\left(\frac{R}{r}\right)^{n+2}\cdot P_n^1(\cos\alpha)P_n^1(\cos\theta),$$

Further R. Forward, using the superposition principle, calculates the orbit characteristics for the satellite in polar orbit and for the satellite in equatorial orbit.

5.0. The similarity and the difference between the GEM equations and EM equations

We will discuss now the similarity and meaning of the differences between the GEM equations and the EM field equations.

5.1 The similarity

1) The complete set of Maxwell's equations for gravitoelectromagnetic (GEM) fields (2.14)-(2.19) up to a numerical factors coincides with the Maxwell equations for electromagnetic fields.

2) General relativity contains a gravitomagnetic field due to mass current, in the same manner as EM theory contains magnetic field due to the electric current.

3) The GEM field equations contain the continuity equation, which corresponds to the equation of continuity for the EM field.

4) We have noted that the geodesic equation for test particle trajectory in space-time, , becomes in GEM the motion equation with the "gravito-Lorentz" force,

5) There is also the analogy between gravitation and electromagnetism in the case of a gravitoelectromagnetic Poynting vector.

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6) In GEM, the propagation velocity of gravitational waves is constant, but in the full theory of general relativity, it changes direction. The magnitude of the wave velocity can be determined directly, if we set gravitomagnetic permeability and gravitoelectric permetivity of free space.

In parallel to this, in the classical electromagnetic theory, the velocity of the wave propagation is constant, and in the strong nonlinear EM field it can change its direction. The velocity of the wave, as it was shown by Maxwell, can be calculated from the values of the magnetic permeability and electric permittivity of vacuum.

5.2 Diffierences between EM and GEM equations

Similarity with Maxwell's electromagnetic equations (Chashchina, Iorio and Silagadze, 2008) is apparent. However, there are several important differences.

1) Gravity is attractive. In contrast, electromagnetism can be both attractive and repulsive. This difference leads to the minus signs in the source terms.

2) It is believed that in the framework of the GEM (i.e., in linear GTR):

a) The gravitational radiation field is quadrupole;

b) The gravitational wave is plane and transverse;

c) The GEM field is quantized and is mediated by quants with spin two, so that the gravitational wave is a superposition of quants of gravity (graviton). However, this theory is not renormalizable due to the fact that the gravitational constant is a dimensional quantity. In addition, gravitons have not been observed yet, and the quantum theory of general relativity does not exists yet.

3) It is known that in the framework of electromagnetic theory in general (Landau and Lifshitz, 1975, p. 190):

a) The total radiation consists of three separate parts; they are called dipole, quadrupole and magnetic dipole radiation;

b) The electromagnetic wave has a linear or circular polarization and is transverse;

c) The electromagnetic field is quantized and is mediated by photons with spin one; the EM wave is a superposition of quants of electromagnetic field - photons. In addition, the quantum theory of electromagnetic fields (QED) has been tested with great accuracy.

4) In framework of LIGT the quantization of the gravitational field can be made similarly to its quantization in GEM. The difference lies in the fact that in the linearized general relativity (Ivanenko and Sokolov, 1949, p. 424), the basic equation of relativistic quantum mechanics, on the basis of which the gravitational field equations are constructed, the scalar Klein-Gordon equation is used:

$$\left(\nabla_\mu \nabla_\mu - k_0^2\right)\psi = 0\,, \qquad (5.1)$$

which in the gravitational field becomes:

$$\left(-g^{\mu\nu}\nabla_\mu \nabla_\nu + g^{\mu\nu}\{\mu\nu,\alpha\}\nabla_\alpha - k_0^2\right)\psi = 0\,, \qquad (5.2)$$

where $\{\mu\nu,\alpha\}$ denote the three-digit Christoffel brackets. In a weak field, using its really emitted transverse part, we can obtain:

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - k_0^2\right)\psi = U\psi\,, \qquad (5.3)$$

where $U = U_1 + U_2$; $U_1 = -h_{ns}\nabla_n \nabla_s$; $U_2 = -\left(\nabla_s h_{ns} h_{nk}\right)\nabla_k - \frac{1}{4}e_\alpha\left(\nabla_\alpha h_{ns}^2\right)\nabla_\alpha$

($e_1 = e_2 = e_3 = -1$; $e_4 = +1$).

In the case of LIGT the source equation of gravitational field is a vector equation of "massive photon" (chapter 2), which is identical in form to (5.1).

5) In equation (2.13) (Ruggiero and Tartaglia, 2002) there is a factor 1/2 which does not appear in standard electrodynamics: the effective gravitomagnetic charge is twice the gravitoelectric one. It is supposed that this is a consequence of the fact that the linear approximation of GR involves a spin-2 field, while ”classical” electrodynamics involves a spin-1 field. (see (Wald, 1984, section 4.4))

We found (Wald, 1984, section 4.4) that to linear order in the velocity of the test body, the geodesic equation yields

$$\vec{a} = -\vec{E}_g - 4\frac{\vec{\upsilon}}{c} \times \vec{B}_g\,, \qquad (5.4)$$

where $\vec{E}_g$ and $\vec{B}_g$ are defined in terms of A_μ by the same formulas as in electromagnetism. (Note, that this missing nemerical factor near the B field in some article is hidden by redefinition of *B*).

6.0. Reconciliation of differences between LIGT and GEM

In order to have LIGT fully in line with the GR, we should primarily explain the difference between LIGT and GEM, as linear approach of GR.

If GR can be reduced to analogues of Maxwell’s equations, are there any possibility to reverse the situation and beginning with Maxwell’s equations, to come to GEM and GR equations?

It is clear that it is impossible to use the theory of Maxwell-Lorentz, to achieve this goal. But the proposed LIGT is built on the basis of NTEP which is a Lorentz-invariant nonlinear theory of elementary particles, built as a mechanical representation of the nonlinear quantum theory of the electromagnetic field. Thus, we must show that its abilities allow to create a theory of gravity, which is equivalent to general relativity, and explains many of the formal aspects of general relativity.

Note that there are two types of differences.

1) the abovementioned differences, which exist between the GEM equations and the equations of EM theory.

2) But a much more important task is to explain the difference between the original basics for constructing LIGT and GTR: physical vector theory and geometrical tensor theory, correspondingly.

First of all, let us consider the differences that we found from the comparison of the equations of GEM theory with the equations of the Maxwell-Lorentz theory and which, of course, are applied to the differences between LIGT and general relativity.

6.1. Reconciliation of differences between equations of EM theory and GEM

(Difference 1) In GR mass is only positive, and gravitation is only attractive. In EM theory the charges have two signs that determine the existence of both attraction and repulsion.

Reconciliation:

In LIGT this difference is adjusted on the basis of the hypothesis Mossotti-Lorentz, according to which the gravitational interaction of massive particles takes place due to the fact that the attraction of opposite charges is somewhat stronger than the repulsion of like charges.

(Difference 2) The speed value of propagation of gravity in the wave equation of general relativity, was considered in a number of articles (see., e.g., ((Wald, 1984; Forrester, 2010; Forward, 1961)). But depending on the chosen initial conditions, it can get different values (note that the direct measurement of the speed is still not available).

Reconciliation:

In LIGT this problem simply does not arise: in theory, built on the basis of EM theory, there can be no other velocity of propagation of the fields. Within the framework of EM theory, the value of this constant can be measured indirectly and directly. However, the non-linearity of the field allows a deviation of the light speed from the exact value that it has in the absence of fields.

Difference 3) In GEM equations there are a factors ½, 2, 4, which does not appear in standard electrodynamics. It is supposed that this is a consequence of the fact that the linear approximation of GR involves a spin-2 field, while ”classical” electrodynamics involves a spin-1 field.

Reconciliation:

Is there another explanation within framework of LIGT?

We did not analyzed this issue in details. Nevertheless, let us pay attention that in NTEP is shown that the amplitude of an electric field of the electron wave is two times less than the magnetic field amplitude, i.e., $\vec{E}_e = \vec{B}_e/2$. This fact demonstrates that the electromagnetic field’s values, which correspond to solution of Dirac equation – equation of nonlinear EM wave, are different in comparison to the fields of a linear wave of Maxwell’s theory, where $\vec{E} = \vec{B}$.

Let us now try to explain the differences in the basics of RG and LIGT.

6.2. Reconciliation of differences between basics of LIGT and GEM theory

Let us list these differences and show that they can be harmonized.

(Difference 1) GR is the geometric theory, which is based on the tensor mathematical basis. EM theory is a theory of the vector, whose elements are only physically measurable quantities. Can the first be replace by the second?

Reconciliation:

Obviously, LIGT, as also the EM theory, can be completely written in tensor form. And, importantly, the energy-momentum tensor of the electromagnetic field in mechanical representation contains the same components as the energy-momentum tensor of GR equations.

On the other hand, the physically tested results of GTR were obtained only in the linear approximation of this theory, which can be represented in terms of vector theory, identical to electromagnetic theory. Therefore, it is difficult to argue that the geometricity and nonlinearity of GR add something to the physical results of GTR in comparision with LIGT.

(Difference 2) GR is nonlinear theory, when in the same time the EM theory is considered as linear. For this reason, it is believed that the EM theory has a less wide range of solutions than GTR. Is it really so?

Reconciliation:

Firstly, it is known that the left side of the equations of general relativity, which imparts non-linearity in the Hilbert-Einstein equations, is a mathematical abstraction, the physical meaning of which we do not know. Therefore, we do not know if the solutions of nonlinear equations have physical meaning, especially because it is impossible to verify them.

Secondly, LIGT, strictly speaking, is not a linear theory, since it is based on NTEP. Moreover, the equation of motion of LIGT - the relativistic Hamilton-Jacobi equation, which is analogue of the geodesic equation - is here nonlinear. Its rigorous solution can also provide a wider range of solutions.

6.3. Difficulties of quantization of GR and the solution of this problem in LIGT

The problem of quantization of GR and/or GEM: is there a quantum theory of gravity, and if so, how can it be built?

In LIGT the problem of quantization of gravity is set differently than in the GR. In GR the quantization is only possible in the linearized theory, for example, in the form of GEM. We will show below that the quantization of LIGT is possible in principle, but does not have a sense, because in this case the classical equations of gravity coincide with the quantum ones, similar to what occurs in the quantum theory of electromagnetic field. This is facilitated by the fact that within LIGT gravity is the residual EM field.

Recall that, according to GR, the source (charge) of the gravitational field is the mass-energy. Its peculiarity lies in the fact that it has almost a sufficiently strong field only if its value is much larger than the mass-energy of the elementary particles (let's call this gravitational charge "effective"). Therefore, because of its value, it can be difficult to characterize by means of the quantum parameters of an elementary particle.

In addition, gravitational charge may have angular momentum (let us say, spin), but its quantization also does not make sense. Therefore, from this point of view, we can not attribute the gravitational charge either to bosons or to fermions. At the same time it has the property of bosons: the superposition of individual masses-energies is possible and creates a new gravitational charge as the sum of mass-energy. In addition, as part of the GEM the gravitational radiation field is considered as composed of bosons - gravitons: particles with spin 2.

If we leave aside the value of the spin of the graviton, all this corresponds to the consequences of LIGT. Since we can conditionally say that the basis of LIGT is EM quantum theory of the "massive photon" (see chapter 2 or (Kyriakos, 2009)), then we can assume that it can be the basis of the quantum theory of gravity. To some extent this is true. But such a theory is almost meaningless because of the size of the effective gravitational charge. However, these quantum equations can be used because they coincide with the classical ones.

Indeed, as it is known, (Feynman, Leighton and Sands, 1964) "…*in the situation in which we can have very many particles in exactly the same state, there is possible a new physical interpretation of the wave functions. The charge density and the electric current can be calculated directly from the wave functions and the wave functions take on a physical meaning which extends into classical, macroscopic situations.*

Something similar can happen with neutral particles. When we have the wave function of a single photon, it is the amplitude to find a photon somewhere… There is an equation for the photon wave function analogous to the Schrödinger equation for the electron. The photon equation is just the same as Maxwell's equations for the electromagnetic field, and the wave function is the same as the vector potential ***A****. The wave function turns out to be just the vector* *potential. The quantum physics is the same thing as the classical physics because photons are noninteracting Bose particles and many of them can be in the same state — as you know, they like to be in the same state. The moment that you have billions in the same state (that is, in the same electromagnetic wave), you can measure the wave function, which is the vector potential, directly.*

Now the trouble with the electron is that you cannot put more than one in the same state".

In general relativity the existence of the graviton and the value of its spin is uncertainty, since the quantum GR does not exist. If we will consider the linear approximation of GR (GEM) as reliable enough, then, because the graviton is a boson, it makes no sense to speak about its spin; in this case it is enough to speak about the classical gravitational waves.

(Note that, as we know, (Fermi, 1950; 1951), equally with the wave function of the electromagnetic field in the form of the vector potential, the wave function in form of the vectors of the EM field can be used, as this is accepted in NTEP).

Chapter 7. The mathematical apparatus of LIGT

1.0. Introduction.

Using the results of chapter 2 in this chapter, we will list the quantum equations and relationships which correspond to our goal - the construction of the LIGT. Then we consider the transition from quantum mechanical equations of motion to the motion equations of classical mechanics.

These mathematical tools will be the base for the solution of specific problems in the theory of gravity, expounded in the following chapters.

1.1. The Bases for selection of LIGT equations

From the equivalence of inertial and gravitational masses follows that the field of gravity is generated simultaneously with inertial mass. This means that the equations of massive elementary particles describe also the gravitational field equations.

Electron is the simplest stable massive particle. Since, according to axioms of the LIGT, the gravitational field is a small part of the electric field, it can be assumed that the simplest candidate for the gravitational equation must be a modification of the nonlinear equation of the electron. In this case, the mass of this equation is the gravitational mass, i.e., the source of the gravitational field.

On the other hand we have the equation of the neutral "massive photon", which we can also - and with a significant reason - consider as a gravitation source equation. The following facts are the arguments in favor of this choice:

1) "massive photon" is the primary massive particle;

2) it is an electrically neutral particle;

3) fermions are not the interaction carriers in the microworld, but bosons are;

4) the "massive photon" equation and the lepton equation are related through operations of decomposition of first equation and squaring of second equation. From this it follows that the first or the second choice of the equations of gravitation is a matter of convenience.

Hence, the "massive photon" equation may be an advantageous variant of the gravitation source equation. Nevertheless, the close relationship of theories of "massive photon", and electron, implies the possibility of the use of the Dirac equation, as the basis of our approach.

There is another indirect argument. As noted by Richard Feynman, a direct transition from the quantum to the classical form of the fermion equation is difficult. In the case of bosons, such a transition is quite simple: we can say that it is the same equation (Feynman, 1964, 21-4. *The meaning of the wave function*).

"…*In the situation in which we can have very many particles in exactly the same state, there is possible a new physical interpretation of the wave functions. The charge density and the electric current can be calculated directly from the wave functions and the wave functions take on a physical meaning which extends into classical, macroscopic situations*" (see also Chapter 6).

2.0. The photon equation

The classical EM wave equation of motion has the form:

$$\left(\frac{\partial^2}{\partial t^2} - c^2\vec{\nabla}^2\right)\Phi = 0, \qquad (2.1)$$

wherein c is light velocity, Φ is a matrix, which contains the components of the wave function of an electromagnetic field $\vec{E},\vec{H}$: $\Phi^+ = \left(\mathrm{E}_x \quad \mathrm{E}_z \quad -i\mathrm{H}_x \quad -i\mathrm{H}_z\right)$

This wave is a superposition of two waves with plane polarization: $\Phi_1 = \begin{pmatrix} E_x \\ H_z \end{pmatrix}$ and $\Phi_2 = \begin{pmatrix} E_z \\ H_x \end{pmatrix}$, which also satisfy (2.1). In this sense, the wave with the flat polarization, and not cyclic polarization, can be regarded as the primary particle of EM field.

3.0. The current and mass of massive particles

Equation (2.1) can be represented as a system of two equations for massless electron and positron:

$$\left(\hat{\alpha}_o \hat{\varepsilon} + c\hat{\vec{\alpha}}\,\hat{\vec{p}}\right)\psi' = 0, \tag{3.1}$$

$$\psi'^{+}\left(\hat{\alpha}_o \hat{\varepsilon} - c\hat{\vec{\alpha}}\,\hat{\vec{p}}\right) = 0, \tag{3.1'}$$

where the wave function of these equations we denoted as ψ'.

Self-interaction of the photon fields leads to the appearance in the photon of two displacement currents of different directions (chapter 2 or (Kyriakos, 2009)). In the mathematical description of this process, in the equations (3.1) an additional term arises.

The emerging particle, which we conditionally call "massive photon", is unstable and breaks into massive particle-antiparticle, particularly, electron and positron (this fact allows us to consider a "massive photon" as an intermediate boson).

$$\left(\hat{\alpha}_o \hat{\varepsilon} + c\hat{\vec{\alpha}}\,\hat{\vec{p}} + \hat{\beta}\, m_e c^2\right)\psi = 0, \tag{3.2}$$

$$\psi^{+}\left(\hat{\alpha}_o \hat{\varepsilon} - c\hat{\vec{\alpha}}\,\hat{\vec{p}} - \hat{\beta}\, m_e c^2\right) = 0, \tag{3.2'}$$

4.0. The equation of “massive photon”

At a time when the system of equations (3.1) obtains current (mass) terms, the photon ceases to move at the speed of light and becomes massive :

$$\left(\frac{\partial^2}{\partial t^2} - c^2\vec{\nabla}^2\right)\Phi = \frac{m_{ph}^2 c^4}{\hbar^2}\Phi, \tag{4.1}$$

Since the currents have a different direction, the photon remains a neutral vector boson.

The equation of neutral "massive photon” (4.1) can be rewritten in the view:

$$\left[\left(\hat{\alpha}_o \hat{\varepsilon}\right)^2 - c^2\left(\hat{\vec{\alpha}}\hat{\vec{p}}\right)^2\right]\Phi = m_{ph}^2 c^4 \Phi, \tag{4.1'}$$

or

$$\left(\hat{\varepsilon}^2 - c^2\hat{\vec{p}}^2 - m_{ph}^2 c^4\right)\Phi = 0, \tag{4.2}$$

From equation (4.1) follows the conservation equation for the elementary particles:

$$\varepsilon^2 - c^2\vec{p}^2 - m_{ph}^2 c^4 = 0, \tag{4.3}$$

Note that this equation is valid both in quantum mechanics and in classical mechanics for all particles.

In connection with general relativity, the different form of equation (4.1) could be interesting for us. The expression for the current was obtained by the rotation transformation, the radius of which was equal to $r_C = \hbar/m_e c$. For this reason, the current (mass) j^e contains curvature $\kappa = 1/r_C$ through which the mass term $m_{ph}^2 c^4/\hbar^2 = 1/4r_C^2 = \kappa^2/4$ can be expressed. In other words, the equation (4.1) can be expressed as:

$$\left(\frac{\partial^2}{\partial t^2} - c^2\vec{\nabla}^2\right)\Phi = \frac{1}{4r_C^2}\Phi, \qquad (4.4)$$

This equation is similar to the equation obtained by Schrödinger as the generalization of the Dirac equation on Riemannian space (see below).

5.0. The generally covariant equation of "massive photon"

The generally covariant equation of "massive photon" Schroedinger (Schroedinger, 1932), was the first to obtain by squaring of Dirac equation, written for the curved space:

$$\frac{1}{\sqrt{g}}\nabla_k\sqrt{g}g^{kl}\nabla_l - \frac{R}{4} - \frac{1}{2}f_{kl}S^{kl} = \mu^2, \qquad (5.1)$$

Here $\mu = \frac{m_e c}{\hbar} = \frac{1}{r_C}$, where r_C is the Compton wave length of electron, R is the invariant curvature.

In the first term is easy to find a regular operator of the Klein second order equation in the Riemann geometry. In the third term on the left is recognized well-known term associated with the spin magnetic and electric moments of the electron (tensor S^{kl}).

"*To me, the second term seems to be of considerable theoretical interest. To be sure, it is much too small by many powers of ten in order to replace, say, the term on the r.h.s. For* μ *is the reciprocal Compton length, about* $10^{11}cm^{-1}$. *Yet it appears important that in the generalised theory a term is encountered at all which is equivalent to the enigmatic mass term*."

This term can be associated with the free term of the equation of Dirac's electron μ. According to Gauss, on a curved surface $R = \kappa_1 \cdot \kappa_2$, where κ_1, κ_2 are the normal curvature of the surface. If, $\kappa_1 = \kappa_2 = \kappa'$ then $R = \kappa'^2$. Assuming by Schrodinger that $R/4 = \mu^2$, we obtain that $\kappa' = 2\mu = \frac{2m_e c}{\hbar} = \frac{m_{ph}c}{\hbar}$

6.0. Quantum equations of particles' motion in the external field

For a complete accordance with the electromagnetic theory of matter (EMTM), the energy ε_{ex} and momentum $\vec{p}_{ex}$ in the equation (3.3) must be expressed as the EM values. We can include the electromagnetic potentials $\varphi(\vec{r},t)$ and $\vec{A}(\vec{r},t)$, using the fact that φ and $(1/c)\vec{A}$ have the same Lorentz-transformation properties as ε and $\vec{p}$ (here φ is scalar potential, $\vec{A}$ is the vector potential of the EM field, and the dimension of $\varphi(\vec{r},t)$ is energy per unit charge, and the dimension of $(1/c)\vec{A}$ is equal to the momentum per unit charge).

As is known, the total momentum and the total energy of a charged particle in an electromagnetic field is determined by the following expressions:

$$\vec{p}_{ful} = \vec{p} + \frac{q}{c}\vec{A}, \quad \varepsilon_{ful} = \varepsilon + q\varphi, \qquad (6.1)$$

where q is charge, $\vec{p} = \frac{m\vec{\upsilon}^2}{\sqrt{1-\vec{\upsilon}^2/c^2}}$ and $\varepsilon = \frac{mc^2}{\sqrt{1-\vec{\upsilon}^2/c^2}}$ are the momentum and energy of a free particle, $\vec{\upsilon}$ is particle velocity, $\vec{p}_{ex} = \frac{q}{c}\vec{A}_{ex}$ and $\varepsilon_{ex} = q\varphi_{ex}$ are the potential momentum and energy of some external source (charged particles), obtained in the EM field.

Hence, (4.1) can be rewritten as the Dirac equation with an external EM field

$$\left[\hat{\alpha}_0\left(\hat{\varepsilon} \mp e\varphi_{ex}\right) + c\hat{\vec{\alpha}} \cdot \left(\hat{\vec{p}} \mp \frac{q}{c}\vec{A}_{ex}\right) + \hat{\beta} m_e c^2\right]\psi = 0, \tag{6.2}$$

The corresponding differential equations for the "massive photon" will be:

$$\left[\left(\varepsilon + \varepsilon_{ex}\right)^2 - c^2\left(\vec{p} + \vec{p}_{ex}\right)^2 - m^2c^4\right]\Phi = 0, \tag{6.3}$$

$$\left[\left(\varepsilon + q\varphi_{ex}\right)^2 - c^2\left(\vec{p} + \frac{q}{c}\vec{A}_{ex}\right)^2 - m^2c^4\right]\Phi = 0, \tag{6.3'}$$

(here and from now on we omit the subscript "ph" in mass of "massive photon")

From this we can obtain the equations of energy-momentum conservation of a particle in an EM field:

$$\left(\varepsilon + \varepsilon_{ex}\right)^2 - c^2\left(\vec{p} + \vec{p}_{ex}\right)^2 - m^2c^4 = 0, \tag{6.4}$$

$$\left(\varepsilon + q\varphi_{ex}\right)^2 - c^2\left(\vec{p} + \frac{q}{c}\vec{A}_{ex}\right)^2 - m^2c^4 = 0, \tag{6.4'}$$

From the above it follows that the values $\frac{q}{c}\vec{A}_{ex}$ and $q\varphi_{ex}$ completely characterize the external field source. Below we will find the expression for the force, with the source acts on the particle.

7.0. The transition from quantum mechanical equations of motion to the motion equations of classical mechanics

There are three main methods of transition from the quantum mechanical equations of motion to the classical equations (Schiff, 1955; Levich, Myamlin and Vdovin, 1973, Landsman, 2005; Anthony, 2014).

a) theorem of Ehrenfest,

b) on the basis of Hamilton's canonical equations, using Poisson brackets,

c) the transition from the wave equation to the Hamilton-Jacobi equation.

We shall illustrate this transition based on the methods a) and b).

7.1. Ehrenfest's theorem in the case of the Lorentz-invariant quantum theory

Let us use the Lorentz-invariant quantum wave equation of "massive photon" in external EM field (6.3), obtained in the above section:

In this case (Anthony, 2014) the wave function has the form

$$\psi = \psi_0 \exp\frac{i}{\hbar}\left[\left(\vec{p} - \frac{q}{c}\vec{A}\right)\vec{r} - \left(\varepsilon + q\varphi\right)t\right], \tag{7.1}$$

Now we want to see whether that equation gives us a description of Reality that conforms to the classical theory. To that aim we will calculate the expectation value of the rate at which a particle's linear momentum changes with the elapse of time.

Using the relativistic formula for the probability density, we have

$$\frac{d}{dt}\langle\vec{p}\rangle = \frac{i\hbar}{2mc^2}\int\left[\psi^+\left(-i\hbar\frac{d}{dt}\vec{\nabla}\right)\frac{\partial\psi}{\partial t} - \psi\left(i\hbar\frac{d}{dt}\vec{\nabla}\right)\frac{\partial\psi^+}{\partial t}\right]d\tau, \tag{7.2}$$

In that equation the operators extract the argument of the wave function and differentiate it, so we have

$$-i\hbar\frac{d}{dt}\vec{\nabla}\frac{\partial\psi}{\partial t}=\frac{\partial\psi}{\partial t}\left[\frac{d}{dt}\vec{\nabla}\left(\vec{p}\cdot\vec{r}-\frac{q}{c}\vec{A}\cdot\vec{r}\right)-\frac{d}{dt}\vec{\nabla}(\varepsilon t+q\varphi t)\right], \tag{7.3}$$

The vector variables $\vec{r}$ and $\vec{p}$ do not represent fields, but rather represent points in phase space that the particle occupies as time elapses, so we take the spatial derivatives of those variables as equal to zero. Further, if we do not want to have the complications with radiation fields, then with respect to the source of the potential fields we must take $d\varphi/dt=0$ and $d\vec{A}/dt=0$.

Carrying out the differentiations thus gives us:

$$\begin{aligned}-i\hbar\frac{d}{dt}\vec{\nabla}\frac{\partial\psi}{\partial t}&=\frac{\partial\psi}{\partial t}\left[q\frac{d\vec{r}}{dt}\times\left(\vec{\nabla}\times\vec{A}\right)-q\left(\frac{d}{dt}\vec{\nabla}\right)\vec{A}-\vec{\nabla}U-q\vec{\nabla}\varphi\right]=\\&=\frac{\partial\psi}{\partial t}\left[q\vec{\upsilon}\times\left(\vec{\nabla}\times\vec{A}\right)-q\left(\frac{d\vec{A}}{dt}-\frac{\partial\vec{A}}{\partial t}\right)-\vec{\nabla}U-q\vec{\nabla}\varphi\right]\end{aligned}, \tag{7.4}$$

Substituting that result and its complex conjugate into Equation 18 then gives us:

$$\frac{d}{dt}\langle\vec{p}\rangle=q\left\langle\vec{\upsilon}\times\left(\vec{\nabla}\times\vec{A}\right)-\frac{\partial\vec{A}}{\partial t}-\vec{\nabla}\varphi\right\rangle+\left\langle-\vec{\nabla}U\right\rangle, \tag{7.5}$$

which describes the Lorentz electromagnetic force plus the force due to any other static potentials of the particle interaction. Thus we gain strong evidence that the relativistic quantum theory, like its non-relativistic version, has the classical limit.

7.2. Derivation of generally covariant classical equation of motion on the base of Ehrenfest theorem

An interesting application of the theory (see chapter 4) is to establish an analogue of Ehrenfest's theorem for the Dirac equation, generalized to the Riemann geometry (Sokolov and Ivanenko, 1952; pp. 650-651). In addition to the results obtained above, by squaring of the Dirac equation, for the center of gravity of the wave packet (provided $\hbar\to 0$), we obtain the equation of relativistic mechanics of point:

$$\frac{d}{dx^4}\left(\gamma^4 p_\alpha\right)=\Gamma^\sigma_{\alpha\rho}p_\alpha+\gamma^\rho\frac{e}{c}F_{\rho\alpha}, \tag{7.6}$$

where γ^4 is the fourth Dirac matrix, γ^ρ corresponds to the particle velocity in fraction of the speed of light c, $\Gamma^\sigma_{\alpha\rho}$ is the Christoffel brackets $\{\mu\nu,\alpha\}=\Gamma^\sigma_{\alpha\rho}=\frac{1}{2}\left(\frac{\partial g_{\mu\sigma}}{\partial x_\nu}+\frac{\partial g_{\nu\sigma}}{\partial x_\mu}+\frac{\partial g_{\mu\nu}}{\partial x_\sigma}\right)$, $F_{\rho\alpha}$ is the electromagnetic field tensor. The first term on the right of equation is the force of gravity, and the second term is the Lorentz force.

7.3. Derivation of classical Hamilton-Jacobi equation of motion on the base of quantum wave equation

The Hamilton-Jacobi equation (HJE) in the classic mechanics is usually obtained by postulating the action in the form of:

$$S=S_{free}+S_{\text{int}}+S_{ext}, \tag{7.7}$$

where S_{free} is the action of a free particle in the absence of other particles; S_{int} is the action of the interaction between the free particle and other particles; S_{ext} is the action of other particles in the absence of the first particle.

In quantum physics HJE can be obtained, if we postulate that the action is equal to phase of the de Broglie wave (as Schrödinger did for the derivation of the Schrödinger equation (Schroedinger, 1932).

The particle wave function, in general, has the form:

$$\psi = \psi_0 \exp i\theta , \tag{7.8}$$

where θ is the phase of the wave function. In the case of a free particle the wave function has the form:

$$\psi = \psi_0 \exp\frac{i}{\hbar}\left(\varepsilon t - \vec{p}\vec{r} + \varphi_0\right), \tag{7.9}$$

Substituting this function in the equation (4.1), we obtain the law of conservation of energy and momentum for a free particle (5.3):

$$\varepsilon^2 - c^2\vec{p}^2 = m^2c^4 , \tag{4.3}$$

In the case of a particle in an external field with the energy and momentum $\varepsilon_{ex}, \vec{p}_{ex}$ the wave function has the form:

$$\psi = \psi_0 \exp\frac{i}{\hbar}\left[(\vec{p} - \vec{p}_{ex})\vec{r} - (\varepsilon + \varepsilon_{ex})t + \varphi_0\right], \tag{7.10}$$

Substituting these functions in the equation (6.3), we obtain the conservation law for a particle in an external field (6.4):

$$(\varepsilon - \varepsilon_{ex})^2 - c^2(\vec{p} - \vec{p}_{ex})^2 = m^2c^4 , \tag{6.4}$$

According to Schrödinger in case of a free particle we take:

$$S = \theta\hbar = \varepsilon t - \vec{p}\vec{r} + \varphi_0 \ , \tag{7.11}$$

and in case of a particle in external field:

$$S = \left[(\vec{p} - \vec{p}_{ex})\vec{r} - (\varepsilon + \varepsilon_{ex})t + \varphi_0\right] , \tag{7.12}$$

Hence we have in the first case for the energy and momentum $\frac{\partial S}{\partial t} = \varepsilon$, $\frac{\partial S}{\partial \vec{r}} = \vec{p}$, and in the second case $\frac{\partial S}{\partial t} = \varepsilon + \varepsilon_{ex}$, $\frac{\partial S}{\partial \vec{r}} = \vec{p} - \vec{p}_{ex}$.

Substituting partial derivatives of first type in the conservation law of energy-momentum without an external field, we obtain the relativistic HJE without an external field:

$$\frac{1}{c^2}\left(\frac{\partial S}{\partial t}\right)^2 - \left(\frac{\partial S}{\partial x}\right)^2 - \left(\frac{\partial S}{\partial y}\right)^2 - \left(\frac{\partial S}{\partial z}\right)^2 = m^2c^2 , \tag{7.13}$$

Substituting second partial derivatives of second type in the conservation law of energy-momentum with an external field, we obtain the relativistic HJE with the external field:

$$\frac{1}{c^2}\left(\frac{\partial S}{\partial t} + \varepsilon_{ex}\right)^2 - \left(\frac{\partial S}{\partial x} - p_{x\,ex}\right)^2 - \left(\frac{\partial S}{\partial y} - p_{y\,\mathrm{ex}}\right)^2 - \left(\frac{\partial S}{\partial z} - p_{z\,\mathrm{ex}}\right)^2 = m^2c^2 , \tag{7.14}$$

In the case of the electromagnetic field we have:

$$\frac{1}{c^2}\left(\frac{\partial S}{\partial t} + q\varphi\right)^2 - \left(\frac{\partial S}{\partial x} - \frac{q}{c}A_x\right)^2 - \left(\frac{\partial S}{\partial y} - \frac{q}{c}A_y\right)^2 - \left(\frac{\partial S}{\partial z} - \frac{q}{c}A_z\right)^2 = m^2c^2 , \tag{7.15}$$

The action for the interaction can be obtained as an instantaneous change of action:

$$dS_{\mathrm{int}} = \frac{\partial S}{\partial t}dt + \frac{\partial S}{\partial \vec{r}}d\vec{r} = \frac{\partial S}{\partial t}dt - \frac{\partial S}{\partial \vec{r}}\frac{d\vec{r}}{dt}dt = \frac{\partial S}{\partial t}dt - \frac{\partial S}{\partial \vec{r}}\vec{\upsilon}dt = (\varepsilon - \vec{p}\vec{\upsilon})dt , \tag{7.16}$$

i.e., $dS_{\text{int}} = (\varepsilon - \vec{p}\vec{\upsilon})dt = L_{\text{int}}dt$; in the case when the external field is organized by electrical charged particles, we have: $dS_{\text{int}} = \left(q\varphi - \frac{q}{c}\vec{A} \right)dt$.

Here

$$L_{\text{int}} = (\varepsilon - \vec{p}\vec{\upsilon}) = q\varphi - \frac{q}{c}\vec{A}\vec{\upsilon}, \tag{7.17}$$

is the interaction Lagrangian (the so-called, minimal connection). As is known, by variation of this action gives the expression for the Lorentz force.

8.0. The interaction law of gravitation field in framework of LIGT

In the case of electrodynamics it is necessary to use not the classical potential energy, but the generalized (and depending on the speed) potential energy (energy of interaction)

$$U = q\varphi - \frac{q}{c}\vec{\upsilon}\cdot\vec{A} = \int \left(\rho\varphi - \frac{1}{c}\vec{j}\cdot\vec{A} \right) dxdydz, \tag{8.1}$$

This interaction energy corresponds to the above interaction Lagrangian (7.17).

From this Lagrangian follows the equation for the Lorentz force. In terms of EM vectors it has the form:

$$\vec{F} = q\vec{E} - \frac{q}{c}\vec{\upsilon}\times\vec{H}, \tag{8.2}$$

Lorentz force in terms of potentials:

$$\vec{F} = q\vec{\nabla}\varphi - \frac{q}{c}\frac{\partial\vec{A}}{\partial t} + \frac{q}{c}\vec{\upsilon}\times\left(\vec{\nabla}\times\vec{A}\right) = q\vec{\nabla}\varphi - \frac{q}{c}\frac{\partial\vec{A}}{\partial t} + \frac{q}{c}\left[\vec{\nabla}\left(\vec{\upsilon}\cdot\vec{A}\right) - \left(\vec{\upsilon}\cdot\vec{\nabla}\right)\vec{A}\right], \tag{8.3}$$

9.0. Conclusion

Thus, we have shown that the Lorentz force occurs at the transition from quantum mechanics of massive particle to classical mechanics of this particle, as a reflection of the unique relation of the inertial mass with internal and external fields of the particle. According to our axioms, we must conclude that the Lorentz force law or its modifications should be responsible for the description of the gravitational force or energy.

In addition, the connection of inertial mass with gravitational charge becomes clear, as well as the relationship between the electric charge and gravity charge (mass), which allow us to proceed from Coulomb equation to the Newton equation of gravitation.

Chapter 8. Geometry and Physic of LIGT and GTR

1.0. Introduction

This chapter is devoted to analysis of the relation of geometrical and physical quantities in the Newtonian theory of gravitation, LIGT and GTR, and also to clarification of the physical meaning of the metric tensor and the space-time interval in the Euclidean, pseudo-Euclidean and pseudo-Riemannian spaces.

We will show the succession of the use of geometric concepts in these three theories.

Another result of this chapter is that the math expression of interval is mutually uniquely associated with physical equations of elementary particles and LIGT.

Further we will show, that in LIGT the metric tensor has the physical meaning of the scale factor, defined by means of the Lorentz-invariant transformations.
In addition the evidences will be given of that the metric tensor in general relativity should have the same meaning.

1.1. Geometry and Physic in the GR

According to general relativity the gravitational field is described by the metric tensor.

The practical side of the Einstein-Hilbert theory (Tonnelat, 1965/1966) is following:

"*All the predictions of general relativity follow from*: **1)** *The solution of the field equations*:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \chi T_{\mu\nu}, \tag{1.1}$$

or

$$G_{\mu\nu}\left(g_{\alpha\beta}, \partial_\rho g_{\alpha\beta}, \partial^2_{\rho\sigma} g_{\alpha\beta}\right) = \chi T_{\mu\nu}(m, \vec{u}) \rightarrow g_{\alpha\beta}, \tag{1.1'}$$

where $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$, $\chi = \frac{8\pi G}{c^4}$, $R_{\mu\nu} = \frac{\Gamma^{\lambda}_{\mu\nu}}{\partial x^{\lambda}} - \frac{\Gamma^{\nu}_{\mu\lambda}}{\partial x^{\nu}} + \Gamma^{\lambda}_{\mu\nu}\Gamma^{\alpha}_{\lambda\alpha} - \Gamma^{\alpha}_{\mu\lambda}\Gamma^{\lambda}_{\nu\alpha}$ is the Ricci curvature tensor, $\Gamma^{\lambda}_{\mu\nu}$ *are the Christoffel symbols*, R is the scalar curvature, γ_N is Newton's gravitational constant, c is the speed of light in vacuum and $T_{\mu\nu}$ is the stress–energy tensor.
and $g_{\mu\nu}$ *is the metric tensor of Riemannian space, and*

2) *The law of motion (geodesic equation)* or the Hamilton-Jacobi equation for a massive body (Landau and Lifshitz, 1951):

$$g^{ik}\left(\frac{\partial S}{\partial x^i}\right)\left(\frac{\partial S}{\partial x^k}\right) + m^2 c^2 = 0, \tag{1.4}$$

The equation (1.1) allows to determine $g_{\mu\nu}$ and to put this value in (1.4).

Since the metric tensor is contained in the square of interval of Riemannian space:

$$(ds)^2 = g_{\mu\nu} dx^{\mu} dx^{\nu}, \tag{1.5}$$

it is often said that the purpose of solution of equation (1.1) is to find the interval (1.5).

Depending on the type of energy-momentum tensor the solutions of (1.1) can be divided into several types. The most important of them are the vacuum solutions, since it was possible to verify some of them experimentally. Such solutions can be obtained from the equation (1.1), if the energy-momentum tensor vanishes: $T_{\mu\nu} = 0$.

These solutions describe the empty space-time around a massive compact source of the gravitational field, down to its surface or singularities. These include the Schwarzschild metric, the Lense-Thirring, Kerr, Reissner - Nordstrom, Kerr - Newman and others metrics.

In general relativity, a vacuum solutions are a Lorentzian manifold, i.e., they relate to asymptotically flat space-time. A Lorentzian manifold is an important special case of a pseudo-Riemannian manifold in which metric is called Lorentzian metric or the pseudo-Euclidian metric of special relativity.

In the vacuum equations of general relativity only the left side is used - purely geometric part of this equation. At the same time, the clarification of the physical meaning of significant elements of the metric tensor (MT) takes place on the basis of a comparison with Newton's theory of gravitation. Hitherto, the question, why a purely geometrical functions produce physical results, has not clarified. In other words, we do not know, how the MT is associated with physics.

The basis for the introduction and use of MT is the interval (often they are identical). Then the question can be reformulated in a different way: how interval and MT in this composition relates to physics?

It is often said that interval in STR is a generalization of interval of Euclidean geometry on pseudo-Euclidean geometry. In turn, the interval in general relativity is a generalization of interval of pseudo-Euclidean geometry on pseudo-Riemannian geometry. But it is easy to make sure, that the introduction of interval in STR and GTR is a postulates rather than a logical conclusion. The intervals in STR and GTR are a generalization of interval of Euclidean geometry. And the reason for the introduction of these new intervals is not geometry, but physics. Then what was postulated and on what basis did it take place in each of these cases?

2. Geometry and Physics of Newtonian Mechanics (Euclidean Space)

Let us begin from the relation of the Euclidean interval with physics. To do this, we need to recall the meaning of the geometrical point and the material point, as well as of the geometric and physical trajectory, as the line of motion of a material point.

The line in geometry is an independent geometric object, almost not related to physics. Not strictly speaking, the line is a continuum (continuous sequence) of dots, for each adjacent pair of which the same relationship is set. If this relationship can always be reduced to a constant number, a line is called Euclidean; if this relationship is a function of the position on the line, the line is called Riemannian.

The line in geometry is defined (described) by specifying coordinates, i.e., some of the numbered lines, which are specified by the location of the material points (objects) of the real world.

In physics, the line is a continuum of points, which a material point passes successively while moving by inertia or under the influence of forces. And this line is determined by the law of motion of a material point with respect to the others, outsider material points which allows to establish a base coordinate system of lines. Namely here, geometry comes in contact with physics.

The interval in Euclidean geometry is a generalized description of Pythagoras theorem for an infinitesimal segment of line (arc): *square of the length of any line segment is equal to the sum of the squares of the projections of the segment on the three coordinate lines*. The objectives of the geometry, which requires the use of this law, has no connection with physics. But for the trajectory of a material point the theorem of Pythagoras is some condition - restrictive law, which must take place in any problem of the motion of material body.

Conditionally speaking, the law of Pythagoras must be contained in the law of motion. Obviously, this one-to-one relationship should allow to restore the movement law by means of the known interval. Approximately in this manner the problem is set on the theory of gravitation of Hilbert and Einstein.

Actually, the interval at any point of the trajectory of motion of a point must be mutually uniquely associated with the solution of the dynamic (physical) problem. Otherwise the decision will be wrong, i.e., the trajectory will not be one that is dictated by the law of motion. But this bond can not be associated with a coordinate system, since the latter is not related to the physical problem, and it can be chosen in many ways. This bond must occur in any coordinate system, in which the law of Pythagoras acts. In this case, the introduction and the choice of the coordinate system is a agreement, required for a quantitative calculation of the physical problem.

Let us demonstrate the correctness of our conclusion in the framework of non-relativistic and then relativistic (i.e., the Lorentz-invariant) mechanics.

2.1. Cartesian system of coordinates

Subject of mechanics (see (Webster, 1912)) is study of motion in space and time of the matter particle or system of particles, as solid body, under the action of forces.

Since the motion description of a material point involves four variables x, y, z, t, kinematics was called by Larange "geometry of four dimensions".

Suppose that we have a system of *n* material points. If they are free to move, a single particle requires 3 coordinates x, y, z, and a system of particles require 3*n* coordinates: $x_1, y_1, z_1, x_2, y_2, z_2, \ldots, x_n, y_n, z_n$

If any particle j at x_j, y_j, z_j is displaced by a small amount, it has the coordinates $x_j + dx_j, y_j + dy_j, z_j + dz_j$

If a number of particles are displaced, we must take the sums like the above for all the particles.

The infinitesimal distance between two points

$$ds = \sqrt{dx^2 + dy^2 + dz^2}\ , \tag{2.1}$$

is a scalar, whereas the geometrical difference in position of the two points is known only when we specify not merely the length, but also the direction of the line joining them. This is usually done by giving its length s and the cosines of the angles λ, μ, ν made by the line with the three rectangular axes, $\cos\lambda, \cos\mu, \cos\nu$, which in virtue of the relation

$$\cos^2\lambda + \cos^2\mu + \cos^2\nu = 1, \tag{2.2}$$

leaves three independent data.

We may otherwise make the specification by giving the three projections of the line upon the coordinate axes:

$$ds_x = s\cos\lambda = dx,\ ds_y = s\cos\mu = dy,\ ds_z = s\cos\nu = dz\ , \tag{2.3}$$

Squaring and adding we have in virtue of relation (2.2):

$ds^2 = ds_x^2 + ds_y^2 + ds_z^2$, (2.4)

The quantities $dx/ds,\ dy/ds,\ dz/ds$ are the direction cosines of the tangent to the arc ds .

The vector denned by the product of the scalar quantity mass by the vector quantity acceleration (vector quantity), whose components are

$$F_x = m\frac{d^2x}{dt^2},\ F_y = m\frac{d^2y}{dt^2},\ F_z = m\frac{d^2z}{dt^2}, \qquad (2.5)$$

is called the force acting upon the body, and is the applied force of the Newton second law. The second and third laws taken together accordingly give us a complete definition and mode of measurement of force.

It is customary to characterize the product of the mass by the vector velocity as the momentum of the body, a vector whose components are

$$p_x = m\frac{dx}{dt} = m\upsilon_x,\ p_y = m\frac{dy}{dt} = m\upsilon_y,\ p_z = m\frac{dz}{dt} = m\upsilon_z, \qquad (2.6)$$

This is the *momentum* whose rate of change measures the force, so that equations (2.5) may be written

$$\frac{dp_x}{dt} = F_x,\ \frac{dp_y}{dt} = F_y,\ \frac{dp_{zx}}{dt} = F_z, \qquad (2.7)$$

These equations are a generalization of equation (2.5), since they may be applied in the case when mass m changes, for example, in the case the engine of the rocket is running.

$$T = \frac{1}{2}m\left\{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2\right\} = \frac{1}{2}m\left(\upsilon_x{}^2 + \upsilon_y{}^2 + \upsilon_z{}^2\right), \qquad (2.8)$$

the half-sum of the products of the mass of particle by the square of its velocity, is called the *kinetic energy* of the particle *T*.

If we have a system of *n* material points then:

$$T = \frac{1}{2}\sum_{i=1}^{n} m_i\left\{\left(\frac{dx_i}{dt}\right)^2 + \left(\frac{dy_i}{dt}\right)^2 + \left(\frac{dz_i}{dt}\right)^2\right\} = \frac{1}{2}\sum_{i=1}^{n} m_i\left(\upsilon_{x_i}{}^2 + \upsilon_{y_i}{}^2 + \upsilon_{z_i}{}^2\right), \qquad (2.9)$$

The kinetic energy may be written, bearing in mind the definition of momentum, as:

$$\begin{aligned} T &= \frac{1}{2}m\left(\upsilon_x{}^2 + \upsilon_y{}^2 + \upsilon_z{}^2\right) = \frac{1}{2}\left(p_x\upsilon_x + p_y\upsilon_y + p_z\upsilon_z\right) = \\ &= \frac{1}{2m}\left(p_x{}^2 + p_y{}^2 + p_z{}^2\right) = \frac{1}{2m}\vec{p}^{\,2} \end{aligned}, \qquad (2.10)$$

It is easily to see:

$$\frac{d}{dt}T = \frac{d}{dt}\left(\frac{1}{2}m\sum_i \upsilon_i{}^2\right) = F_x\frac{dx}{dt} + F_y\frac{dy}{dt} + F_z\frac{dz}{dt},$$

whence

$$dT = F_x dx + F_y dy + F_z dz$$

is the work done upon the particle at relocation it on infinitesimal distance.

The equation

$$T_{t_1} - T_{t_0} = \int_{t_0}^{t_1} \left(F_x dx + F_y dy + F_z dz \right) , \qquad (2.11)$$

is called the equation of energy, and states that the gain of kinetic energy is equal to the work done by the forces during the motion.

In the case that the forces acting on the particles depend only on the positions of the particles, and that the components may be represented by the partial derivatives of a single function of the coordinates $U(x, y, z)$ so that

$$F_x = \frac{dU}{dx}, F_y = \frac{dU}{dy}, F_{zx} = \frac{dU}{dz} , \qquad (2.12)$$

the equation of energy then is

$$T_{t_1} - T_{t_0} = U_{t_1} - U_{t_0} , \qquad (2.13)$$

The function *U* is called the force -function, and its negative *W*= —*U* is called the *potential energy* of the system. Inserting *W* in (2.13) we have

$$T_{t_1} + W_{t_1} = T_{t_0} + W_{t_0} , \qquad (2.14)$$

the *principle of conservation of energy*.

Suppose that the particle instead of being free is constrained to lie on a given surface. The path described must then be an arc of a shortest or geodesic line of the surface. The calculus of variations enables us to find the differential equations of such a line.

The principle of least action says that in the natural or unconstrained motion it will go from P to Q along the shortest path, that is, an arc of a great circle.

2.2. Generalized system of coordinates

As was shown by Beltrami (Beltrami, 1869), and worked out in detail by Hertz, that the properties of Laginterval's equations have to do with a quadratic form, of exactly the sort that represents the *arc of a curve* in geometry.

For instance if a particle is constrained to move on the surface of a sphere of radius r , we may specify its position by giving its longitude φ and colatitude ϑ. These are two independent variables.

The potential energy depending only on position will be expressed in terms of cp and #. The kinetic energy will depend upon the expression for the length of the arc of the path in terms of ϑ and φ:

$$ds^2 = r^2 \left(d\theta^2 + \sin^2 \theta \, d\varphi^2 \right)$$

Dividing by dt^2 and writing $\dot{\theta} = d\theta/dt, \dot{\varphi} = d\varphi/dt$, we have

$$T = \frac{1}{2} m r^2 \left(d\dot{\theta}^2 + \sin^2 \theta \, d\dot{\varphi}^2 \right), \qquad (2.15)$$

The parameters ϑ and φ are coordinates of the point, since when they are known the position of the point is fully specified. Their time -derivatives $\dot{\vartheta}$ and $\dot{\varphi}$ being time-rates of change of coordinates may be termed velocities, and when they together with ϑ and φ are known, the velocity of the particle may be calculated. The kinetic energy in this case involves both the coordinates ϑ and φ and the velocities $\dot{\vartheta}$ and $\dot{\varphi}$. Inasmuch as the particle in any given position may have any given velocity, the variables ϑ, φ, $\dot{\vartheta}$, $\dot{\varphi}$ are to be considered in this sense as independent, although in any given actual motion they will all be functions of a single variable t.

The form of the function *T* is worthy of attention. It is a homogeneous quadratic function of the velocities $\dot{\vartheta}$ and $\dot{\varphi}$, the coefficients of their squares being functions of the coordinates ϑ and φ, the product term in $\dot{\vartheta}$ and $\dot{\varphi}$ being absent in this case. We may prove that if a point moves on any surface the kinetic energy is always of this form. We may prove that if a point moves on any surface the kinetic energy is always of this form.

In the geometry of surfaces it is convenient to express the coordinates of a point in terms of two parameters q_1 and q_2. Suppose

$$x = f_1(q_1, q_2), y = f_{21}(q_1, q_2), z = f_3(q_1, q_2),$$

from these three equations we can eliminate the two parameters q_1, q_2, obtaining a single equation between *x, y, z,* the equation of the surface. The parameters q_1 and q_2 may be called the coordinates of a point

We may obtain the length of the infinitesimal arc of any curve in terms of q_1 and q_2. We have

$$dx = \frac{\partial x}{\partial q_1} dq_1 + \frac{\partial x}{\partial q_2} dq_2, \; dy = \frac{\partial y}{\partial q_1} dq_1 + \frac{\partial y}{\partial q_2} dq_2, \; dz = \frac{\partial z}{\partial q_1} dq_1 + \frac{\partial z}{\partial q_2} dq_2, \tag{2.16}$$

Squaring and adding,

$$ds^2 = dx^2 + dy^2 + dz^2 = Edq_1^{\,2} + 2Fdq_1 q_2 + Gdq_2^{\,2}, \tag{2.17}$$

where

$$E = \left(\frac{\partial x}{\partial q_1}\right)^2 + \left(\frac{\partial y}{\partial q_1}\right)^2 + \left(\frac{\partial z}{\partial q_1}\right)^2$$

$$F = \frac{\partial x}{\partial q_1}\frac{\partial x}{\partial q_2} + \frac{\partial y}{\partial q_1}\frac{\partial y}{\partial q_2} + \frac{\partial z}{\partial q_1}\frac{\partial z}{\partial q_2}, \tag{2.18}$$

$$G = \left(\frac{\partial x}{\partial q_2}\right)^2 + \left(\frac{\partial y}{\partial q_2}\right)^2 + \left(\frac{\partial z}{\partial q_2}\right)^2$$

Thus the square of the length of any infinitesimal arc is a homogeneous quadratic function of the differentials of the coordinates q_1 and q_2, the coefficients *E*, *F*, *G* being functions of the coordinates q_1, q_2 themselves.

If the coordinate lines cut each other everywhere at right angles we shall have

$$ds^2 = Edq_1^{\,2} + Gdq_2^{\,2}, \tag{2.19}$$

The coordinates q_1, q_2 are then said to be orthogonal *curvilinear* coordinates.

In general we have the equations of change of coordinates,

$$x = r\sin\theta\cos\varphi, \; y = r\sin\theta\sin\varphi, \; z = r\cos\theta, \tag{2.20}$$

from which

$$\frac{\partial x}{\partial\theta}=r\cos\theta\cos\varphi,\ \frac{\partial y}{\partial\theta}=r\cos\theta\sin\varphi,\ \frac{\partial z}{\partial\theta}=-r\sin\theta,$$
$$\frac{\partial x}{\partial\varphi}=r\sin\theta\sin\varphi,\ \frac{\partial y}{\partial\theta}=r\sin\theta\cos\varphi,\ \frac{\partial z}{\partial\theta}=0$$

and

$$E=r^2,\ F=0,\ G=r^2\sin^2\theta$$

Employing the expression (2.17) for the length of the arc, dividing by dt^2 and writing

$$\frac{dq_1}{dt}=\dot q_1,\frac{dq_2}{dt}=\dot q_2$$

we find for the kinetic energy,

$$T=\frac{1}{2}m\left(Ed\dot q_1{}^2+2Fd\dot q_1\dot q_2+Gd\dot q_2{}^2\right), \tag{2.21}$$

This is a typical example of the employment of the *generalised coordinates* introduced by Lagrange interval, q_1 and q_2 being the coordinates, $\dot q_1$, $\dot q_2$, the velocities corresponding, and T being a homogeneous quadratic function or quadratic form in the velocities q[, q2 , the coefficients of the squares and products of the velocities being functions of the coordinates alone. We shall show that this is a characteristic property of the kinetic energy for any system depending upon any number of variables.

In the case of a single free particle we may express the coordinates *x, y, z* in terms of three parameters q_1, q_2, q_3 , and we shall then have as in (2.16) and (2.17)

$$ds^2=E_{11}dq_1^2+E_{22}dq_2^2+E_{33}dq_3^2+2E_{12}q_1q_2+2E_{12}q_1q_2+2E_{31}dq_3dq_1, \tag{2.22}$$

where

$$E_{ij}=\frac{\partial x}{\partial q_i}\frac{\partial x}{\partial q_j}+\frac{\partial y}{\partial q_i}\frac{\partial y}{\partial q_j}+\frac{\partial z}{\partial q_i}\frac{\partial z}{\partial q_j}, \tag{2.23}$$

(here $i, j=1,2,3$)

Proceeding now to the general case of any number of particles, whether constrained or not, let us express all the coordinates as functions of m independent parameters, $q_1,q_2,...q_m$, the generalized coordinates of the system,

$$x_k=x_r(q_1,q_2,...q_m),\ y_k=y_r(q_1,q_2,...q_m),\ z_k=z_r(q_1,q_2,...q_m), \tag{2.24}$$

where $k=1,2,3,\dots,n$

Differentiating, squaring and adding, we obtain

$$ds_k^2=E_{11}^{(k)}dq_1^2+E_{22}^{(k)}dq_2^2+...+E_{mn}^{(k)}dq_m^2+2E_{12}^{(k)}dq_1dq_2+2E_{13}^{(k)}dq_1dq_3+..., \tag{2.25}$$

where

$$E_{ij}^{(k)}=\frac{\partial x_k}{\partial q_i}\frac{\partial x_k}{\partial q_j}+\frac{\partial y_k}{\partial q_i}\frac{\partial y_k}{\partial q_j}+\frac{\partial z_k}{\partial q_i}\frac{\partial z_k}{\partial q_j} \tag{2.26}$$

Thus the square of each infinitesimal arc is a quadratic form in the differentials of all the coordinates *q*. Dividing by dt^2, multiplying by $m_k/2$ and taking the sum for all the particles, we obtain

$$T = \frac{1}{2} m_k \left(E_{11}^{(k)} d\dot{q}_1^2 + E_{22}^{(k)} d\dot{q}_2^2 + \ldots + E_{mn}^{(k)} d\dot{q}_m^2 + 2E_{12}^{(k)} d\dot{q}_1 d\dot{q}_2 + 2E_{13}^{(k)} d\dot{q}_1 d\dot{q}_3 + \ldots \right), \qquad (2.27)$$

Thus the kinetic energy possesses the characteristic property mentioned above of being a quadratic form in the *generalized velocities* $\dot{q}$, the coefficients E_{ij} being functions of only the generalized coordinates q. They must satisfy the conditions necessary, in order that for all assignable values of the q's T shall be positive.

It is sometimes convenient to employ the language of multidimensional geometry. This signifies nothing more than that when we speak of a point as being in n dimensional space we mean that it requires n parameters to determine its position.

Inasmuch as in motion along a curve, that is in a space of *one dimension* we have for the length of arc

$$ds^2 = \left(\frac{ds}{dq} \right)^2 dq^2$$

on a surface, that is in a space of *two dimensions*,

$$ds^2 = Edq_1{}^2 + 2Fdq_1q_2 + Gdq_2{}^2,$$

and in space of *three dimensions*

$$ds^2 = \sum_{i=1}^{3} \sum_{j=1}^{3} F_{ij} dq_i q_j$$

so by analogy , in space of n *dimensions*,

$$ds^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} F_{ij} dq_i q_j \quad , \qquad (2.28)$$

That is to say a quadratic form in n differentials may be interpreted as the square of an arc in n dimensional space. Thus we may assimilate our system depending upon m coordinates to a single point moving in space of n dimensions.

To each possible position of this point corresponds a possible configuration of our system. No matter what be taken as the mass of the point, n, its kinetic energy, $T = (m/2)(ds/dt)^2$ is equal to the kinetic energy of our system, the coefficients in the quadratic form for ds and T being proportional.

The advantage of this mode of speaking (*for it is no more*) may easily be seen from the many analogies that arise, connecting the dynamical theory of least action with the purely geometrical theory of geodesic lines.

This method is adopted by Hertz in his (Hertz, 1894). The ideas involved were first set forth by Beltrami. (Beltrami, 1869).

Hamilton showed that the function S, which is named *action*

$$S = \int_{t_0}^{t} L dt, \qquad (2.29)$$

where $L = T - W$ is Lagrange function, satisfies a certain partial differential equation, a solution of which being obtained, the whole problem is solved:

$$\frac{\partial S}{\partial t} + H\left(t, q_1, .. q_m, \frac{\partial S}{\partial q_1}, \ldots, \frac{\partial S}{\partial q_m} \right) = 0, \qquad (2.30)$$

where $H = T + W$ is the Hamilton function

The equation is of the first order since only first derivatives of *S* appear, and, from the way in which *T* contains the momenta, is of the second degree in the derivatives. Since *S* appears only through its derivatives an arbitrary constant may be added to it.

Hamilton's equation (2.30) assumes a somewhat simpler form when the force-function and consequently *H* are independent of the time that is when the system is conservative. We may then advantageously replace the principal function S by another function called by Hamilton the *characteristic function*, which represents the action *A*. Making use of the equation of energy, $T + W = h$, to eliminate W, we have

$$A = \int_{t_0}^{t} 2T dt = S + h(t - t_0), \tag{2.31}$$

and the above partial differential equation (2.30) becomes merely

$$H\left(q_1, .. q_m, \frac{\partial A}{\partial q_1}, ... \frac{\partial A}{\partial q_m}\right) = h, \tag{2.32}$$

In these system a new variational principle will work; this principle was obtained in 1837 by Jacobi (Encyclpedia of matematics, 2011).

The kinetic energy of a system may be expressed in generalized coordinates q_i as follows:

$$T = \frac{1}{2}\sum_{i,j=1}^{n} a_{ij}\dot{q}_i\dot{q}_j, \tag{2.33}$$

The metric of the coordinate space is given by the formula

$$ds^2 = \frac{1}{2}\sum_{i,j=1}^{n} a_{ij} q_i q_j \tag{2.34}$$

The initial and final positions r_0 and r_1 of the system in some actual motion are also given.

Jacobi's principle of stationary action: *if the initial and final positions of a holonomic conservative system are given, then the following equation is valid for the actual motion*:

$$\delta\int_{r_0}^{r_1} 2(h - W)ds = 0, \tag{2.35}$$

as compared to all other infinitely near motions between identical initial and final positions and for the same constant value of the energy h as in the actual motion.

Jacobi's principle reduces the study of the motion of a holonomic conservative system to the geometric problem of finding the extremals of the variational problem (2.35) in a Riemannian space with the metric (2.34) which represents the real trajectories of the system. Jacobi's principle reveals the close connection between the motions of a holonomic conservative system and the geometry of Riemannian spaces.

If the motion of the system takes place in the absence of applied forces, i.e., $U = 0$, the system moves along a geodesic line of the coordinate space $(q_1, ... q_n)$ at a constant rate. This fact is a generalization of Galilei's law of inertia. If $U \neq 0$, determining the motion of a holonomic conservative system is also reduced to the task of determining the geodesics in a Riemannian space with the metric

$$ds_1^2 = 2(U + h)ds^2 = \frac{1}{2}\sum_{i,j=1}^{n} b_{ij} q_i q_j \tag{2.36}$$

In the case of a single material point, when the line element ds is the element of three-dimensional Euclidean space, Jacobi's principle is the mechanical analogue of Fermat's principle in optics.

These results prove that in a Riemannian form we can write all classical potential fields, not just gravity field.

3. Geometry and Physics of Theory of Elementary Particles (Pseudo-Euclidean or Lorentz-invariant Space)

Let us now consider the connection of interval with physics in the case of the pseudo-Euclidean geometry.

A study of the literature shows that the pseudo-Euclidean coordinates and interval of the four-dimensional space-time are introduced into physics by analogy with the interval of Euclidean geometry (Landau and Lifshitz, 1973)

"*It is frequently useful for reasons of presentation to use a fictitious four-dimensional space, on the axes of which are marked three space coordinates and the time*".

3.1. Interval and square of 4-distance differential

In Cartesian coordinate system of the Euclidean geometry an interval is the distance s between two points on a straight line in space, which is calculated according to the Pythagorean theorem. Since in physics trajectories are often curved lines, the Pythagorean theorem in this case is valid only for the infinitelisemal distances. Therefore, an interval is defined here according to (2.1) as the square root of the square of the distance differential in Euclidean space.

In the pseudo-Euclidean geometry an interval is defined as the square root of the square of the 4- distance differential and is given by the sum (taking into account the summation of Einstein)

$$ds = \sqrt{dx_\mu dx_\mu},$$

where $\mu = 0,1,2,3$ $dx_0 = icdt$.

The square of the interval looks like:

$$(ds)^2 = (ic\,dt)^2 + (d\vec{r})^2 = -c^2(dt)^2 + (dx)^2 + (dy)^2 + (dz)^2$$

Note that currently the imaginary time coordinate is rarely used (although it is by no means a mistake and has certain advantages), and the square of the interval is written as:

$$(ds)^2 = (c\,dt)^2 - (d\vec{r})^2 = c^2(dt)^2 - (dx)^2 - (dy)^2 - (dz)^2, \tag{3.1}$$

$$(ds)^2 = dx_\mu dx_\mu, \tag{3.1'}$$

where $\mu = 1,2,3,4$, and $dx_4 = cdt$. In addition, the squares of differentials are often written without parentheses: ds^2, dx^2, instead of $(ds)^2$, $(dx)^2$, etc ..

Thus, the use of characteristics of the 3-dimensional space in the case of 4-dimensional space – time is a postulate, i.e., some chosen mathematical expression, which is necessary for the construction of special relativity by Minkowski . It also follows from the fact that in nature the length of the arc in the 4- space-time is not measurable.

Therefore the question of the physical meaning of the 4-interval arises. Let's try to answer it.

3.2. Derivation of pseudo-Euclidean interval from the physical equations

The vectors of the Lorentz-invariant (i.e., relativistic) theories necessarily depend on the 4-coordinate: one time coordinate and three space coordinates. In other words, these equations are "working" in a 4-dimensional space-time. Does this theory contain the equations, which have a sum of terms, each of which is associated with one of the four coordinates, like the square of the interval?

As we know, in the first time such equations in classical electrodynamics appear, and then in quantum field theory. The wave equations of these theories include a sum of terms, each of which is associated with one of the variables *t*, *x*, *y*, *z*. It would be logical, to seek the cause and the meaning of the appearance of 4-interval in them, instead of introducing them artificially, as did Minkowski.

Recall that our study of the gravitational field is based on an inhomogeneous wave equation of the so-called "massive photon" (which in mathematical notation is similar to the Klein-Gordon equation). It is an equation for the two vectors of the electric and magnetic fields that give this photon a mass. From this equation follows the well-known equation of conservation of energy and momentum for massive particles, which is easy to obtain also from the definitions of 4-vectors of momentum and energy (see above). (Landau and Lifshitz, 1973)

From (3.1) we can easily obtain:

$$(ds)^2 = c^2(dt)^2 - (d\vec{r})^2 = c^2(dt)^2\left(1-\frac{(dr/dt)^2}{c^2}\right) = c^2(dt)^2\left(1-\frac{\upsilon^2}{c^2}\right), \tag{3.2}$$

At the same time interval is associated with proper time $d\tau$ by relation:

$$ds = c\sqrt{1-\upsilon^2/c^2}\,dt = cd\tau\,, \tag{3.3}$$

For a free material point the concept of the 4-momentum is introduced:

$$p_\mu = mcu_\mu \quad \text{or} \quad p_\mu = (p_0, p_i), \qquad (3,4)$$

where $p_0 = i\dfrac{\varepsilon}{c} = \dfrac{mc}{\sqrt{1-\upsilon_i^2/c^2}}$, $p_i = \dfrac{m\upsilon_i}{\sqrt{1-\upsilon_i^2/c^2}}$, $\varepsilon == \dfrac{mc^2}{\sqrt{1-\upsilon^2/c^2}}$; u_μ is the 4-velocity.

From this:

$$\frac{\varepsilon^2}{c^2} - p_i^2 = m^2c^2\,, \tag{3.5}$$

where the energy and momentum is rewritten for convenience as follows: $p_0 \equiv \varepsilon = mc^2\gamma_L$, $p_i = m\upsilon_i\gamma_L = m(dx_i/dt)\gamma_L$ (where $\gamma_L = 1/\sqrt{1-\upsilon^2/c^2}$ and $\gamma_L^{-1} = \sqrt{1-\upsilon^2/c^2}$ are the Lorentz factor and antifactor, respectively). Hence, in the Cartesian coordinate system:

$$\frac{\varepsilon^2}{c^2} - p_x^2 - p_y^2 - p_z^2 = m^2c^2\,, \tag{3.5'}$$

Since $p_i = m\upsilon_i\gamma_L = m(dx_i/dt)\gamma_L$, a $\varepsilon = mc^2\gamma_L$, this relation can be rewritten as:

$$\gamma_L^2c^2(dt)^2 - \gamma_L^2(dx)^2 - \gamma_L^2(dy)^2 - \gamma_L^2(dz)^2 = c^2(dt)^2\,, \tag{3.6}$$

Multiplying it by γ_L^{-2} , we get:

$$c^2(dt)^2\gamma_L^{-2} = c^2(dt)^2 - (dx)^2 - (dy)^2 - (dz)^2\,, \tag{3.7}$$

Since (see above (3.2)) we got $c^2(dt)^2\gamma_L^{-2} = c^2(dt)^2(1-\upsilon^2/c^2) = (ds)^2$, the expression (3.7) can be written as square of a 4-interval:

$$(ds)^2 = c^2(dt)^2 - (dx)^2 - (dy)^2 - (dz)^2 , \qquad (3.1')$$

In general case of use in Euclidean space of any other, than the Cartesian, coordinate system for recording of the relation (3.5'), particularly, the orthogonal curvilinear coordinates, this interval takes the form:

$$(ds)^2 = g^{\mu\nu}dx_\mu dx_\mu , \qquad (3.8)$$

where $g^{\mu\nu}$ is a so-called metric tensor, whose elements take into account the changes in the projections of the segments of the trajectory of the body on the coordinate axes, at the transition from the Cartesian coordinate system to any other. In a Cartesian system, all elements $g^{\mu\nu}$ are equal to unities.

Obviously, if we go in the opposite direction, we can obtain the equation (3.5') from the square of the interval. This implies, firstly, that these equations - (3.1) and (3.5') - closely bind the massive elementary particles physics and geometry. Secondly, the equation of "massive photon" is derived from Maxwell's equations of a massless photon as a result of his self-interaction of fields (chapter 2).

This non-linearity of a self-acting fields of the "massive photon" does not mean transition from Euclidean to some new geometry. From this it follows that (3.1) is not a metric of pseudo-Euclidean geometry, but it is a metric of Euclidean geometry that describes the Lorentz-invariant field equations. The only change in the geometry, which we can observe in this case is the transition from rectilinear to curvilinear geometry.

In addition, another link between the interval (2.1) and the physical equation is detected. As we have shown in a previous chapters,, using the Schroedinger definition of action ($p_\mu = \partial S/\partial x_\mu$), from the equation (3.5') it is easy obtain Lorentz-invariant Hamilton-Jacobi equation in general view. For this it is enough to write the equation (3.5') in a form, suitable for any of the Euclidean coordinate system:

$$g_{\mu\nu}p^\mu p^\nu = m^2c^2 , \qquad (3.9)$$

where, we recall, $g_{\mu\nu}$ is the metric tensor of geometrical space, but not of the gravitational space-time of general relativity (in other words, in this case the tensor $g_{\mu\nu}$ does not include the physical characteristics of the field). In this case the Hamilton-Jacobi equation of free particles obtains the form:

$$g^{\mu\nu}\left(\frac{\partial S}{\partial x^\mu}\right)\left(\frac{\partial S}{\partial x^\nu}\right) - m^2c^2 = 0 , \qquad (3.10)$$

Recall that the physical field (e.g., electromagnetic field) is included in Hamilton-Jacobi equation in the following way:

$$g^{\mu\nu}\left(\frac{\partial S}{\partial x^\mu} - p_{\mu\ ex}\right)\left(\frac{\partial S}{\partial x^\nu} - p_{\nu\ ex}\right) - m^2c^2 = 0 , \qquad (3.11)$$

Thus, we conclude that the three equations: (3.1) (3.5') and (3.10) are bonded to each other one-to-one and, in fact, are equivalent. From this follows that the interval (3.1) within a relativistic physics is the physical law, and not a geometric relation.

Next, we consider how the 4-interval is introduced in the transition from Euclidean geometry to *Riemann geometry*.

4. Geometry and Physics in the Pseudo-Riemannian Space

In general relativity an interval similar to (3.8) is introduced (postulated), where $g^{\mu\nu}$ takes into account the peculiarities of the Riemann geometry. But the most important thing here is other: in general relativity, it is postulated that, due to the transition to the Riemann geometry, the metric tensor $g^{\mu\nu}$ is a function of the gravitational field $g^{GR}_{\mu\nu}$.

Whether this is proved by experiment, we do not know because all the experimental confirmation of general relativity are obtained for problems in the pseudo-Euclidean metric.

Another fact also raises the question about the significance of Riemann geometry in physics. As we know, all theories of physics, except the GTR, are built in a Euclidean space, although mathematically, relativistic theories can be constructed in the pseudo-Euclidean space. But there is no such theory, which needs the introduction of the Riemann geometry.

Let us write the interval GTR as follows:

$$(ds)^2 = g^{GR}_{\mu\nu} dx^\mu dx^\nu , \qquad (4.1)$$

where the metric tensor $g^{GR}_{\mu\nu}$ contains the characteristics of the gravitational field.

In addition, instead of the equation for the external field (3.11) in general relativity the equation of external field of type (3.10) is taken, but with the appropriate metric tensor $g^{GR}_{\mu\nu}$:

$$g^{GR}_{\mu\nu}\left(\frac{\partial S}{\partial x^\mu}\right)\left(\frac{\partial S}{\partial y^\nu}\right) - m^2c^2 = 0 , \qquad (4.2)$$

The question is, why is there such a difference between (3.10) and (4.2), as well as between (3.8) and (4.1), and why is the external field in GTR inserted through the metric tensor?

To answer this question, we will try to find out the physical sense of the metric tensor. Let us turn first to Euclidean geometry.

4.1. The physical sense of the metric tensor of curvilinear coordinates' system of the Euclidean geometry

Recall the generalized coordinate system and particularly, curvilinear coordinates. (Korn and Korn, 1968)

Let us introduce a new set of coordinates q_1, q_2, q_3, so that among x, y, z and q_1, q_2, q_3 there are some relations

$$x = x(q_1, q_2, q_3),\ y = y(q_1, q_2, q_3),\ z = z(q_1, q_2, q_3), \qquad (4.3)$$

The differentials are then

$$dx = \frac{\partial x}{\partial q_1} dq_1 + \frac{\partial x}{\partial q_2} dq_2 + \frac{\partial x}{\partial q_3} dq_3 , \qquad (4.4)$$

and the same for *dy* and *dz*.

In Cartesian coordinates the measure of distance, or metric, in a given coordinate system is the arc length ds, which is defined by

$$ds^2 = dx^2 + dy^2 + dz^2, \tag{4.5}$$

In general, taking into account (4.4), from (4.5) we obtain

$$ds^2 = g_{11}dq_1^{\ 2} + g_{12}dq_1dq_2 + \ldots = \sum_{ij} g_{ij}dq_idq_j \ , \tag{4.6}$$

where g_{ij} is the metric tensor. Thus in orthogonal system we can write

$$ds^2 = (H_1dq_1)^2 + (H_2dq_2)^2 + (H_3dq_3)^2, \tag{4.7}$$

where the H_i's are

$$H_i = \sqrt{\left(\frac{\partial x}{\partial q_i}\right)^2 + \left(\frac{\partial y}{\partial q_i}\right)^2 + \left(\frac{\partial z}{\partial q_i}\right)^2}\ , \tag{4.8}$$

are called Lame coefficients or scale factors, and are 1 for Cartesian coordinates.

Thus, the Riemann metric tensor, recorded in coordinates q_i, is a diagonal matrix whose diagonal contains the squares of Lame coefficients:

For example, in the case of spherical coordinates, the bond of spherical coordinates with Cartesian is given by (2.20).

The Lame coefficients in this case are equal to: $H_r = 1$, $H_\theta = r$, $H_\varphi = r\sin\theta$, and the square of the differential of arc (interval) is:

$$ds^2 = dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\varphi^2$$

Since the metric tensor is determined by means of Lame coefficients, let us recall the geometric meaning of the latter: the Lame coefficients show how many units of length are contained in the unit of length of coordinates of the given point, and used to transform vectors when transition from one system to another takes place.

This means that the metric tensor in Euclidean geometry defines rescaling of three coordinates r, θ, φ, and in the pseudo-Euclidean or pseudo-Riemannian geometry it determines rescaling of four coordinates t, r, θ, φ.

As we will show in the folloing chapter, from the solution of the Kepler problem within LIGT, the relativistic corrections within LIGT correspond to changes of scales t and r, caused by the Lorentz-invariant effects (time dilation and Lorentz-Fitzgerald length contraction). In the next article, we will show that the same thing occurs in problems of a moving source. Note that this is the case for problems of stationary and moving source.

Thus, we conclude that relationships (4.1) and (4.2) have metric tensor $g_{\mu\nu}^{GR}$ as a factor that takes into account the change of scales of time and distance due to relativistic effects associated with motion of bodies.

5. Consequences

From the foregoing analysis follows that by regular way the interval of a 4-space-time can be obtained only for the pseudo-Euclidean space, as a variant of the physical law of motion of elementary particles.

Since there is no other law of motion for massive particles, we can assume that the hypothesis of Einstein that the gravitational field is created by the curvature of space-time, which requires a transition to a pseudo-Riemannian geometry, needs considerable adjustment.

Following the theory of mass generation (chapter 2), we have to conclude that the gravitational field arises from the self-action of massless fields. It is indeed accompanied by the transformation of the linear movement of fields in curvilinear motion (mathematically, this is the transition from linear equations to nonlinear equations). But this has nothing to do with the Riemann geometry.

It can be assumed that the use of the Riemann geometry in GR is possible for the reason that math in the case of the Riemann geometry is very close to the math physics using generalized coordinates of Euclidean geometry..

Chapter 9. The equivalence principle and metric tensor of LIGT

1.0. Equivalence of inertial and gravitational masses and its consequences

Interpretation of the equivalence of inertial and gravitational masses by Einstein led him to assertion that the theory of gravity can not be a Lorentz-invariant theory, but it should be a general relativistic theory in a Riemannian – non-flat - space-time.

At the same time a characteristic feature of the Lorentz-invariant theory is a flat space-time. Can we solve this contradiction between our approach and the approach of general relativity?

1.1. Is GTR an L-invariant theory?

It is known that general relativity is considered a relativistic theory, but it is not a L-invariant theory (see "Chapter 1", or (Katanaev, 2013, pp. 777-778))

The general relativity principle, according to Einstein's hypothesis, should be a generalization of the Lorentz-invariance of the special relativity theory. As such principle, Einstein proclaimed the requirement of general covariance. As is known, most physicists - see, e.g., Hilbert, Fock, Logunov (Polak, 1959; Fock, 1964; Logunov, 2002) - do not consider the general covariance to be equivalent with some type of relativity, which generalizes the Lorentz-invariance. This follows from the fact that any Lorentz-invariant theory can always be written in covariant form.

Thus, the absence of such a generalization makes the Lorentz- invariance a basic requirement for any relativistic theory. The real space of such theories is Euclidian (or, conditionally, taking into account time, it is pseudo-Euclidian). Obviously, this is also valid for the gravitation theory. Hence, the Riemannian space is not a real space, but a mathematical model. Indeed, the assertion that the real space is Riemannian is not supported by theory or experiment.

As Jacobi have theoretically shown (see about Jacobi's geometrization of Newton's theory of gravitation in chapter 8 or in (Polak, 1959; Encyclopaedia of matematics, 2011), any conservative potential fields of Newtonian mechanics can be written in the form of Riemann geometry. Thus, in this case the Riemannian space is not a geometric space, but, rather, a physical field in a geometric shape.

As is known, there are numerous representations of the physical characteristics in the geometric form, and they are often very useful and productive. For example, the use of the n-dimensional configuration space provides a new mathematical apparatus. In the modern theory space of different measurements is widely used (including fractional measurements). But hardly anyone would agree that these spaces exist in nature.

Attempts to prove experimentally, that the space of the universe is Riemannian space, gave negative results. Recent measurements (Plank collaboration, 2013a, b) show that the space of the universe is flat with an accuracy to one tenth of a percent. This means that in fact the space-time of universe is Euclidean, or if we operate with 4-forms, pseudo-Euclidean. Thus, we must recognize that in terms of a physical theory the space is always Euclidean space, but physical problems can be formulated mathematically in terms of geometric forms, including the Riemannian space.

Let us analyze the possibility of describing the gravitational interaction without the involvement of a Riemannian space.

1.2. Einstein interpretation of the equivalence of inertial and gravitational masses in building a theory of gravitation

Let us see first of all how in modern physics is described the transition of Einstein from Euclidean space to Riemann, the starting point of which was the principle of equivalence. Is it possible to give a different interpretation of the equivalence of gravitational and inertial mass?

In an inertial reference system, the free motion of all bodies is uniform and rectilinear, and if, say, at the initial time their velocities are the same, they will be the same for all times. Clearly, therefore, if we consider this motion in a given non-inertial system, then relative to this system all the bodies will move in the same way.

The fundamental property of gravitational fields that all bodies, independently of mass, move in them in the same way, remains valid also in relativistic mechanics. The properties of the motion in a non-inertial system are the same as those in an inertial system in the presence of a gravitational field. In other words, a non-inertial reference system is equivalent to a certain gravitational field. This is called the principle of equivalence.

A somewhat more general case is a non-uniformly accelerated motion of the reference system: it is clearly equivalent to a uniform but variable gravitational field (Landau and Lifshitz, 1951).

"*Consequently, their acceleration depends only on the point in space where they happen to be. Can we, therefore, attribute the gravitational characteristics (acceleration) to the points in space, where the bodies are, rather than to the bodies themselves? However, Minkowski's flat space-time doesn't have the properties needed to implement this idea: it is homogeneous, that is, everywhere uniform and isotropic (the same in all directions). This means that the components of the metric (the metric tensor)* $\bar{g}_{\alpha\beta}$ *are constant '(their individual moduli are either zero or unity). Consequently, we need space-time whose metric tensor has components* $g_{\alpha\beta}(x)$ *that change from point to point, i.e. the space-time should be curved This enables us to consider geometrical properties of space-time that change at different points*" (Vladimirov et al, 1987, pp. 40-41)

«*Universal gravitation does not fit into the framework of uniform Galilean space. The deepest reason for this fact was given by Einstein. It is that not only the inertial mass, but also the gravitational mass of a body depends on its energy.*

It proved possible to base a theory of universal gravitation on the idea of abandoning the uniformity of space as a wholef and attributing to space only a certain kind of uniformity in the infinitesimal. Mathematically, this meant abandoning Euclidean, or rather pseudo-Euclidean, geometry in favour of the geometry of Biemann » (Fock, 1964) .

Let us see first of all how in modern physics is described the transition of Einstein from Euclidean space to Riemann, the starting point of which was the principle of equivalence. Is it possible to give a different interpretation of the equivalence of gravitational and inertial mass?

Why is the dependence of the gravitational field on the coordinates and time in Newton's theory $f = f(x,t)$, not suitable to describe the gravity, and the dependence $g_{\alpha\beta} = g_{\alpha\beta}(x,t)$ in general relativity is the only possibility? The use of the heterogeneity of space-time is not tenable. Firstly, it is, in fact, Einstein's postulate who has identified this heterogeneity with Riemannian space-time. Secondly, in a heterogeneous space the conservation laws are not valid, since they are only valid in a homogeneous and isotropic space.

In other words, the authors try to justify a posteriori the introduction of the Riemann geometry by Einstein. This means that we have the right to try to give a different interpretation to the fact of the equivalence of masses.

But above all, it is not clear how is Einstein's equivalence principle related to the Riemann space.

Let us begin with the formulation of the principle of equivalence which Einstein gave himself:

"*A little reflection will show that* ***the law of the equality of the inertial and gravitational mass is equivalent to the assertion that the acceleration imparted to a body by a gravitational field is independent of the nature of the body****. For Newton's equation of motion in a gravitational field, written out in full, it is:*

(Inertial mass) .(Acceleration) =(Intensity of the gravitational field) .(Gravitational mass).

It is only when there is numerical equality between the inertial and gravitational mass that the acceleration is independent of the nature of the body" (Einstein, 2005).

Let us consider the mathematical basis of the principle of Einstein's equivalence and try to give this mathematics another form.

As we can see, Einstein relied on the Newtonian law of motion of a particle with inertial mass m_{in} in a gravitational field of source with a mass M:

$$m_{in}\frac{d\vec{\upsilon}}{dt} = \gamma_N \frac{m_{gr}M}{r^2}\vec{r}^{\,0} \;, \tag{1.1}$$

where m_{gr} is gravitational mass. Since $m_{in} = m_{gr} = m$, then dividing (1.1) by m we obtain in the case of gravitation the movement equation of the form:

$$\frac{d\vec{\upsilon}}{dt} = \gamma_N \frac{M}{r^2}\vec{r}^{\,0} \;, \tag{1.1'}$$

where acceleration is on the left and the Newton force per unit mass is on the right.

It is easy to see that this equation is the mathematical expression of Einstein's abovementioned principle of equivalence: the power (ie, action) of Newton's gravity exerted on the unit mass (i.e., local point mass), coincides with the acceleration of the moving body in this field (i.e., with the force of inertia acting per unit mass).

Our second question was whether it is possible to give another explanation to this principle.

1.3. Interpretation of the equivalence of inertial and gravitational masses in framework of LIGT

In the GTR we assume that in the (pseudo-) Euclidean reference frame, "*the noninertial frames possessed spatial and temporal inhomogeneities that show up as inertial forces, that depend on the specific characteristics of the reference frame. Obviously, the inertial forces have to have a noticeable effect on the physical processes in these reference frames*" (Vladimirov et al, 1984, p. 38).

In this case, this heterogeneity does not generate the real Riemannian space-time (although, following the example of Jacobi, this heterogeneity can be displayed mathematically as a Riemannian space).

Indeed, this heterogeneity can be considered as inhomogeneity of the field in space and time of the real Euclidean space and time, but not as heterogeneity of spacetime itself. In accordance with this our interpretation of the principle of equivalence is as follows.

The gravitational field, and not the space and/or time, sets the variable speed of body motion. Therefore, if the field depends on space and time, it is not necessary to bind the body velocity with time and space; it is enough to relate this speed with field itself. **Thus, all we need is to describe the action of force on the movement of the body, to find this relation between field and speed.**

It appears, that based on mathematics, we can actually find this connection. As is known, the equation (1.1 ') can be represented in the energy form.

For this let us rewrite the Newton's motion law in the form:

$$d\vec{\upsilon} = \gamma_N \frac{M}{r^2}\vec{r}^{\,0}dt \;, \tag{1.2}$$

Multiplying the left and right hand side of equation (1.2) on the speed $\vec{\upsilon}$, and taking into account that $\vec{\upsilon} = d\vec{r}/dt$ and $d\vec{r}/r^2 = -d(1/r)$, we have from (1.2) after integration:

$$\frac{\upsilon^2}{2} + \gamma_N \frac{M}{r} = const \,, \tag{1.3}$$

where $\frac{\upsilon^2}{2} = \varepsilon_\kappa / m$ is the kinetic energy of the moving particle per unit mass, and $\gamma_N \frac{M}{r} = \varepsilon_{pot} / m$ is the potential energy of a particle per unit mass at a given point of the gravitational field.

Thus, taking into account the postulate of equivalence and the expression for the potential of the gravitational field $\varphi_N = \gamma_N M / r$, we obtain from (1.1), the relationship between the velocity of the particle and potential of the gravitational field at the position of the particle:

$$\upsilon^2 = 2\varphi_N + const, \tag{1.4}$$

Obviously, we can assume that constant "*const*" is either the square of the particle velocity or the double potential at the initial point of reference of the particle motion.

If at the initial moment a particle was at rest, and the motion is only carried out via the potential energy outlay, then during the whole period of motion *const* = 0. For example, this occurs when the reference frame, that is related to the observer, falls freely to the center of gravity source along the radius (*radial infall*) from infinity, where it had a zero velocity. In this case, we have:

$$\upsilon^2 = 2\varphi_N = \frac{2\gamma_N M}{r}, \tag{1.5}$$

Thus, as a mathematical consequence of Newton's theory of gravity, we have received another interpretation of the fact of the equality of inertial and gravitational mass. Following the example of Einstein's equivalence principle, it can be expressed as follows: the potential of the gravitational field is equivalent to the square of the velocity of the motion of particles in this field.

In addition, (see chapter 4) the electromagnetic basis of gravitational equations allows one to write the vector potential of the gravitational field through the scalar potential.

2.0. Peculiarities of metric tensor of LIGT

As is known, Einstein came to the metric tensor of the pseudo-Riemannian space on the basis of interpretation of the experimental fact of the equality of inertial and gravitational mass as a so-called Einstein equivalence principle.

Above we have given a different interpretation of the equivalence of masses, in order to obtain an expression for the metric tensor of L-invariant theory of gravitation.

As we mentioned (Chapter 8), in differential geometry, the metric tensor elements are equal to the squares of the Lame scale coefficients. The Lame coefficients indicate how many units of length are contained in the unit coordinates in a given point and are used to transform vectors in the transition from one coordinate system to another.

At the same time, in the framework of general relativity, these two coordinate systems represent basically two dissimilar geometric coordinate systems from a number of well-known rectangular, oblique, or any other coordinates.

In contrast, in the L-invariant transformation is examined the transition between two identical from geometric point of view, coordinate systems, which are attached to two reference frames moving relative to each other. Moreover, it was found that a simultaneously this transition requires to take into account the transformation of time.

It is clear that this is not about geometric relationships which may not affect the final results of the solution of physical problems. We proved it by showing that the square of the arc element (interval) in this case is a consequence of the well-known relation between the energy, momentum and mass of the moving particle. Thus, these changes are purely physical. They contribute to the correction of physical problems non-relativistic physics.

However, conventionally this interval can be seen as a geometric object that generates a pseudo-Euclidean geometry, which has in addition to three spatial coordinates, one time coordinate (4-Minkowski geometry, in which SRT is often formulated).

From this geometrical point of view, coordinates and time undergo the change of the scales. These changes can be considered, along with changes of coordinates that take place during the transition between two different coordinate systems. But we should not forget that from the physical point of view it is a completely different transformations and changes of scales.

3.0. Calculation of metric tensor of LIGT

The linear arc element in the 3-dimensional mechanics is expressed through Lame's scale factors in the form of linear elements:

$$ds = \sum_{i=1}^{3} h_i dx_i = h_1 dx_1 + h_2 dx_2 + h_3 dx_3 , \qquad (3.1)$$

where $x_i = \vec{r} = (x_1, x_2, x_3)$, $i = 1, 2, 3$. . In a Cartesian coordinate system $x_i = \vec{r} = (x, y, z)$, and all the Lame coefficients equal to one.

In the L-invariant mechanics it is impossible to enter the line element of the arc since the physical equation, which shows the magnitude of the arc, connects the squares of the energy, momentum and mass, and not the first degrees of these values. The exact expression is obtained in the form of the square of length of arc element, which is often referred to simply as an interval. In the 4-geometry it is of the form:

$$(ds)^2 = \sum_{\mu-0}^{3} (h_\mu dx_\mu)^2 = (h_0 dx_0)^2 + (h_1 dx_1)^2 + (h_2 dx_2)^2 + (h_3 dx_3)^2 , \qquad (3.2)$$

or, taking into account that $g_{\mu\mu} = h_\mu h_\mu$, (3.2) receives the form:

$$(ds)^2 = \sum_{\mu-0}^{3} g^L_{\mu\mu} (dx_\mu)^2 = g^L_{00} (dx_0)^2 + g^L_{11} (dx_1)^2 + g^L_{22} (dx_2)^2 + g^L_{33} (dx_3)^2 , \qquad (3.2')$$

where $x_\mu = (ict, \vec{r}) = (ict, x_i) = (x_0, x_i)$ $\mu = 0, 1, 2, 3$, $g^L_{\mu\mu}$ is metric tensor in LIGT.

3.1. The Lorentz-Fitzgerald length contraction and time dilation as a change of the scales of coordinates of space and time in LIGT

Using the definition of the metric tensor in LITG given above, let us calculate it in the simplest case. Consider (Pauli, 1958) Lorentz transformation in the transition from the coordinate system *K* to *K*', which is currently moving at a speed υ along the axis x. In this case only the coordinates and time t undergo transformations.

The Lorentz effects of length contraction and time dilation are the simplest consequences of the Lorentz transformation formulae, and thus also of the two basic assumptions of SRT.

$$x = \frac{x' - \upsilon t'}{\sqrt{1 - \upsilon^2 / c^2}} , \; y = y' , \; z = z' , \; t = \frac{t' - \frac{\upsilon}{c^2} x'}{\sqrt{1 - \upsilon^2 / c^2}} , \qquad (3.3)$$

The transformation which is the inverse of (1) can be obtained by replacing υ by $-\upsilon$:

$$x = \frac{x' + \upsilon t'}{\sqrt{1 - \upsilon^2 / c^2}} , \; y = y' , \; z = z' , \; t = \frac{t' + \frac{\upsilon}{c^2} x'}{\sqrt{1 - \upsilon^2 / c^2}} , \qquad (3.3a)$$

Take a rod lying along the x-axis, at rest in reference system K'. The position coordinates of its ends, x_1' and x_2' are thus independent of t' and $x_2' - x_1' = l_0$ is the rest lengtl of the rod. On the other hand, we might determine the length of the rod in system K' in the following way. We find x_1 and x_2 as functions of t. Then the distance between the two points which coincide simultaneously with the end points of the rod in system K will be called the length l of the rod in the moving system: $x_2(t)-x_1(t)=l$

Since these positions are not taken up simultaneously in system K', it cannot be expected that l equals l_0. In fact, it follows from (3.3) :

$$x_2' = \frac{x_2(t)-\upsilon t'}{\sqrt{1-\upsilon^2/c^2}}; \quad x_1' = \frac{x_1(t)-\upsilon t'}{\sqrt{1-\upsilon^2/c^2}}$$

for infinitesimal time intervals of length dx has form $dx' = \frac{dx}{\sqrt{1-\upsilon^2/c^2}}$.

From here the scaling factor of the Lorentz transformation of coordinates (denote it as k_x^L) will be equal to:

$$k_x^L = \frac{dx'}{dx} = \frac{1}{\sqrt{1-\upsilon^2/c^2}} = \gamma_L, \tag{3.4}$$

where $\gamma_L = 1\big/\sqrt{1-\upsilon^2/c^2}$ is the Lorentz factor.

The corresponding element λ_{xx} of the metric tensor of the Lorentz transformation will be:

$$\lambda_{xx} = \frac{dx'}{dx}\frac{dx'}{dx} = (\gamma_L)^2, \tag{3.5}$$

The rod is therefore contracted in the ratio $\sqrt{1-\upsilon^2/c^2}:1$, as was already assumed by Lorentz…. It therefore follows that the Lorentz contraction is not a property of a single measuring rod taken by itself, but is a reciprocal relation between two such rods moving relatively to each other, and this relation is in principle observable.

Analogously, the time scale is changed by the motion. Let us again consider a clock which is at rest in K'. The time t' which it indicates in x' is its proper time, τ and we can put its coordinate x' equal to zero. It then follows from (3.3a) that $t = \frac{\tau}{\sqrt{1-\upsilon^2/c^2}}$, which for infinitesimal time intervals dt give: $dt = \frac{dt'}{\sqrt{1-\upsilon^2/c^2}}$.

From here the scaling factor of the Lorentz transformation of time (denote it as k_t^L) will be equal to:

$$k_t^L = \frac{dt'}{dt} = \sqrt{1-\upsilon^2/c^2} = \gamma_L^{-1}, \tag{3.6}$$

The corresponding element λ_{xx} of the metric tensor of the Lorentz transformation will be:

$$\lambda_{tt} = \frac{dt'}{dt}\frac{dt'}{dt} = (\gamma_L)^{-2}, \tag{3.7}$$

Measured in the time scale of K, therefore, a clock moving with velocity υ will lag behind one at rest in K in the ratio $\sqrt{1-\upsilon^2/c^2}:1$. While this consequence' of the Lorentz transformation was already implicitly contained in Lorentz's and Poincare's results, it received its first clear statement only by Einstein.

Then, in framework of LITG the square interval will be as follows:

$$(ds)^2 = \sum_{\mu-0}^{3} \lambda_{\mu\mu}\eta_{\mu\mu}(dx_\mu)^2 = \lambda_{00}\eta_{00}(dx_0)^2 + \lambda_{11}\eta_{11}(dx_1)^2 + \lambda_{22}\eta_{22}(dx_2)^2 + \lambda_{33}\eta_{33}(dx_3)^2, \tag{3.8}$$

where $\eta_{\mu\mu}$ is the geometric metric tensor in LIGT (tensor of pseudo-Euclidian space); $\lambda_{\mu\mu}$ is the physical metric tensor in LIGT. Using the values $\lambda_{00} = \lambda_{tt}$ and $\lambda_{11} = \lambda_{xx}$, according to (3.5) and (3.7), we obtain in the Cartesian system of coordinates:

$$(ds)^2 = -(\gamma_L)^{-2}(dt)^2 + (\gamma_L)^2(dx)^2 + (dy)^2 + (dz)^2, \qquad (3.9)$$

Taking into account (1.5) it is easy to see that (3.9) corresponds to the Schwarzschild-Droste solution.

4.0. Relation between Lorentz factor and characteristics of the Newton gravitational field

The main characteristic of the Lorentz transformation is the Lorentz factor γ_L: $\gamma_L = 1/\sqrt{1-\beta^2}$ (where $\vec{\beta} = \vec{\upsilon}/c$), which is determined by the speed of motion of the body $\vec{\upsilon} = \vec{\upsilon}(\vec{r}, t)$.

The vector of speed of the particle motion can be considered as its main component, by which its trajectory, acceleration and some other quantities are determined.

On the base of our interpretation of the principle of equivalence of mass, we found relation between field and speed.

In the case of Newton's theory, probably the first, that found this relationship was E.A.Milne (Milne, 1934). Later, independently, and from an other primary bases, this was also done by Arnold Sommerfeld assistant - Wilhelm Lenz. He took advantage of this connection to find a solution to the Kepler problem, which coincides with the results of the Schwarzschild-Droste solution of Einstein-Hilbert equation (Sommerfeld, 1952a).

Below we will expand this relationship to the case of the Lorentz-invariant mechanics, to obtain the next approximations in the form of a power series.

Using (1.5) it is easy to find an expression for the Lorentz-factor due to the gravitational field of Newton:

$$\gamma_L = \frac{1}{\sqrt{1-2\varphi_g/c^2}} \text{ or } \gamma_L = \frac{1}{\sqrt{1-r_S/r}}, \qquad (4.1)$$

where $r_S = \frac{2\gamma_N M}{c^2}$ is the, so-called, Schwarzschild radius.

5.0. Connection between the characteristics of the gravitational field and the Lorentz factor, taking into account the Lorentz-invariant generalization of mechanics

Within the framework of the Lorentz-invariant theory, the particle mass is a function of velocity and position in the field: $m = m(\vec{r}, \vec{\upsilon})$

Let us recall the relativistic relations, binding the energy and momentum in relativistic mechanics.

The law of conservation of the energy-momentum is valid for each of the material particles:

$$\varepsilon_f^2 = c^2 p^2 + m_0^2 c^4, \qquad (5.1)$$

where c is the speed of light, m_0 means the particle rest mass, and the full energy ε_f and momentum $\vec{p}$ are defined via relativistic expressions:

$$\varepsilon_f = m_0 c^2 \gamma_L, \quad \vec{p} = m_0 \vec{\upsilon} \gamma_L = \frac{\varepsilon}{c^2}\vec{\upsilon}, \qquad (5.2)$$

where, $\gamma_L = 1/\sqrt{1-\beta^2}$ called Lorentz-factor, where $\beta = \vec{\upsilon}/c$, $\vec{\upsilon}$ is speed of a particle. Besides, in the relativistic mechanics the kinetic energy ε_κ is entered by the following expression:

$$\varepsilon_\kappa = \varepsilon_f - m_0c^2 = m_0c^2\gamma_L - m_0c^2 = m_0c^2(\gamma_L - 1), \tag{5.3}$$

Since $\upsilon < c$, the expressions, containing δ, can be expanded to Maclaurin series (we take here into account only 4 terms):

$$\gamma_L = 1 + \left\{\frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \frac{5}{16}\beta^6 + \frac{35}{128}\beta^8 + ...\right\}, \tag{5.4}$$

$$\gamma_L^{-1} = 1 - \frac{1}{2}\beta^2 - \frac{1}{8}\beta^4 - \frac{1}{16}\beta^6 - \frac{5}{128}\beta^8 + \ldots, \tag{5.5}$$

$$\beta\gamma_L = 0 + \beta + 0 + \frac{1}{2}\beta^3 + \ldots \tag{5.6}$$

Thus we can obtain for energy and momentum the following expressions:

$$\varepsilon_f = m_0c^2 + m_0c^2\left\{\frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \frac{5}{16}\beta^6 + \frac{35}{128}\beta^8 + ...\right\}, \tag{5.7}$$

$$\begin{aligned}\varepsilon_\kappa = \varepsilon_f - m_0c^2 = m_0c^2\left\{\frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \frac{5}{16}\beta^6 + \frac{35}{128}\beta^8 + \ldots\right\} = \\ = \frac{1}{2}m_0\upsilon^2 + \frac{3}{8}m_0\left(\frac{\upsilon^4}{c^2}\right) + \frac{5}{16}m_0\left(\frac{\upsilon^6}{c^4}\right) + \frac{35}{128}m_0\left(\frac{\upsilon^8}{c^6}\right) + \ldots\end{aligned}, \tag{5.8}$$

$$p = m_0\upsilon + \frac{1}{2}m_0\frac{\upsilon^3}{c^2} + ..., \tag{5.9}$$

At $\beta << 1$ we obtain from (5.7)-(5.9) as first approximation the non-relativistic expressions:

$$\varepsilon_f \approx m_0c^2 + \frac{1}{2}m_0\upsilon^2; \quad p \approx m_0\upsilon; \; \varepsilon_k \approx \frac{1}{2}m_0\upsilon^2; \tag{5.10}$$

As we can see, the energy and momentum of a particle can be represented as an expansion in powers of the parameter υ^2/c^2.

Let us consider the problem of motion of a particle of mass m_0 in a gravitational field of a source mass $M >> m_0$. It is obvious that under the influence of gravitational field the particle changes its position and speed. Due to a change in position, the corresponding potential of the gravitational field, changes also and hence the potential energy of the particle and the force of gravity acting on the particle.

According to Newton's theory of gravity, the potential energy of a particle is equal to $\varepsilon_g = m_0\varphi_N = \gamma_N m_0 M/r$. A change of the speed of the particle is accompanied by a change in its kinetic energy (5.3):

$$\begin{aligned}\frac{\gamma_N m_0 M}{r} = m_0\varphi_N = \varepsilon_k = \frac{1}{2}m_0\upsilon^2 + \frac{3}{8}m_0\left(\frac{\upsilon^4}{c^2}\right) + \frac{5}{16}m_0\left(\frac{\upsilon^6}{c^4}\right) + \frac{35}{128}m_0\left(\frac{\upsilon^8}{c^6}\right) + \ldots = \\ = m_0c^2\left\{\frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \frac{5}{16}\beta^6 + \frac{35}{128}\beta^8 + \ldots\right\}\end{aligned}, \tag{5.11}$$

In the case of sufficiently small velocities, we obtain a first approximation, $\gamma_N M/r \approx \varphi'_N = \upsilon^2/2$ or; $2\gamma_N M/r \approx 2\varphi'_N = \upsilon^2$, from where:

$$\frac{2\gamma_N M}{c^2 r} = \frac{r_S}{r} \approx \frac{2\varphi'_N}{c^2} = \frac{\upsilon^2}{c^2} = \beta^2, \tag{5.12}$$

where $r_S = 2\gamma_N M/c^2$ is called the gravitational radius of the body of mass M (Schwarzschild radius).

We can use the expressions (5.4) and (5.6), which determine the energy (kinetic and full) and the momentum to give more precisely the motion of a particle in a gravitational field. For this it is possible to clarify the relation between φ_N and r_S by using the following term of the expansion in (5.11) to obtain a second approximation:

$$\frac{2\gamma_N M}{c^2 r} \approx \frac{2\varphi''_N}{c^2} = \frac{\upsilon^2}{c^2} + \frac{3}{4}\frac{\upsilon^4}{c^4} = \frac{2\varphi'_g}{c^2} + \frac{3}{4}\left(\frac{2\varphi'_g}{c^2}\right)^2 = \frac{r_S}{r} + \frac{3}{4}\frac{r_S^2}{r^2}, \tag{5.13}$$

Using the first approximation (5.4.14), we obtain for the second approximation:

$$\frac{2\varphi''_N}{c^2} = \frac{\upsilon^2}{c^2} + \frac{3}{4}\frac{\upsilon^4}{c^4} = \frac{2\varphi'_g}{c^2} + \frac{3}{4}\left(\frac{2\varphi'_g}{c^2}\right)^2 \approx \frac{r_S}{r} + \frac{3}{4}\frac{r_S^2}{r^2}, \tag{5.14}$$

6.0. Calculation of gravitation field potentials

Thus, according to our axioms, it is sufficient to calculate the gravitational field of the potentials to be able to enter the L-invariant amendments to the gravitational theory of Newton.

In particular, for the calculation of the metric tensor elements of LIGT (see formulae (3.8), (3.9) and (4.1)) we use the potential $\varphi_g = \varphi_N$ of the Newtonian gravitation theory. It is remarkable that this non-relativistic potential gives the relativistic corrections to the solution of the Kepler problem.

Since LITG is based on electromagnetic theory, this calculation is not difficult. We only briefly recall the results of this approach, adequately set out in the chapter 4.

As we have seen, the Maxwell-Lorentz equations can be written in potentials in the form of equations of the electromagnetic field propagation. Using $\vec{E} = -grad\varphi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}$, $\vec{B} = rot\vec{A}$, in the case of the Lorentz condition $\frac{1}{c}\cdot\frac{\partial \varphi}{\partial t} + \nabla\vec{A} = 0$, we have:

$$\frac{1}{c^2}\cdot\frac{\partial^2 \varphi}{\partial t^2} - \nabla^2\varphi = 4\pi\rho , \tag{6.1}$$

$$\frac{1}{c^2}\cdot\frac{\partial^2 \vec{A}}{\partial t^2} - \nabla^2\vec{A} = \frac{4\pi}{c}\rho\vec{\upsilon} , \tag{6.2}$$

From a mathematical point of view, these equations are the d'Alembert non-homogeneous equation. Its solution is known (see chapter 4). Additionally, it turns out that the vector potential associates with the scalar potential by expression:

$$\vec{A} = \varphi\frac{\vec{\upsilon}}{c}, \tag{6.3}$$

In this case, the main characteristics of the electromagnetic vector field are the scalar and vector potentials φ and $\vec{A}$, respectively, or the 4-potential $A_\mu = \left(\frac{\varphi}{c}, -\vec{A}\right)$. ($A_\mu = \frac{\varphi}{c^2}u_\mu$, where $u_\mu = dx_\mu/d\tau$ is 4-speed , dx_μ is 4-movement , τ is the proper time of the particle).

If the system contains a set of particles, each of which generates its own potential, then the potentials φ and $\vec{A}$ of the system of particles depend mainly on the general system parameters – the dimensions of the system, the total charge, etc. It is very important that the calculation of the system potential is defined by the superposition principle, i.e., by summation of the potentials of all the particles. Thus we can determine all the main characteristics of the system’s electromagneticfield with the help of the 4-potential.

But before we will find the 4- potential of the system, we need to determine the potentials of a single particle.

As it is known, it is the centrally symmetric potential of Newton that defines the field of a point particle. As we have seen (see chap. 4), the calculation of the potential of a system of point sources - i.e., of a body with known charge density requires the integration of the potential of al l particles over the volume of the body.

Turning to gravitation, it can be expected that in the relativistic theory of gravity, along with the scalar potential φ_g there should be also a vector potential $\vec{A}_g$. There is also no doubt that the relationship (6.3) is valid in the case of the gravitational field. General relativity confirms this, and a great achievement for GR was to demonstrate this characteristic.

Thus, the solutions of equations (6.1) and (6.2) allow us to calculate both Lorentz factor and relativistic corrections to Newton's theory of gravitation.

Chapter 10. The solution of non-cosmological problems in framework of LIGT

1.0. Introduction.

In this chapter we will look at the problems arising at description of a test particle motion in the gravitational field of some motionless or moving source, as well as their solutions under the Lorentz-Invariant Gravitation Theory (LIGT). We will show that the solution for the moving body is connected with the solution for the fixed body on the basis of the Lorentz transformations.

Under the moving masses (gravitational charges) we will still, understand the two interacting bodies, one of which we call the source of the gravitational field, and the other - the test particle.

2.0. Universal gravitational problem

The Kerr metric is the generalization of Lense-Thirring and Schwarzschild-Droste metrics. In this regard, the question arises whether the other metrics are more common than Kerr-metric. Obviously, in framework of LITG the answer to this question must be sought in the electromagnetic theory.

The EM theory has one source of electromagnetic fields - electric charge. At the same time in LIGT it is a mass. And the charge is invariant with respect to L-transformation, as well as the rest mass.

The fixed charge is the source of the Coulomb field (Newton's gravitational field, respectively). The rotation of this charge leads to appearance of a magnetic field (gravimagnetic field, respectively).

Rotation of test charge around the rotating charge in the framework of classical physics is described in the same way as the rotation of one mass around another. All Lorentz (relativistic) effects (L-effects) in the EM case have parallels in GR: the precession of the orbit, changing of the frequency of light and the deviation of the trajectory of photons (taking into account the non-linear theory of Heisenberg-Euler; see (Heisenberg and Euler, 1936).

Since the Maxwell-Lorentz electromagnetic theory is a universal description of the motion of the charges and the rays of light, no other issues can occur here.

If we apply this conclusion to the gravitation theory in the framework of LITG, we must conclude that the universal problem must describe the motion of the test particle with spin around the quick-rotating source of any mass.

3.0. Amendments to the Newton gravitation theory as a manifestation of the L-invariant effects

In this chapter, we show how the effects of L- transformation, determine the metric.

First we will find the solution of the Kepler problem: the test particle rotation around a stationary massive source. We have already discussed this solution based on the Hamilton-Jacobi equation (see chapter 7)). In a previous article (chapter 8) we have shown that in the framework of LIGT the equation of square of the interval is equivalent to the equation of conservation of energy and to the Hamilton-Jacobi equation. Here, we will use the Lenz approach for obtaining the corresponding square of the interval, which is also convenient for solution of other tasks of the gravitation theory within the framework LITG.

Assume that only one compact spherically symmetric mass exists in the Universe and that space-time is asymptotically characterized (at the spatial infinity) by the pseudo-Euclidean metric (the square of infinitesimal interval):

$$(ds)^2 = ds_\mu ds^\nu = H_\mu H_\nu x^\mu x^\nu \ , \qquad (3.1)$$

where $\mu,\nu = 0,1,2,3$, H_μ are the Lamé coefficients or scale factors (conditionally accepting here $H_0 = c$, where c is velocity of light), and $H_\mu H_\nu = g_{\mu\nu}$ (the summation is done over μ,ν).

In spherical coordinates it can be given as

$$(ds')^2 = (cdt)^2 - (d\vec{r})^2 - r^2 d\theta^2 - r^2 \sin^2\theta (d\varphi)^2, \tag{3.2}$$

It is easier to operate on linear differential forms (linear element of interval) rather than on separate components of square of interval. These are defined as

$$ds_\mu = H_\mu x^\mu \quad , \tag{3.3}$$

In our case

$$ds_0' = cdt,\ ds_1' = dr,\ ds_2' = rd\theta,\ ds_3' = r\sin\theta d\varphi, \tag{3.4}$$

so that for a distant observer, the coordinates t, r, θ, φ have the sense of standard spherical coordinates (and can for instance be measured by standard methods).

3.1. The Lenz solution of the Kepler problem

It turns out that it is sufficient to take into account the two Lorentz effects: time dilation and lengthening distances. The result coincides with the solution of the equations of general relativity by Schwarzschild-Droste.

We shall show (Sommerfeld, 1952) that this equivalence principle suffices for the elementary calculation of the $g_{\mu\nu}$ in a specific case (on the basis of an unpublished paper of W. Lenz, 1944).

Consider a centrally symmetric gravitational field, e.g. that of the sun, of mass M, which may be regarded as at rest. Let a reference frame (box) K' fall in a radial direction toward M. Since it falls freely, K' is not aware of gravitation (as the consequence of the equivalence principle, i.e., $m_{grav} = m_{inert}$) and therefore carries continuously with itself the Euclidean metric valid at infinity ∞. Let the coordinates measured within it be x_∞ (longitudinal, i.e. in the direction of motion), y_∞ , z_∞ (transversal), and t_∞. K' arrives at the distance r from the sun with the velocity υ. υ and r are to be measured in the reference frame K of the sun, which is subject to gravitation. In it we use as coordinates r, ϑ, φ, and t. Between K' and K there exist the relations of the special Lorentz transformation, where K' plays the role of the system “moving" with the velocity $\upsilon = \beta c$, K that of the system "at rest".

Since the time and space scales are essentially the basis for the frame relative to which the measurements are done the freely falling basis carried by the observer from infinity is related to the basis of reference frame (the one the observer passes at a given instant) as follows:

$$ds'_0 = dt\sqrt{1-\beta^2} \quad \text{(Lorentz dilatation)}, \tag{3.5}$$

$$ds'_1 = \frac{dr}{\sqrt{1-\beta^2}} \quad \text{(Lorentz contraction)}, \tag{3.5'}$$

$$\left.\begin{aligned} ds'_2 &= rd\vartheta \\ ds'_3 &= r\sin\vartheta d\varphi \end{aligned}\right\} \quad \text{(Invariance of the transversal lengths)} \tag{3.5''}$$

Hence the Euclidean world line element

$$ds^2 = ds'^2_0 + ds'^2_1 + ds'^2_2 + ds'^2_3, \tag{3.6}$$

passes over into

$$ds^2 = -c^2 dt^2 (1-\beta^2) + \frac{dr^2}{1-\beta^2} + r^2\left(d\vartheta^2 + \sin^2\vartheta d\varphi^2\right), \tag{3.7}$$

The factor $\left(1-\beta^2\right)$, which occurs here twice, is meaningful so far only in connection with our specific box experiment. In order to determine its meaning in the system of the sun we write down the energy equation for K, as interpreted by an observer on K. Let m be the mass of K', m_0 its rest mass . The equation then is:

$$(m - m_0)c^2 - \frac{\gamma_N mM}{r} = 0 \ , \tag{3.8}$$

At the left we have the sum of the kinetic energy and of the (negative) potential energy of gravitation, i.e.. T + V = 0. The energy constant on the right was to be put equal to zero since at infinity $\infty \quad m = m_0$ and $r = \infty$. We have computed the potential energy from the Newtonian law, which we shall consider as a first approximation. We divide (3.8) by mc^2 and obtain then, since $m = m_0\sqrt{1-\beta^2}$:

$$1 - \sqrt{1-\beta^2} = \frac{\alpha}{r}, \quad \alpha = \frac{GM}{c^2} = \frac{\kappa M}{8\pi}, \tag{3.9}$$

where κ is the Einstein constant. It follows from (3.9) that

$$\sqrt{1-\beta^2} = 1 - \frac{\alpha}{r}, \quad 1 - \beta^2 \approx 1 - \frac{2\alpha}{r} \ , \tag{3.10}$$

From (3.7) we have:

$$ds^2 = -c^2 dt^2 \left(1 - \frac{2\alpha}{r}\right) + \frac{dr^2}{1 - \frac{2\alpha}{r}} + r^2\left(d\vartheta^2 + \sin^2\vartheta d\varphi^2\right), \tag{3.11}$$

This is the line element derived by K. Schwarzschild from the GTR equation. In Eddington's presentation the 40 components $\Gamma^{\sigma}_{\mu\nu}$ of the gravitational field are computed and (3.11) is shown to be the exact solution of the ten equations contained in the GTR equation. Our derivation claims only to yield an approximation, since it utilises the Newtonian law as first approximation and 'neglects, in the second equation (3.10), the term $(\alpha/r)^2$; nevertheless, our result is, as shown by Schwarzschild and Eddington, exact in the sense of Einstein's theory.

For the single point mass it is completely described by the four coefficients of the line element (3.11) and the vanishing of the remaining $g_{\mu\nu}$.

The difference between the Lenz approach and GTR is that in the first is not used the hypothesis about geometrical origin of gravitation.

Perhaps the only book, in which the authors, to obtain the results of GTR, have used the Lenz approach, is the review of the problems of gravitation in book (Vladimirov et al, 1987)

In this book, along with the Schwarzschild solution, by means of Lenz method are obtained the solutions of Lense-Thirring and Kerr for the metric around a rotating body, and solutions of Reissner-Nordstrom and Kerr-Newman, when this source has an electric charge. These solutions we will present below.

3.2. Gravitational fields around rotating source

To begin with it (Vladimirov et al, 1987) is worth discussing some general properties of rotation. In order to describe the rotation of a rigid body an angular velocity Ω is introduced in addition to the conventional (linear) velocity V of a point of the body, because the angular velocity is constant for a rigid rotation, whereas the linear velocity of any point of the body is proportional to the distance between the point and the axis of rotation.

The relationship between angular and linear velocities in cylindrical coordinates is

$$V = \Omega\rho, \tag{3.12}$$

and in spherical coordinates (here $\rho = r\sin\theta$).

$$V = \Omega r\sin\theta, \tag{3.13}$$

However, a body can rotate not as a rigid one (for example, Jupiter's atmosphere rotates with different angular velocities at different latitudes as a result having different periods of rotation). The rotation period is related to angular velocity thus: $T = 2\pi/\Omega$. Hence the angular velocity may depend on position (coordinates) of point.

A reference frame may be rotating, too; though a rigid body rotation is even less natural for such a system than a rotation with different angular velocities at different points. Also, if a reference frame extended to infinity could rotate as a rigid body, that is, with a constant angular velocity Ω, then a linear velocity at a finite distance from its axis (on a cylinder $\rho = c/\Omega$) would reach the velocity of light c, and outside of this "light cylinder" would surpass it. Obviously, this kind of reference frame is impossible to simulate for any material bodies, therefore, the angular velocity of any realistic reference frame must change with distance from the axis. The slowdown must not be less than inversely proportional to that distance. But there should be a domain, well within the light cylinder, where the reference frame would rotate as a rigid body.

A rotating physical body possesses an angular momentum L as a conserved characteristic, which in certain respects is related to energy and momentum, which are also subject to the conservation laws.

In Newton's theory, mass (or energy, divided by the velocity of light squared) is the source of a gravitational field, while linear and angular momenta have no such a role. In the GR, however, a gravitational field is generated by a combination of distributions of energy, and linear and angular momenta, and the stress, too.

Let us examine the angular momentum of an infinitely thin ring (which has, however, a finite mass M_r) rotating around its axis. This angular momentum is a vector which is directed along the axis of rotation and has an absolute value of

$$L = M_r VR = M_r \Omega R^2 \ , \tag{3.14}$$

where M_r is the mass of the ring, V is its linear velocity, Ω is its angular velocity, and R is the radius of the ring.

Now, let us try to obtain a metric which describes the gravitational field around the rotating ring using a technique like the above W. Lenz technique.

3.3. The Lense-thirring metrics in framework of LIGT

To account for rotation effects (Vladimirov et al, 1987) using the equivalence principle, we start from a rotating reference frame (it rotates not as a rigid body, but so that at large distances the effect of the rotation weakens; the nature of the frame rotation will be examined at the final stage of this analysis). In this rotating frame, we let a box with an observer (test particle) fall towards the gravitating centre and in the box we take into account the slowing-down of the clock and the contraction of the scales in the direction of the fall. Assume that the box falls radially in the rotating frame. Then, we shall get back to the initial, non-rotating reference frame and consider the result.

We begin with the Euclidean space-time in which we introduce spherical coordinates in a non-rotating frame; we assume the basis is, thus relative to it the flat space-time metric will be

$$(ds')^2 = (cdt)^2 - (d\vec{r})^2 - r^2 (d\theta)^2 - r^2 \sin^2\theta (d\varphi)^2, \tag{3.15}$$

A transition to a non-uniformly rotating reference frame is done by locally applying Lorentz transformations so that every point has its own speed of motion directed towards an increasing angle φ. The absolute value of this velocity is a function V which depends, generally speaking, on the coordinates r and θ: $V = V(r,\theta)$.

Such a local Lorentz transformation is not equivalent to the transformation of the coordinates in the domain studied (in practice this domain is the whole of space) but is limited only to the transformation of the basis at each point.

Thus, we have:

$$d\tilde{s}_0 = \left(ds'_0 - \frac{V}{c} ds'_3 \right) \Big/ \sqrt{1 - V^2/c^2}\,,\ d\tilde{s}_1 = ds'_1$$
$$d\tilde{s}_2 = ds'_2\,,\ d\tilde{s}_3 = \left(ds'_3 - \frac{V}{c} ds_0' \right) \Big/ \sqrt{1 - V^2/c^2}\,, \qquad (3.16)$$

Since the motion is assumed to be slow, we will henceforth ignore the value V^2/c^2 in comparison with unity.

Now let the box with the observer be released from infinity. In this case we can write a new basis in which time has slowed down, and the lengths in radial direction have shortened. This is equivalent to the substitution of the ds'_0 in (3.5) by the basis linear elements from (3.16)

$$ds''_0 = d\tilde{s}_0 \sqrt{1 - \upsilon^2/c^2}\,,\ ds''_1 = d\tilde{s}_1 \Big/ \sqrt{1 - \upsilon^2/c^2}\,,$$
$$ds''_2 = d\tilde{s}_2\,,\ ds''_3 = d\tilde{s}_3 \qquad (3.17)$$

Thus, we have assumed that the observer makes his measurements in the rotating frame and notices the relativistic changes in his observations. (We can not neglect by the value of υ^2/c^2 in comparison with unity; see the derivation of the Schwarzschild metric).

Now let us do the reverse transformation to the non-rotating reference frame by applying Lorentz transformations (inverse to (3.16)) to the basis (3.17):

$$ds_0 = ds''_0 + \frac{V}{c} ds''_3\,, ds_1 = ds''_1\,,$$
$$ds_2 = ds''_2\,, ds_3 = ds''_3 + \frac{V}{c} ds''_0\,, \qquad (3.18)$$

We now insert into (3.18) the ds''_μ $\omega'^{(\alpha)}$ basis, which is expressed in terms of the $d\tilde{s}_\mu$ $\tilde{\omega}^{(\alpha)}$ from (3.17), and then write this expression in terms of the ds'_μ $\omega_\infty^{(\alpha)}$ from (3.16), after a few manipulations, we obtain:

$$ds^2 = \left(1 - \frac{\upsilon^2}{c^2}\right) c^2 dt^2 - \frac{dr^2}{\left(1 - \frac{\upsilon^2}{c^2}\right)} - r^2\left(\sin^2\theta\, d\varphi^2 + d\theta^2\right) + \frac{2V\upsilon^2}{c^2} r \sin^2\theta\, d\varphi dt\,, \qquad (3.19)$$

The principle of correspondence with Newtons theory gives $\upsilon^2 = 2\gamma_N M_s/r$.

It remains to clear out the dependence V from r and θ. On the one hand, according to equation (3.13), $V = \Omega r \sin\theta$. However, it is clear that the reference frame can not rotate as a solid body. Therefore, the angular velocity Ω must be a function of the point. Since the reason for the existence of this velocity is eventually the rotation of central mass, we can assume that it decreases in all directions away from the center. For a rough estimate, it can be assumed that Ω depends only on r. Then from (3.14) :

$$L = M_r \Omega(R) R^2\,, \qquad (3.20)$$

(because the ring lies in a plane $\theta = \pi/2$). If we now require that the field does not depend on the choice of the radius of the ring, but only on its angular momentum, it is natural to take for a function Ω the expression

$$\Omega = (L/M_r) r^{-2}\,, \qquad (3.21)$$

Let us introduce the notation for "parameter Kerr" $a = L/M_r$, so that

$$V = \frac{a \sin\theta}{r}\,, \qquad (3.22)$$

The substitution of the values V and expression for υ^2/c^2 into the formula (3.19) finally gives the metric of Lense-Thirring

$$ds^2 = \left(1-\frac{r_s}{r}\right)c^2dt^2 - \frac{dr^2}{\left(1-\frac{r_s}{r}\right)} - r^2\left(\sin^2\theta\, d\varphi^2 + d\theta^2\right) + \frac{4\gamma_N M_s a}{c^2 r}\sin^2\theta\, d\varphi dt, \qquad (3.23)$$

$r_s = \dfrac{2\gamma_N M_s}{c^2}$ is gravitational radius, M_s is a mass of central body (a field source), $a = L/M_s$ represents the angular momentum of the source per unit mass; more precisely stated, it is the projection of the angular momentum three-vector on the direction of the rotation axis, divided by the mass).

The obtained metric is the approximate metric in the sense that the dimensionless quantities km/c^2r and $a/c\,r$ are considered as small values of first order, and we have neglected their higher degrees. But at the beginning, for simplicity we have made the assumption about a coordinate system as the normal spherical coordinate system, which is, of course, not suitable for a rotating body because its gravitational field should have the symmetry of an oblate spheroid.

3.4. The Kerr Metric in framework of LIGT

Now (Vladimirov et al, 1987), let us try to obtain a metric which describes the gravitational field around the rotating ring using a technique like the one we used above for the Lense-Thirring metric.

To do this, at first we must pass to the ellipsoidal coordinates, and secondly, use Newtonian potential source (ring). If in accordance with what has been said we minimally modify the formula (3.22) without discarding any terms (of the type V^2/c^2 in (3.18), we can directly come to the exact Kerr metric.

We will begin with (3.16):

$$d\tilde{s}_0 = \left(ds'_0 - \frac{V}{c}ds'_3\right)\Big/\sqrt{1-V^2/c^2}\,,\ d\tilde{s}_1 = ds'_1$$
$$d\tilde{s}_2 = ds'_2\,,\ d\tilde{s}_3 = \left(ds'_3 - \frac{V}{c}ds'_0\right)\Big/\sqrt{1-V^2/c^2}\,, \qquad (3.16)$$

and then pass to (3.17)

$$ds''_0 = d\tilde{s}_0\sqrt{1-\upsilon^2/c^2}\,,\ ds''_1 = d\tilde{s}_1\Big/\sqrt{1-\upsilon^2/c^2}\,,$$
$$ds''_2 = d\tilde{s}_2\,,\ ds''_3 = d\tilde{s}_3\,, \qquad (3.17)$$

Thus, we have assumed that the observer makes his measurements in the rotating frame and notices the relativistic changes in his observations. Now let us do the reverse transformation to the nonrotating reference frame by applying Lorentz transformations (inverse to (3.16)) to the basis (3.17):

$$ds_0 = \left(ds''_0 + \frac{V}{c}ds''_3\right)\Big/\sqrt{1-V^2/c^2}\,, ds_1 = ds''_1\,,$$
$$ds_2 = ds''_2\,, ds_3 = \left(ds''_3 + \frac{V}{c}ds''_0\right)\Big/\sqrt{1-V^2/c^2}\,, \qquad (3.24)$$

We now insert into (3.24) the basis ds''_μ, which is expressed in terms of the $d\tilde{s}_\mu$ from (3.17), and then write this expression in terms of the ds'_μ from (3.16). We postulate, as we did previously, that the resulting basis (3.24) remains orthonormalized. A few manipulations yield:

$$ds^2 = \left(1 - \frac{\upsilon^2}{c^2 - V^2}\right)c^2 dt^2 - \frac{dr^2}{\left(1 - \frac{\upsilon^2}{c^2}\right)} - r^2 d\theta^2 - \left(1 - \frac{\upsilon^2 V^2}{c^2 - V^2}\right) r^2 \sin^2\theta \, d\varphi^2 + \qquad (3.25)$$
$$+ \frac{2V\upsilon^2}{c^2 - V^2} r \sin^2\theta \, d\varphi dt$$

Here the Newton's potential φ_N represents a solution of the Laplace equation, though under the new symmetry, that is rotational and not spherical. Therefore it is now worth considering oblate spheroidal coordinates in flat space. These coordinates, ρ, θ, and φ are defined as

$$x + iy = (\rho + ia)e^{i\varphi}\sin\varphi, z = \rho\cos\theta$$
$$\frac{x^2 + y^2}{\varphi^2 + a^2} + \frac{z^2}{\rho^2} = 1, r = \sqrt{x^2 + y^2 + z^2} \quad , \qquad (3.26)$$

We know that $\Delta(1/r) = 0$ when $r \neq 0$, and this equality holds under any translation of coordinates. Let this translation be purely imaginary and directed along the z axis, i.e., $x \to x, y \to y,$ and $z \to z - ia/c$. Then we easily find that $cr \to \left(c^2 r^2 - a^2 - 2iacz\right)^{1/2} = c\rho - ia\cos\theta$. From here the expression for Newton's potential follows,

$$\varphi_N = -\frac{\gamma_N M_s}{c^2 r} \to \varphi_N = \frac{\gamma_N M_s}{c} \mathrm{Re} \frac{1}{c\rho - ia\cos\theta} = \frac{\gamma_N M_s \rho}{c^2\rho^2 + a^2\cos^2\theta}, \qquad (3.27)$$

since the Laplace equation is satisfied simultaneously by both the real and imaginary parts of the potential. Hence we can get with the help of $\upsilon^2 = 2\gamma_N M_s / r = 2\varphi_N$:

$$\frac{\upsilon^2}{c^2 - V^2} = \frac{\gamma_N M_s \rho}{c^2\rho^2 + a^2\cos^2\theta} \quad , \qquad (3.28)$$

We determine the velocity V using a model of rotating ring of some radius ρ_0 for the source of the Kerr field, this ring being stationary relative to the rotating reference frame (3.17).

On the one hand, $V = \Omega\left(x^2 + y^2\right)^{1/2} = \left(\rho^2 + a^2/c^2\right)^{1/2}\Omega\sin\theta$ corresponds to the relation (3.13) . On the other hand, it is clear that the reference frame cannot rotate as a rigid body, otherwise the frame wouldn"t be extensible beyond the light cylinder as we dropped our box from infinity. Therefore the angular velocity Ω has also to be a function of position.

The ring lies naturally in the equatorial plane, so that its angular momentum is

$$L = mV\sqrt{\left(\rho^2 + a^2/c^2\right)}, \quad \left(\rho = \rho_0, \theta = \frac{\pi}{2}\right)$$

We now introduce an important hypothesis which establishes a connection between the angular momentum and the Kerr parameter *a*, which is also a characteristic for spheroidal coordinates (3.26), namely we put $a = L/M_s$. These last three statements yield

$$\Omega\left(\rho = \rho_0, \ \theta = \pi/2 = c^2 a / \left(c^2\rho_0^2 + a^2\right)\right)$$

If we now add a second hypothesis, that the field is independent of the choice of the ring radius (depending only on its angular momentum), then naturally we can get for Ω:

$$\Omega = c^2 a / \left(c^2\rho^2 + a^2\right)$$

and finally

$$V = ca\left(c^2\rho^2 + a^2\right)^{-1/2}\sin\theta, \qquad (3.29)$$

It only remains for us to choose the expression for a basis ds'_μ which would correspond to the assumed rotational symmetry (i.e., to the oblique spheroidal coordinates). We may substitute the coordinates x, y and z from (3.26) into the pseudo-Euclidean squared interval, $ds^2 = cdt^2 - dx^2 - dy^2 - dz^2$, hence getting a quadratic form with a non-diagonal term. This term, which contains $d\rho d\varphi$, can be excluded by a simple change of the azimuth angle: $d\varphi \to d\varphi + ca\left(c^2\rho^2 + a^2\right)^{-1} d\rho$ thus leading to a diagonal quadratic form. If now the square roots of the separate summands are taken, we get the final form of the initial basis ds'_μ:

$$\begin{aligned} &ds'_0 = c^2 dt, \quad ds'_1 = \sqrt{\left(c^2\rho^2 + a^2\cos^2\theta\right)/\left(c^2\rho^2 + a^2\right)}\,d\rho, \\ &ds'_2 = \sqrt{\rho^2 + a^2\cos\theta/c^2}\,d\theta, \qquad , \\ &ds'_3 = \sqrt{\rho^2 + a^2/c^2}\,\sin\theta d\varphi, \end{aligned} \tag{3.30}$$

A mere substitution of these expressions into (3.25) yields the standard form of the Kerr metric in terms of the Boyer-Lindquist coordinates,

$$\begin{aligned} ds^2 = &\left(1 - \frac{r_s r}{\rho^2}\right)c^2 dt^2 - \frac{\rho^2}{\Delta}dr^2 - \rho^2 d\theta^2 - \\ &-\left(r^2 + a^2 + \frac{r_s r a^2}{\rho^2}\sin^2\theta\right)\sin^2\theta\, d\varphi^2 + \frac{2r_s r a}{\rho^2}\sin^2\theta\, d\varphi dt \end{aligned} \quad , \tag{3.31}$$

where we have introduced the notation

$$\Delta = r^2 - r_s r + a^2, \quad \rho^2 = r^2 + a^2\cos^2\theta, \quad a = L/M_S, \tag{3.32}$$

The resulting metric is a solution of Einstein's gravitational field equations, and the method does give some hint as to how to understand the Kerr metric and its sources, and it lets us look at the structure of the latter.

If we assume in the calculations that $(V/c)^2 << 1$, thus dropping the corresponding terms in (3.16) and (3.24). This is the assumption of slow rotation (more exactly, of the smallness of L, the angular momentum of the source) and it leads to $V = a\sin\theta/r$ instead of (3.29). Thus instead of the Kerr metric (3.31) we will get the approximate the Lense-Thirring metric:

$$ds^2 = \left(1 - \frac{r_s}{r}\right)c^2 dt^2 - \frac{dr^2}{\left(1 - \frac{r_s}{r}\right)} - r^2\left(\sin^2\theta\, d\varphi^2 + d\theta^2\right) + \frac{4\gamma_N M_s a}{c^2 r}\sin^2\theta\, d\varphi dt,$$

(we have written in it r instead of ρ and taken into account the approximate sense of the expressions).

3.5. The Reissner-Nordstroem metric in framework of LIGT

Besides the Schwarzschild (non-rotating) and Kerr (rotating) black holes, which have no electric charge, we also have exact solutions of Einstein's equations when the source has an electric charge. These solutions are referred to as the Reissner-Nordstroem and Kerr-Newman metrics..

The Reissner-Nordstrem and Kerr-Newman black holes are mostly of academic interest, but they are important for the theory.

The Reissner-Nordstroem metric can be "derived" (Vladimirov et al, 1987) using the same technique we used for the Schwarzschild metric. The only difference between the two is that the Newtonian potential for a point mass should be replaced by a solution of the Poisson equation for a distributed source,

$$\Delta\varphi_N = 4\pi\gamma_N \rho_m, \tag{3.33}$$

The point mass remains at the origin and yields the same potential $(-\gamma_N m/r)$, but the electrostatic source has a trick of its own. In Newton's theory, the ρ_m on the right-hand side of equation (3.33) is usually interpreted as the density of mass (or energy, since from special relativity mass and energy are equivalent). That was the case, however, only for nonrelativistic matter, whereas an electromagnetic, or even an electrostatic field is always relativistic though it might appear at rest. It can be rigorously shown that for such a field we have to take instead of $\rho_m c^2$ *double* the energy density

$$\Delta\varphi_N = 8\pi\gamma_N \, w/c^2 , \tag{3.34}$$

The density of the energy of a Coulomb electrostatic field (i.e. of $E = q/r^2$) is

$$w = q^2/8\pi r^4 , \tag{3.35}$$

and the Laplace operator Δ in a spherically symmetric case (when $\varphi_N \neq \varphi_N(\varphi,\theta)$) takes the form

$$\Delta\varphi_N = \frac{1}{r}\frac{d^2}{dr^2}(r\varphi_N), \tag{3.36}$$

Bearing this in mind, we have, as a complete solution of equation (3.34), the Newtonian potential

$$\varphi_N = -\gamma_N m/r + \gamma_N q^2/2c^2 r^2 , \tag{3.37}$$

which enters the 00-component of the metric tensor in the form:

$$g_{00} = 1 + 2\varphi_N/c^2 = 1 - \gamma_N m/r + \gamma_N q^2/2c^2 r^2 , \tag{3.38}$$

Hence, by doing exactly what we did in above sections, we finally obtain the Reissner-Nordstroem field in the form:

$$ds^2 = \left(1 - \frac{r_s}{r} + \frac{\gamma_N q^2}{c^4 r^2}\right)c^2 dt^2 - \frac{dr^2}{\left(1 - \frac{r_s}{r} + \frac{\gamma_N q^2}{c^4 r^2}\right)} - r^2\left(\sin^2\theta\, d\varphi^2 + d\theta^2\right), \tag{3.39}$$

Conclusion

Thus, within the framework of the non-geometric gravitational theory - LITG – we have obtained in framework of LIGT all the exact noncosmological solutions of the equations of GR, which were verified experimentally. The objective of our next article will be to receive in framework of LIGT the cosmological solutions of the equations of GR.

Chapter 11. Solution of the Kepler problem in the framework of LIGT

Introduction

In present chapter, based on results of previous chapter 9, we consider the solution of the Kepler problem, i.e., the solution of the problem of motion of two bodies in a centrally symmetric gravitational field of a stationary source. It is shown that this solution coincides with that obtained in GR.

As the motion equation of LITG we use the Hamilton-Jacobi equation (Chapter 8). According to Chapter 7, the equation of motion of Hamilton-Jacobi has a one-to-one connection with the square of the interval (square of arc element of trajectory) in framework of LITG. Therefore, as we will show below, it is not necessarily to find an appropriate interval to write the corresponding Hamilton-Jacobi equation for particle motion in gravitation field.

1.0. Effects of Lorentz transformation

A consequence of the previously adopted axiomatics (chap. 3) of Lorentz-invariant gravitation theory (LIGT) is the assertion that all features of the motion of matter in the gravitational field owed their origin to effects associated with the Lorentz transformations. This means that the elaboration of the equations of Newton's gravitation must follow from considering of these effects.

As is well known (Becker, 2013), these questions can be considered without special relativity theory, using only the Maxwell equations.

Effects, that owe their existence to the Lorentz transformations are discussed in many textbooks devoted to the EM theory or SRT (Becker, 2013; Pauli, 1981; et al.). We will not dwell on their withdrawal, and we will only briefly mention some of them.

From the Lorentz transformations follows the velocity transformation, showing that no body can overcome the speed of light. From the Lorentz speed transformations follow the time dilation and length contraction in a moving frame of reference, as well as the transformation of energy and momentum. The use of invariance properties of the wave phase with respect to the Lorentz transformations, allows to obtain the relativistic formula of Doppler effect, aberration, reflection from a moving mirror, Wien's displacement law, etc.

1.1. The transition from Newtonian mechanics to the Lorentz-invariant mechanics

Let us try (Becker, 2013) to alter the Newtonian equations so that they satisfy the Lorentz transformations. We begin by considering the motion of a particle in a given force field (e.g., electromagnetic or gravitational). Newtonian equations of motion read as follows:

$$m\frac{d\vec{\upsilon}}{dt} = \vec{F}_L\,, \tag{1.1}$$

where $\vec{F}_L$ is, e.g., the Lorentz force :

$$\vec{F}_L = q\vec{E} - \frac{q}{c}\vec{\upsilon}\times\vec{H}\,, \tag{1.2}$$

Now we will try to give this equation the Lorentz-invariant form. Obviously, the Lorentz-invariant version of the equation (1.1) instead of the classical time t must contain the proper time $\tilde{t}$:

$$m\frac{d\vec{\upsilon}}{d\tau} = \vec{F}_L\,, \tag{1.1'}$$

In order to find this version of the equation, we replace in (1.1') its proper time in line with the ratio for the Lorentz time dilation $d\tilde{t} = dt\sqrt{1-\beta^2}$ on $dt\sqrt{1-\beta^2}$:

$$m_0 \frac{d}{dt}\frac{\vec{\upsilon}}{\sqrt{1-\beta^2}} = q\vec{E} - \frac{q}{c}\vec{\upsilon}\times\vec{H} , \qquad (1.3)$$

As is known, the equation (1.3) is the Lorentz-invariant equation of motion of a charged particle in an EM field.

Below we will consistently apply this method to obtain the relativistic equations of gravitation in the form of Hamilton-Jacobi equations.

2.0. Solution of the Kepler problem in the framework of LIGT

Two of the most important effects from the point of view of mechanics that arise due to the Lorentz transformations, are the Lorentzian time dilation and contraction of lengths:

$$d\tilde{t} = dt\sqrt{1-\beta^2} , \quad d\tilde{r} = \frac{dr}{\sqrt{1-\beta^2}} , \qquad (2.1)$$

where, as shown previously, $\beta^2 = r_S/r$, and r_S is the Schwarzschild radius.

The free particle motion is described by the Hamilton-Jacobi equation Landau and Lifshitz, 1971):

$$\frac{1}{c^2}\left(\frac{\partial S}{\partial t}\right)^2 - \left(\vec{\nabla}S\right)^2 = m^2c^2 , \qquad (2.2)$$

In a spherical coordinate system (taking into account both relativistic effects) it takes the form:

$$\frac{1}{c^2}\left(\frac{\partial S}{\partial \tilde{t}}\right)^2 - \left(\frac{\partial S}{\partial \tilde{r}}\right)^2 - \frac{1}{r^2}\left(\frac{\partial S}{\partial \theta}\right)^2 - \frac{1}{r^2}\frac{1}{\sin^2\theta}\left(\frac{\partial S}{\partial \varphi}\right)^2 = m^2c^2 , \qquad (2.3)$$

where $\tilde{t}$ and $\tilde{r}$ are measured in a fixed coordinate system associated with a stationary spherical mass M .

We will start with the account of the first effect

2.1. The equation of motion of a particle in a gravitational field, taking into account the relativistic effect of time dilation

Taking into account that the motion of a particle around the source occurs in the plane, we define this plane by condition $\theta = \pi/2$. In this case, the equation (2.3) takes the form:

$$\frac{1}{c^2}\left(\frac{\partial S}{\partial \tilde{t}}\right)^2 - \left(\frac{\partial S}{\partial \tilde{\vec{r}}}\right)^2 - \frac{1}{r^2}\left(\frac{\partial S}{\partial \varphi}\right)^2 = m^2c^2 , \qquad (2.4)$$

Taking into account only the transformation of time $d\tilde{t} = dt\sqrt{1-\beta^2}$ (see (2.1)), equation (2.4) can be rewritten as follows:

$$\frac{1}{1-\beta^2}\left(\frac{\partial S}{\partial t}\right)^2 - c^2\left(\frac{\partial S}{\partial \vec{r}}\right)^2 - \frac{1}{r^2}\left(\frac{\partial S}{\partial \varphi}\right)^2 = m^2c^4 , \qquad (2.5)$$

Substituting $1-\beta^2 = 1 - r_s/r$, we obtain:

$$\frac{1}{1-\frac{r_s}{r}}\left(\frac{\partial S}{\partial t}\right)^2 - c^2\left(\frac{\partial S}{\partial \vec{r}}\right)^2 - \frac{1}{r^2}\left(\frac{\partial S}{\partial \varphi}\right)^2 = m^2c^4 , \qquad (2.6)$$

Let us simplify this equation, taking into account the expansion $1/(1-x)=1+x+x^2+...+x^n$ for $x<<1$. Since for the actual sizes of the planets and Sun and the distances between them, value $r_s/r<<1$, we can be limited by first two terms of the expansion. At the same time $\frac{1}{1-r_s/r}\cong 1+r_s/r$, and the equation (2.4) takes the form:

$$\left(1+\frac{r_s}{r}\right)\left(\frac{\partial S}{\partial t}\right)^2-c^2\left(\frac{\partial S}{\partial \vec{r}}\right)^2-\frac{1}{r^2}\left(\frac{\partial S}{\partial \varphi}\right)^2=m^2c^4\ , \qquad (2.7)$$

We will show that L-invariant time dilation leads to the appearance of Newton's gravitational field.

2.1.1 *Newton's approximation*

Let us present this equation to the nonrelativistic mind, using the transformation $S=S'-mc^2t$:

$$\left(\frac{\partial S}{\partial t}\right)^2=\left(\frac{\partial S'}{\partial t}\right)^2-2mc^2\frac{\partial S'}{\partial t}+m^2c^4\,.$$

Substituting this in (7), we find

$$\left(1+\frac{r_s}{r}\right)\left[\left(\frac{\partial S'}{\partial t}\right)^2-2mc^2\frac{\partial S'}{\partial t}+m^2c^4\right]-c^2\left(\frac{\partial S'}{\partial r}\right)^2-\frac{1}{r^2}\left(\frac{\partial S}{\partial \varphi}\right)^2=m^2c^4\,.$$

Expanding the brackets, we obtain:

$$\left(\frac{\partial S'}{\partial t}\right)^2-2mc^2\frac{\partial S'}{\partial t}+\frac{r_s}{r}\left(\frac{\partial S'}{\partial t}\right)^2-2mc^2\frac{r_s}{r}\frac{\partial S'}{\partial t}+\frac{r_s}{r}m^2c^4-c^2\left(\frac{\partial S'}{\partial \vec{r}}\right)^2-\frac{1}{r^2}\left(\frac{\partial S}{\partial \varphi}\right)^2=0\,.$$

Dividing this equation by $2mc^2$, we find:

$$\frac{1}{2mc^2}\left(\frac{\partial S'}{\partial t}\right)^2-\frac{\partial S'}{\partial t}+\frac{1}{2mc^2}\frac{r_s}{r}\left(\frac{\partial S'}{\partial t}\right)^2-\frac{r_s}{r}\frac{\partial S'}{\partial t}+\frac{1}{2}\frac{r_s}{r}mc^2-\frac{1}{2m}\left(\frac{\partial S'}{\partial \vec{r}}\right)^2-\frac{1}{r^2}\left(\frac{\partial S}{\partial \varphi}\right)^2=0\,, \qquad (2.8)$$

Taking into account that $r_s=\frac{2\gamma M}{c^2}$, we obtain $\frac{1}{2}\frac{r_s}{r}mc^2=\frac{\gamma mM}{r}=m\varphi_N=-U$, where U is the energy of the gravitational field in the Newtonian theory. In the nonrelativistic case we put $c\to\infty$. Furthermore, for real distances r of the body movement around source with Schwarzschild radius r_s, we have $\frac{r_s}{r}<<1$ and $\frac{r_s}{r}\frac{\partial S'}{\partial t}<<\frac{\partial S'}{\partial t}$, and then we can ignore the term $\frac{r_s}{r}\frac{\partial S'}{\partial t}$.

In the limit as $c\to\infty$, equation (2.8) goes over into the classical Hamilton-Jacobi equation for Newton gravitation field

$$\frac{\partial S'}{\partial t}+\frac{1}{2m}\left(\frac{\partial S'}{\partial r}\right)^2-\frac{1}{r^2}\left(\frac{\partial S}{\partial \varphi}\right)^2=-U\ , \qquad (2.9)$$

As is known, the solution of this problem leads to a closed elliptical (not preccesing) satellite orbit around the spherical central body.

From this it follows that the inclusion only of Lorentz time dilation into the free Hamilton-Jacobi equation leads to the Kepler problem in non-relativistic theory of gravitation.

Note also that equation (2.9) is a consequence of the L-invariant HJE with the Newton potential field:

$$\frac{1}{c^2}\left(\frac{\partial S}{\partial t}+U\right)^2-\left(\frac{\partial S}{\partial r}\right)^2-\frac{1}{r^2}\left(\frac{\partial S}{\partial \theta}\right)^2-\frac{1}{r^2}\frac{1}{\sin^2\theta}\left(\frac{\partial S}{\partial \varphi}\right)^2=m^2c^2, \qquad (2.10)$$

Thus, the equations (2.6), (2.9) and (2.10) are equivalent from point of view of their results.

2.2. The equation of motion of a particle in a gravitational field with the Lorentz time dilation and length contraction

Now in order to take into account the length contraction effect along with the effect of time dilation, we will use the Hamilton-Jacobi equation (2.3) in form:

$$\left(\frac{\partial S}{\partial \tilde{t}}\right)^2-c^2\left(\frac{\partial S}{\partial \tilde{r}}\right)^2-\frac{c^2}{r^2}\left[\left(\frac{\partial S}{\partial \theta}\right)^2+\frac{1}{\sin^2\theta}\left(\frac{\partial S}{\partial \varphi}\right)^2\right]=m^2c^2, \qquad (2.11)$$

Substituting in (2.11) not only $d\tilde{t}=dt\sqrt{1-\beta^2}$, but also $d\tilde{r}=dr\big/\sqrt{1-\beta^2}$, we obtain:

$$\frac{1}{1-\beta^2}\left(\frac{\partial S}{\partial t}\right)^2-c^2\left(1-\beta^2\right)\left(\frac{\partial S}{\partial r}\right)^2-\frac{c^2}{r^2}\left[\left(\frac{\partial S}{\partial \theta}\right)^2+\frac{1}{\sin^2\theta}\left(\frac{\partial S}{\partial \varphi}\right)^2\right]=m^2c^4, \qquad (2.12)$$

Taking into account that in our theory $1-\beta^2=1-r_s/r$, we obtain from (2.12) the well-known Hamilton-Jacobi equation for general relativity in the case of the Schwarzschild-Droste metric (Schwarzschild, 1916; Droste, 1917):

$$\frac{1}{1-\frac{r_s}{r}}\left(\frac{\partial S}{\partial t}\right)^2-c^2\left(1-\frac{r_s}{r}\right)\left(\frac{\partial S}{\partial r}\right)^2-\frac{c^2}{r^2}\left[\left(\frac{\partial S}{\partial \theta}\right)^2+\frac{1}{\sin^2\theta}\left(\frac{\partial S}{\partial \varphi}\right)^2\right]=m^2c^4, \qquad (2.13)$$

As is well known (Landau and Lifshitz, 1971), the solutions of this equation are three well-known effects of general relativity, well confirmed by experiment: the precession of Mercury's orbit, the curvature of the trajectory of a ray of light in the gravitational field of a centrally symmetric source and the gravitational frequency shift of EM waves.

As we noted, in the Kepler problem solution, based on this equation, there is an additional term in the energy, which is missing in Newton's theory:

$$U(r)=-\frac{\gamma_N mM_s}{r}+\frac{M^2}{2mr^2}-\frac{\gamma_N M_s M^2}{c^2mr^3}, \qquad (2.14)$$

which is responsible for the precession of the orbit of a body, rotating around a spherically symmetric stationary center. From the above analysis it follows that the appearance of this term is provided by Lorentz effect of the length contraction.

We found above that the term $\frac{1}{1-r_s/r}\left(\frac{\partial S'}{\partial t}\right)^2$ containing the Lorentz time dilation effect in the classical approximation leads to the equation of Newton gravitation with Newton's gravitational energy. From this it follows that the precession of the orbit ensure the introduction of an additional term $c^2\left(1-\frac{r_s}{r}\right)\left(\frac{\partial S}{\partial r}\right)^2$.

2.3. Gravitational deflection of light ray trajectory

The path of a light ray (Landau and Lifshitz, 1971, p. 308-309) in a centrally symmetric gravitational field is determined by the eikonal equation which differ from Hamilton-Jacobi equation only in having $m=0$, at the same time, in place of the energy $\varepsilon_p = -\partial S/\partial t$ of the particle we must write the frequency of the light $\omega_\lambda = -\partial \Psi/\partial t$.

$$g^{ik}\frac{\partial \Psi}{\partial x^i}\frac{\partial \Psi}{\partial x^k}=0, \qquad (2.15)$$

The solution show that under the influence of the field of attraction the light ray is bent: its trajectory is a curve, which is concave toward the center (the ray is 'attracted' toward the center), so that the angle between its two asymptotes differs from π by

$$\delta\varphi = \frac{2r_s}{\rho} = \frac{4\gamma_N M_S}{c^2\rho}, \qquad (2.16)$$

In other words, the ray of light, passing at a distance ρ from the center of the field, is deflected through an angle $\delta\varphi$.

2.4. Gravitational time dilation and red shift of the frequency

Within the framework of general relativity, these gravitational effects are took into consideration on the basis of the Schwarzschild-Droste metric. In the framework of LIGT, this solution is based on the account of effects resulting from the Lorentz transformations, and has no relation to the metric. Nevertheless, the indicated effects are easily solved here.

We are able to prove a general statement regarding the influence of a gravitational field on clocks (Pauli, 1981). Let us take a reference system *K* which rotates relative to the Galilean system K_0 with angular velocity ω. A clock at rest in *K* will then be slowed down the more, the farther away from the axis of rotation the clock is situated, because of the transverse Doppler effect. This can be seen immediately by considering the process as observed in system K_0. The time dilatation is given by

$$t = \frac{\tau}{\sqrt{1-\frac{\upsilon^2}{c^2}}} = \frac{\tau}{\sqrt{1-\frac{\omega^2 r^2}{c^2}}}, \qquad (2.17)$$

The observer rotating with *K* will not interpret this shortening of the time as a transverse Doppler effect, since after all the clock *is* at rest relative to him. But in *K* a gravitational field (field of the centrifugal force) exists with potential $\varphi = -\frac{1}{2}\omega^2 r^2$.

Thus the observer hi *K* will come to the conclusion that the clocks will be slowed down the more, the smaller the gravitational potential at the particular spot. In particular, taking into account that $\upsilon^2/c^2 = \beta^2 = r_s/r = 2\varphi/c^2$, the time dilatation Δt is given, to a first approximation, by

$$t = \frac{\tau}{\sqrt{1+\frac{2\varphi}{c^2}}} \sim \tau\left(1-\frac{\varphi}{c^2}\right); \quad \frac{\Delta t}{\tau} = -\frac{\varphi}{c^2}, \qquad (2.18)$$

Einstein[293] applied an analogous argument to the case of uniformly accelerated system. We thus see that the transverse Doppler effect and the time dilatation produced by gravitation appear as two different modes of expressing the same fact, namely that a clock will always indicate the proper time $\tau = \frac{1}{ic}\int ds$.

Relation (2.18) has an important consequence which can be checked by experiment. The transport of clocks can also be effected by means of a light ray, if one regards the vibration process of light as a clock.

If, therefore, a spectral line produced in the sun is observed on the earth, its frequency will, according to (2.18), be shifted towards the red compared with the corresponding terrestrial frequency. The amount of this shift will be

$$\frac{\Delta\nu}{\nu} = -\frac{\varphi_E - \varphi_S}{c^2}, \qquad (2.19)$$

where φ_E is the value of the gravitational potential on the earth, φ_S that on the surface of the sun. The numerical calculation gives $\frac{\Delta\nu}{\nu} = 2{,}12 \cdot 10^{-6}$, corresponding to a Doppler effect of 0,63 km/sec.

Einstein (Einstein, 1911) applied an analogous argument to the case of uniformly accelerated system.

Let us assume (Sivukhin, 2005) that the clock *A* relatively to the system *S* is moving with constant acceleration a. We will count the time t from the moment when the velocity was zero. Then $\upsilon = \sqrt{2ax}$, where *x* is the distance that the clock A covered during the time t. Therefore:

$$dt = dt_0 \Big/ \sqrt{1 - 2ax/c^2}, \qquad (2.20)$$

Now let us introduce an accelerated reference frame S_0, which moves together with the clock *A*. In this system the clock *A* is immobile, but there are inertial forces. If all the phenomena will be described, taking S_0 as a reference frame, then as the cause of time dilation t_0 the inertial forces should be considered. The inertial force per unit mass of the moving body is $-a$. But, according to the principle of equivalence, the inertial forces are indistinguishable from the gravitational field, the intensity of which in our case is $\vec{g} = -\vec{a}$. Then the gravitational potential is $\varphi = -gx$ and the formula (2.20) becomes:

$$dt = dt_0 \Big/ \sqrt{1 - 2\varphi/c^2} \approx dt_0 \left(1 - \varphi/c^2\right), \qquad (2.21)$$

or

$$\frac{dt - dt_0}{dt_0} = -\frac{\varphi}{c^2}, \qquad (2.22)$$

As zero gravitational potential, the potential of point is considered, at which the moving and stationary clocks run equally fast. Therefore, in formulas (2.21) and (2.22), the time interval dt can be counted not by the clock of the inertial system *S*, but by the clock that is in rest in system S_0, which is located at the point *B* with zero potential. In general, we can set the initiation of count of gravitational potential at any point, if the formula (2.22) has the form:

$$\frac{dt_{0A} - dt_{0B}}{dt_{0A}} = -\frac{\varphi_B - \varphi_A}{c^2}, \qquad (2.23)$$

where the time intervals dt_{0A} and dt_{0B} are counted by two clocks, which are in rest in an accelerated reference frame S_0 at points *A* and *B* with gravitational potentials φ_A and φ_B.

Conclusion

Thus, we can say that, in the case of centrally symmetric gravitational field, within the framework of LIGT we get the same results as in the framework of general relativity. It is noteworthy that in order to obtain these results minor adjustments in equation of motion are required, which are ensured by two effects following from the Lorentz transformations.

Chapter 12. The cosmological solutions in the framework of LIGT

1.0 Cosmological solutions of GTR

All solutions of the equations of General Relativity concerning the movement of single massive bodies relative to each other (planets, stars, etc.) and which are tested experimentally, were obtained by us within the framework of LIGT in the previous article.

Moreover, there are solutions that are interpreted as cosmological, that is, related to the entire universe. At the moment, as a tested solution is also considered the solution, obtained by means of the postulates of the homogeneity and anisotropy of Universe, jointly with the results of general relativity and thermodynamics.

The question of the legality of such description of the Universe that contains, along with an almost infinite number of stars, planets and smaller bodies also an almost infinite number of other objects (microwave cosmic background, gases, dust, supernovae, neutron and many other types of stars, different types of galaxies and so forth.), will be left outside the limits of this article. Also, we will not consider the contribution of electromagnetic field (in particular, its lower state - physical vacuum) and elementary particles, although their presence in the universe is primary. Thus, according to the Hans Alfven theory (Alfven, 1942; Alfven and Arrhenius, 1976) (for which he received the Nobel Prize), electric and magnetic fields play a crucial role in the formation of the solar and other star systems.

Let us only note that direct experimental proofs of correctness of cosmological postulates and solutions do not exist (Baryshev, 1995). However, under the current cosmological paradigm are accepted interpretations of observational data, which was recognized as confirmation of abovementioned solutions. At the same time, there are numerous alternative explanations for these observational data, which, as was repeatedly noted by Alfven (Alfven, 1984).) and other scientists, are not taken into account (which means that they can not be published in official publications):

«Perhaps never in the history of science has so much quality evidence accumulated against a model so widely accepted within a field. Even the most basic elements of the theory, the expansion of the universe and the fireball remnant radiation, remain interpretations with credible alternative explanations. One must wonder why, in this circumstance, that four good alternative models are not even being comparatively discussed by most astronomers» (Flandern, 2002). (see also Krogdahl, W.S. «A Critique of General Relativity» (Krogdahl, 2007)).

2.0. Formulation of the problem

It is obvious that if we want to fully confirm the equivalence of general relativity and LIGT, it seems necessary to obtain the corresponding cosmological solution in framework of LIGT. The present article will be dedicated to this subject. At the same time, our paper bears a feature which the previous article also bore. We have practically no need to present this solution since it has long been known, and is even taken into consideration at the pedagogical level.

Basic cosmological solutions of general relativity (for three types of curvature of space-time Universe) were obtained by Friedman (1922). Their derivation is reported in numerous textbooks, lectures and monographs; See, for example. (Bogorodsky, 1971; Dullemond et al. 2011, Ch. 4.).

The basis upon which the solution of Friedman is built (Dullemond et al . 2011, Ch. 4) are the two postulates mentioned above about the state of the universe. Besides that, it was proven by Robertson and Walker that the only one choice of metric exists, that satisfies these postulates.

Let us consider this metric (Dullemond et al . 2011, Ch. 4):

2.1 The Robertson-Walker geometry of space

The Universe is homogeneous and isotropic. Isotropy means that the metric must be diagonal. Because, as we shall see, space is allowed to be curved, it will turn out to be useful to use spherical coordinates (r,θ,φ) for describing the metric. The center of the spherical coordinate system is us (the observers) as we look out into the cosmos. Let us focus on the spatial part of the metric. For flat space the metric is given by the following line element:

$$ds^2 = dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \qquad (2.1)$$

where θ is now measured from the north pole and is π at the south pole. It is useful to abbreviate the term between brackets as

$$d\omega^2 = d\theta^2 + \sin^2\theta d\varphi^2, \qquad (2.2)$$

because it is a measure of angle on the sky of the observer. Because the universe is isotropic the angle between two galaxies as we see it is in fact the true angle fromour vantage point: The expansion of the universe does not change this angle. Therefore we can use $d\omega$ for the remainder of this lecture. So, for flat space we have

$$ds^2 = dr^2 + r^2 d\omega^2, \qquad (2.3)$$

It was proven by Robertson and Walker that the only alternative metric that obeys both isotropy and homogeneity is:

$$ds^2 = dr^2 + f_K(r)^2 d\omega^2, \qquad (2.4)$$

where the function $f_K(r)$ is the curvature function given by

$$f_K(r) = \begin{cases} K^{-1/2}\sin\left(K^{1/2}r\right) & for\, K > 0 \\ r & for\, K = 0 \\ K^{-1/2}\sinh\left(K^{1/2}r\right) & for\, K < 0 \end{cases}, \qquad (2.5)$$

The constant K is the curvature constant. We can also define a "radius of curvature"

$$R_{curv} = K^{-1/2}, \qquad (2.6)$$

which, for our 2-D example of the Earth's surface, is the radius of the Earth. In our 3- D Universe it is the radius of a hypothetical (!) 3-D "surface" sphere of a 4-D"sphere" in 4-D space.

Note that the metric given in Eq. (2.4) can be written in another way if we define an alternative radius r as $r \equiv f_K(r)$. The metric is then:

$$ds^2 = \frac{dr^2}{1 - Kr^2} + r^2 d\omega^2 , \qquad (2.7)$$

Note that this metric is different only in the way we choose our coordinate r; it is not in any physical way different from Eq. (2.4)".

The Robertson-Walker metric allows to build a solution Friedman. The Friedmann Equations are two simple first order ordinary differential equations. Solutions to these equations yield the cosmological model we are interested in (Dullemond et al. 2011, Ch. 4).

2.2. The Robertson-Walker Universe metric in framework of LIGT

As we have shown (chapter 8), the square of the interval, which in SRT and GTR is considered as a geometry object, in the physics of elementary particles and within LIGT is a mathematical notation of the Lorentz-invariant energy-momentum conservation law.

That is why the Lorentz transformation can be found formally as a group of transformations preserving invariant the squared interval.

In the presence of a gravitational field this quadratic form contains a metric tensor, in which the amendment of changing the scale of coordinates derived due to the effects of the Lorentz transformation, is taken into account. As we have shown, this tensor is identical to the one obtained from the solution of equations of general relativity. Thus there is no need to interpret this interval as belonging to a Riemann space. It may be written in any (including rectangular) coordinate system.

Thus, all above arguments in section 2.1 may be repeated in LIGT as well as the further calculations of Friedman. Since Newton's equation is a first approximation of the equations of gravitation LIGT, you can expect that the results of Friedman's (at least to a first approximation) can be derived from Newton's theory of gravitation.

Such solutions were indeed found in 1934 (Milne, 1934; McCrea and Milne, 1934). Moreover, it appears that these solutions are the same as the solutions of Fridman. Later they were refined (Milne, 1948; Krogdahl, 2004).

"*A Lorentz-invariant cosmology based on E. A. Milne's Kinematic Relativity is shown to be capable of describing and accounting for all relativistic features of a world model without space-time curvature. It further implies the non-existence of black holes and the cosmological constant. The controversy over the value of the Hubble constant is resolved as is the recent conclusion that the universe's expansion is accelerating. "Dark matter" and "dark energy" are possibly identified and accounted for as well*" (Krogdahl, 2004).

A modern formulation of this solution in Russian can be found, for example, in the presentation of the expert in the field of general relativity, academician Ya.B.Zeldovich; see Appendix I to the book (Weinberg, 2000), p. 190, titled "The classical non-relativistic cosmology" , who note here:

"All the calculations could have been made not only in the nineteenth century, but also in the eighteenth century".

The lecture 2 from the modern cosmology course ((Dullemond et al. 2011, Ch. 2) is dedicated to this subject.

Closing notes

This concludes our presentation of LIGT itself. It would be interesting to analyze the question of whether the Hilbert-Einstein's general relativity has some advantages over non-geometrical approach, besides the fascinating mathematical interpretation that goes beyond the usual physics. Some thoughts on this matter will be set out in independent articles.

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