

# THURALS & INPOLARS

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## Introduction

The aim of this review is firstly, to present again a new family of polar curves (e.g. *thurals* [1]) and secondly, to introduce their so called *inpolars* as main objects of one original geometrical transformation [2]. *Addendum* is completely new with a brief analysis of s-thural.

## 1. Thurals

Four new, quite original, transcendental curves (let us call them *thurals*) will be presented in this chapter. The very first one (Fig. 1) would be super-spiraling curve given with the polar formula

$$r = a \theta^{b\theta} \quad (1.1)$$

or

$$r = e^{\theta \ln \theta}, \quad (1.1a)$$

assuming  $a, b = 1$ . The second *thural* (Fig. 2) is formally analogous to the above and defined with the formula

$$r = \theta^{-\theta}. \quad (1.2)$$

The idea for its name comes naturally: *c-curve*. Finally, the last two spirals (Figs. 3. and 4) in this review would be defined with the formulae

$$r = \theta^{1/\theta} \quad (1.3)$$

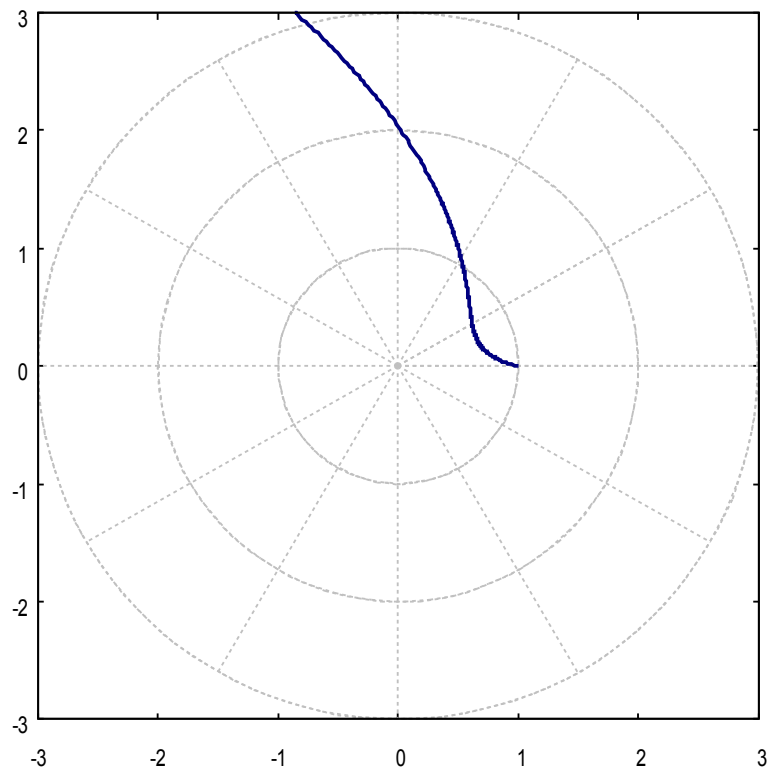
and

$$r = \theta^{-1/\theta}. \quad (1.4)$$

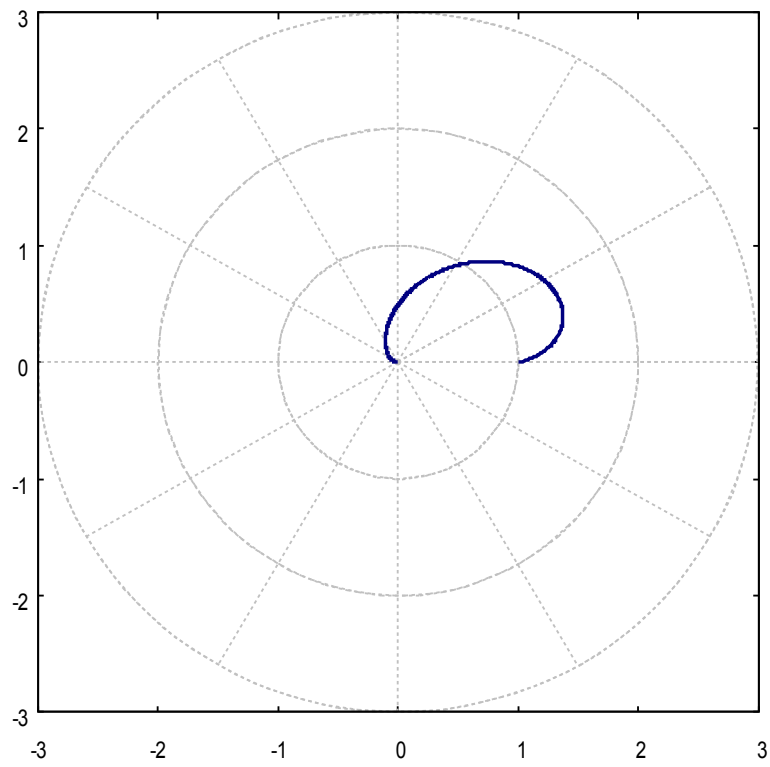
*Loop* is what comes in one's mind when one takes a look at both curves, respectively.

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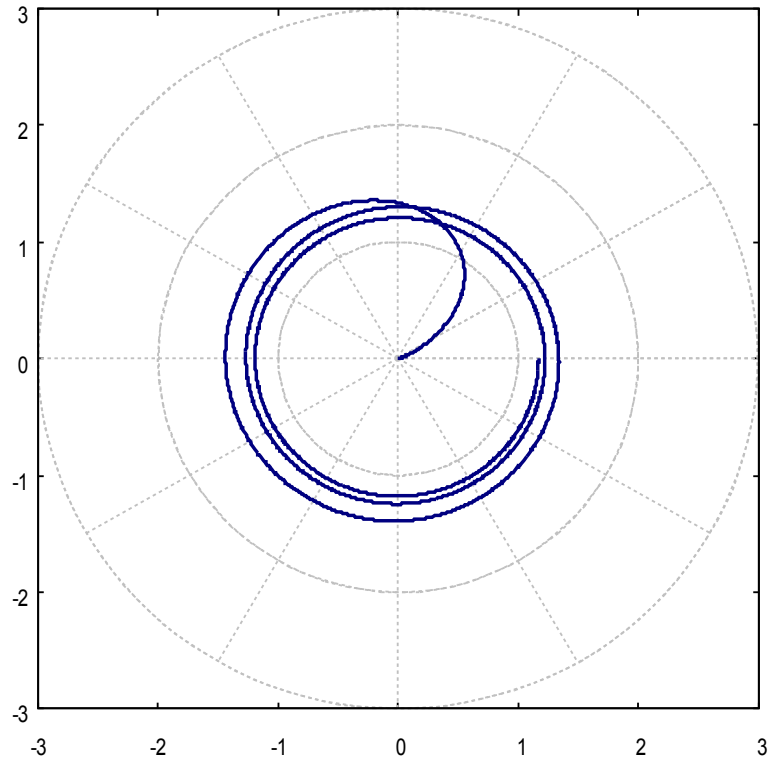
\* Belgrade, Serbia, web: [wavespace.webs.com](http://wavespace.webs.com)



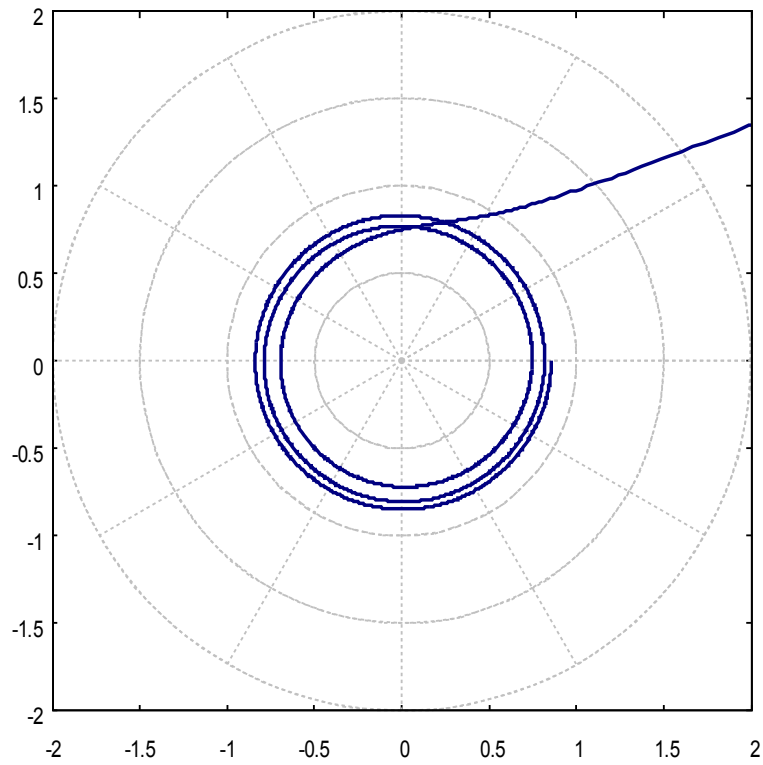
**Fig. 1:** *s-spiral*,  $0 < \theta \leq 4\pi$



**Fig. 2:** *c-curve*,  $0 < \theta \leq 4\pi$



*Fig. 3: super-loop,  $0 < \theta \leq 6\pi$*



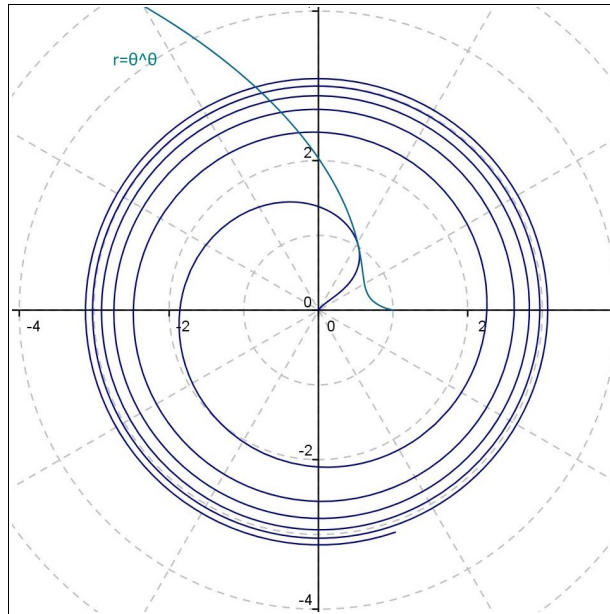
*Fig. 4: sub-loop,  $0 < \theta \leq 6\pi$*

## 2. Inthurals

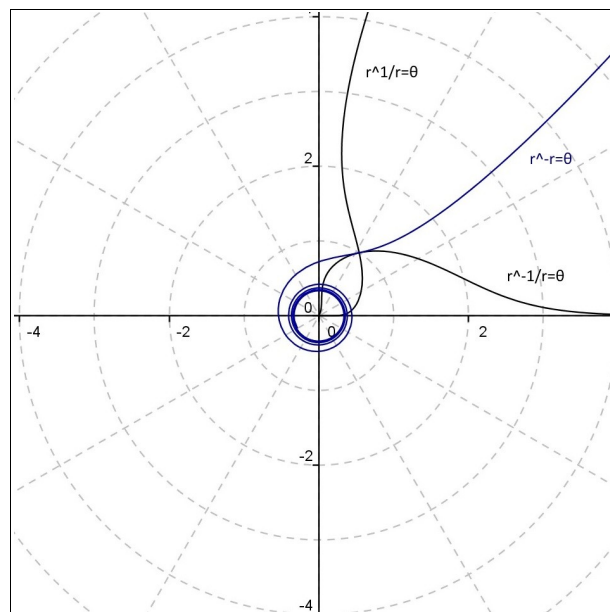
An initial, almost naive, question: Does exist (and how looks like) the *inspiral* of the form

$$r^r = \theta, \quad (2.1)$$

has surprisingly forced us to accept a change in our usual geometric intuition of  $r$  and  $\theta$  mutual dependency in general [2]. The positive answer via an *inpolar* transformation leads (among other *inpolars*) towards the inpolar thurals, i.e. *inthurals*, as the consequence (Figs. 5 and 6).



**Fig. 5:** *s-inthural*  $r^r = \theta$



**Fig. 6:** *The rest three inthurals*

## Acknowledgments

And the heaven departed as a *scroll* when it is *rolled together*...

*Revelation 6:14*

## References

- [1] Turanyanin, D. A Gallery of Unusual Spirals, General Science Journal, (2013)  
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- [2] Turanyanin D., Jovičin S., On Inpolars, General Science Journal, (2015)  
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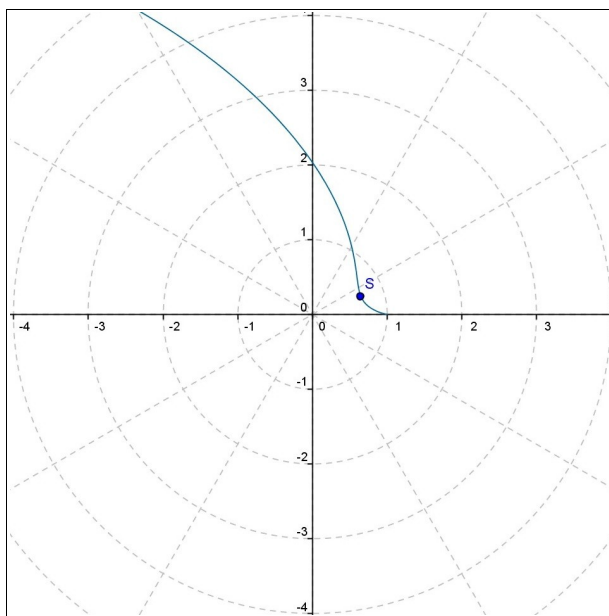
## Literature

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Bronstein I.N., Semendyaev K.A., Spravochnik po Matematike (Handbook of Mathematics), Nauka, Moskva (1964), in Russian

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## Addendum



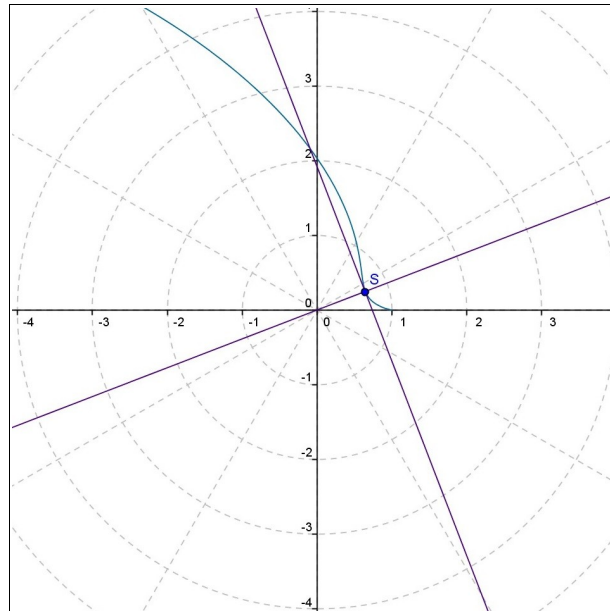
**Fig. 7:** *s-thural's S point*

The point S is the prominent point of the *s-thural* (Fig. 7). It can be seen as analogous to the minimum of the Cartesian function  $y=x^x$ . Its polar coordinates we deduce by examining the standard condition  $r' = 0$ , thus

$$r' = (\theta \ln \theta)' e^{\theta \ln \theta} = (\ln \theta + 1) e^{\theta \ln \theta} = 0 .$$

The condition is satisfied with  $\ln \theta = -1$ , hence  $\theta = 1/e$  in radians. Writing in degrees S coordinates reads (0.692, 21.078).

Finally, it is interesting to notice the curves' S tangent<sup>1</sup> as well as the polar line through S.



*Fig. 8: S tangent and normal*

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<sup>1</sup>In [GeoGebra](#) solution reads `Tangent[S, curve ]`