An Introduction to New Foundations:

The Ordual Calculus for Ultimate Search

By Arthur Shevenyonov

Abstract

The calculus being proposed can be seen as an illustrative extension of, or a "constructive" backing for, the Ordinalcy paradigm. The dual account can be generalized to the m-ality setup while lending itself to a host of reduction or bridge type calculi as forthcoming.

Prior Considerations & The "Unutterable"

It would be at odds with an intuitive or constructivist cut-off to shun the issue of what it is that can be discerned from the New Foundations. Put differently, if anything, what could an Ordual or *relational calculus* look like?

Apart from the Twelve Axioms as set forth in Shevenyonov (2016b), one may want to further rationalize the basis over which relationships can be defined, other than the complete set of objects as a generic domain. From 2002 onwards, I have made heavy use of the seemingly arcane **Ψ** metric[1] whose careful semantic depiction could count as a daring enterprise in its own right. I now attempt it, by maintaining a host of *mutually inclusive* and *collectively supportive* propositions outlining the nature of this construct—which list is near complete even if open-ended, by the very nature of the notion in question.

(P1) This is a generalized depiction of the Residuale

(P2) It serves as a relational basis—or domain captured by objects rather than abstract dimensions

(P3) It could be visualized as a structured distribution[2] resolving the form-substance duality by positing no distinction between the "surface" or hull versus the "interior" (in line with there being no function-argument hierarchy)

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[1] This should not, however, be taken for a Psi letter, as the symbol in use is but a rough proxy for the actual graphic whose etymology and allusions count in the dozens. For starters, it could connote a cross-over of two *relational lines*, referring to dual CES or Lame rho metrics that will be put to heavy use shortly. Alternatively, I have referred to it as the Slavonic for "*youse*" (properly read as a nasal [j]$o^{n}s$ or [j]$e^{n}s$ standing for dual linkage).

[2] From 1999, I have visualized this as the Gradiency or lambda-omega snapshot, a Schroedinger density function, a non-linear local representation of time and space (as an alternative to hidden dimensions as in string theory), a vertices- or ribs-only or phase-diagram like depiction of a solid polyhedron, or indeed an asymmetric polytope (as of the early spring of 2007, months ahead of Garrett Lisi publishing his seminal work). Incidentally,

(P4) It may, among other things, mark the Gradiency *(A, G)* path as "surface" with reference to its *(B, F)* azimuthality trajectory (Shevenyonov, 2016a) standing for the "interior," comparison as distance, or inner local transfers as the simplices of a generalized polyhedron as relationships in their own right

(P5) The whole is related to its representative parts, such that a transition between any special and general representations is facilitated

(P6) By the same token, any structure having a sufficient set of qualifiers will exhibit comparability to similarly qualified, or ordual structures

(P7) The same holds for automorphous traits (as in P5, beyond conventional fractality or such like symmetry)

(P8) In particular, P7 can be the case in light of simplicity rather than complexity obtaining as an outcome

(P9) In Residuale terms, this is one reason why *marginal* propagation is irrelevant (yet can prove most relevant in a complexity or catastrophe setup)

(P10) Its perceived circularity, or otherwise dialectic behavior, could be due to the inherent inseparability between completeness and duality

A Formal Exposition of the Propositions

A starting point, as in Shevenyonov (2016b), could be to formalize a relationship as $R(A,B)$, which in the higher-order case would yield, $R^{[n]}(A,B)$. For simplicity's sake as well as toward greater ease of further algebraic manipulation, from the outset as in March 2008, this has been modeled by means of rho as the relational parameter or upper index (resembling the Lame or CES metric yet manipulable much like conventional powers)[3]:

$$R(A,a) \equiv (A,a)^{\rho}$$

From basic duality rationale (yet on careful empirical scrutiny which cannot be adduced here and will be made part of forthcoming findings), it has been maintained that:

$$(1)\ \ (A,a)^{\rho} = [(A,a),X] = [(A,a),(A,a)]$$

$$(2)\ \ (A,a)^{\rho} \sim (a,A)^{\frac{\rho}{\rho-1}}$$

by the time I saw any mention of his paper in June 2008, I had my Ordinal calculus evolved as of early March 2008. As before, any parallels between the related results might be about as promising as they too could prove shallow and irrelevant—in light of the per-element direct comparison having questionable merit.

[3] A detailed account of scaling or the [n]-rho correspondence will not be provided at this stage, even though this was an early step and the core original intent behind the entire enterprise. It is of interest to address the general, *m-al* setup in its entirety though, prior to collapsing to the special cases.

Whereas the right-hand side of identity (1) refers to an early instance of *ordinalization* (reaching a *higher-level* relationship), the middle part represents the same relationship in terms of the *(A,a) basis* being mediated with respect to *X* acting as the *third object* (akin to distant point), which (as will be shown in subsequent research) essentially amounts to the basis itself (thus hinting at the "cardinalcy" notion of a domain mapping into self) while also informing the relationship in a way that can be visualized[4] as a pulling of the *(A,b)* relationship "string," in shaping the rho curve as defined between two extremes: straight line (or perfect substitution in CES terms) versus rectangular kink (perfect complementarity). Any interim shapes would be arc-like, with the rho and its dual defined as,

$$\frac{\rho}{\rho-1} \in (-\infty, 0), \rho \in [0,1]$$

It is straightforward that the above domains can be induced from:

$$\frac{1}{\rho} + \frac{\rho-1}{\rho} \equiv 1 \equiv \alpha + (1-\alpha)$$

For this reason, it will occasionally be instrumental to embark on an informal equivalence of the sort:

$$(3)\ \rho \sim \frac{1}{\alpha}\ or\ \frac{1}{1-\alpha}$$

Among other things, this could provide an early glance at a linkage between probabilistic equivalents as the "shadow" reduction of the ultimate relationships. What is more, this could be one straightforward way of dealing with negative or above-unity odds as well as similar weights (e.g. in financial short-selling, which cannot quite be "bought into" for lack of genuinely feasible earnings expectations as secured by this peculiar investment position or mixed strategy). As it happens, switching to relationships as opposed to probabilism and risk taking could render the entire analysis more intuitive as well as Gradiency compliant (referring to the speed of arriving at the cross-implications).

The core equivalence as in (2) can be rendered simpler as well as more operational:

$$(2')\ \ (A,a)^{\rho-1} \sim (a,A)$$

At this stage, the entire calculus can be caste in experimental terms. To begin with, (1) would suggest that the rho may act to generalize the zero (its dual generalizing 2), as long as ordinalization could be seen as an action subtracting from the initial power index:

[4] Incidentally, it is by placing any two objects, A and a anywhere close by on such curved relational lines (the graphic constituents of the Ψ) that their short distance—acting as a marginal differential—yields a good linear approximation, which is moreover symmetric with respect to the dual relational line or inverse path as long as both are marked by a zero rho value (i.e. neutrality, orthogonality, or "cardinalcy"). Whilst irrelevant compared to the generalized treatment, this narrowing comes at the prohibitive cost of overlooking the latter altogether. This framework or semi-formal metaphor first occurred to me in 2002, and it is unfortunate that its extended implications will not be fully recovered due to the bulk of the pre-2003 notes having been lost irrevocably.

$$(A, a)^{\rho} = [(A, a), (A, a)] \sim (A, a)^{1-1}$$

In other words, this can be induced toward:

$$(4) \;\; [(A, a)^{\rho}, (A, a)] \sim (A, a)^{\rho-1} = (a, A)$$

One way to intuitively appreciate this would be to consider as a special case the strong symmetry rendering the values of 0 and 2 as the corners solutions of:

$$(4') \;\; \rho = \frac{\rho}{\rho - 1}$$

In this light, the *implied commutator* in (*2'*) would imply strong commutativity, or otherwise non-commutativity of the conventional form:

$$(a, A) = (A, a)^{-1}$$

Likewise, largely the same special reduction could refer to the functional inverse:

$$x(y) = y^{-1}(x)$$

Overall, it can be surmised that "cardinalcy" or conventional functionality could be captured alternatively by the following narrowed-down representations:

(*4'*) OR $(A \to a)$ OR both

Having assumed this as the "right action," what would the "left action" (or cardinalization) possibly be, with an eye toward asymmetry in the general case? One minimalist notation of path invariance working either way could mnemonically posit:

$$(5) \; xYx \equiv Y$$

On prior grounds, the dual of index decrement could be a tantamount increment—yet that would only hold in the strongly symmetric case. In other words, for $Yx \equiv (A, a)^{\rho-1}$, a left action recovering $Y \equiv (A, a)^{\rho}$ would plausibly be about multiplying the index times the dual or (2) right-hand side power (rather than just adding 1 to it):

$$(5') \;\; xYx \equiv Y \equiv (A, a)^{\rho} = (Yx)^{\frac{\rho}{\rho-1}}$$

After all, it is straightforward to appreciate how $\rho - 1$ and $\frac{1}{\rho-1}$ generalize +1 and -1 as per the corner cases[5].

In hindsight, it is for this reason that the left action on any construct, *(a, A)* included, yields

$$[(a, A), (a, A)] = (a, A)^{\frac{\rho}{\rho-1}} = (A, a)^{(\rho-1)\frac{\rho}{\rho-1}}$$

[5] Bottlenecks apparently may arise at this point marking a possibly inconclusive rather than dual "convention or widget hunt."

QED, referring to the original (2).

Apart from the ordinalization and cardinalization duality, it could be straightforward to approach counterparts of the regular *differentiation* and *integration*. Irrespective of how these are to be arranged vis-à-vis the aforementioned dyad, the key juxtaposition would build on a relationship of the basis with its element or any sub-basis, e.g.:

$$[(A,a),a]\ vs.[a,(a,A)]\ vs.[a,(A,a)\ vs.\ [A,(a,A)]\ vs.\ [A,(A,a)]\ \ etc.]$$

As opposed to,

$$[(A,a),(A,a)]\ vs.\ [(a,A),(A,a)]\ vs.\ [(A,a)^{\rho},(A,a)]\ etc.$$

In fact, all of these mixed cases can be tackled by means of (*5'*). Alternatively, one other auxiliary bridge could be invoked, referring to a quasi-cardinal analogue of (2) and (*2'*):

$$(6)\ \ A^{\rho-1}{\sim}a$$

Though highly arbitrary and somewhat exogenous in nature, the above still appears in line with the overall rationale as in (*2'*) while enabling some extra reduction to, or sense-making of, cardinal-like notions akin to integro-differential or operational calculus. At this rate, all conceivable resultant structures can be compared based on the implied power index.

What is more, such structures can be decomposed as a matter of *characteristic indices*—somewhat akin to binary value representations[6].

Forward Looking Afterthoughts

The scope of applications accommodating this calculus has shown to be virtually boundless (yet structurally bound by the similarity shining through)—let alone the family of related calculi that have been inspired by its logic. Future research will dwell on these while proposing some detail on candidate uses in social sciences and (*caveat emptor!*) theological accounting. For now, one straightforward corollary could be to stretch the basic duality to the *m-ality* case as follows:

$$(7)\ \ \{A_{k_i}\}^{\rho_k}{\sim}\{A_{l_i}\}^{\rho_l}\ \forall k\neq l$$

$$s.t.\sum_{i=1}^{m}1/\rho_i\equiv 1$$

The full-blown basis as maintained has been treated as well as acted on with respect to its reshufflings or rotations.

[6] Any Yi Ching exercises or quantum computational ventures along these lines shall be blamed on the desperate onlooker solely.

References

Lisi, G. (2007). An exceptionally simple theory of everything. *arXiv: 0711.0770*

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