The physical nature of the basic concepts of physics

1. Linear Momentum

Guido F. Nelissen

guido.nelissen@gmail.com

Abstract

Since all macroscopic matter is made of extremely small particles (electrons, protons, neutrons), all macroscopic bodies are in reality multi-particle systems that are built up from an extremely large number of particles.
The present physics defines the linear momentum of such a multi-particle system as the mathematical product of its total mass times the velocity of its center of mass, and its conservation is explained as a consequence of Newton's first law of motion.
In this paper the author reveals the physical nature of the velocity of a particle system, as the congruent translational velocity with which all the particles of the particle system move in unison, that is with exactly the same speed in exactly the same direction. This allows him to demonstrate that the 'linear momentum' of a moving particle system is a mathematical expression of its total amount of congruent translational velocity.
This leads to the conclusion that the conservation of linear momentum of a mass particle system is a mathematical expression of the physical fact that a particle system cannot change its congruent velocity by its own.

0. Introduction

The principles of the conservation of 'linear momentum' and the conservation of "energy" are the corner stones of the present theory of physics. The true nature of these concepts and the underlying physical mechanisms of their conservation have, however, never been properly cleared out. Even the great European physicists, Descartes, Leibniz and D'Alembert had lengthy discussions [1] on whether 'kinetic energy' or 'linear momentum' were the true property considered by the conservation laws.
In the present physics the linear momentum of a body is mathematically defined as the product of its mass and its velocity and its conservation is explained as a consequence of Newton's first law of motion.
In this paper the author reveals the physical nature of the linear momentum of a multi-particle system and the physical reason for its conservation in the absence of external forces.

1. The present law of conservation of linear momentum

1.1 The linear momentum of a single particle/body

The notion of "linear momentum" of a single particle/body finds its origin in Newton's second law of motion, which states that "a force acting on a body causes an acceleration of that body in the direction of the force, which has a magnitude that is inversely proportional to the mass of the body" ($\mathbf{a} = \mathbf{F}/m$). So that:

$$\mathbf{F} = m.\mathbf{a} = m.d\mathbf{v}/dt = d(m.\mathbf{v})/dt = d\mathbf{p}/dt$$

It follows from this second law, that in the absence of external forces:

$$\mathbf{F} = d\mathbf{p}/dt = d(m\mathbf{v})/dt = 0$$

the linear momentum $\mathbf{p} = m\mathbf{v}$ = a constant

This makes Newton's second law of motion at the same time a quantitative expression of his first law, which states that in the absence of a force on it ($\mathbf{F} = 0$), the velocity of a body remains constant.

1.2 The linear momentum of multi-particle systems

Since all macroscopic matter is made of extremely small particles (electrons, protons, neutrons), all macroscopic bodies are in fact multi-particle systems, that are built up from an extremely large number of particles.

1.2.1 The conservation of the linear momentum of a multi-particle system

Newton's third law of motion gives the quantitative relationship between the mutual forces on colliding particles: "*Whenever a body exerts a force on another body, the latter exerts a force of equal magnitude and opposite direction on the former*".

Since 'force' equals the rate of change of momentum ($\mathbf{F} = d\mathbf{p}/dt$), Newton's third law can also be expressed in terms of momentum as: "*Whenever a body exert a force on another body, the rate of change of momentum of both bodies are of equal magnitudes and opposite directions".* Which means that if:

$$d\mathbf{p}_1/dt = \mathbf{F}$$

then:

$$d\mathbf{p}_2/dt = -\mathbf{F}$$

so that:

$$d\mathbf{p}_1/dt + d\mathbf{p}_2/dt = d(\mathbf{p}_1 + \mathbf{p}_2)/dt = 0$$

So that the total linear momentum of two colliding particles/bodies remains necessarily constant: $\mathbf{p}_1 + \mathbf{p}_2$ = constant
This demonstrates that Newton's third law of motion means that for a multi-particle system (on which there are no external forces), the internal collisions between particles cannot change the total linear momentum of the particle system, because at each internal collision the linear momentum $\mathbf{p}$ remains constant.

1.2.2 The motion of the center of mass of a multi-particle system

It follows from the former section that only external forces can change the momentum of a multi-particle system. To calculate the change of motion of such a system by means of an external force exerted on it, the present physics introduces the concept of the "center of mass".
Therefore, the position vector of the center of mass of a system of 'n' particles is defined as the weighted average of the position vectors of all the particles:
$\mathbf{r_{cm}} = \Sigma m_i\mathbf{r_i}/\Sigma m_i = \Sigma m_i\mathbf{r_i}/m$
(m_i is the mass of one particle, and $m = \Sigma m_i$, is the total mass of the particle system).

The change per unit time of the position vector of the center of mass, gives us the velocity vector of the center of mass:
$\mathbf{v_{cm}} = d\mathbf{r_{cm}}/dt = \Sigma m_i\,(d\mathbf{r_i}/dt)/m = \Sigma m_i\mathbf{v_i}/m$

so that the total momentum of a multi-particle system is a vector which has its origin in the center of mass and of which magnitude is equal to the total mass of the multi-particle system, times the velocity of its center of mass: $\mathbf{p} = \Sigma m_i\mathbf{v_i} = m.\mathbf{v_{cm}}$

This means that under the influence of an external force $\mathbf{F_{ex}}$, the center of mass of a particle system moves as though it were a single particle of mass 'm':

$\mathbf{F_{ex}} = d\mathbf{p}/dt = d(m\mathbf{v_{cm}})/dt = m(d\mathbf{v_{cm}}/dt) = m\mathbf{a_{cm}}$

2. The mathematical character of Newton's laws of motion

2.1 The mathematical character of the linear momentum of multi-particle systems

Newton's laws of motion don't specify the internal structure of the considered 'bodies' which means that they are tacitly considered as monolithic bodies without internal structure. In reality, all physical 'objects' are however composite, dynamic vibrating particle systems. To analyze the conservation of linear momentum for such dynamic structures, the present physics decomposes the velocities of their individual particles into their fundamental velocity components:

- the velocity of the CM in the chosen reference frame (which is called the CM-velocity '$\mathbf{v_{CM}}$') and,
- the individual velocities of the particles relative to the CM (which are called the 'internal' or 'thermal' velocity components '$\mathbf{q_j}$').

According to Jay Orear [2] the center of mass is merely a point in space, that indicates the average position of all the elements of a mass particle system and that allows us to calculate the motion of that particle-system as if its whole mass were concentrated in that given point.
In that way, the center of mass is not a material, but a calculated, virtual point in space in which there is even no mass at all, like this is clearly the case in a hollow cylinder or a hollow sphere.

On the other hand, the velocity vector of the center of mass is the calculated average of all the velocity vectors of the particles. This means that the velocity of the center of mass is also a mathematical concept, in the way that it doesn't correspond to any of the velocities of the individual particles of the considered particle system.

This means that in the present physics, the concept of the linear momentum of a multi-particle system is above all a mathematical tool that allows us to calculate the motion of a multi-particle system (i).

2.2 The mathematical character of Newton's laws of motion

Besides this mathematical character of the position and the velocity of the linear momentum of multi-particle systems, Newton's second law of motion contains an intrinsic ambiguity about the real nature of the concepts of 'force', 'mass' and 'acceleration', because it defines these three concepts in function of one another, that is:
- force, as the acceleration of a given mass ($\mathbf{F} = m\mathbf{a}$),
- acceleration, as the force per unit mass: ($\mathbf{a} = \mathbf{F}/m$)
- mass, as the force per unit acceleration: ($m = \mathbf{F}/\mathbf{a}$).

This means that, even if one can define acceleration independently from Newton's second law, namely as the rate of change of the velocity per unit time ($\mathbf{a} = d\mathbf{v}/dt = d^2\mathbf{r}/dt^2$), force' (ii) and 'mass' (iii) are still ambiguously defined in function of each other.
Therefore it is actually only possible to define the mass of a body in proportion to a standard mass. In that procedure, that was first proposed by Ernst Mach [3], an unknown mass 'm' and a standard mass 'm_s' exert forces on each other. The forces acting on these two masses are then, according to Newton's second law, of equal magnitudes: $ma = m_s a_s$
so that the unknown mass can be expressed as a fraction of the standard mass: $m = (a_s/a)m_s$.
But it is clear that this procedure doesn't really explain what 'mass' is and why it opposes itself to changes in velocity.

It is thereby important to realize that Newton's laws of motion are empirical laws, based on macroscopic observations and they are in that way comparable to the laws of classical thermodynamics. According to A. B. Pippard [4] "*In classical thermodynamics, the method of approach takes no account of the atomic constitution of matter, but seeks rather to derive from certain basic postulates, the laws of thermodynamics"*.
Just like the laws of classic thermodynamics, Newton's laws of motion for macroscopic bodies are not exhibited as a logical consequence of their microscopic constitution. Rather, their sole function is to explain their observable, macroscopic behavior as the consequence of a few empirical 'laws'. A. B. Pippard observes thereby that "*Not all practitioners of the physical sciences (in which term we may include without prejudice chemists and engineers) have this particular ambition to probe the ultimate mysteries of their craft, and many who have are forced by circumstances to forgo their desire. .. For often enough in the pure sciences, and still more in the applied sciences, it is more important to know the relations between the properties of substances than to have a clear understanding of the origin of these properties in terms of their molecular constitution*."
In this paper I will make an attempt to reveal the physical nature of the linear momentum of a moving particle system.

3. The physical nature of linear momentum

3.1 Congruent translational velocity

(i) This will be analyzed in my paper on the physical nature of 'velocity'.
(ii) This will be analyzed in my paper on the physical nature of 'force".
(iii) This will be analyzed in my paper on the physical nature of 'mass'.

As we have seen in section 1.2.2, the velocity of the center of mass ($\mathbf{v_{cm}}$) is classically obtained by differentiating the position vector of the CM ($\mathbf{r}_{\rm cm}$) with regard to time:

$$\mathbf{r}_{\rm cm} = \sum m_j\, \mathbf{r_j}/\sum m_j = \sum m_j \mathbf{r_j}/m$$

$$\mathbf{v_{cm}} = d\mathbf{r}_{\rm cm}/dt = \sum m_j\, (d\mathbf{r_j}/dt)/\sum m_j$$

So that: $\mathbf{v_{cm}} = \sum m_j \mathbf{v_j}/m$

In this equation (in which 'm' is the total mass of the particle system) the individual mass (m_j) of each particle is used as a weighting factor, which means in fact that each particle with a mass 'm_j' and a velocity '$\mathbf{v_j}$' is implicitly considered as a sub-particle system consisting of 'm_j' (basic) particles with unit mass, that all move in a congruent way, that is with exactly that same speed 'v_j' in exactly that same speed in exactly that same direction.

This means that the factor 'm**v**' which, in the present textbooks is generally denoted as the 'linear momentum', is actually a physical indication of the vector sum of the velocities of all the (basic) particles with unit mass, so that the CM-velocity is in fact the resultant velocity of all the particles with unit mass divided by the total number of particles with unit mass (m).

In nowadays physics, the 'unity of mass' is the arbitrarily chosen SI-unit of 'kilogram'. If we want to unveil the true nature of linear momentum, we have to put these arbitrarily chosen mass units aside and have to define the mass unit in a natural way, that is the mass of the smallest possible component, which we can define as the basic quantity of mass. In that way, the mass of a material body is expressed as the total amount of these mass units. This viewpoint is as a matter of fact also in line with Newton's view, who already defined "mass" as "the quantity of matter" [5].

From this viewpoint, the velocity of the center of mass ($\mathbf{v_{cm}}$) of a particle system is not the velocity of a point in space, but it is in fact the 'common', 'congruent' velocity ($\mathbf{v_c} = \mathbf{v_{cm}}$) with which all the unit particles of that particle system move in a coherent way, that is with exactly the same speed in exactly the same direction (Fig. 1.1) which corresponds to P. W. Atkins' view [6].

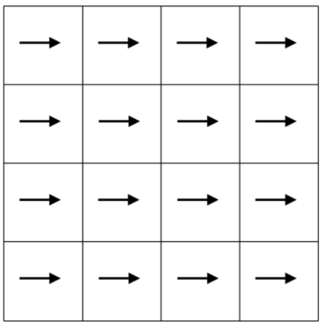

Fig. 1.1

This 'congruent', 'common' or 'coherent' velocity $\mathbf{v_c}$ is sometimes called the 'external' or 'bulk' velocity of the particles of a particle system.

One important conclusion from this definition of the CM-velocity as the 'common', or 'congruent' velocity ($\mathbf{v_c}$) of all the particles of a particle system, is that it cannot be affected by the internal motions or the internal collisions of the particles within the particle system, so that, without external intervention, the congruent velocity of a particle system is bound to remain constant.

$\mathbf{v_c} = \sum m_j \mathbf{v_j}/m = \mathbf{v_{cm}} =$ a constant

This means that the congruent velocity of all the individual elements of a particle system can be represented by the only one vector with a given length and a given direction.

3.2 The total amount of congruent translational velocity

Since in a closed system, the total mass 'm' (which may be regarded as the total number 'm' of basic particles with unit mass) is constant, this means that the sum of the congruent velocities of all the basic particles with unit mass is also constant:

$\sum m_j \mathbf{v_j} = m\mathbf{v_c} = \mathbf{p} =$ a constant

In this equation, the expression '$m\mathbf{v_c}$' is generally known as the 'linear momentum' of a particle system with total mass 'm' and a congruent CM-velocity '$\mathbf{v_c}$' and this equation is therefore generally known as 'the conservation of linear momentum'.

In the former section I have demonstrated that the linear momentum of a moving particle system, is equal to the sum of the (identical) congruent velocities of all of its particles with unit mass ($\sum m_j \mathbf{v_j} = m.\mathbf{v_c}$). This means that the conservation of linear momentum is a mathematical expression for the conservation of the total amount of congruent translational velocity/motion of the particle system.

This definition of the physical nature of the conservation of linear momentum corresponds exactly to Newton's point of view who, in his "Principia Mathematica" called it the conservation of the total "quantity of motion" [7]. In French the linear momentum is therefore still named '*quantité de mouvement*'.

Since a 'quantity' or an 'amount' is generally expressed as a number, we will in this paper use the concept of the conservation of the total 'quantity of motion' of a particle system to indicate the conservation of the sum of the magnitudes of the velocities of all its basic components.

We will on the other hand use the general concept of the conservation of 'velocity/motion' if all the individual velocities of these basic particles, their magnitudes as well as their directions, remain unchanged.

In this way the conservation of linear momentum:

$\sum m_j \mathbf{v_j} = m\mathbf{v_c} = \mathbf{p} =$ a constant

means that in an isolated mass particle system, the common, congruent translational velocity of the individual particles '$\mathbf{v_c}$' is conserved, in both magnitude and direction and that as well for each unit mass particle in particular as for the particle system as a whole!

Newton's first law of motion as well as the conservation of linear momentum can therefore be expressed in a general way as "the conservation of the congruent translational motion of the particles of a particle system", and this conservation is independent of any chosen inertial reference frame.

4. Internal translational motion

The velocity components ($\mathbf{q_j}$) with which the individual particles of an ideal particle system move relative to the center of mass of the particle system to which they belong, are defined as the 'internal' or 'thermal' velocities of the particles. They are by definition the remaining translational velocity components of the particles, when we subtract their common (or congruent) velocity component ($\mathbf{v_c}$) from their individual velocities ($\mathbf{v_j}$) in the chosen reference frame:

$$\mathbf{q_j} = (\mathbf{v_j} - \mathbf{v_c}) = (\mathbf{v_j} - \sum m_j\mathbf{v_j}/m)$$

It follows from this definition, that these thermal velocities are isotropicly distributed over all possible directions (Fig 1.2), which corresponds to P. W. Atkins' view [6].

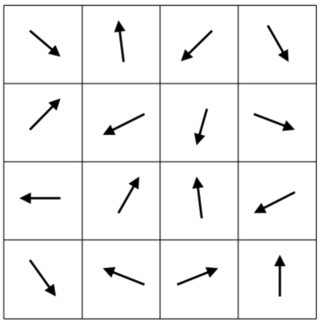

Fig. 1.2

The isotropic character of these velocities means that they cannot in any way produce a resultant velocity:

$$\mathbf{q} = \sum m_j\mathbf{q_j}/m = 0$$

This means that the total thermal linear momentum of a particle system is bound to be zero:

$$\mathbf{p_q} = \sum m_j\mathbf{q_j} = 0$$

and that the total amount of internal translational motion, which we have defined as the sum of the magnitudes of the thermal velocities of the basic particles with unit mass, is however not zero:

$$p_q = \sum m_j q_j \neq 0.$$

And neither their average internal speed is zero:

$$q = \sum m_j q_j/m \neq 0$$

In the present textbooks of physics, the notion of "the total amount of internal translational motion" of an ideal gas is not commonly used, instead the notion of internal or thermal kinetic 'energy' is used (iv):

$$K_T = mv^2/2 = (3/2)NkT$$

and the average thermal speed of the molecules 'q' is defined on the basis of this thermal energy, as the root mean square speed of the molecules:

$$q = v_{rms} = \sqrt{3kT/(m/N)}$$

(iv) This will be analyzed in my paper on the physical nature of pressure, temperature and thermal energy.

5. The flow characteristics of congruent translational velocity

5.1 Mass flow

In hydraulic engineering "the amount that flows across a given section per unit time" is defined as "the flow rate" or shortly "the flow" (Q). In electro-magnetism; the word 'flux' (Φ) is used (electric flux, magnetic flux) to indicate the same characteristic.
In principle the amount of any physical quantity can be used to define the flow or the flux: mass flow/flux, momentum flow/flux, heat flow/flux, charge flow/flux, energy flow/flux, etc.
In fluid dynamics one of the basic applications of the concept of 'flow' is the equation of the mass flow (Q_m) which gives the amount of mass (m) that flows across a given section per unit time, which is equal to the number (N) of unit mass (m_1) particles per unit time:
$Q_m = Nm_1/t = m/t = \rho V/t = \rho AL/t = \rho Av$

It is similar to the electric current (I) which is in fact "the charge flow" (Q_e) which is the amount of unit-charge particles (e^-) that flow across a given section per unit time, which in metallic conductors is the number of electrons (N_e) per unit time:
$Q_e = N_e/t = I$

5.2 Total amount of mass flow

In a general way, "mass per second" indicates a growth or a decrease of mass in time, rather than a motion with a given magnitude and a given direction.
But the mathematical expression of 'mass flow' is also a typical engineering characteristic that expresses the mass per unit time that flows through a specific arrangement of pipes or ducts. In the specific case of a fluid with N unit particles, each with mass 'm_1', that moves with a velocity '$\mathbf{v}$' in a piping system with a total length '$\mathbf{L}$', the mass flow of the fluid can be expressed in function of the length of the piping system:

$$Q_m = Nm_1/t = Nm_1(\mathbf{v_c}/\mathbf{L}) = m.(\mathbf{v_c}/\mathbf{L}) = (m/\mathbf{L}).\mathbf{v_c}$$

In this expression, the factor '$m/\mathbf{L}$' represents the (average) mass per unit length in its direction of motion, whereas '$\mathbf{v}$' represents the congruent velocity of the particles of the fluid. In that way, the mass flow, or 'the mass per unit time' is equal to 'the mass per unit length' in its direction of motion ($m/\mathbf{L}$), times its congruent velocity ($\mathbf{v_c}$).

This viewpoint allows us to reveal the true nature of the 'linear momentum' of a moving mass particle system: if the mass of the particles in a unit length times their congruent velocity is equal to the congruent mass flow 'Q_m', then the total mass of the particles over the whole length of the particle system, times their congruent velocity must be equal to the 'total amount (or capacity) of (congruent) mass flow' ($\mathbf{Q_{mL}}$) of that particle system:

$$\mathbf{Q_{mL}} = Q_m.\mathbf{L} = (m\mathbf{v_c}/\mathbf{L}).\mathbf{L} = m.\mathbf{v_c} = \mathbf{p} \quad \text{(expressed in M = kg.m/s = N/s)}$$

This demonstrates that the 'linear momentum' of a mass particle system is in fact nothing else than a mathematical expression of its total amount of congruent mass flow (or its total amount of congruent translational velocity/motion).

6. Conclusion: the physical nature of linear momentum

In the present physics the linear momentum of a particle system is arbitrarily defined as the product of its total mass times the vector velocity of its center of mass, and its conservation is explained as a consequence of Newton's first law of motion.

In this paper I have explained the physical meaning of the velocity of the center of mass of a particle system, as the 'common' translational velocity ($\mathbf{v_c}$) with which all the particles of the particle system move in a congruent way, that is with the same speed in the same direction, and which cannot in any way be affected by the internal/thermal motions of the particle system, so that without external interaction, the congruent velocity of a particle system must remain constant.

This allowed me to demonstrate that the 'linear momentum' of a moving particle system is a mathematical expression of its total amount of mass flow, which is an engineering term that expresses the total amount of congruent translational motion.

This leads us to the conclusion that the conservation of linear momentum of a mass particle system is a mathematical expression of physical fact that a particle system cannot change its total amount of congruent translational velocity/motion at the given velocity level by its own.

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