

Planets and Suns and Their Corresponding Sphere Packed Average Particles

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Abstract

When one talks about the density of a planet or star, one normally talks about the average density, despite the fact that the core is much more dense and the surface much less dense than the average density. Here we will link the notion of an average density to a new concept of a hypothetical planetary average subatomic particle. We will define this hypothetical particle as a particle if, when sphere-packed according to the Kepler conjuncture, it matches both the volume and the mass of the planet or sun in question. Even if this type of average particle may not actually exist, we still feel it gives us some new insight into how the average density could be linked to a hypothetical average particle. Take the question of how such a particle would be compared to an electron, for example. The answer is in the analytical solution presented.

Keywords: Planet average density, sphere packing, reduced Compton wavelength, averages planet particle.

1 A Planet or Sun's Average Subatomic Particle

We will assume that a subatomic particle occupies a spherical space that is defined by using its reduced Compton wavelength as its radius. The volume of a single subatomic particle sphere with "radius" $\bar{\lambda}$ will then be given by

$$V = \frac{4}{3}\pi\bar{\lambda}^3 \quad (1)$$

In 1831, Gauss [1] proved that the most densely one could pack spheres amongst all possible lattice packings was given by

$$\frac{\pi}{3\sqrt{2}} \approx 0.74048 \quad (2)$$

In 1611, Johannes Kepler suggested that this was the maximum possible density for both regular and irregular arrangements; this is known as the Kepler conjecture. The Kepler conjecture was supposedly finally proven in 2014 by Hale et. al. [2].

Here we will try to find a subatomic particle that we will sphere pack according to the Kepler conjecture, so that it matches both the volume and mass of the planet of interest. The mass of any subatomic particle is given by its reduced Compton wavelength through the formula

$$m = \frac{\hbar}{\bar{\lambda} c} \quad (3)$$

and if we assume a planet consists of only one uniform subatomic particle, then its mass has to be related by

$$M = Nm = N \frac{\hbar}{\bar{\lambda} c} \quad (4)$$

where N is the unknown number of average particles, and $\bar{\lambda}$ is the unknown reduced Compton wavelength of this particle. Further, \hbar is the reduced Planck constant, and M is the known mass of the planet in question. The radius of the planet and the subatomic radius of the average particle (we assume the

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radius of the particle is equal to its reduced Compton wavelength) have to be linked by sphere packing and must be¹

$$R = \bar{\lambda}^3 \sqrt[3]{\frac{N}{\pi}} \sqrt[6]{18} \quad (5)$$

In other words, we have two equations with two unknowns

$$M = N \frac{\hbar}{\bar{\lambda}} \frac{1}{c} \quad (6)$$

$$R = \bar{\lambda}^3 \sqrt[3]{\frac{N}{\pi}} \sqrt[6]{18} \quad (7)$$

and solving with respect to the unknown number of particles N and the unknown reduced Compton wavelength of the average particle, we get

$$N = \left(RM \frac{c}{\hbar} \right)^{\frac{3}{4}} \pi^{\frac{1}{4}} \left(\frac{1}{18} \right)^{\frac{1}{8}} \quad (8)$$

and the reduced Compton wavelength of this Planetary average density particle must be

$$\bar{\lambda}_A = \frac{N \hbar}{M c} \quad (9)$$

We know that the Earth has a radius $R \approx 6,371,000$ meters and a mass of 5.972×10^{24} kg. What is the number of average particles that, if sphere packed, will match the mass and radius if the Earth? It must be

$$N = \left(6,371,000 \times 5.972 \times 10^{24} \times \frac{c}{\hbar} \right)^{\frac{3}{4}} \pi^{\frac{1}{4}} \left(\frac{1}{18} \right)^{\frac{1}{8}} \approx 3.11 \times 10^{55}$$

and from this we get the reduced Compton wavelength of the average particle:

$$\bar{\lambda}_A = \frac{3.11 \times 10^{55} \hbar}{5.972 \times 10^{24} c} \approx 1.83 \times 10^{-12} \text{ m}$$

Further, the mass of this average Earth particle must be

$$m_A = \frac{\hbar}{\bar{\lambda}_A} \frac{1}{c} \approx 1.92 \times 10^{-31} \text{ kg}$$

and we have that $Nm = 3.11 \times 10^{55} \times 1.92 \times 10^{-31} \approx 5.972 \times 10^{24}$.

The radius of all the larger spheres of all of the average particles sphere-packed together must be

$$R = 1.83 \times 10^{-12} \sqrt[3]{\frac{3.11 \times 10^{55}}{\pi}} \sqrt[6]{18} \approx 6,371,000 \text{ m}$$

This means that the formulas for N and $\bar{\lambda}$ given above allow us to find the average particle that matches the mass and the volume as well as the radius and the mass density of the Earth. For comparison, the mass of the hypothetical average density particle of the Earth is only about 78% of the mass of an electron. The electron mass is about 9.10938×10^{-31} which is about 4.75 times the mass of the hypothetical earth average density-particle. However, since the electron has a considerably smaller radius, the mass density of the average particle is only about 0.197% of the electrons mass density, assuming they both are sphere-packed in relation to the Kepler conjuncture.

Again this is related to a purely hypothetical average particle from which the matter on Earth is built. We can not totally exclude that a particle with such a mass actually exists somewhere in the Earth or solar system, but this is not the main point here. The focus is on the mathematical properties of a hypothetical average particle, and the fact that we can calculate it easily. Just like the average density, we do not really know what the different parts of a planet consist of in terms of density. Similarly, the average particle is not really what the subatomic planet would consist of. Still, we feel this gives us some new insight about the average properties of a planet.

The same equations can be used for any type of planet or star where we know the volume and the mass as illustrated in table 1.

Interestingly, the average particle for all of the planets in the solar system, including the Sun, are all roughly 80% less massive than the electron². If this can lead us to any new progress in the field is

¹See [3] for the full derivation of this formula.

²Calculated as: $\frac{m_e - m_A}{m_e}$ where m_e is the electron mass and m_A is the hypothetical average particle mass..

	Earth	Sun	Moon	Mercury	Venus	Mars	Jupiter	Saturn
Radius	6,371,000	696,342,000	1,737,000	2,439,700	6,051,800	3,389,500	69,911,000	58,232,000
Mass	5.97E+24	1.99E+30	7.35E+22	3.30E+23	4.87E+24	6.42E+23	1.90.E+27	5.68E+26
Number of particles	3.11E+55	1.46E+61	4.34E+53	1.73E+54	2.57E+55	3.64E+54	1.41E+58	4.98E+57
R-Compton wavelength	1.83E-12	2.58E-12	2.08E-12	1.84E-12	1.86E-12	1.99E-12	2.62E-12	3.08E-12
Particle mass	1.92E-31	1.36E-31	1.69E-31	1.91E-31	1.90E-31	1.76E-31	1.34E-31	1.14E-31
Relative to electron	21.07%	14.97%	18.60%	20.99%	20.81%	19.37 %	14.76 %	12.52 %
$\frac{m_e - m_A}{m_e}$	78.93%	85.03%	81.40%	79.01%	79.19%	80.63%	85.24%	87.48%

Table 1: The table gives the reduced Compton wavelength and the mass of the hypothetical average particle relative to the Sun and some of the planets in the solar system. When sphere-packed, the average particle matches the mass and the volume of the planet from which it is calculated.

unknown at this point. While that remains to be seen, for now, one can look at it simply as a theoretical curiosity.

References

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