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Mathematical Theory of Black Holes – Its Infinite Equivalence Class

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The simplest solution to Einstein's 'field equations' that purportedly leads to a black hole universe is the so-called 'Schwarzschild solution' for the system $R_{\mu\nu} = 0$:

$$ds^2 = \left(1-\frac{\alpha}{r}\right)dt^2 - \left(1-\frac{\alpha}{r}\right)^{-1} dr^2 - r^2 d\Omega^2$$
$$d\Omega^2 = \left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \quad 0 \le r \qquad (1)$$
$$r = \sqrt{x^2+y^2+z^2}$$

The quantity r in (1) can be replaced by any analytic function of r without violating spherical symmetry or the 'field equations' $R_{\mu\nu} = 0$. However, not any analytic function of r is admissible because the solution must satisfy Einstein's prescription [1]:

1. It must be static.
2. It must be spherically symmetric.
3. It must satisfy $R_{\mu\nu} = 0$.
4. It must be asymptotically flat.

There exists an infinite equivalence class of solutions for the equations $R_{\mu\nu} = 0$, thereby constituting all admissible 'transformations of coordinates' [2, 3]. If any element of this infinite equivalence class cannot be extended then none can be extended, owing to equivalence. It has been proven [2, 3] that the infinite equivalence class is given by,

$$ds^2 = \left(1-\frac{\alpha}{R_c}\right)dt^2 - \left(1-\frac{\alpha}{R_c}\right)^{-1} dR_c^2 - R_c^2 d\Omega^2 \qquad (2)$$
$$R_c = \left(|\,r-r_0|^n + \alpha^n\right)^{1/n} \quad r, r_0 \in \Re, \quad n \in \Re^+$$

It follows immediately that no element of (2) can be extended because $|r - r_0|^n \ge 0$. This is amplified by the case $r_0 = 0$, $n = 2$. The 'Schwarzschild' universe is recovered by selecting $r_0 = \alpha$, $n = 1$, $r_0 \le r$. The line-element (1) cannot be extended to $0 \le r$ to produce a black hole because it is an element of the class (2). Consequently, $0 \le r$ in (1) is incorrect because it implies a violation of the rules of pure mathematics: *viz.* the positive real power of the absolute value of a real number must take on values less that zero.

According to cosmology there are four types of black hole. All must be elements of an infinite equivalence class that reduces to the solution for $R_{\mu\nu} = 0$ in accordance with (2). It has been proven [2, 3] that the overall infinite equivalence class, in the Boyer-Lindquist ground-form, is given by,

$$ds^2 = -\frac{\Delta - a^2\sin^2\theta}{\rho^2}dt^2 - \frac{2a\sin^2\theta\left(R_c^2+a^2-\Delta\right)}{\rho^2}dt\,d\varphi +$$
$$+\frac{\left(R_c^2+a^2\right)^2 - a^2\Delta\sin^2\theta}{\rho^2}\sin^2\theta\, d\varphi^2 + \frac{\rho^2}{\Delta}dR_c^2 + \rho^2 d\theta^2$$
$$\Delta = R_c^2 - \alpha R_c + a^2 + q^2, \quad \rho^2 = R_c^2 + a^2\cos^2\theta,$$
$$R_c = \left(|r-r_0|^n + \xi^n\right)^{1/n}, \quad \xi = \frac{\alpha}{2} + \sqrt{\frac{\alpha^2}{4} - q^2 - a^2\cos^2\theta},$$
$$a^2 + q^2 < \frac{\alpha^2}{4}, \quad r, r_0 \in \Re, \quad n \in \Re^+ \qquad (3)$$

It is immediately evident that no element of (3) can be extended to $0 \le R_c$ to produce a black hole because $|r - r_0|^n \ge 0$, again amplified by the case $r_0 = 0$, $n = 2$. The production of a black hole by 'extension' of any element of (3) once again constitutes a violation of the rules of pure mathematics. It is therefore invalid. Selecting $r_0 = \xi$, $r_0 \le r$, $n = 1$ in (3) yields the Kerr-Newman universe, which cannot be extended to produce a black hole universe because it is an element of the infinite equivalence class (3).

The geometric basis for the non-extendibility of (1) is that in (1) cosmology has incorrectly specified $r = 0$ as the 'origin' for the spherical symmetry of the line-element. The correct specification of r in (1) is fixed by the non-extendible infinite equivalence class (2), of which the line-element of (1) is a member, and is given by [2, 3],

$$r = \sqrt{x_0^2+y_0^2+z_0^2} +$$
$$+\sqrt{\left(x-x_0\right)^2+\left(y-y_0\right)^2+\left(z-z_0^2\right)} = \qquad (4)$$
$$= r_0 + r' = \alpha + r'$$

By means of (2) and (4), $r = r_0 = \sqrt{x_0^2+y_0^2+z_0^2} = \alpha$ marks the centre of spherical symmetry at (x_0, y_0, z_0) in (1), not at the origin $x = y = z = 0$. Although $r = 0 = x = y = z$ is the origin of a Euclidean coordinate system, it does not denote the centre of spherical symmetry of (1). When a sphere originally located at the origin of a coordinate system is moved to some other place in the same space, it necessarily takes its centre with it, as (4) states. The black hole is the result of unwittingly moving a Euclidean sphere, originally centred at the origin of a coordinate system, to some other place in the same space, but leaving its centre behind, which is a violation of the rules of pure mathematics, as revealed analytically by the infinite equivalence class.

References

[1] Einstein, A., The Meaning of Relativity, Princeton University Press, 1988
[2] Crothers, S.J., A Critical Analysis of LIGO's Recent Detection of Gravitational Waves Caused by Merging Black Holes, *Hadronic Journal*, Vol. 39, 2016, http://vixra.org/abs/1603.0127
[3] Crothers, S.J., On Corda's 'Clarification' of Schwarzschild's Solution, *Hadronic Journal*, Vol. 39, 2016, http://vixra.org/abs/1602.0221