Modeling the Flow of Yield-Stress Fluids in Porous Media

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Abstract

Yield-stress is a problematic and controversial non-Newtonian flow phenomenon. In this article, we investigate the flow of yield-stress substances through porous media within the framework of pore-scale network modeling. We also investigate the validity of the Minimum Threshold Path (MTP) algorithms to predict the pressure yield point of a network depicting random or regular porous media. Percolation theory as a basis for predicting the yield point of a network is briefly presented and assessed. In the course of this study, a yield-stress flow simulation model alongside several numerical algorithms related to yield-stress in porous media were developed, implemented and assessed. The general conclusion is that modeling the flow of yield-stress fluids in porous media is too difficult and problematic. More fundamental modeling strategies are required to tackle this problem in the future.

1 Introduction

Yield-stress or viscoplastic fluids are characterized by their ability to sustain shear stresses, that is a certain amount of stress must be exceeded before the flow initiates. So an ideal yield-stress fluid is a solid before yield and a fluid after. Accordingly, the viscosity of the substance changes from an infinite to a finite value. The physical situation, however, indicates that it is more realistic to regard yield-stress substances as fluids whose viscosity as a function of applied stress has a discontinuity as it drops sharply from a very high value on exceeding a critical stress.

There are many controversies and unsettled issues in the non-Newtonian literature about yield-stress phenomenon and yield-stress fluids. In fact, even the concept of a yield-stress has received much recent criticism, with evidence presented to suggest that most materials weakly yield or creep near zero strain rate. The supporting argument is that any material will flow provided that one waits long enough. These conceptual difficulties are backed by practical and experimental complications. For example, the value of yield-stress for a particular fluid is difficult to measure consistently and it may vary by more than an order of magnitude depending on the measurement technique [1, 2, 3, 4].

Among the difficulties in working with yield-stress fluids and validating the experimental data is that yield-stress value is usually obtained by extrapolating a plot of shear stress to zero shear rate [2]. This can result in a variety of values for yield-stress, depending on the distance from the shear stress axis experimentally accessible by the instrument used. The vast majority of yield-stress data reported results from such extrapolations [5, 6], making most values in the literature instrument-dependent [2]. Another method used to measure yield-stress is by lowering the shear rate until the shear stress approaches a constant. This may be described as a dynamic yield-stress [7]. The results obtained using such methods may not agree with the static yield-stress measured directly without disturbing the

microstructure during the measurement. The latter seems more relevant to the flow initiation under gradual increase in pressure gradient as in the case of porous media flow experiments. Consequently, the accuracy of the predictions made using flow simulation models in conjunction with such experimental data is limited.

Another difficulty is that while in the case of pipe flow the yield-stress value is a property of the fluid, in the case of flow in porous media it may depend on both the fluid and the porous medium itself [8, 9]. One possible explanation is that yield-stress value may depend on the size and shape of the pore space when the polymer macromolecules become comparable in size to the pore. The implicit assumption that yield-stress value at pore level is the same as the value at bulk may not be evident. This makes the predictions of the models based on analytical solution to the flow in a uniformly-shaped tube combined with the bulk rheology less accurate. When the duct size is small, as it is usually the case in porous media, flow of macromolecule solutions normally displays deviations from predictions based on corresponding viscometric data [10]. Moreover, the highly complex shape of flow paths in porous media must have a strong impact on the actual yield point, and this feature is lost by modeling these paths with ducts of idealized geometry.

Various attempts to model the flow of yield-stress fluids in porous media can be found in the literature. Most of these are based on extending the existing continuum non-yield-stress flow models to incorporate yield-stress. For example, Pascal [11] modified Darcy's law by introducing a threshold pressure gradient to account for the yield-stress. This threshold gradient is directly proportional to the yield-stress and inversely proportional to the square root of the absolute permeability. Vradis and Protopapas [9] extended the 'capillary tube' and the 'resistance to flow' models to describe the flow of Bingham fluids in porous media and presented a solution in which the flow is zero below a threshold head gradient and Darcian above it. Chase and Dachavijit [12] modified the Ergun equation to describe the flow of yield-stress

fluids through porous media by applying a bundle of capillary tubes approach. Other attempts include Wu *et al* [13] who used an integral analytical method to obtain an approximate analytical solution for single-phase flow of Bingham fluids through porous media. Chaplain *et al* [14] also modeled the flow of Bingham fluids through porous media by generalizing Saffman [15] analysis for the Newtonian flow to describe the dispersion in a porous medium by a random walk. Recently, Balhoff and Thompson [16] used their three-dimensional network model, which is based on a computer-generated random sphere packing, to investigate the flow of Bingham fluids in packed beds. Network modeling was also used by several researchers to investigate the generation and mobilization of foams and yield-stress fluids in porous media. These include Rossen and Mamun [17] and Chen *et al* [18, 19, 20].

2 Modeling Yield-Stress in Porous Media

Several constitutive equations to describe the fluids with yield-stress are in use; the most popular ones are Bingham, Casson and Herschel-Bulkley. Some have suggested that the yield-stress values obtained via such models should be considered model parameters and not real material properties.

There are four main approaches for modeling the flow through porous media in general. These are: continuum models, bundle of tubes models, numerical methods and pore-scale network modeling. It seems that all these approaches suffer from difficulties when applied to the flow of yield-stress fluids in complex porous media.

The failure of the continuum models should not be a surprise because these models do not account for the complex geometry and topology of the void space. As the yield point depends on the fine details of the pore space structure, no continuum model is expected to predict the threshold yield pressure. The continuum models also fail to predict the flow rate, at least at transition stage where the medium is partly conducting, because according to these models the medium is either fully blocked or fully conducting whereas in reality the yield of porous medium is a gradual and pressure-dependent process. For instance, these models predict for Bingham fluids a linear relationship between Darcy velocity and pressure gradient with an intercept at the threshold yield gradient whereas network modeling results, supported by experimental evidence, predict a nonlinear behavior at transition stage [9]. Some authors have concluded that in a certain range the macroscopic flow rate of Bingham plastic in a network depends quadratically on the departure of the applied pressure difference from its minimum value [20]. Our network simulation predictions confirm this quadratic correlation.

Regarding the capillary bundle models, the situation is similar to the continuum models as they predict a single universal yield point at a particular pressure drop if a uniform bundle of capillaries is assumed, whereas in reality porous medium

yield occurs gradually as the pressure gradient increases. Moreover, because all capillary bundle models fail to capture the topology and geometry of complex porous media, they cannot predict the yield point and describe the flow rate since yield is highly dependent on the fine details of the void space. An important aspect of the geometry of real porous media, which strongly affects the yield point and flow rate of yield-stress fluids, is the converging-diverging nature of the flow paths. This feature is not reflected by the bundle of uniform capillaries models. Another feature is the connectivity of the flow channels where bond aggregation (i.e. how the throats are distributed and arranged) strongly affects yield behavior.

As for the application of numerical methods to yield-stress fluids in porous media, very few studies can be found on this subject. Moreover, the results, which are reported only for very simple cases of porous media, cannot be fully assessed. As a result, network modeling seems to be the most viable candidate for modeling yield-stress fluids in porous media. However, the research in this field is limited, and hence no final conclusion on the merit of this approach can be reached. In the followings we present our modeling strategies with a brief assessment of the results and some general conclusions.

In our attempt to investigate the flow of yield-stress fluids in porous media we use pore-scale network modeling. Network modeling has been fully described in a number of references (e.g. [21, 22, 23]). Here, we focus only on the part of network modeling related to yield-stress fluids. The interested reader should refer to those references for other details. In our non-Newtonian model, yield-stress is described by a Herschel-Bulkley fluid [24]:

$$\tau = \tau_o + C\dot{\gamma}^n \qquad (\tau > \tau_o) \tag{1}$$

where τ is the shear stress, τ_o is the yield-stress above which the substance starts flowing, C is the consistency factor, $\dot{\gamma}$ is the shear rate and n is the flow behavior

index. For yield-stress fluids, the threshold pressure drop above which the flow in a single tube starts is given by the relation

$$\Delta P_{th} = \frac{2L\tau_o}{R} \tag{2}$$

where R and L are the tube radius and length respectively. For Herschel-Bulkley fluids, the volumetric flow rate in a cylindrical capillary at yield is given by

$$Q = \frac{8\pi}{C^{\frac{1}{n}}} \left(\frac{L}{\Delta P}\right)^3 (\tau_w - \tau_o)^{1+\frac{1}{n}} \left[\frac{(\tau_w - \tau_o)^2}{3+1/n} + \frac{2\tau_o(\tau_w - \tau_o)}{2+1/n} + \frac{\tau_o^2}{1+1/n}\right] \quad (\tau_w > \tau_o) \tag{3}$$

where τ_w $(= \frac{\Delta P R}{2L})$ is the shear stress at the tube wall.

The implementation of yield-stress in the network model is based on the yield condition (i.e. Equation 2) of its conducting ducts which are assumed to be circular cylinders. The verification of the yield condition in the individual ducts associates the process of solving the pressure field in the network. Moreover, when flow occurs the volumetric flow rate of yield-stress fluids in the network elements is described by the Herschel-Bulkley relation, i.e. Equation (3). Another condition is imposed before any duct in the network is allowed to yield, that is the duct must be part of a non-blocked path spanning the network from the inlet to the outlet. The reason is that any conducting duct should have a source on one side and a sink on the other, and this will not happen if either or both sides are blocked.

In our model, the substance before yield is assumed to be fluid with very high but finite viscosity so the flow virtually vanishes. What justifies this assumption is that the pressure across the network must communicate. Accordingly, the pressure field in the case of yield-stress fluids is solved as in the case of non-yield-stress fluids. In both cases, to find the pressure field a set of simultaneous equations representing the capillaries and satisfying mass conservation have to be solved subject to the

boundary conditions which are the pressures at the inlet and outlet of the network. This unique and consistent solution, which should mimic the unique physical reality of the pressure field in the porous medium, is the only mathematically acceptable solution to the problem. It should be remarked that the assumption of very high but finite zero-stress viscosity for yield-stress fluids is realistic and supported by experimental evidence.

The yield-stress flow model, as described above, was implemented in a non-Newtonian computer code [25]. Two randomly-distributed networks representing two different porous media, a sand pack and a Berea sandstone [23], were used to investigate and assess the model. Computer-generated cubic networks were also used in this investigation. Theoretical analysis has been carried out using a simple bundle of capillary tubes of uniform radius. The threshold yield path was visualized and analyzed to investigate the possibility of aggregation of extreme elements in a favorable combination that can compromise the reliability of predictions. The continuity of flow across the network was tested by numerical and visual inspections. Comparison to experimental data sets found in the literature [6, 12, 26] was also carried out. A representative sample of these data sets with the corresponding simulation results are given in Figures (1, 2, 3). As seen, the results are mixed with obvious failure in some cases.

The theoretical analysis and comparison revealed sensible trends of behavior and sound implementation. However, as compared to the available experimental data the network model results for yield-stress fluids are less satisfactory than the results of non-yield-stress fluids. This, with similar outcome obtained by other researchers, indicates that network modeling in its current state is not fully capable of describing yield-stress phenomenon. Although the quality of some experimental data sets is questionable, this reason alone cannot fully explain these failures. One possible reason is the relative simplicity of the rheological models, such as

Bingham, used in these investigations. These models may be capable of providing a phenomenological description of yield-stress in simple flow situations. However, they cannot accommodate the underlying physics at pore level in complex porous media. Consequently, yield-stress as a model parameter obtained in bulk viscometry may not be appropriate to use in this complex situation.

Another reason is the experimental difficulties associated with non-Newtonian fluids in general. This can affect the experimental results and introduce complications, such as viscoelasticity and retention, that may not be accounted for in the network model. A third reason is the relative simplicity of the current network modeling approach to yield-stress fluids. This may be supported by the fact that much better results are normally obtained for non-yield-stress fluids using the same network modeling techniques. One major limitation of the current network models with regard to yield-stress fluids is the use of analytical expressions for cylindrical tubes based on the concept of equivalent radius. This is far from reality where the void space retains highly complex shape and connectivity. Consequently, the yield condition for cylindrical capillaries becomes invalid approximation to the yield condition for the intricate flow paths in porous media. The concept of equivalent radius, which is used in network modeling, though is completely appropriate for Newtonian fluids and reasonably appropriate for purely viscous non-Newtonian fluids with no yield-stress, seems inappropriate for yield-stress fluids as yield depends on the actual shape of the void space rather than the equivalent radius and flow conductance. The simplistic nature of the yield condition in porous media is highlighted by the fact that in almost all cases of disagreement between the network and the experimental results the network produced a lower yield value.

In summary, yield-stress fluid results are extremely sensitive to how the fluid is characterized, how the void space is described and how the yield process is modeled. In the absence of a comprehensive and precise incorporation of all these

factors in the network model, pore-scale modeling of yield-stress fluids in porous media remains a crude approximation that may not produce quantitatively sensible predictions.

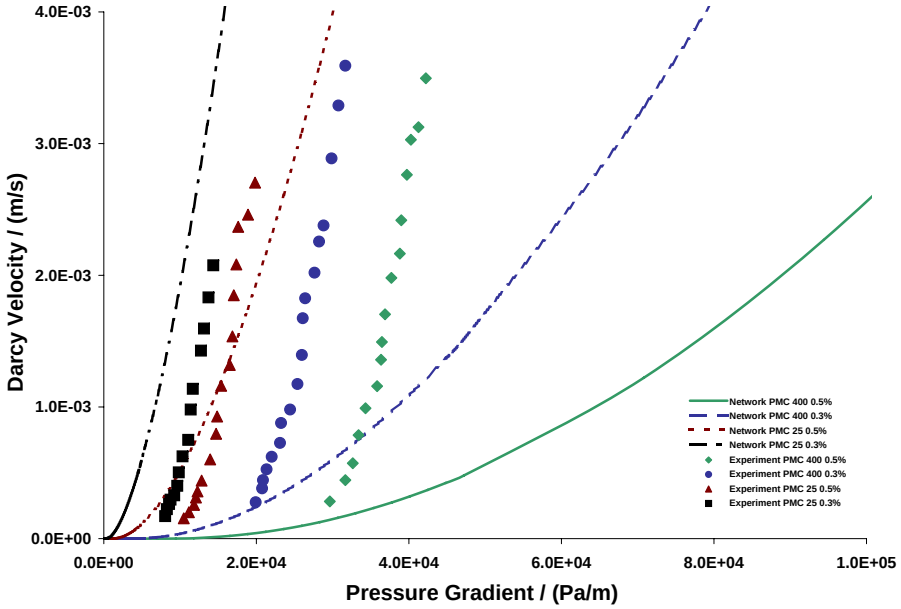

Figure 1: Park experimental data group for Herschel-Bulkley aqueous solutions of PMC 400 and PMC 25 with 0.5% and 0.3% weight concentration flowing through a fine packed bed of glass beads having $K = 366\,\text{Darcy}$ and $\phi = 0.39$ alongside the simulation results obtained with a scaled sand pack network having same K.

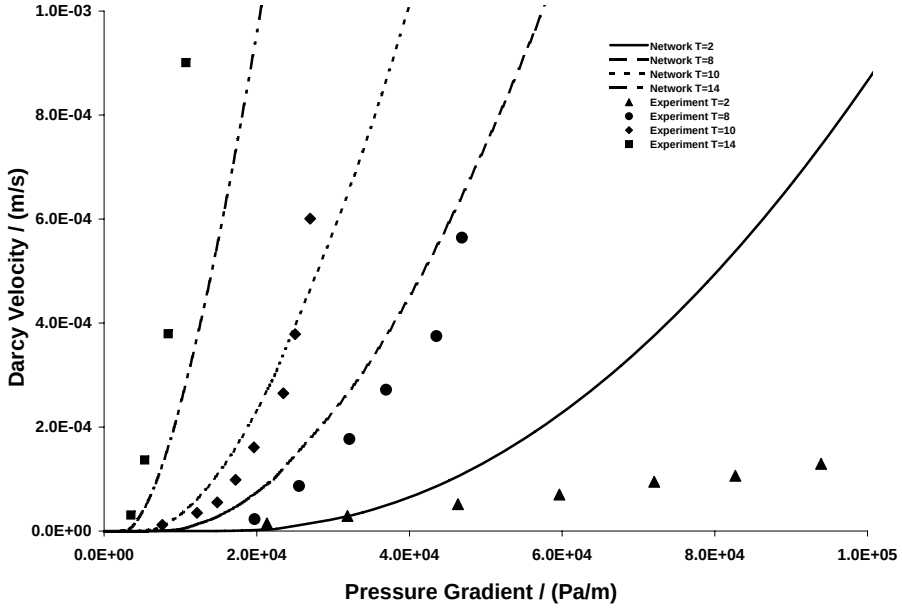

Figure 2: Al-Fariss and Pinder experimental data group for Herschel-Bulkley waxy crude oils flowing through a column of sand having $K = 1580\,\text{Darcy}$ and $\phi = 0.44$ alongside the simulation results obtained with a scaled sand pack network having the same K. The temperatures, T, are in °C.

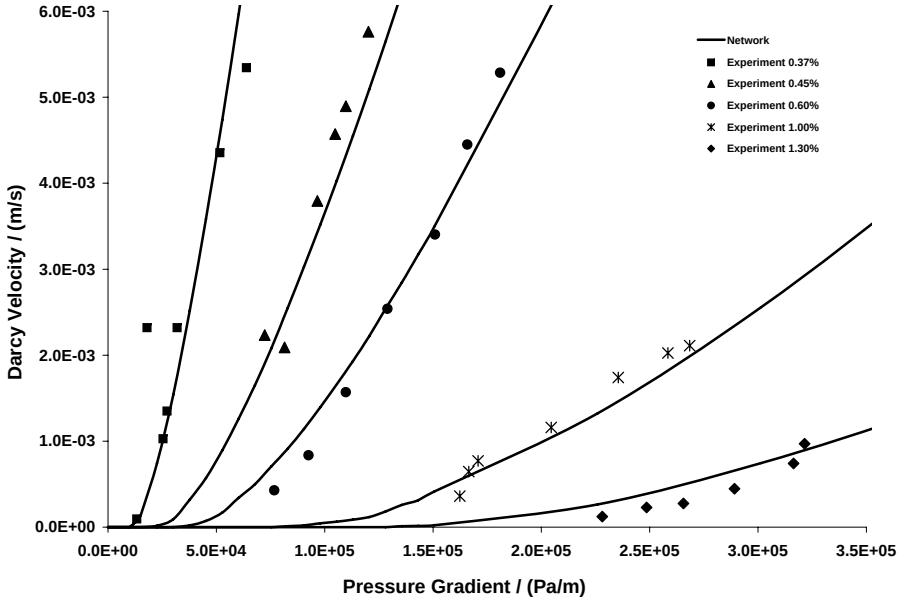

Figure 3: Network simulation results with the corresponding experimental data points of Chase and Dachavijit for Bingham aqueous solutions of Carbopol 941 with various concentrations (0.37%, 0.45%, 0.60%, 1.00% and 1.30%) flowing through a packed column of glass beads.

3 Predicting Threshold Yield Pressure

Here we discuss the attempts to predict the yield point of a complex porous medium from the void space description and yield-stress value of an ideal yield-stress fluid without modeling the flow process. In the literature of yield-stress we can find two well-developed methods proposed for predicting the yield point of a morphologically-complex network that depicts a porous medium; these are the Minimum Threshold Path (MTP) and the percolation theory algorithms. In this regard, there is an implicit assumption that the network is an exact replica of the medium and the yield-stress value reflects the real yield-stress of the fluid so that any failure of these algorithms can not be attributed to mismatch or any factor other than flaws in these algorithms. It should be remarked that the validity of these methods can be tested by experiment.

3.1 Minimum Threshold Path Algorithms

Predicting the threshold yield pressure of a yield-stress fluid in porous media in its simplest form may be regarded as a special case of the more general problem of finding the threshold conducting path in disordered media that consist of elements with randomly distributed thresholds. This problem was analyzed by Roux and Hansen [27] in the context of studying the conduction of an electric network of diodes by considering two different cases, one in which the path is directed (no backtracking) and one in which it is not. They suggested that the minimum overall threshold potential difference across the network is akin to a percolation threshold and investigated its dependence on the lattice size. Kharabaf and Yortsos [28] noticed that a firm connection of the lattice-threshold problem to percolation appears to be lacking and the relation of the minimum threshold path to the minimum path of percolation, if it indeed exists, is not self-evident. They presented a new algorithm, Invasion Percolation with Memory (IPM), for the construction of the

MTP to study its properties. The Invasion Percolation with Memory was further extended by Chen *et al* [20] to incorporate dynamic effects due to viscous friction following the onset of mobilization. In the course of our investigation, another MTP algorithm called Path of Minimum Pressure (PMP), which is computationally more efficient than IPM, was developed by the author. Both algorithms were implemented in the non-Newtonian code. The results of these algorithms, which are identical in most cases, were analyzed and compared to the experimental data and the network flow simulation predictions. In the following we outline and assess these algorithms.

The IPM is a way for finding the inlet-to-outlet path that minimizes the sum of the values of a property assigned to the individual elements of the network, and hence finding this minimum. For a yield-stress fluid, this reduces to finding the inlet-to-outlet path that minimizes the yield pressure. The yield pressure of this path is taken as the network threshold yield pressure. An algorithm to find the threshold yield pressure according to IPM is outlined below:

1. Initially, the nodes on the inlet are considered to be sources and the nodes on the outlet and inside are targets. The inlet nodes are assigned a pressure value of 0.0. According to the IPM, a source cannot be a target and vice versa, i.e. they are disjoint sets and remain so in all stages.

2. Starting from the source nodes, a step forward is made in which the yield front advances one bond from a single source node. The condition for choosing this step is that the sum of the source pressure plus the yield pressure of the bond connecting the source to the target node is the minimum of all similar sums from the source nodes to the possible target nodes. This sum is assigned to the target node.

3. This target node loses its status as a target and obtains the status of a source.

4. The last two steps are repeated until the outlet is reached, i.e. when the target is an outlet node. The pressure assigned to this target node is regarded as the threshold yield pressure of the network.

A flowchart of the IPM algorithm is given in Figure (4).

The PMP is based on a similar assumption to that upon which the IPM is based, that is the network threshold yield pressure is the minimum sum of the threshold yield pressures of the individual elements of all possible paths from the inlet to the outlet. However, it is computationally different and is more efficient than the IPM in terms of the CPU time and memory. According to the PMP, to find the threshold yield pressure of a network all possible paths of serially-connected bonds from inlet to outlet are inspected. For each path, the threshold yield pressure of each bond is computed and the sum of these pressures is found. The network threshold yield pressure is then taken as the minimum of these sums. The implementation of this algorithm can be outlined as follows:

1. The nodes on the inlet are initialized with zero pressure while the remaining nodes are initialized with infinite pressure (very high value).

2. A source is an inlet or an inside node with a finite pressure while a target is a node connected to a source through a single bond in the direction of pressure gradient.

3. Looping over all sources, the threshold yield pressure of each bond connecting a source to a target is computed and the sum of this yield pressure plus the pressure of the source node is found. The minimum of this sum and the current yield pressure of the target node is assigned to the target node.

4. The loop in the last step is repeated until a stable state is reached when no target node changes its pressure during looping over all sources.

5. A loop over all outlet nodes is used to find the minimum pressure assigned to the outlet nodes. This pressure is taken as the network threshold yield pressure.

The PMP algorithm is graphically depicted in a flow chart in Figure (5).

As implemented in the non-Newtonian code, the IPM and PMP produce very similar results. However, they both predict lower threshold yield pressure compared to the network flow simulation results and the experimental data.

There are two possibilities for defining yield stress fluids before yield: either solid-like substances or liquids with very high viscosity. According to the first, the most sensible way for modeling a presumed pressure gradient inside a medium before yield is to be a constant, that is the pressure drop across the medium is a linear function of the spatial coordinate in the flow direction. The reason is that any other assumption requires justification. In the second case, the fluid should be treated like non-yield-stress fluids and hence the pressure field inside the porous medium should be subject to the consistency criterion given in § 2. The logic is that the magnitude of the viscosity should have no effect on the flow behavior as long as the material is assumed to be fluid.

Several arguments can be presented against the MTP algorithms for predicting the yield point of a network representing a medium. Although some arguments may be more obvious for a network with cylindrical ducts, they are valid in general for regular and irregular geometries of flow channels. Some of these arguments are outlined below

- The MTP algorithms are based on the assumption that the threshold yield pressure (TYP) of an ensemble of serially connected bonds is the sum of their yield pressures. This assumption can be challenged by the fact that for a non-uniform ensemble (i.e. an ensemble whose elements have different TYPs) the pressure gradient across the ensemble should reach the threshold

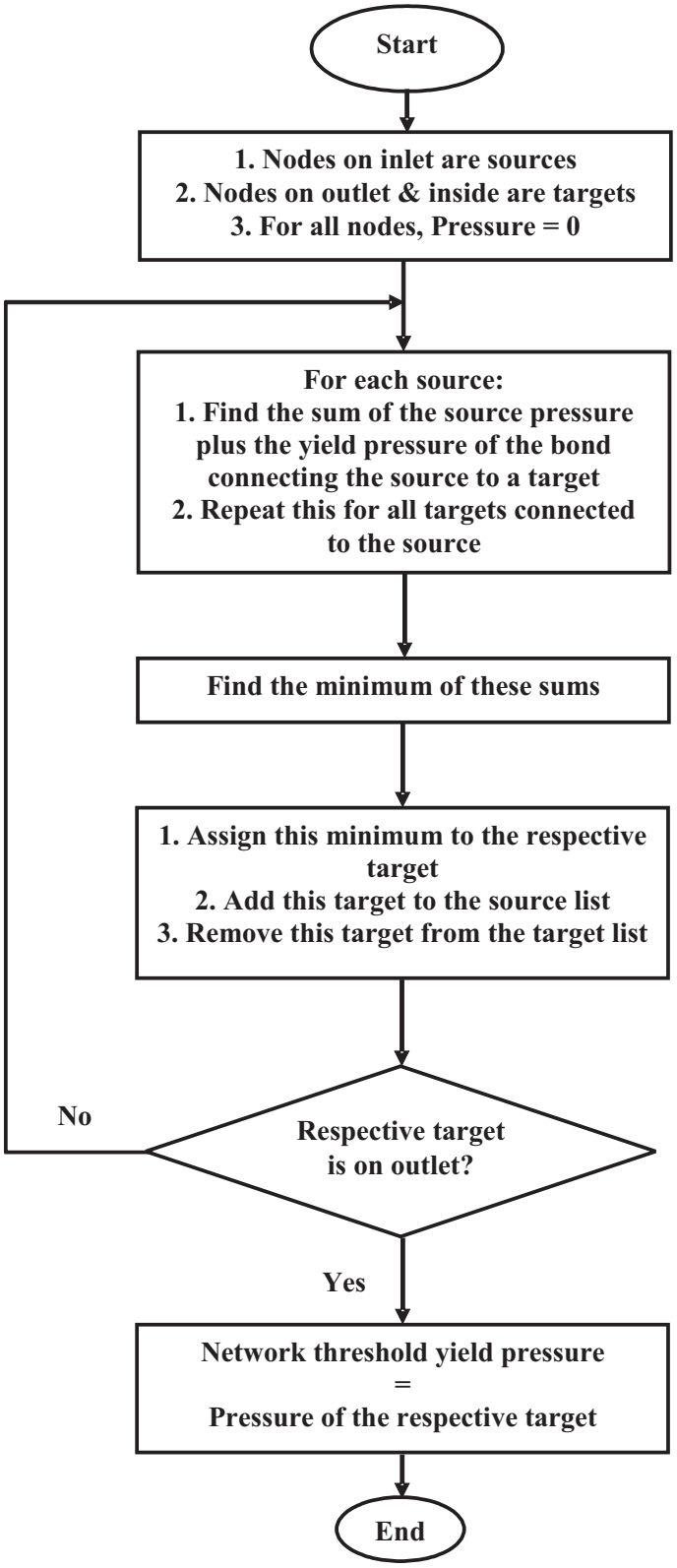

Figure 4: Flowchart of the Invasion Percolation with Memory (IPM) algorithm.

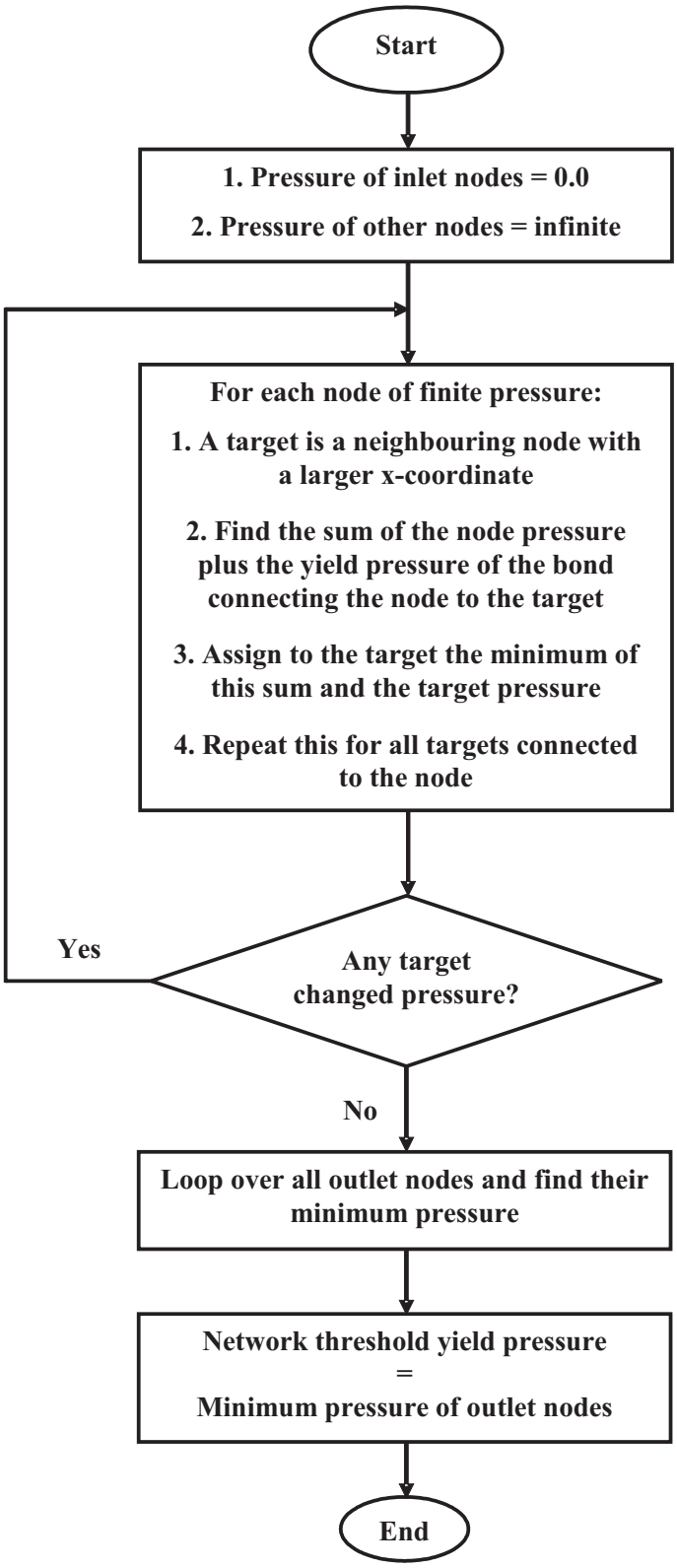

Figure 5: Flowchart of the Path of Minimum Pressure (PMP) algorithm.

yield gradient of the bottleneck (i.e. the element with the highest TYP) of the ensemble if yield should occur. Consequently, the TYP of the ensemble will be higher than the sum of the TYPs of the individual elements. This argument may be more obvious if yield-stress fluids are regarded as solids and a linear pressure drop is assumed.

- Assuming that yield-stress substances before yield are fluids with high viscosity, the dynamic aspects of the pressure field are neglected because the aim of the MTP algorithms is to find a collection of bonds inside the network with a certain condition based on the intrinsic properties of these elements irrespective of the pressure field. The reality is that the bonds are part of a network that is subject to a pressure field, so the pressure across each individual element must comply with a dynamically stable and mathematically consistent pressure configuration over the whole network. The MTP algorithms rely for justification on a hidden assumption that the minimum sum condition is identical or equivalent to the stable configuration condition. This assumption is disputable because it is very unlikely that a stable configuration will require a pressure drop across each element of the minimum threshold path that is identical to the threshold yield pressure of that element. Therefor, it is not clear that the actual path of yield must coincide, totally or partially, with the path of the MTP algorithms let alone that the actual value of yield pressure must be predicted by these algorithms.

- The MTP algorithms that allow backtracking have another drawback that is in some cases the minimum threshold path requires a physically unacceptable pressure configuration. This is more obvious if the yield-stress substances are assumed to be solid before yield.

- The effect of tortuosity is ignored in the MTP algorithms since they implicitly

assume that the path of yield is an ensemble of serially-connected and linearly-aligned tubes, whereas in reality the path is generally tortuous as it is part of a network and can communicate with the global pressure field through the intermediate nodes. The effect of tortuosity, which is more obvious for the solid assumption, is a possible increase in the external threshold pressure gradient and a possible change in the bottleneck.

3.2 Percolation Theory

Concerning the percolation approach, it is tempting to consider the conduct of yield-stress fluids in porous media as a percolation phenomenon to be analyzed by the classical percolation theory. However, three reasons, at least, may suggest otherwise

1. The conventional percolation models can be applied only if the conducting elements are homogeneous, i.e. it is assumed in these models that the intrinsic property of all elements in the network are equal. However, this assumption cannot be justified for most kinds of media where the elements vary in their conduction and yield properties. Therefore, to apply percolation principles, a generalization to the conventional percolation theory is needed as suggested by Selyakov and Kadet [29].

2. The network elements cannot yield independently as a spanning path bridging the inlet to the outlet is a necessary condition for yield. This contradicts the percolation theory assumption that the elements conduct independently.

3. The pure percolation approach ignores the dynamic aspects of the pressure field, that is a stable pressure configuration is a necessary condition which may not coincide with the simple percolation requirement. The percolation condition, as required by the classic percolation theory, determines the stage

at which the network starts flowing according to the intrinsic property of its elements as an ensemble of conducting bonds regardless of the dynamic aspects introduced by the external pressure field, as explained earlier.

In a series of studies on generation and mobilization of foams in porous media, Rossen *et al* [17, 30] analyzed the threshold yield pressure using percolation theory concepts and suggested a simple percolation model. In this model, the percolation cluster is first found, then the minimum threshold path was approximated as a subset of this cluster that samples those bonds with the smallest individual thresholds [20]. This approach relies on the validity of applying percolation theory to yield-stress, which is disputed. Moreover, it is a mere coincidence if the yield path is contained within the percolation sample. Yield is an on/off process which critically depends on factors other than smallness of individual thresholds. These factors include the particular distribution and configuration of these elements, being within a larger network and hence being able to communicate with the global pressure field, and the dynamic aspects of the pressure field and stability requirement. Any approximation, therefore, has little meaning in this context.

4 Conclusions and Discussions

In this study, we developed and implemented a non-Newtonian flow simulation model for the flow of yield-stress fluids in porous media, and compared the simulation results to a number of experimental data sets found in the literature. We also investigated two minimum threshold path algorithms and the percolation theory as a basis for predicting the threshold yield pressure of a network depicting a porous medium. General results and conclusions that can be drawn from this study are

- In several cases, the simulation results as compared to the experimental data are not satisfactory. One possible reason is inadequate representation of the pore space structure with regard to the yield process, i.e. the yield in a network depends on the actual shape of the void space rather than the flow conductance of the pores and throats. Other reasons include experimental errors and the occurrence of other physical phenomena which are not accounted for in our model such as precipitation and adsorption.

- The analysis of minimum threshold path algorithms (i.e. IPM and PMP) revealed that these algorithms are too simplistic and hence cannot produce reliable predictions for the threshold yield pressure of a network. Compared to the experimental data and the network simulation results, these algorithms predict lower threshold yield pressure.

- The assessment of the percolation theory suggested that percolation may not be suitable for modeling yield-stress in porous media.

The final conclusion is that the current flow modeling methodologies cannot cope with the complexity of yield-stress fluids in porous media. More elaborate strategies are required to make progress on this front.

Nomenclature

$\dot{\gamma}$	strain rate (s^{-1})
τ	stress (Pa)
τ_o	yield-stress (Pa)
τ_w	stress at tube wall (Pa)
ϕ	porosity
C	consistency factor in Herschel-Bulkley model ($Pa.s^n$)
K	absolute permeability (m^2)
L	tube length (m)
n	flow behavior index
P	pressure (Pa)
ΔP	pressure drop (Pa)
ΔP_{th}	threshold pressure drop (Pa)
Q	volumetric flow rate ($m^3.s^{-1}$)
R	tube radius (m)
T	temperature (K, °C)
IPM	Invasion Percolation with Memory algorithm
MTP	Minimum Threshold Path
PMP	Path of Minimum Pressure algorithm
TYP	Threshold Yield Pressure

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