# About The Geometry Of Cosmos (Revised)

**D.Mastoridis – 1**

Vyzantiou 24, Nea Smirni, Athens, Greece, $P.O.Box$17121, d.mastoridis@prv.ypeka.gr

**K.Kalogirou – 2**

Eleytheriou Venizelou 187, Ilioupoli, Athens, Greece, $P.O.Box$16341, k.kalogirou@prv.ypeka.gr

## Contents

**Abstract**

The current paper presents a new idea that it might lead us to the Grand Unified Theory. A concrete mathematical framework has been provided that could be appropriate for one to work with. Possible answers were given concerning the problems of dark matter and dark energy as well as the "penetration" to vacuum dominant epoch, combining Quantum Physics with Cosmology through the existence of Higg's boson. A value for Higg's mass around 125.179345 Gev/$c^2$ and a value for vacuum density around $4.41348x10^{-5}Gev/cm^3$ were derived . Via Cartan's theorem a proof regarding the number of bosons existing in nature (28) has been presented. Additionally, the full Lagrangian of our Cosmos (including Quantum Gravity) was accomplished.

# 1 Introduction

The recent results from CERN concerning the discovery of Higg's boson made many scientists enthusiastic; such a result convinced us that Higg's mechanism (HM) is essential for the deep understanding of Cosmos. Due to this discovery, many scientists hold the belief that now we can and must make the great step to present a Grand Unified Theory (GUT). Grand Unified Theories must present a fundamental scheme where Quantum Physics and General Relativity can be combined into one unique theory. On the other hand such a formulation is difficult to be presented due to the fact that General Relativity, which describes gravity, is a classical theory. However, on the other, electromagnetic, weak nuclear and strong nuclear fields can be presented and unified by Quantum theories. Standard Model (SM) presented a great breakthrough in the history of Physics; it gave us a huge amount of information about the interactions between particles, the nature of fields and, most important, a way to unify. Unfortunately, all particles described by SM were massless, which was contrary to the experimental facts; and it was HM that filled this gap. However, despite the success of HM, it not only remains an ad-hoc mechanism, but rather a classical one, as well. We do not know what mathematics are hidden behind or how Yang-Mills theories include this mechanism by formulation. Moreover, even if we combine SM with HM, there still remain many questions unanswered, such as :

1. Mass gap.
2. There are totally 18 individual parameters.
3. Neutrinos remain massless. On the contrary, experiments gave us the opposite.
4. Are gluons massless? Are there gluon balls?
5. It doesn't answer us if there are other bosons with spin 1 and if there are other Fermion families.
6. We are in total darkness regardingg dark matter and dark energy.
7. We still have the major issue of the cosmological constant which is related to vacuum, where the theoretical value with experimental one differs by a factor of $10^{122}$.

All the above questions and problems can be compactified to only one and are well described by Arthur Jaffe and Edward Witten [9] in the following quote. "On the other hand one does not yet have a mathematical complete example of a quantum Gauge theory in a four dimensional space time, nor even a precise definition of a quantum Gauge theory in four dimensions. Will this change in the $21^{st}$ century? We hope so!" In this paper, we will present a new idea which we hopefully believe could not only answer the big questions of Physics, but be a good candidate as a Grand-Unified theory. First of all our starting belief was that differential Geometry is the key for a G.U.T because we could have a concrete representation on a well-established scheme to work with. Furthermore, we were convinced that a Kaluza-Klein type of theory was in the right direction. As a consequence, we have started to build the necessary space by having in mind that we need more than four dimensions and that the origin of mass would be presented in an evolutionary way. Our approach is unsimilar to Strings theories but similar to the point of many dimensions and their "compactifications". To begin with, we start up with an 8 dimensional real space or a four dimensional complex one. We will show that such a choice is more economical compared to the eleven dimensions of the M-theory in Strings, and that the property of Triality (Cartan's theorem) comes naturally. We will construct our space step-by-step in paragraph 1 by defining almost all the necessary properties. We have to point out that we choose to work in $C^4$ in order to provide and remain in the standard formulation of Quantum theories, but in certain occasions for convenience we will choose to work with $R^8$. This is due to the fact that differential Geometry in $C^4$ is not so well written and studied in detail, compared to the one of $R^n$.

In paragraph 3, we will work with Cartan's theorem of Triality. Truly, this marvelous theorem was the key in order to decide in a definite way, about the signature of metric tensor. Moreover, it opened the way to unify the fermions and bosons and gave us an independent signature geometry. The unification of fields as well as, the number of bosons and fermions existing in our Cosmos, is succeeded through this theorem.

In paragraph 4, we present the curved $C^4$ combined with the results of paragraph 2. When we will define the "Levi- Civita" connections of $C^4$, we will show that this connection is exactly the same with the covariant derivative of Standard model (SM), plus the definition of the dark matter. We will easily justify that dark matter and dark energy are totally irrelevant.

In paragraph 6, we will explain the nature of Higg's mechanism. Considering the flat-case of $C^4$ or $R^8$ HM is represented by the analogous Klein-Gordon equation in this dimension, and by solving the differential equation we will have a mass for Higg's boson at 125.179345 Gev/$c^2$ and a cosmological constant at $4.41348x10^{-5}Gev/cm^3$. Moreover, it will be the first time that quantum physics meets cosmology and we will see that in the period of cosmological constant domination we have a De-Sitter space.

In paragraph 7, we present the case of fermions by answering about the number of families and the number of fermions existing in Cosmos.

In paragraph 8, we formulate Quantum Gravity and we see, as many others have, that gravity is unified with the other fields in a different way than we expected; this is due to the fact that it is a field which connects and it is already existing in those fields. As we suspected of course, we have a boson of spin 2 and it is related to the metric tensor of $C^4$. Furthermore we will see how dark matter leads to the formulation of Galaxies. As a conclusion, we will write the full Lagrangian of this theory and we will have one promising G.U.T.

Finally, in paragraph 9 we examine one of the most puzzling mathematical problems: "Why are there fermions and bosons in our cosmos?" Cartan's theorem gave us an almost satisfied answer.

We wanted to find a more geometrical one. By combining functional analysis and differential geometry we concluded, with some new mathematical structures, that we named f(d)-Geometries (maybe functional differential geometry is more accurate).

Those f(d) Geometries gave us the opportunity not only to answer why there are fermions and bosons in our cosmos, but even how other cosmoses can be created or which type of cosmoses can exist. The only fact that we are sure is that:

GOD LOVES GEOMETRY

$AEIO\Theta EO\Sigma OME\Gamma A\Sigma\Gamma E\Omega METPEI$

## 2 Describing our Cosmos

In this chapter we will present a new theory which seems to provide answers to the problems that were presented in SM and also the Quantum Gravity. We will see that the current theory not only reproduces SM and GR but also fills smoothly the gaps existing in our present knowledge. The key for the development of this theory is as many expected Geometry. It is nothing else than a theory of differential geometry which is capable to explain in depth the essence of Quantum Physics and mass. In this paragraph we will develop our theory using the conventional way in regards to the so far development of physical theories and we will deal with a strict mathematical basis in paragraph 3. Our effort will be to define a mathematical space or manifold and a Lagrangian that will hopefully give us the Physics of our Cosmos. Let us consider that K is the true Cosmos that we live, Θ the space of positions and M the space of masses. Θ, M are four dimensional real spaces while K is a four dimensional complex space:

$$K = \Theta + iM \equiv R^4 + iR^4 \equiv C^4$$

K is also algebrically equivalent to an 8 dimensional real space.

1. Θ has four coordinates of positioning with measurement units in meters (m). To be more precise three of them are "pure" coordinates of positioning $x_1$, $x_2$, $x_3$, and the fourth is a dynamic parameter of positioning T. T will play the role of "time" for Θ and will be expressed in meters (m). $x_1, x_2, x_3$ are the familiar to us coordinates of space which we experience as observers (meaning: Length, Width, Height). We will explain the exact meaning and purpose of T to the next paragraphs.
2. M has four coordinates of mass with measurement units in kilos (kgr). There are three mass coordinates $m_1, m_2, m_3$ (In analogy to $x_1, x_2, x_3$) and a dynamic parameter t with units in seconds (sec) which will appear as kilograms after using appropriate constants.
3. Finally, K will appear in meters(m) for our convenience through a constant B which will transform M in meters(m):

   $$\overrightarrow{K} = \overrightarrow{\theta} + i\overrightarrow{m} = (x_1, x_2, x_3, T) + iB\Big(m_1, m_2, m_3, \frac{ct}{B}\Big) =$$

   $$= (x_1, x_2, x_3, T) + i(Bm_1, Bm_2, Bm_3, ct)$$

   As we can observe we consider T as "time" of Θ and t as "time" of M, where t is the well known to us time. We have to admit that in the beginning of this theory we considered t in Θ and T in M. But this alternation gave us vast properties and the true picture of our Cosmos. Our initial consideration looked like a type of Ptolemy paradox. We have to question ourselves which are the reasons that shows us the current placement of the two times in the correct spaces.

(a) The belief of many physicists like Lev.Landau in regards to the connection of space with Minkowski's metric. The common belief is that space-time's vectors have the form

$$(x_1, x_2, x_3, ct)$$

and Minkowski's metric $ds^2 = dx_1^2 + dx_2^2 + dx_3^2 - c^2dt^2$. So some physicists claim that it will be more appropriate to hypothesize vectors of the form

$$(x_1, x_2, x_3, ict)$$

so that it corresponds correctly with Minkowski's metric.

(b) The definition of proper time and coordinate time in GR and co-kinetic coordinates in Cosmology are confused.

(c) Economy in dimensions. It was the main reason that we have considered this placement of times. The magical property of Triality and Cartan's theory convinced us for such a hypothesis. The Nature is inexpensive.

(d) Playing schematically with $ds^2$.

$dk^2 = dx_1^2 + dx_2^2 + dx_3^2 + dT^2 - dm_1^2 - dm_2^2 - dm_3^2 - dt^2 =$

$= (dx_1^2 + dx_2^2 + dx_3^2 - dt^2) + (dT^2 - dm_1^2 - dm_2^2 - dm_3^2)$

If we consider that T does not play any role in the motion of a particle and that the quantity of $dm_1^2 + dm_2^2 + dm_3^2$ corresponds to the mass of the particle we have:

$dk^2 = ds^2 = d(spacetime) = dx_1^2 + dx_2^2 + dx_3^2 - dt^2$

In reality we live in $C^4$ or K, although human beings experience the well known spacetime, because as we shall see M is "small" enough compared to our energy scale. Furthermore, we can not experience T because is the "time" that **cosmos experiences**. It is obvious that T will be linked with the Cosmic expansion. Moreover our theory looks like a string theory (Kaluza-Klein type) as for the addition of dimension to the already existing four, but it changes totally the definition of those additional dimensions plus the fact that we have the hypothesis of "2 times" or "2 dynamical parameters" or "2 clocks". As we have mentioned above, we will show that our hypothesis guarantees that we need only 8 dimensions, in contrast with string theory that the final M theory needs 11 dimensions. The key is the signature of $dk^2$ which is $(1, 1, 1, 1, -1, -1, -1, -1)$ or simply $(4, 4)$ consisting a pseudo-eucleidian space which finally shrinks to the signature of $(1, 1, 1, -1)$ or $(3, 1)$. We have considered for our convenience that all constants of Physics are equal to one.

## 3 Triality

It is true that from the beginning of this theory we were convinced that we actually live in a 8 dimensional real space or 4 dimensional complex one, but with a signature (6,2) or (1,1,1,-1,1,1,1,-1) and an elementary length:

$dk^2 = dx_1^2 + dx_2^2 + dx_3^2 - dt^2 + dm_1^2 + dm_2^2 + dm_3^2 - dT^2$

We thought, as it was natural, that t is an element of Θ and T an element of M, defining in that way Θ as the usual Lorentzian spacetime. It was logical to assume that time parameters t, T go with minus signs in the signature. However, during our effort to present the fermions, we found the annoying matter of signatures. I can remember some discussions with Konstantinos where he was asking consistently the definition of spinor and I was unable to answer, due to the fact of signature. According to the signature of metric tensor, we have Majorana-Weyl representations, Majorana, Dirac, Symplectic Majorana-Weyl etc. That was unacceptable for a theory of physics. We needed an independent signature scheme. We have started thinking that simple definitions from linear algebra could help to solve the mystery. Specifically, we made a parallelism between the definition of basis and signature. The question was if we could find a "minumum" signature in $R^8$, so that all the other possible signatures would be more structurally complicated. The key was the dimension of $R^8$. In 8 dimensions the magic property of Triality arises if we choose the right signature. For example, if we choose a Lorentz one time signature we have to jump to 10 dimensions in order to find the property of Triality in a subspace of a 10 dimensional space. Thus, we wanted not only the "minumum" signature, but the one that comes naturally with Triality. Fortunately, the answer was given by the great E.Cartan 80 years ago. It was then when we finally managed to rip off a certain belief that the real world is Lorentzian or Minkwowskian and time parameters come with a minus sign. We have changed places to time parameters by putting t in M and T in $\Theta$ finding in that way the signature $(4,4)$:

$$dk^2 = dx_1^2 + dx_2^2 + dx_3^2 + dT^2 - dm_1^2 - dm_2^2 - dm_3^2 - dt^2$$

Then, the most important information came from [1, 2, 3] were we discovered that the signatures $(4,4)$, $(8,0)$, $(0,8)$ are all interrelated. If those three signatures are equivalent and all possible others produce more "expensive" structures, then we have found our independent signature system. Therefore, we have a Triality property between $(4,4)$, $(8,0)$, $(0,8)$ signatures that we will call Triality A (or external Triality, or signature's Triality) (figure 26 from)[3] which it is a $S_3$ symmetry, and has the most beautiful and symmetrical Dykin diagram $D_4$.

Nature, seems to appreciate beauty and the Greeks were right once again. However, there is one more Triality property. Let us consider a vector space V, $S^+$ chiral spinor space and $S^-$ antichiral spinor space. Then, we have a second Triality (we will call it Triality B, or internal Triality) which unifies $V, S^+, S^-$ to one form, giving us the ability to define representations from one space to the other; every two of them, automatically concludes the other. Once more again, there is a $S_3$ symmetry and $D_4$ Lie algebra with a Dykin diagram(figure 13 from [3]).

We have to admit that the second diagram was one of our favorite moments because it looks like Feynman's diagrams. The combination of all above diagrams give us (figure 28 [3]).

Following, we will borrow a passage from [3] because it concludes in an elegant way the essence of Triality.

"Let us conclude that Triality can be seen not only as a source of duality-mappings, but as an invariance property. In the original Cartan's formulation this is seen as follows. At first, a group G of invariance is introduced as the group of linear homogeneous transformations acting on the $8x3 = 24$ dimensional space, leaving invariant, separately, the bilinears $B_V, B_{S^+}, B_{S^-}$ for vectors, chiral and antichiral spinors respectively (the spinors are assumed commuting in this case) plus a trilinear term T. Next, the Triality group $G_{Tr}$ is defined by relaxing one condition as the group of linear homogeneous transformations leaving invariant T and the total bilinear $B_{sum}$:

$$B_{sum} = B_V + B_{S^+} + B_{S^-}$$

it can be proven that $G_{tr}$ is given by the semidirect product of G and $S_3$ :

$$G_{tr} = G \otimes S_3$$

"

Let us consider $V = R^8 \equiv C^4$ then the signatures $(4,4) \leftrightarrow (0,8) \leftrightarrow (8,0)$ concludes a Majorana-Weyl representation for $S^+, S^-$.

Moreover, $S^+, S^-$ are necessary 8 dimensional real spaces. As a result $B_V, B_{S^+}, B_{S^-}$ are each one invariant under $SO(8)$ creating the product $SO(8) \times SO(8) \times SO(8)$ for the $B_{sum}$. Consequently, in our case $G = SO(8)$ and the group that leaves invariant T is $SO(8) \otimes S_3$ or $Spin(8)$. The big issue that arises is where the $T_{Tr}$ comes from. It is true that Triality is an algebric-group property and we cannot see where geometry is. This will be fixed in a preliminary basis in paragraph 7. Let us consider as p the number of plus(+) in signature, q the number of minus(-) and d the dimension of K:

$$(p,q) = (4,4) \quad d = p + q = 8$$

Then $d = 0mod8$ and $p - q = 0mod8$

From d and p-q we can conclude that we have a real 8 dimensional Majorana-Weyl representation for the spinor spaces and that the group of automorphisms is $SO(8)$.

Let us consider $(V,G), (S^+,s), (S^-,s)$ where $V = C^4 \equiv R^8, G$ the hermitian metric tensor $S^+, S^-$ 8 dimensional spinor spaces and s spin invariant inner-product:

$$s = \overline{(\psi,\varphi)} = (\psi^c, \varphi^c) \; \forall \; \varphi, \; \psi \in S$$

Moreover $(V,G)$ , $(S^+,s)$ , $(S^-,s)$ are isomorphical as orthogonal spaces and the Triality B ensures the isometry (because of $S_3$) between the spaces. Once again, s will be the charge-conjucation operator C, which preserves the spinor spaces and it can be used to raise and low indices. In order to unify the scheme between $V, S^+, S^-$ we could "bosonise" $S^+, S^-$ or "fermionise" V through [7, 8]

$$B_V = V_m^T (g^{-1})^{mn} V_n$$
$$B_{S^+} = \Psi^T C^{-1} \Psi$$
$$B_{S^-} = X^T C^{-1} X$$
$$T_{Tr} = \Psi^T C \Gamma^m \Psi = 2(\Psi^T C^{-1} \sigma^m X V_m)$$

where $\Psi = \begin{pmatrix} \Psi_a \\ X_a \end{pmatrix}, a = 1, ..., 8$

## 4 Bosons

**Definition 1:** A hermitian or complex manifold is the complex analogous of a Riemann manifold equipped with a smooth hermitian inner-product which concludes a hermitian metric tensor $G_{ij}$

**Definition 2(alternative):** A hermitian or complex manifold is a real manifold equipped with a Riemann metric which preserves a complex or almost complex structure which concludes a unitary structure $U(n)$

**Definition 3:** A hermitian metric tensor $G_{ij}$ in a complex or almost complex manifold defines a Riemann metric $g_{ij}$ on the underlying smooth manifold which is a symmetric bilinear form on $TX^c$ and a complex form $I_{ij}$ of degree (1,1):

$$G_{ij} = g_{ij} + I_{ij}$$

**Definition 4:** Choosing a hermitian metric on an almost complex manifold X is equivalent to a choice of U(n)- structure on X, that is a reduction of the structure group of the frame bundle of X from GL(4,C) to U(n).

**Definition 5:** A unitary frame on an almost hermitian manifold is a complex linear frame which is orthogonal with respect to the hermitian metric. The unitary frame bundle of X is the principal U(n) bundle of all unitary frames.

**Definition 6:** If $C^n$ is a complex Euclidean space with a standard hermitian metric, then $C^n$ is a Kahler manifold. A Kahler manifold is a hermitian manifold :

$$dI_{ij} = 0 \quad (I_{ij} \text{ is closed})$$

then $I_{ij}$ is called Kahler form which is symplectic and so Kahler manifold is a symplectic manifold. The closed hermitian form $I_{ij}$ is called Kahler metric.

**Proposition:** $GL(n, C)$ is a non compact group while $U(n)$ is the maximal compact subgroup of $GL(n, C)$.

**Definition 7:** Let us consider $\alpha_i$ the vector components in $R^4, i = 0, 1, 2, 3$. Then the contravariant derivative of a vector with respect to $\mu$ is :

$$\alpha_{i,\mu} = \frac{\partial \alpha_i}{\partial x^\mu} - \Gamma^s_{i\mu}\alpha_s \tag{1}$$

Where $\Gamma^s_{i\mu}$ are the Christoffel sympols of second type and $\Gamma_{k,ij}$ the Christoffel sympols of first type:

$$\Gamma_{k,ij} = \frac{1}{2}\Big(\frac{\partial g_{ki}}{\partial x^j} + \frac{\partial g_{kj}}{\partial x^i} - \frac{\partial g_{ij}}{\partial x^k}\Big)$$

$$\Gamma^k_{ij} = g^{kl}\Gamma_{l,ij}$$

If we would like to find the operator of Christoffel sympols we could suppose:

$$, \mu = \frac{\partial}{\partial x^\mu} - \Gamma^{\bullet}_{\bullet\mu}$$

Where ($\bullet$) we symbolize the empty places of indices which arise from the vector components. Furthermore, for a vector $\alpha$ (1) would be

$$\alpha_{,\mu} = \frac{\partial \alpha}{\partial x^\mu} - \Gamma^{\bullet}_{\bullet\mu}\alpha$$

$$\alpha_{,\mu} = \frac{\partial \alpha}{\partial x^\mu} - g^{\bullet\bullet}\Gamma_{\bullet,\bullet\mu}\alpha$$

Now, let us consider $(V, G)$ , $V = C^4$,G the hermitian metric tensor:

$G_{ij} = g_{ij} + I_{ij}$ and if $k \in C^4$. We could have:

$$k_{,\mu} = \frac{\partial^c k}{\partial z^\mu} - G^{\bullet\bullet}\Gamma^c_{\bullet,\bullet\mu}k \tag{2}$$

Where $\dfrac{\partial^c k}{\partial z^\mu}$ the Cauchy derivative :

$$\partial^c_\mu = \frac{\partial}{\partial z^\mu} = \frac{\partial}{\partial \Theta} - i\frac{\partial}{\partial M} = \partial_\mu - i\eth_\mu$$

$$\partial_\mu = \left(\frac{\partial}{\partial x^1}, \frac{\partial}{\partial x^2}, \frac{\partial}{\partial x^3}, \frac{\partial}{\partial T}\right) = (\partial_1, \partial_2, \partial_3, \partial_T)$$

$$\eth_\mu = \left(\frac{\partial}{\partial m^1}, \frac{\partial}{\partial m^2}, \frac{\partial}{\partial m^3}, \frac{\partial}{\partial t}\right) = (\eth_1, \eth_2, \eth_3, \eth_t)$$

Besides from (2) we could write

$$, \mu = \partial^c_\mu - (\Gamma^{\bullet}_{\bullet\mu})^c$$

$$, \mu = \partial^c_\mu - G^{\bullet\bullet}(\Gamma_{\bullet,\bullet\mu})^c$$

and if we call $, \mu = D_\mu$

$$D_\mu = \partial_\mu^c - \Gamma^{\bullet}_{\bullet\mu}{}^c = \partial_\mu^c - G^{\bullet\bullet}\Gamma_{\bullet,\bullet\mu}{}^c \tag{3}$$

$$(\Gamma^{\bullet}_{\bullet\mu})^c = \frac{1}{2}\Big(\frac{\partial_c G^{\bullet\bullet}}{\partial k^{\bullet}} + \frac{\partial_c G^{\bullet\bullet}}{\partial k^{\bullet}} - \frac{\partial_c G^{\bullet\bullet}}{\partial k^{\bullet}}\Big) = \Gamma^{\bullet}_{\bullet\mu} - i\Delta^{\bullet}_{\bullet\mu}$$

where $\Gamma^{\bullet}_{\bullet\mu}$ the Christoffel symbols with respect to $\Theta$ and $\Delta^{\bullet}_{\bullet\mu}$ with respect to M. Moreover:

$$(3) \rightsquigarrow D_\mu = \partial_\mu - i\eth_\mu - G^{\bullet\bullet}(\Gamma_{\bullet,\bullet\mu} - i\Delta_{\bullet,\bullet\mu})$$

$$= (\partial_\mu - G^{\bullet\bullet}\Gamma_{\bullet,\bullet\mu}) - i(\eth_\mu - G^{\bullet\bullet}\Delta_{\bullet,\bullet\mu}) \tag{4}$$

If we set $\Gamma_{\bullet,\bullet\mu} = 0$ (we could say that reminds gravity so we do not want gravity now) and $\Delta^{\bullet}_{\bullet\mu} = \Omega^n_\mu$ , $\Delta_{\bullet,\bullet\mu} = \Omega_\mu$

$(4) \rightsquigarrow D_\mu = \partial\mu - i(\eth_\mu - G\Omega^n_\mu) = \partial_\mu - i(\eth_\mu - \Omega_\mu)$

where n will be explained in the followings. It is obvious that $G \in GL(4, C)$ and it is invariant under the general linear group in $C^4$. Accordingly to [10] we can analyze G in the 16-dimensional Dirac-Gellman basis through the matrices $\lambda_n$

$n = 0, 1, 2, ..., 15$ (including I as $\lambda_0$). So:

$G = \alpha^n\lambda_n = f^n(G_{ij})\lambda_n \quad n = 0, 1, 2, ..., 15$

where $\alpha^n = f^n(G_{ij})$ are functions of the hermitian metric tensor's components. Then

$\Omega_\mu = \Delta_{\bullet,\bullet\mu} = G\Delta^{\bullet}_{\bullet,\mu} = \alpha^n\lambda_n\Delta^{\bullet}_{\bullet,\mu} = \lambda_n(\alpha^n\Delta^{\bullet}_{\bullet,\mu})$

$= \lambda_n\Omega^n_\mu$

finally we have

$$D_\mu = \partial_\mu - i\eth_\mu + i\lambda_n\Omega^n_\mu, \;\; n = 0, 1, 2, ...., 15 \tag{5}$$

$$D^*_\mu = \partial_\mu + i\eth_\mu - i\lambda_n\Omega^n_\mu, \;\; n = 0, 1, 2, ...., 15 \tag{6}$$

We have to remark that (5), (6) looks like the usual SM's covariant derivative. To be honest, maybe the symbolization $\Gamma_{\bullet,\bullet\mu} = \Omega_\mu$ was premature, because the familiar fields $\Omega_\mu$ (meaning $A_\mu, W_\mu, G_\mu$) would arise if we reduct $C^4$ to the usual spacetime. Unfortunately, since many things in SM are not so mathematically established, we have proceeded with this symbolization and clear out everything in next paragraph. However, it is obvious that the Christoffel symbols of M are related to the usual fields of Quantum Physics. One interesting element is the sixteen $\lambda_n$ matrices. We could say that corresponds to the sixteen bosons, and that would be true if we lived in a Cosmos with only bosons. We will see that bosons are related to $B_V$ and due to the existence of $B_{S^+}, B_{S^-}$ we could find the number of bosons from $B_{Tr}$, which is invariant under $SO(8) \otimes S_3$ or Spin(8). Thus, in reality the number of bosons existing in nature, would be found from SO(8) and not $GL(4, C)$. The number of generators of $SO(8)$ is $\frac{7*8}{2} = 28$, resulting in 28 bosons in nature. We can see that there is a broken symmetry from GL$(4, C)$ to $SO(8)$ and backwards. We choose not to work with $SO(8)$ (that would be the correct representation), but to remain in complex representation and work with an equivalent group to $SO(8)$ by expanding or inflating GL$(4, C)$. It is a move in order to stay in SM's representation. GL$(4, C)$ is algebrically isomorphic to $U(n)$, and as we saw, a complex manifold has a $U(n)$ structure. Moreover, $U(n)$ is the maximal compact group of GL$(4, C)$, so locally $GL(4, C)$ falls naturally to $SU(n)$, whose chain is $SU(n) \supseteq .. \supseteq SU(2) \supseteq U(1)$. Then ,$U(4) = SU(4)xU(1)$ and by expanding according to the chain, we have

$$GL(4, C) \equiv U(4) \subseteq U(1) \times SU(2) \times SU(3) \times U(4) = Z$$

we can claim that Z is algebrically isomorphic to $SO(8)$, because the rank of $SO(8)$ is $8-1 = 7$ and the rank of Z is $1+1+2+3 = 7$. Let us process those $\lambda_n$ matrices of $GL(4, C)$ in the spirit of Z.

$$\lambda_1 + \lambda_2 + ..... + \lambda_{15} + I = g_1 I_1 + g_2(\lambda_1 + \lambda_2 + \lambda_3) + g_3(\lambda_1 + .... + \lambda_8) + g_4(I_4 + \lambda_1 + ..... + \lambda_{15})$$

Where $g_i$ $i = 1, 2, 3, 4$ are appropriate proportional fractions that could be (it is true) our familiar coupling constants of $U(1)$, $SU(2)$, $SU(3)$, $U(4)$. Then we could write (5) in the form of

$$D_\mu = \partial_\mu - i\eth_\mu + ig\lambda_n\Omega^n_\mu$$

$$= \partial_\mu - i\eth_\mu + ig_1 I\Omega_\mu + ig_2\lambda_i\Omega^i_\mu + ig_3\lambda_j\Omega^j_\mu + ig_4\lambda_w\Omega^w_\mu \tag{7}$$

where $n = 0,1,2,....28,\ i = 1,2,3,\ j = +1,2,...8,\ w = 0,1,2,....15$ and g coupling or structure constant coming from $SO(8)$

It is easy to see that $\Omega_\mu, \Omega_\mu^i, \Omega_\mu^j$ are related with $A_\mu, W_\mu, G_\mu$ of SM. The problem is the meaning of $\Omega_\mu^w$. Someone could say that is related to gravity, but this is not true. Gravity is a totally different field (actually it is already inside Z). The answer is :

## $U(4)$ describes dark matter

and because $U(4)$ has 16 generators there exist 16 dark-bosons or skoteenons. In conclusion (7) describes-produces the 28 bosons of spin 1, of our Cosmos.

Now, let us go back to the hermitian metric tensor which will have the following form :

$$G = \begin{pmatrix} g_{11} & g_{12} - iI_{12} & g_{13} - iI_{13} & g_{14} - iI_{14} \\ g_{21} + iI_{21} & g_{22} & g_{23} - iI_{23} & g_{24} - iI_{24} \\ g_{31} + iI_{31} & g_{23} + iI_{32} & g_{33} & g_{34} - iI_{34} \\ g_{41} + iI_{41} & g_{42} + iI_{42} & g_{43} + iI_{43} & g_{44} \end{pmatrix}$$

$= \alpha_n \lambda^n = \alpha^n \lambda_n \quad , n = 0,1,....,15:$

$\alpha_0 = \frac{1}{4} trG \rightarrow I$

$\alpha_1 = g_{12} \rightarrow \lambda_1$

$\alpha_2 = I_{12} \rightarrow \lambda_2$

$\alpha_3 = \frac{g_{11} - g_{22}}{2} \rightarrow \lambda_3$

$\alpha_4 = g_{13} \rightarrow \lambda_4$

$\alpha_5 = I_{13} \rightarrow \lambda_5$

$\alpha_6 = g_{23} \rightarrow \lambda_6$

$\alpha_7 = I_{23} \rightarrow \lambda_7$

$\alpha_8 = \frac{\sqrt{3}}{6}(g_{11} + g_{22} - 2g_{33}) \rightarrow \lambda_8$

$\alpha_9 = g_{14} \rightarrow \lambda_9$

$\alpha_{10} = I_{14} \rightarrow \lambda_{10}$

$\alpha_{11} = g_{24} \rightarrow \lambda_{11}$

$\alpha_{12} = I_{24} \rightarrow \lambda_{12}$

$\alpha_{13} = g_{34} \rightarrow \lambda_{13}$

$\alpha_{14} = I_{34} \to \lambda_{14}$

$\alpha_{15} = \frac{\sqrt{6}}{12}(trG - 4g_{44}) \to \lambda_{15}$

We will call $I, \lambda_3, \lambda_8, \lambda_{15}$ main generators and all the others escort ones. Let us suppose that we have only $I, \lambda_1, \lambda_2, \lambda_3$ then :

$$\alpha_0 = \frac{1}{4}(g_{11} + g_{22})$$

$$\alpha_3 = \frac{1}{2}(g_{11} - g_{22})$$

then

$$\alpha_0 + \alpha_3 = \frac{1}{4}((g_{11} + g_{22}) + 2(g_{11} - g_{22}))$$

$$= \frac{1}{4}(g_{11}^2 - g_{22}^2)\Big(\frac{1}{g_{11} - g_{22}} + \frac{2}{g_{11} + g_{22}}\Big) \qquad (8)$$

or

$$\alpha_0 + \alpha_3 = 2\Big(\frac{3}{8}g_{11} - \frac{1}{8}g_{22}\Big)$$

$$\alpha_0 - \alpha_3 = 2\Big(-\frac{1}{8}g_{11} + \frac{5}{8}g_{22}\Big)$$

we suppose from (8) that $I, \lambda_3$ will necessarily communicate, which means that for sure we will have the results of $SU(2)xU(1)$ concerning the building of photon in Z boson. If we proceed this way we can see exactly how the generators are going to communicate, in order to produce bosons. In strong nuclear field $\lambda_3$ will communicate with his escorts $\lambda_1, \lambda_2$ and will be bonded with $\lambda_8$ and of course $\lambda_8$ will communicate with $\lambda_4, \lambda_5, \lambda_6, \lambda_7$. Moreover, $\lambda_4, \lambda_5$ will produce 2 $\pm$ charged bosons, and $\lambda_6, \lambda_7$ too. Similarly, in the dark field bondages will be created between $I, \lambda_3, \lambda_8, \lambda_{15}$ where 4 bosons from this bondage will be created and all the other escort ones will behave in a similar way with strong field.

We can come to the conclusion that the electromagnetic and weak nuclear fields are “free”, while strong nuclear and dark are “bonded”. Main generators are bonded in pairs between them and escort generators are bonded to the main ones in a way depending to the dimension. This bondage that strong nuclear and dark field present is the main reason why we meet “matter prisons” (we are already familiar with hadron prisons QCD). This property will reflect to the analogous fermions as well. Let us consider the mixed Riemann-Christoffel tensor in $C^4$:

$$R^{l}_{i\mu\nu} = \begin{vmatrix} \frac{\partial}{\partial k^{\mu}} & \frac{\partial}{\partial k^{\nu}} \\ (\Gamma^{l}_{i\mu})^{c} & (\Gamma^{l}_{i\nu})^{c} \end{vmatrix} + \begin{vmatrix} (\Gamma^{l}_{\alpha\mu})^{c} & (\Gamma^{l}_{i\mu})^{c} \\ (\Gamma^{l}_{\alpha\nu})^{c} & (\Gamma^{l}_{i\nu})^{c} \end{vmatrix}$$

$$R^{\bullet}_{\bullet\mu\nu} = \begin{vmatrix} \frac{\partial}{\partial k^{\mu}} & \frac{\partial}{\partial k^{\nu}} \\ (\Gamma^{\bullet}_{\bullet\mu})^{c} & (\Gamma^{\bullet}_{\bullet\nu})^{c} \end{vmatrix} + \begin{vmatrix} (\Gamma^{\bullet}_{\bullet\mu})^{c} & (\Gamma^{\bullet}_{\bullet\mu})^{c} \\ (\Gamma^{\bullet}_{\bullet\nu})^{c} & (\Gamma^{\bullet}_{\bullet\nu})^{c} \end{vmatrix}$$

$\dfrac{\partial}{\partial k^{\mu}} = \partial_{\mu} - i\eth_{\mu}$ and $(\Gamma^{\bullet}_{\bullet\mu})^{c} = \Gamma^{\bullet}_{\bullet\mu} - i\Delta^{\bullet}_{\bullet\mu} = -i\Delta^{\bullet}_{\bullet\mu} = -i\Omega_{\mu}$

we can have also:

$$\begin{aligned} R^{\bullet}_{\bullet\mu\nu} &= (\partial_{\mu} - i\eth_{\mu})(-i\Omega_{\nu}) - (\partial_{\nu} - i\eth_{\nu})(-i\Omega_{\mu}) \\ &\quad -(\Omega_{\mu}\Omega_{\nu} + \Omega_{\nu}\Omega_{\mu}) \\ &= -i(\partial_{\mu}\Omega_{\nu} - \partial_{\nu}\Omega_{\mu}) - (\eth_{\mu}\Omega_{\nu} - \eth_{\nu}\Omega_{\mu}) \\ &\quad -[\Omega_{\mu}, \Omega_{\nu}] \end{aligned} \tag{9}$$

Once again ,$R^{\bullet}_{\bullet\mu\nu}$ looks like the strength field tensor, as defined in SM. Finally, we have to clarify some things. Actually, (8)(9) are the correct formulas, concerning the covariant derivative and field strength tensor, of our cosmos. The usual ones defined in SM are somewhere between $C^4$ and $R^4$, a mixture or meanings mixed together. Let us consider for now that $\partial_c$ transforms to $\partial_R$ according to

$$\begin{aligned} &\partial_c = \partial_{\mu} - i\eth_{\mu} = \nabla_{\theta} + \partial_T - i\nabla_M - i\partial_t \\ &\to \partial_R = (\nabla_{\theta}, \partial_t) \end{aligned}$$

and the transformed $\Delta_{\mu} = \Omega_{\mu}$

and $\eth_{\mu}\Omega_{\nu} - \eth_{\nu}\Omega_{\mu} = 0$

because $\eth_{\mu}(\eth_{\mu} - i\Omega_{\mu}) = \eth_{\mu}\eth_{\mu} - i\eth_{\mu}\Omega_{\mu}$

and the eigenvalues discussed in next paragraph.

Then (8), (9) are just like the usual definition of covariant derivative and fields strength tensor of SM in every detail which is marvelous. If we proceed this way (things will be explained in next paragraph for such a consideration) and remembering that from SM

$$\Omega_{\mu\nu} = i[D_\mu, D_\nu] = \partial_\mu \Omega_\nu - \partial_\nu \Omega_\mu \pm i[\Omega_\mu, \Omega_\nu]$$

we have

$$R^{\bullet}_{\bullet\mu\nu}(transformed) = -i\Omega_{\mu\nu}$$

$$R^{\bullet}_{\bullet\mu\nu} R^{\bullet\mu\nu}_{\bullet} = (-i\Omega_{\mu\nu})(-i\Omega^{\mu\nu}) = -\Omega_{\mu\nu}\Omega^{\mu\nu}$$

the trace of $R^{\bullet}_{\bullet\mu\nu}$ is $G_{\bullet\bullet}R^{\bullet}_{\bullet\mu\nu}$ combined with $G_{\bullet\bullet} = \alpha^n \lambda_n = \alpha_n \lambda^n$

and the properties of $\lambda^n$

$$R_{\bullet\bullet\mu\nu} R^{\bullet\bullet\mu\nu} = tr(R^{\bullet}_{\bullet\mu\nu} R^{\bullet\mu\nu}_{\bullet})$$

$$= -tr(\Omega_{\mu\nu}\Omega^{\mu\nu}) = -\Omega^n_{\mu\nu}\Omega^{n\mu\nu}$$

Where $n = 1, ...28$, so

$$\Omega^n_{\mu\nu}\Omega^{n\mu\nu} = B_{\mu\nu}B^{\mu\nu} + W^j_{\mu\nu}W^{j\mu\nu} + G^k_{\mu\nu}G^{k\mu\nu} + \Sigma^w_{\mu\nu}\Sigma^{w\mu\nu}$$

$$\Omega_{\mu\nu}\Omega^{\mu\nu} = B_{\mu\nu}B^{\mu\nu} + W_{\mu\nu}W^{\mu\nu} + G_{\mu\nu}G^{\mu\nu} + \Sigma_{\mu\nu}\Sigma^{\mu\nu}$$

Where $j = 1, 2, 3$ , $k = 1...8$ , $w = 0, ...15$

## 5 Lie brackets

The usual anti commutating relations in our case of $R^8 = C^4$ are:

1. $[x^i, m_j] = 0$
   this relation reflects to $R^4$ as $\Delta x^i \Delta m_i \geq \frac{1}{2}\frac{\hbar}{c}$
2. $[x^i, p_{r_j}] = i\hbar\delta^i_j \Longrightarrow \Delta x^i \Delta p_{r_j} \geq \frac{1}{2}\hbar$
3. $[x^i, p_{m_j}] = 0$
4. $[m^i, p_{r_0}] = 0$
5. $[m^i, p_{m_j}] = -im_P^2 c\delta^i_j \Longrightarrow \Delta m^i \Delta p_{m_j} \geq \frac{1}{2}m_P^2 c$
6. $[T, H_T] = i\hbar c \Longrightarrow \Delta T \Delta E_T \geq \frac{1}{2}\hbar c$
7. $[t, H_t] = -i\hbar \Longrightarrow \Delta t \Delta E_t \geq \frac{1}{2}\hbar$
8. $[T, H_t] = 0$
9. $[t, H_T] = 0$
10. $[t, T] = 0$
11. $[x^i, x^j] = 0$
12. $[m^i, m^j] = 0$
13. $[p_{r_i}, p_{r_j}] = 0$
14. $[p_{m_i}, p_{m_j}] = 0$

These are 14 anti commutating relations.In our usual spacetime the Poincare group is the group of Minkowski spacetime isometry and it is a ten dimensional noncompact Lie group $R^{1,3} \times SO(1,3)$ where ten dimensional comes from the number of the Killing's vectors of the Minkowski metric.In our case the analogous Poincare group is $R^{4,4} \times SO(4,4)$ and it is a 36 dimensional Lie group where 36 are the Killing's vectors of our flat metric in $R^{(4,4)}$.Actually we have 28 generators from $SO(4,4)$ and 8 translations(displacements).The 28 generators are presented above in the anti commutating relations.Moreover as we saw in the previous chapter we used the group G instead of $SO(8) \backsimeq SO(4,4)$ in order to represent the Standard Model and so someone could say that our usual coordinate transformations of $S0(8)$ are in perfect match with the gauge transformations of Standard model's.

## 6 The origin of mass

Finally, it is time to clear the origin of mass. HM is not only a marvelous attempt to explain how particles gain their masses, but it is a classical mechanism which mixes quantum properties, such mass eigenvalues and eigenvectors, as well. Now, let us start from the beginning.

$$C^4 : \overrightarrow{K} = (x_1, x_2, x_3, T) + i(m_1, m_2, m_3, t)$$

where $i(m_1, m_2, m_3, t)$ belongs to the mass space M and is [small] enough compared to $\Theta$.

Then, what does and observer from our usual spacetime observers from $C^4$? First of all, he can not feel T, as he studies the motion of a particle, or even a car.T is observable only for cosmological events. Secondly, he knows and sees $(x_1, x_2, x_3)$ From M, what does he see? From $(m_1, m_2, m_3)$ he sees the scalar mass, as a eigenvalue of the mass operator, just like it happens in Quantum Physics for energy or momentum. From [it] we see the proper time of general relativity. Then if (it) is coordinate time, we observe $\tau = it$, the usual time that clocks count. In reality , we have a first type of quantization of $M^4$ which explains to us how mass and $< time >$ are produced. Afterwards, we have a second quantization in our usual spacetime, which tells us how the products of $M^4$ are going to move in usual spacetime:

$$C^4 \qquad\qquad\qquad R^4$$

$$(\overrightarrow{x}, T) + i(\overrightarrow{m}, t) \longrightarrow (\overrightarrow{x}, \tau)$$

By passing the eigenquantities of $M^4$ to $R^4$ is like behaving to mass and time in a classical way. Of course we can not see their eigenvectors,but just the mass eigenvalues and proper time. The operator that describes mass is:

$$\widehat{p_m} = im_p^2 c \nabla_M$$

where A is a unified constant:

$A = \frac{1}{6}\sqrt{\frac{2}{3}}\sqrt{\frac{G}{G_F}\frac{h}{c}} = \frac{\sqrt{2}}{2}\sqrt{\frac{1}{28-1}}\sqrt{\frac{G}{G_F}\frac{h}{c}} = 3.26297x10^{-18}$ and is dimensionless.

Moreover the operator of dark energy is

$$T = -i\hbar c \partial_T$$

Starting from $dk^2$, we will solve the general differential equation of cosmos.

$$ds^2 = d\overrightarrow{r}^2 + dT^2 - \frac{G^2}{c^4} d\overrightarrow{m}^2 - c^2 dt^2$$

then if we define $\overrightarrow{v} = \frac{d\overrightarrow{r}}{dT}$( is dimensionless) and $\overrightarrow{u} = \frac{d\overrightarrow{m}^2}{dt^2}$

we have

$ds^2 = (v^2+1)dT^2 - \Big(\frac{G^2}{c^6}u^2+1\Big)c^2dt^2$ where $\frac{G^2}{c^6}u^2$ is dimensionless and we define as:

$L = \sqrt{D^2c^2(v^2+1) - B^2c^4\Big(\frac{G^2}{c^6}u^2+1\Big)}$ where constant D has dimensions of usual momentum.

The canonical momentums are:

$p_r = \frac{\partial L}{\partial v}$ and $p_m = \frac{\partial L}{\partial u}$

$$p_r = \frac{D^2 v}{\sqrt{D^2(v^2+1) - B^2c^2\Big(\frac{G^2}{c^6}u^2+1\Big)}}$$

$$p_m = -\frac{B^2\frac{G}{c}u}{\sqrt{D^2(v^2+1) - B^2c^2\Big(\frac{G^2}{c^6}u^2+1\Big)}}$$

the Legendre's transformation of L is

$$H = -\frac{D^2 v}{\sqrt{D^2(v^2+1) - B^2c^2\Big(\frac{G^2}{c^6}u^2+1\Big)}} + \frac{B^2\frac{G}{c}u}{\sqrt{D^2(v^2+1) - B^2c^2\Big(\frac{G^2}{c^6}u^2+1\Big)}}$$

and $H^2 = (D^2c^2 - B^2c^4)\Big(1 + \frac{1}{B^2c^2}p_m^2 - \frac{1}{D^2}p_r^2\Big)$

but we define $c_0 = \frac{D}{B}$ with velocity dimension so

$$H^2 = \Big(1 - \frac{c^2}{c_0^2}\Big)c^2p_r^2 + \Big(1 - \frac{c_0^2}{c^2}\Big)c^2p_m^2 + \Big(\frac{c_0^2}{c^2} - 1\Big)B^2c^4$$

we define as operators

$$H_T = -i\hbar c\frac{\partial}{\partial T}$$

$$H_t = i\hbar\frac{\partial}{\partial t}$$

$$p_\theta = -i\hbar\nabla_r$$

$$p_m = im_P^2 c\nabla_m$$

we set $H^2 = H_T^2 - H_t^2$ and the differential equation is

$$\hbar^2 c^2\frac{\partial^2\Psi}{\partial T^2} - \hbar^2\frac{\partial^2\Psi}{\partial t^2} = -\Big(1-\frac{c_0^2}{c^2}\Big)\hbar^2 c^2\nabla_r^2\Psi - \Big(1-\frac{c_0^2}{c^2}\Big)m_P^4 c^2\nabla_m^2\Psi \quad +\Big(1-\frac{c_0^2}{c^2}\Big)B^2c^2\Psi$$

we split parameters as $\Psi = \sigma(t,T)\ \psi(\overrightarrow{r},\overrightarrow{m})$

$$\hbar^2 c^2\frac{\partial^2\sigma}{\partial T^2} - \hbar^2\frac{\partial^2\sigma}{\partial t^2} = \omega^2$$

$$-\Big(1-\frac{c_0^2}{c^2}\Big)\hbar^2 c^2\nabla_r^2\psi - \Big(1-\frac{c_0^2}{c^2}\Big)m_P^4 c^2\nabla_m^2\psi \quad +\Big(1-\frac{c_0^2}{c^2}\Big)B^2c^2\psi \quad = \omega^2\psi$$

where $\omega$ a constant that be defined at the end.

**Equation of times**

we split $\sigma(t,T) = \varphi(t)g(T)$ then

$c^2\frac{1}{g}\frac{\partial^2 g}{\partial T^2} - \frac{1}{\varphi}\frac{\partial^2\varphi}{\partial t^2} = \Big(\frac{\omega}{\hbar}\Big)^2$ and if we call $\frac{1}{\varphi}\frac{\partial^2\varphi}{\partial t^2} = k^2$

$$c^2\frac{1}{g}\frac{\partial^2 g}{\partial T^2} - k^2 = \Big(\frac{\omega}{\hbar}\Big)^2$$

$\frac{1}{g}\frac{\partial^2 g}{\partial T^2} = \Big(\frac{\omega}{\hbar c}\Big)^2 + \Big(\frac{k}{c}\Big)^2$ and if we call $\rho^2 = \Big(\frac{\omega}{\hbar c}\Big)^2 + \Big(\frac{k}{c}\Big)^2$

$$\frac{1}{g}\frac{\partial^2 g}{\partial T^2} = \rho^2$$

so we get two equation $\frac{1}{\varphi}\frac{\partial^2\varphi}{\partial t^2} = k^2$ and $\frac{1}{g}\frac{\partial^2 g}{\partial T^2} = \rho^2$ with general solutions

$\varphi(t) = a_1e^{-kt} + a_2e^{kt}$ and $g(t) = b_1e^{-\rho T} + b_2e^{\rho T}$

A good combination to examine is $\sigma(t,T) = Ne^{-\rho T}e^{kt}$ where N constant.Moreover $e^{-\rho T}$ can be normalised if we set $\int_0^\infty N^2e^{2\rho t}dT = 1$ which means $N = \sqrt{2\rho}$.Furthermore we can calculate the mean value of T as:

$< T >_t = \int_0^\infty T(\sqrt{2\rho}e^{-\rho T}e^{kt})^2 = \frac{1}{2\rho}e^{2kt} = \frac{1}{2\sqrt{\Big(\frac{\omega}{\hbar c}\Big)^2 + \Big(\frac{k}{c}\Big)^2}}e^{2kt}$ and if we set $2k = H$ and $\omega = k\hbar$

$< T >_t = \frac{\sqrt{2}}{2}\frac{c}{H}e^{Ht}$ which is the universe of De-Sitter in a cosmos with vacuum domination.It is very pleasing than finally quantum mechanics has meet cosmology and as we can presume this is happened in vaccuum epoch as we suspected.H at this time is a constant.In order to obtain the usual Hublle's parameter we need a curved space and not a flat one as in our present case.

**Time independed**

$$-\Big(1-\frac{c_0^2}{c^2}\Big)\hbar^2c^2\nabla_r^2\psi-\Big(1-\frac{c_0^2}{c^2}\Big)m_P^4c^2\nabla_m^2\psi \quad +\Big(1-\frac{c_0^2}{c^2}\Big)B^2c^2\psi \quad =\omega^2\psi$$

$$-\hbar^2c^2\nabla_r^2\psi+A^2m_P^4c^4\nabla_m^2\psi-A^2B^2c^4\psi=\frac{A^2\omega^2}{1-A^2}\psi$$

where we set $A=\frac{c_0}{c}$ (the above mentioned universal constant).

If we split:

$\psi(\overrightarrow{r},\overrightarrow{m})=\zeta(\overrightarrow{r})\xi(\overrightarrow{m})$ and set one more time for our convenience

$A_1^2=(1+\frac{1}{2(A-1)})\omega^2$ and $B_1^2=A^2B^2c^4+\frac{\omega^2}{2(A+1)}$ we can get

$-\hbar^2c^2\frac{1}{\zeta(r)}\nabla_r^2\zeta(r)+A_1^2-[-A^2m_p^4c^4\frac{1}{\xi(m)}\nabla_m^2\xi(m)+B_1^2]=0$

then we call $-A^2m_P^4c^4\nabla_m^2\xi(m)=(\mu^2-B_1^2)\xi(m)$ and

$-\hbar^2c^2\nabla_r^2\zeta(r)=(\mu^2-A_1^2)\zeta(r)$.

Furthermore if we move to a spherical coordinate system we will have of solution of the form $\xi(m,\vartheta,\varphi)$ where $\vartheta,\varphi$ angles and $\xi$ can be written as $\xi(m,\vartheta,\varphi)=\Upsilon_m^l(\vartheta,\varphi)R(m)$ where $\Upsilon_m^l(\vartheta,\varphi)$, the spherical harmonic functions and $R(m)$ the radial part of the solution. For a spherical infinite well of radius $m_0$ the radial part R(m) is presented by the spherical Bessel functions where $m=\sqrt{m_1^2+m_2^2+m_3^2}$ If we symbolise the zero points of Bessel functions as $u_{l,k}$ we can prove that

$$\mu^2-B_1^2=\frac{u_{l,k}^2m_P^4c^4}{m_0^2}.$$

For $l=0$ the Bessel function has the form $\frac{sinx}{x}$ and the zero points are $k\pi, k\neq 0$ then

$\mu^2-B_1^2=(\frac{k\pi Am_P^2c^2}{m_0})^2$ .By setting $m_p^2=\frac{\hbar c}{G}$ and $m_0=m_p$ we have the eigenvalue

$$m=k\pi Am_p$$

and for $k = 1$

$$m_H = \pi A m_p \simeq 125.179345 \ Gev/c^2$$

The above result presents the mass of Higg's boson . It is obvious that the vacuum energy or vacuum ground state comes from the free wave equation.Let us consider as : $V = m_H^4 = 2.4534x10^8 Gev^4$ the potential of vacuum and because $(1Gev)^3 = 1.3014x10^{41} cm^{-3}$ we have $V = 3.1929x10^{49} Gev/cm^3$ .If we multiple V by $A^3$ in order to recover the scale then:

$\rho_\Lambda = \dfrac{1}{8\pi} A^3 V = (m_H A)^3 m_H = A^6 m_P^3 A m_p = A^7 m_P^4 = 4.41348x10^{-5} Gev/cm^3$

or $\Lambda = \dfrac{A^5}{l_H^2} = \dfrac{A^7}{l_P^2}$

The above values seems to be valid but in order to prove them we need to solve not the flat metric case but the curved one.It looks surprising how these values looks correct. Therefore, when we pass from $C^4$ to $R^4$ we get the Higg's boson in the flat case. The Higg's field will "fill" all $R^4$ by giving existence to the vacuum of $R^4$. The most impressive element is that we only solved the flat case and we have "earned" the mass Higg's boson as an eigenvalue. If we get the full Hermitian metric tensor we pass to curved space and as we saw curvature in $M^4$:

Curvature in $M^4 \longrightarrow$ Christofell symbols in $M^4 \longrightarrow \Omega_\mu$ fields $\longrightarrow$ 28 bosons of spin 1

Furthermore, we know that always in a curved space we can find appropriate coordinate system where Christoffel symbols vanishes. But, coordinate transformation in $C^4$ means Gauge transformation under $U(1), SU(2), SU(3), U(4)$. As a result, in the curved $S^4$, locally $\Omega_\mu$ fields vanishes or bosons ($spin1$) are just like Higg's boson. In reality bosons at the null point they are exactly like Higg's boson, but when they move under their geodesics they gain a mass proportional to Higg's one. The boson's masses can be calculated if $G_{ij}$ is known by solving the analogous eigenvalue problems. If $G_{ij}$ of our Cosmos is unique or is a standard one is a question that must be answered. All boson's masses can be calculated in this spirit. We have to remark that all bosons gain the property of mass. If a boson has mass equal to zero this does not mean that is without mass, on the contrary it has mass, with a mass eigenvalue equal to zero. All bosons interact with the vacuum and so all will gain the property of mass, even gluons and skoteenons and if some of them have mass equal to zero is a matter of eigenvalue problem. As concerns the constants D,B are just scales that we can use in order to see the evolution of the energy scale of Cosmos.We could put A inside the operator of $p_m$ but we decided to remain in a standard notation and leave A as a scale.The same could happen also for the other constants.As an example if we include A wherever is necessary in the anti commutating relations we can get an uncertainty principle for Higg's field instead of Planch mass.

## 7 Fermions

We have to admit that the fermionic structure were more puzzling for us, due to the fact that for bosons, we had all the geometrical information we wanted. We started form the $dk^2$ and the differential geometry led the way.

Michael Atiyah wrote:

"No one fully understands spinors. Their algebra is formally understood but their general significance is mysterious. In some sense they describe the square root of geometry and just as understanding the square root of -1 took centuries, the same might be true for spinors"

In that sense , if $dk^2$ gave us the geometry of bosons the same could have happened if we could write $\sqrt{dk^2} \to f(dk)$ where f represents some type of function. Our ideas on this subject will be presented in the last paragraph, while in the current paragraph we will assume the standard fermion's formulation. Of course, this formulation will be presented in 8-dimensions. Once again Triality properties and assumptions, as were written and presented in paragraph 2, will be our basic lead.Most of the following information were taken and processed from papers [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]. Particularly from [3] we have:

"The most suitable basis is the Majorana-Weyl(MWR) basis where all spinors are either real or imaginary. In such a representation the following set of data underlines any given theory:

(i) The spinor fields specified by chiral and antichiral indices $\alpha, \widehat{\alpha}$ respectively.

(ii) The diagonal pseudo-orthogonal spacetime metric $(g^{-1})^{mn}$ and $g_{mn}$ which we will assume to be the flat (or curved if it is necessary).

(iii) The A matrix, used to define barred spinors, coincides with the $\Gamma^0$-matrix in the Minkowski case : In a MWR basis is decomposed in an equal size block diagonal form such as $A = A \bigoplus \overline{A}$ with structure of indiced $(A)_a^b$ and $(\overline{A})_{\widehat{\alpha}}^{\widehat{b}}$ respectively

(iv) the charge-conjucation matrix C which also appear in an equal-sizeblock diagona form $C = C^{-1} \bigoplus \overline{C}{-1}$ it is invariant under bispinorial transformatives and it can be promoted to be a metric in the space of chiral (and respectively antichiral) spinors, used to raise and lower spinoral indices. Indeed we can set $(C^{-1})^{\alpha b}$, $(C)_{\alpha b}$, $(\overline{C}^{-1})^{\overline{\alpha}\overline{b}}$, $(C)_{\overline{\alpha}\overline{b}}$

(v) The $\Gamma$- matrices which are decomposed in equal-size blocks, $\sigma^m$'s is the upper right blocks and $\overline{\sigma}^m$'s is the lower-left blocks having structure of indices $(\sigma^m)_{\alpha}^{\widehat{b}}$, $(\overline{\sigma}^m)_{\widehat{\alpha}}^{b}$ respectively choices for C.The special case d=8 is the fundamental MWR representation. So a, $\widehat{\alpha} = 1, ...8$"

Let us give some useful remarks:

1. in an even d=p+q dimensions metric tensor

   $n^{\mu v} = diag(++++,----)$gamma matrices $\gamma^\mu$ satisfies the Clifford algebra

   $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\psi = 2n^{\mu\nu}$

2. In $p - q = 0mod8$ we have a Clifford algebra $C(4, 4)$ and because $p + q = 2x4$ we have a real $2^4 = 16$ representation.
3. For $p - q = 0mod8$, $p + q = 0mod8$ we have a unique irreducible representation which is of course the MWR one.
4. The MW spinors satisfy both of the following conditions:

   $\gamma^{(d+1)}\psi = \psi$, $\overline{\psi} = \psi^{+}C_{\pm}$

   and exist only if $p - q = 0mod8$ for us $d = p + q = 8$
5. If $d = p + q = 0mod8$ we have only kinetic terms in the Lagrangian of the form $K_{xy}$.

   Where $K_{xy}$:

   $K_{xy} = \Psi_R^T C\Gamma^{\mu}\partial_{\mu}\Psi_L + \lambda\Psi_L^T C\Gamma^{\mu}\partial_{\mu}\Psi_R$
6. For the $(4, 4)$ signature the $(4_s + 4_A)$- representation of $\Gamma$ matrices has to be employed for both values on $n = \pm 1$ in order to provide a MW basis.
7. The gamma matrices are given by (3)

$$\Gamma^9 = \begin{pmatrix} 1_8 & 0 \\ 0 & -1_8 \end{pmatrix},$$

$$\Gamma^i = \begin{pmatrix} 0 & \sigma^c \\ \overline{\sigma}^c & 0 \end{pmatrix} i = 1...8$$

   (as presented in appendix 2 in [3])

   Where $\overline{\sigma}_i = -\overline{\sigma}_i^T$, $i = 1...4$ $(antisymmetric)$

   $\overline{\sigma}_i = \overline{\sigma}_i^T$, $i = 5, 6, 7, 8$ $(symmetric)$ and the diagonal charge-conjucation matrices are given by:

$$C^{-1} = 1_4 \oplus -1_4$$

$$\overline{C}^{-1} = -nC^{-1}$$

After all these notations, definitions and properties concerning the frame of description regarding fermions it would be wise to look back to Triality once again. The wisest move that someone could follow, is to "bosonise" the fermions structure [7, 8]. This way we could either go back to the usual differential geometry's structure, or we could do the opposite. Specifically we could take all the work from bosons and translate it to fermionic language. This is an extreme task, that is above and beyond this first approaching paper. Instead we can think, and at times guess how things should be.From the above we can have the Lagrangian term in $R^8$[1]:

$$\Psi_R^T C\Gamma^\mu(\partial_\mu)_c\Psi_L + \lambda\Psi_L^T C\Gamma^\mu(\partial_\mu)_c\Psi_R$$

Let us take the first term

$\Psi_R^T C\Gamma^\mu(\partial_\mu)_c\Psi_L = \Psi_R^T C\Gamma^\mu(\partial_\mu, \eth_\mu)\Psi_L = (\Psi_R^T C\Gamma^\mu\partial_\mu\Psi_L, \Psi_R^T C\Gamma^\mu\eth_\mu\Psi_L)$

where $\partial_\mu = (\nabla_R, \partial_T)$ and $\eth_\mu = (\nabla_M, \partial_t)$

and we have a split between space Θ and M concerning the operators.Once again,we have to solve this Dirac-like equation (in the same way we did with bosons) and because it is in flat space this will give us a ground state in the free wave case.It is not difficult to assume that this ground state is the same as in the case of bosons meaning that we will have the same vacuum and the constant of separation will be $\omega$(than $\omega^2$ in boson case). Thus in both bosons and fermions we have the vacuum energy as ground state and this vacuum will produce the masses of fermions through the appearance of its eigenvalue and eigenvectors inside the Lagrangian.However we need to make fermions show their faces since from the above mentioned only the vacuum has appeared.The strategy is always the same with boson case.Particularly when we took the full Hermitian metric tensor in curved space,we decomposed accordingly to Dirac-Gellman basis and through its matrices we produced boson fields.In fermionic case we have to find the analogous curved charge conjugate matrix C,so let us symbolize as $C_{cur}$ , and we will decompose it again.This decomposition will lead us to the fermionic fields,as well as to, the number of fermions and the number of families.We could write:

$\Psi^T C_{cur}\Psi = \Psi_n^T C_{cur}\Psi_n = \Psi_e^T C^e\Psi_e + \Psi_\mu^T C^\mu\Psi_\mu + \Psi_\tau^T C^\tau\Psi_\tau + ....$

Of course in the Lagrangian in curved space we have to replace C with $C_{cur}$ and the partial derivative $(\partial_\mu)_c$ with the covariant $(D_\mu)_c$ completing this way the full Lagrangian.Definitely the number of fermions will be increased from the 12 known ones due to the presence of scoteinons (dark matter bosons) which will give us the corresponding dark fermions (preetzonions). In the beginning ,we were considering that there must be 16 fermions, meaning that there are 4 preetzonions. During the process,however, we realized that fermions must be equal to the number of bosons resulting in 28 fermions in our Cosmos.The big question is how these 28 fermions are organized. Although we do not proceeded to the definition of the form of $C_{cur}$ we believe it is reasonable to organize them as follows:

1. We have 4 families
2. They are separated through the 4 existing fields meaning that we have the corresponding fermions for each field under $U(1)$, $SU(2)$, $SU(3)$, $U(4)$ respectively
3. Preetzonions can be found in three states ,just as quarks have 2 states up and down, we can say that we have an up a middle and a down state as:

   $(Pr)_u, (Pr)_m, (Pr)_d$
4. Usual matter consists of 16 fermions and dark matter of 12 fermions
5. The total number of particles of the usual matter is 12 bosons plus 16 fermions = 28 particles.

The total number of particles of dark matter is 16 bosons plus 12 fermions=28 particles

6. Maybe the reason why dark matter is dominant to usual matter is the fact that it consists of a total 28 particles in one structure. Moreover ,the way that dark matter is organized in order to produce matter structures (which is of the same logic as quarks-gluons producing mesons ,hadrons e.t.c) is another one.Beside 16 skoteenons which could create enormous scoteenon-balls giving us an enormous number of combinations in order to do so.The ground state will interact with all these 28 fermion states $\Psi_n, n = 1, 2, ....28$ in the same way that bosons do.All these $\Psi_n$ states that came out of a curved fermion metric tensor $C_{cur}$ will always locally find the ground state of vacuum that we came across in the flat case.In addition ,the existence of gamma matrices in the part of M space will lead us to introduce some new physics.Usual gamma matrices in the Dirac equation were connected with spin.It is tempting to connect the gamma matrices of M space with weak isospin ,isospin e.t.c.Concerning the Yukawa constants we have to worked some formulas ,but they are still phenomenological and we have decided not to present them on this paper.Though we can mention that the key must be the volumes of the groups $U(1)$, $SU(2)$, $SU(3)$, $SU(4)$.

# 8 Quantum gravity

In paragraphs 4,5 we set $\Gamma^{\bullet}_{\bullet\mu}, \Gamma_{\bullet,\bullet\mu}$ equal to zero with the excuse that somehow they are not only connected with gravity,but because we were studying only the fields $A_\mu, W_\mu, G_\mu, \Sigma_\mu$ we did mot wanted to mix all the fields together.The $\Gamma^{\bullet}_{\bullet\mu}, \Gamma_{\bullet,\bullet\mu}$ Christoffel symbols describe the curved space $\Theta$ ,which consists of the coordinates of x,y,z (our usual coordinates) plus the "time" that Cosmos experiences. Therefore, it was natural to hypothesize ,that $\Gamma^{\bullet}_{\bullet\mu}, \Gamma_{\bullet,\bullet\mu}$ are related to the usual gravity of spacetime. To continue ,if we put back those $\Gamma^{\bullet}_{\bullet\mu}, \Gamma_{\bullet,\bullet\mu}$ we will need just one more step to visualize Quantum Gravity. We need to remark that the existence of $\Delta^{\bullet}_{\bullet\mu}, \Delta_{\bullet,\bullet\mu}$ automatically means the existence of $\Gamma^{\bullet}_{\bullet\mu}, \Gamma_{\bullet,\bullet\mu}$ and because the Christoffel symbols of M are connected with the fermions and bosons the analogous Christoffel symbols of $\Theta$ are related with their corresponded "gravitational fields". Moreover, we have to point out that Christoffel symbols of $\Theta$ are the "gravitational fields" as they exist in $C^4$ or $R^8$ and not our usual gravitational ones,that an observer of spacetime experiences.For one more time ,we will have to transformate the tangent vectors of $C^4$ to the tangent vectors of spacetime (with Minkowski metric tensor) and then the Christoffel symbols of $\Theta$ will be transformed to the Christoffel symbols of spacetime,meaning of course, gravity.Thus all we need is a Lagrangian ,whose Euler-Lagrange equations will give us quantum gravity.The only question that arises is where graviton is.The answer is surprising,because we have already mentioned it several times,through out this paper. Graviton is connected to G the hermitian metric tensor in $C^4$, which as we have seen, splits in 28 bosons. G is a second rank tensor ,opposite to $\Omega_\mu$ and as we were suspected all these years ,means a boson (graviton) with spin 2. The full Lagrangian of the G.U.T is:

$$L = G^{\bullet\bullet}(D_c\Phi)(D_c\Phi) + (\Psi_R^T C\Gamma^\mu(\partial_\mu)_c\Psi_L + \lambda\Psi_L^T C\Gamma^\mu(\partial_\mu)_c\Psi_R) +$$

$$+ G_{\bullet\mu}G_{\bullet\nu}R^{\bullet\bullet\mu\nu} + R_{\bullet\bullet\mu\nu}R^{\bullet\bullet\mu\nu}$$

where all Riemann-Christoffel tesnors are meant to be in $C^4$ and $\Phi$ locally differential functions of $C^4$. The following cases exist:

1. Euler-Lagrange with respect to $\Phi \to$ Klein-Gordon equation in $C^4$.
2. Euler-Lagrange with respect to $\Psi \to$ Dirac equation in $C^4$.
3. Euler-Lagrange with respect to $\Omega_\mu \to A_\mu, W_\mu, G_\mu, \Sigma_\mu$ equations unified in $C^4$. (The Christoffel symbols of $\Theta$ will be vanished)
4. Euler-Lagrange with respect to hermitian metric tensor G $\to$ Quantum gravity in $C^4$.

all $1, 2, 3, 4$ describes the physics in $C^4$. After the first quantization (means that we take the eigenvalues of M) we have which we are calling Quantum Physics, where 4 will be transformed accordingly to what we expect as Quantum Gravity (where all particles are formed). Subsequently the second quantization, and by taking the mean value of all fields "we take Classic Physics". Therefore in the case of gravity after the first and second quantization all the terms of the Lagrangian (except the Ricci one) will be mixed to formulate our usual energy-momentum tensor. Of course, as mentioned in paragraph 5, we saw that T,t are connected giving us the possibility to express the Physics of spacetime with only one time parameter, as expected.

One more aspect we have to analyze is the one of dar matter. As we mentioned, dark matter is described by $U(4)$ Lie group. If we have not expanded the hermitian metric tensor, gravity boson would be connected to dark matter as concerned with the strength or coupling constant. We could say that they share the same coupling constant, which means that dark matter would act extremely weakly with the fields $A_\mu, W_\mu, G_\mu$ but strongly with gravity. As a result the use of dark matter in our cosmos is to help formulate the big structures of matter. Actually, dark matter is a strong gravitational trap. Let us consider some usual matter that moves somewhere in cosmos and suddenly meets dark matter(trap). Since, matter and dark matter will interact extremely weak, matter will fall in this gravitational trap(which is formed by dark matter). Following, matter will contribute to the gravitationed field, which means that matter will be more trapped. This can happen several times during hundred millions of years. Consequently, this trap is continuously enriched with a lot of matter creating a very strong gravitational field (formulated by both matter and dark matter), big pressure and temperature; this is exactly how stars or Galaxies are formulated.

# 9 f(d) Geometries

The issue of metricity constitutes one of the most central ones and has puzzled mathematicians from ancient Greek's times till today. In physics it is definitely the most major issue. From ancient years great mathematicians such as Hippocrates, Thales, Pythagoras, Eucleides, Ptolemy, Heron, Eudoxus, Archimedes, the great Plato and Aristotle and many others settled and studied it. The meaning of metric-distance has till today three basic properties:

1. Positive defined

2. Homogeneity

3. Triangle inequality

Let us consider a normed space $(N, \|\cdot\|)$. A norm $\|\cdot\|$ is defined by three properties

1. $\|x\| \geq 0, \|x\| = 0 \Leftrightarrow x = 0, \forall x \in N$
2. $\|x + y\| \leq \|x\| + \|y\| \; \forall x, y \in N$
3. $\|\lambda x\| = \mid \lambda \mid \|x\| \; \forall x \in N \; \forall \lambda \in K$ Where $K = (R$ or $C)$

Similar properties can be seen in more generalized metric spaces. During the 19th century Riemann, Christoffel deleted the positive defined, while they preserved the others. This was a brilliant move, in order to formulate or express electromagnetic theory, general relativity and field theory. Let us consider that in space N, exists to two or more norms i.e $\|\|_1, \|\|_2$ and $a, b > o$ then it is easy to see that the function $a\|\|_1 + b\|\|_2$ is still a norm in N. Further, if we consider the function $f(\xi, \zeta) = a\xi + b\zeta \; \xi, \zeta \in R$ the above mentioned expression will occur if we place $\xi = \|x\|_1 \; \zeta = \|x\|_2$. The above can be generalized if we consider a polynomial of first degree. We have to mention that in functional analysis, we use functions of the form $f(x) = \mid x \mid^{\frac{1}{p}} \; p \geq 1$ or even replace x by series $\sum \mid x_n \mid^p$ or integrals $\int \mid f(x) \mid^p$. Now, let us consider a real function that only implies the property $f(0) = 0$ i.e $f(x) = x^2 + x$. If we replace x by $\|x\|$ then we have $f(\|x\|) = \|x\|^2 + \|x\|$ in which is obvious that $2, 3$ properties are not fulfilled. Let us suppose that $f(\|x\|) = d(x)$, then the only property from $1, 2, 3$ is $d(x) \geq 0$ and $d(x) = 0 \Leftrightarrow x = 0$. Now, the question that arises is "can we with this type of "metric" make analysis?" maybe if f is continuous we could form a sort of analysis. The more interesting question is if we can formulate or define some new form (analogous to norm, metric or topology) that could have some similar properties with $1, 2, 3$. By this new form we could make the synthesis of different types of geometries, giving us the possibility to create a vast number of cosmoses. The idea came from the synthesis of boson and fermion geometry. Bosons and fermions are put together in the Lagrangian, meaning that two geometries are mixed together. It is like in $C^4$ we run multiple norms or metrics simultaneously. It is obvious that if this is true, we could create cosmoses (by using a series of norms or metric or whatever else) with fermions + bosons + somethingons + elseons + anyons + ... . In this spirit the hidden geometry behind Triality could be those mixed geometries that we name f(d) geometries. We had a lot of internal discussions regarding the properties, the meaning, the use of those geometries which we would not present in this paper. The only thing we could say, is that with f(d) geometries we could create many types of cosmoses. Anyhow, this "heretical" hypothesis may become the key to the mathematics of the $21^{st}$ century.

## 10 Conclusion

In this paper, we have introduced some new ideas and an entire mathematical structure that one could work with, without inserting many hypothesis. Just only one, the structure of $C^4$ or $R^8$ and afterwards mathematics will "work" to give us all the details.We could construct M space with other units in order to obtain the momentum in M to mass units or we could work with octonions , split octonions, quartenonions but we kept everything as simple we could. We feel very pleased that the current work can answer all-at-once the major problems in physics. If this work is truly the grand unified theory remains a question that can be answered by the scientific community. It is evident that at times we have not proceeded in full detail (like in the case of fermions), but instead, we gave a scheme of the work that must be accomplished, and the results that we could gain. We have worked in depth in all the matters of this theory, but we have decided not to present any indefinite and uncertain conclusion. In those cases, we have presented you with our thoughts. We could have waited and worked, for a couple of more months in order to present *almost everything* in detail, but our major goal is to share the idea rapidly with other scientists.

## 11 References

## References


[1] M.A. De Andrade, F. Toppan, *Real Structures in Clifford Algebras and Majorana Conditions in Any Space-time*, Mod.Phys.Lett. A14 (1999) 1797-1814.

[2] Ivan Todorov, *Clifford Algebras and Spinors*, Bulg.J.Phys.38:3-28, 2011.

[3] M.A. De Andrade ,M. Rojas, F. Toppan , *The Signature Triality of Majorana-WeylSpacetimes* , Int.J.Mod.Phys. A16 (2001) 4453-4480.

[4] Jeong-Hyuck Park, *Lecture note on Clifford algebra*, Modave Summer School in Mathematical Physics, Belgium, june, 2005.

[5] A. Pashnev,F. Toppan, *On the Classification of N-extended Supersymmetric Quantum Mechanical Systems*, J.Math.Phys. 42 (2001) 5257-5271.

[6] H.L. Carrion, M. Rojas, F. Toppan, *Quaternionic and OctonionicSpinors. A Classification*, JHEP 0304 (2003) 040.

[7] Liu Yufen, *Triality and Quantization of Singularities in Massive Fermion*, Commun. Theor. Phys. (Beijing, China) 34 (2000) pp 705-714.

[8] C. Wetterich, *Spinors in euclidean field theory, complex structures and discrete symmetries*, Nuclear Physics B, Volume 852, Issue 1, 1 November 2011, Pages 174–234.

[9] Arthur Jaffe , Edward Witten, *Quantum Yang-Mills Theory* .

[10] Victor M. Red'kov, Andrei A. Bogush, Natalia G. Tokarevskaya, *On Parametrization of the Linear GL(4,C) and Unitary SU(4) Groups in Terms of Dirac Matrices*, SIGMA 4 (2008), 021, 46 pages.

[11] David Miralles, JosepManel Parra (1), Jayme VazJr (2), *Signature Change and Clifford Algebras*, Int.J.Theor.Phys. 40 (2001) 229-242.