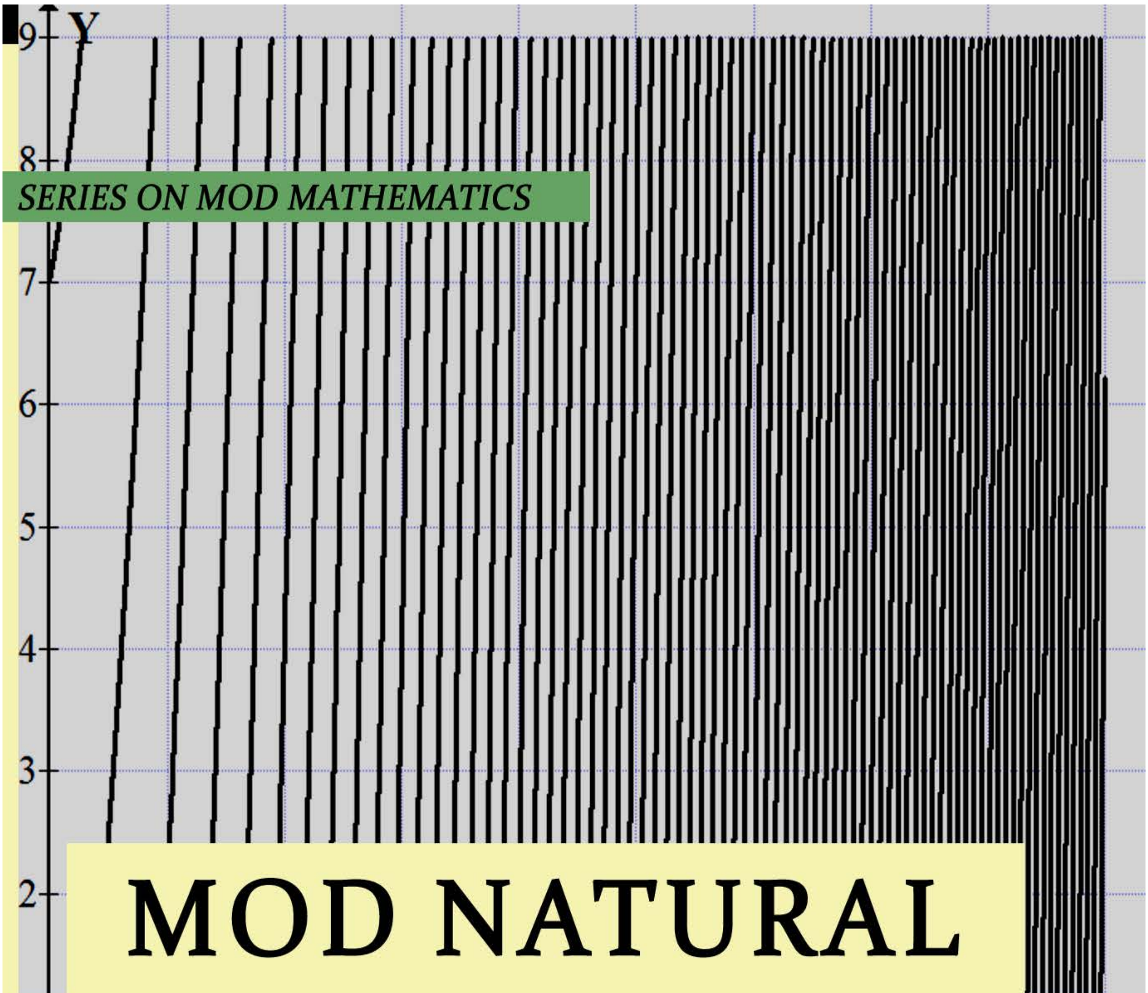

SERIES ON MOD MATHEMATICS

MOD NATURAL NEUTROSOPHIC SEMIRINGS

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EuropaNova
2016

ISBN-13: 978-1-59973-470-5
EAN: 9781599734705

Printed in the United States of America

CONTENTS

ABOUT THE BOOK

In this book for the first time authors describe and develop the new notion of MOD natural neutrosophic semirings using Z_n^I, $C^I(Z_n)$, $\langle Z_n \cup I\rangle_I$, $\langle Z_n \cup g\rangle_I$, $\langle Z_n \cup k\rangle_I$ and $\langle Z_n \cup h\rangle_I$.

Several interesting properties about this structure is derived. Using these MOD natural neutrosophic semirings MOD natural neutrosophic matrix semirings and MOD natural neutrosophic polynomial semirings and defined and described.

Special elements of these structures are analysed. When MOD intervals [0, n) and MOD natural neutrosophic intervals $^I[0, n)$ are used we see the MOD semirings do not in general satisfy the associative laws and the distributive laws leading to the definition of pseudo semirings of infinite order. These are also introduced in this book. We also define and develop MOD subset pseudo semiring and MOD subset natural neutrosophic pseudo semirings. This study is innovative and interesting by providing a large class of MOD pseudo semirings. Special elements in them are analysed.

Using these MOD subset matrix pseudo semirings and MOD subset polynomial pseudo semirings and developed.

They enjoy very many special features. Several problems are suggested and these notions will certainly attract semiring theorists.

We wish to acknowledge Dr. K Kandasamy for his sustained support and encouragement in the writing of this book.

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Chapter One

SEMIRINGS USING NATURAL NEUTROSOPHIC ELEMENTS

In this chapter for the first time authors fully analyse the MOD natural neutrosophic semiring which we also describe as natural neutrosophic semiring Z_n^I. However for basic definition and properties about natural neutrosophic numbers of Z_n refer [66].

Barring distributive lattices of finite order, lattice groups and lattice semigroups one is not in a position to give examples of finite semirings.

However in this book we give an infinite class of finite semirings constructed using Z_n^I which are never semifields. So finding finite semifields other than distributive lattices and the above said structures happens to be a challenging problem.

We first describe finite semirings built on Z_n^I by some examples.

Example 1.1: Let $Z_2^I = \{0, 1, I_0^2, 1+I_0^2\}$ by the set on which we can define $\times$ and $+$.

Clearly Z_2^I under + is only semigroup. $I_0^2 + I_0^2 = I_0^2$.

$$1 + I_0^2 + 1 + I_0^2 = I_0^2.$$

Thus Z_2^I is a semigroup under + and 0 is the trivial idempotent under + and I_0^2 is the natural neutrosophic element which is an idempotent of Z_2^I.

$$o(Z_2^I) = 4.$$

We see $Z_2 \subseteq Z_2^I$. Z_2 is a group under +. So Z_2^I is a Smarandache semigroup.

Clearly $\{Z_2^I, \times\}$ is a semigroup for $1. I_0^2 = I_0^2$, $0. I_0^2 = I_0^2$, $I_0^2 \times I_0^2 = I_0^2$, $1 + I_0^2 \times I_0^2 + 1 = 1 + I_0^2$ as $I_0^2 + I_0^2 = I_0^2$ [66].

Now Z_2^I under × is a semigroup and this has I_0^2 and $1 + I_0^2$ to be nontrivial idempotents. Infact as $1 \times 1 = 1$, $0 \times 0 = 0$ we see Z_2^I is an idempotent semigroup.

$\{Z_2^I, +, \times\}$ is a semiring. We see this is an example of a finite semiring. However characteristic of Z_2^I is not two as $I_0^2 + I_0^2 = I_0^2$ so for Z_2^I we cannot associate the characteristic to be two.

But we have a subset $Z_2 \subseteq Z_2^I$ which is of characteristic two.

In view of all these we define the notion of quasi characteristic for these semirings.

Example 1.2: Let $Z_3^I = \{\{0, 1, 2, I_0^3, 1 + I_0^3, 2 + I_0^3\}, +, \times\}$ be a natural neutrosophic semiring. $o(Z_3^I) = 6$.

Clearly $1 + I_0^3 \times 2 + I_0^3 = 2 + I_0^3$ and

$2 + I_0^3 \times 2 + I_0^3 = 1 + I_0^3$.

$1 + I_0^3 \times 1 + I_0^3 = 1 + I_0^3$

$I_0^3 \times I_0^3 = I_0^3$.

So the additive idempotents of Z_3^I are 0 and I_0^3.

The multiplicative idempotents are I_0^3, $I_0^3 + 1$, 1 and 0.

Clearly characteristic of Z_3 is not 3 as $I_0^3 + I_0^3 + I_0^3 \neq 0$ only $I_0^3 + I_0^3 + I_0^3 = I_0^3$.

We see Z_3^I is a S-semigroup under + as well as under $\times$.

Example 1.3: Let $Z_4^I = \{0, 1, 2, 3, I_0^4, I_2^4, 1 + I_0^4, 2 + I_0^4, 3 + I_0^4, 1 + I_2^4, 2 + I_2^4, 3 + I_2^4, I_0^4 + I_2^4, 1 + I_0^4 + I_2^4, 2 + I_0^4 + I_2^4, 3 + I_0^4 + I_2^4\}$ be the MOD natural neutrosophic semiring.

Clearly $o(Z_4^I) = 16$.

Clearly 0, I_0^4, I_2^4, $I_0^4 + I_2^4$ are additive idempotents of Z_4^I.

0, 1, I_0^4, $1 + I_0^4$ are multiplicative idempotents of Z_4^I.

$(2 + I_0^4)^2 = I_0^4$ is a MOD natural neutrosophic nilpotent of order two.

$(I_0^4 + I_2^4) = I_0^4$,

$(2 + I_0^4 + I_2^4) = I_0^4$ and $I_2^4 \times I_2^4 = I_0^4$ are all MOD natural neutrosophic nilpotent elements of order.

Thus $P_1 = \{I_0^4, I_0^4 + I_2^4\} \subseteq Z_4^I$ is a subsemiring of order two.

$P_2 = \{0, I_0^4, I_0^4 + I_2^4\} \subseteq Z_4^I$ is a subsemiring of order three.

$P_3 = \{0,2, 2+I_0^4\} \subseteq Z_4^I$ is a subsemiring of order three.

$P_4 = \{0, I_0^4, I_2^4, I_0^4 + I_2^4\} \subseteq Z_4^I$ is again a subsemiring of order four.

$P_5 = \{0, 2, I_0^4, I_2^4, 2 + I_0^4, 2 + I_2^4, I_0^4 + I_2^4, 2 + I_0^4 + I_2^4\} \subseteq Z_4^I$ is again a subsemiring of order 8.

Thus we can work to get subsemirings of very many different orders.

$P_6 = \{0, I_0^4, I_2^4, 2+I_0^4, I_2^4+2, I_0^4+I_2^4, 2 + I_0^4+I_2^4\} \subseteq Z_4^I$ is again a subsemiring of order 7 and so on.

$P_7 = \{0, I_0^4, I_2^4, 2 + I_0^4, I_0^4 + I_2^4, 2 + I_2^4, 2 + I_0^4 + I_2^4, 1 + I_0^4 + I_2^4, 3 + I_0^4 + I_2^4\} \subseteq Z_4^I$ is a subsemiring of order 9.

$P_8 = \{0, 1, 2, 3, I_0^4, I_0^4 + 1, 2 + I_0^4, 3 + I_0^4\} \subseteq Z_4^I$ is again a subsemiring of order 8.

Thus we get several subsemirings of various orders.

The order of the subsemiring may divide the order of Z_4^I or may not divide the order of Z_4^I.

Example 1.4: Let
$Z_5^I = \{0, 1, 2, 3, 4, I_0^5, 1 + I_0^5, 2 + I_0^5, 3 + I_0^5, 4 + I_0^5, +, \times\}$ is a natural neutrosophic semiring of order 10.

This has no nontrivial zero divisors or nilpotents.

But $I_0^5 \times I_0^5 = I_0^5$ is the natural neutrosophic zero.

Example 1.5: Let $Z_6^I = \{0, 1, 2, 3, 4, 5, I_0^6, I_2^6, I_3^6, I_4^6, 1 + I_0^6, 2 + I_0^6, 3 + I_0^6, 4 + I_0^6, 5 + I_0^6, 1 + I_2^6, 2 + I_2^6, 3 + I_2^6, 4 + I_2^6, 5 + I_2^6, 3 + I_3^6, 1 + I_3^6, 2 + I_3^6, 4 + I_3^6, 5 + I_3^6, 4 + I_4^6, 1 + I_4^6, 2 + I_4^6, 3 + I_4^6, 5 + I_4^6, +, \times\}$ be the natural neutrosophic semiring of finite order.

$o(Z_6^I) = 30$.

Clearly this has zero divisors for 2.3 = 0 and 4.3 = 0.

Further this has idempotents 3.3 = 3 and 4.4 = 4.

Now we see Z_6^I has neutrosophic nilpotent which is trivial as $I_0^6 \times I_0^6 = I_0^6$.

But Z_6^I has natural neutrosophic idempotents and zero divisors given by the following.

$I_2^6 \times I_3^6 = I_0^6$,

$I_3^6 \times I_4^6 = I_0^6$,

$I_3^6 \times I_3^6 = I_3^6$ and

$I_4^6 \times I_4^6 = I_4^6$ are the natural neutrosophic zero divisors and natural neutrosophic idempotents of Z_6^I respectively.

$(3 + I_3^6)\,(I_3^6 + 3) = 3 + I_3^6$

is again a natural of neutrosophic idempotent of Z_6^I.

$4 + I_4^6 \times 4 + I_4^6 = I_4^6 + 4$ is again a natural neutrosophic idempotent of Z_6^I.

$4\,(3 + I_3^6) = I_3^6$
$2\,(3 + I_3^6) = I_3^6$ and so on.

Clearly Z_6^I has zero divisors, units and idempotents apart from natural neutrosophic zero divisors and idempotents.

Example 1.6: Let $Z_7^I = \{0, 1, 2, 3, 4, 5, 6, I_0^7, 1 + I_0^7, 2 + I_0^7, 3 + I_0^7, 4 + I_0^7, 5 + I_0^7, 6 + I_0^7\}$.

$\{Z_7^I, +, \times\}$ is a semiring of order 14 has no nontrivial zero divisors or idempotents or nilpotents other than 0.0 = 0.

$1 \times 1 = 1$, $I_0^7 \times I_0^7 = I_0^7$.

In view of all these we put forth the following theorem.

THEOREM 1.1: *Let $Z_p^I = \{0, 1, 2, \ldots, p–1, I_0^p, 1 + I_0^p, 2 + I_0^p, \ldots, p–1 + I_0^p\}$ be the MOD natural neutrosophic semiring (p a prime).*

(i) $o\,(Z_p^I) = 2p$.

(ii) Z_p^I has no nontrivial zero divisors, idempotents or nilpotents.

(iii) Z_p^I has no non trivial natural neutrosophic zero divisors or idempotents or nilpotents.

(iv) Z_p^I is not a semifield.

Proof is direct and hence left as an exercise to the reader.

Now we proceed onto define a new notion of quasi characteristic of a natural neutrosophic semiring.

DEFINITION 1.1: *Let { Z_n^I, ×, +} be the semiring of natural neutrosophic elements.*

Clearly $Z_n \subseteq Z_n^I$ is such that Z_n is a ring of characteristic n, $2 \leq n < \infty$ as nx = 0 for all $x \in Z_n$.

Thus Z_n^I is a quasi characteristic semiring of characteristic Z_n^I has a substructure which is a ring of characteristic n.

We will provide examples of them.

Example 1.7: Let { Z_{20}^I, +, ×} = {0, 1, 2, …, 19, a + I_0^{20}, a + I_2^{20}, …, a + I_{18}^{20}, a + I_5^{20}, a + I_{15}^{20}, a + I_0^{20} + I_2^{20}, …, a + I_0^{20} + I_2^{20} + … + I_{18}^{20} + I_5^{20} + I_{15}^{20} where $a \in Z_{20}$} be a MOD natural neutrosophic semiring of finite order. The quasi characteristic of the semiring Z_{20}^I is 20.

Example 1.8: Let { Z_{16}^I, +, ×} = {0, 1, 2, 3, …, 15, a + I_0^{16}, a + I_2^{16}, …, a + I_2^{16} + … + I_{14}^{16}, +, ×} be the MOD natural neutrosophic semiring whose quasi characteristic is 16.

Example 1.9: Let { Z_{12}^I, +, ×} = {0, 1, 2, …, 11, a + I_0^{12}, 0a + I_2^{12}, …, a + I_{10}^{12}, a + I_3^{12}, a + I_9^{12}, a + I_0^{12} + I_2^{12}, …, a + I_0^{12} + I_2^{12} + I_4^{12} + I_6^{12} + I_8^{12} + I_{10}^{12} + I_3^{12} + I_9^{12}; $a \in Z_{12}$, +, ×} be the MOD natural neutrosophic semiring. The quasi characteristic of Z_{12}^I is 12.

Clearly $I_2^{12} \times I_6^{12} = I_0^{12}$ is a natural neutrosophic zero divisors.

$I_4^{12} \times I_3^{12} = I_0^{12}$, $\qquad$ $I_4^{12} \times I_6^{12} = I_0^{12}$,

$I_6^{12} \times I_8^{12} = I_0^{12}$, $\qquad I_4^{12} \times I_4^{12} = I_0^{12}$,

$I_9^{12} \times I_9^{12} = I_0^{12}$, $\qquad I_9^{12} \times I_4^{12} = I_0^{12}$

are natural neutrosophic zero divisors, idempotents and nilpotents.

This has also zero divisors, idempotents and nilpotents.

Example 1.10: Let $\{Z_{34}^I, +, \times\} = \{0, 1, 2, \ldots, 33, I_0^{34}, I_2^{34}, \ldots, I_{32}^{34}, I_{17}^{34}, a + I_0^{34}, a + I_2^{34}, \ldots, a + I_0^{34} + I_2^{34} + I_4^{34} + \ldots + I_{32}^{34} + I_{17}^{34}, +, \times\}$ be the MOD natural neutrosophic semiring. The quasi characteristic of Z_{34}^I is 34.

We try to find ideals of Z_n^I. This will first be illustrated by some examples.

Example 1.11: Let $\{Z_{10}^I, +, \times\} = \{0, 1, 2, \ldots, 9, I_0^{10}, I_2^{10}, I_4^{10}, I_6^{10}, I_5^{10}, I_8^{10}, a + I_0^{10}, \ldots, a + I_0^{10} + I_2^{10} + \ldots + I_5^{10}, +, \times\}$ be the MOD natural neutrosophic semiring,.

Consider $I_1 = \{0, I_5^{10}, I_0^{10}, I_0^{10} + I_5^{10}\}$.

$I_2 = \{0, I_0^{10}, I_2^{10}, I_4^{10}, I_6^{10}, I_8^{10}, I_2^{10} + I_4^{10}, I_2^{10} + I_6^{10}, I_2^{10} + I_8^{10}, I_4^{10} + I_6^{10}, I_4^{10} + I_8^{10}, I_6^{10} + I_8^{10}, I_0^{10} + I_2^{10}, I_4^{10} + I_0^{10}, I_0^{10} + I_6^{10}, I_0^{10} + I_8^{10}, I_0^{10} + I_2^{10} + I_4^{10}, I_0^{10} + I_2^{10} + I_6^{10}, \ldots, I_0^{10} + I_2^{10} + I_4^{10} + I_6^{10} + I_8^{10}\} \subseteq \{Z_{10}^I, +, \times\}$ is an ideal of the semiring. $o(I_2) = 32$.

$I_3 = \{Z_{10}, +, \times\} \subseteq \{Z_{10}^I, +, \times\}$ is a subsemiring and is not an ideal of $\{Z_{10}^I, +, \times\}$.

$I_4 = \{\langle 0, I_2^{10}, I_4^{10}, I_6^{10}, I_8^{10}, I_5^{10}\rangle\}$ is an ideal of Z_{10}^I.

Example 1.12: Let S = $\{Z_7^I, +, \times\}$ be the MOD natural neutrosophic semiring.

$\{Z_7^I, +, \times\} = \{0, 1, 2, 3, 4, 5, 6, I_0^7, a + I_0^7, a \in Z_7 \setminus \{0\}\}$.

Clearly $P_1 = \{0, 1, 2, 3, \ldots, 6\} \subseteq Z_7^I$ is only a field of characteristic seven and is not an ideal.

$P_2 = \{I_0^7, a + I_0^7, a \in Z_7 \setminus \{0\}\} \subseteq Z_7^I$ is a subsemiring which is an ideal.

$P_3 = \{I_0^7, 0\} \subseteq Z_7^I$ is also a subsemiring which is an ideal.

Does Z_7^I contain subsemirings which are not ideals other than P_1?

Example 1.13: Let $\{Z_8^I, +, \times\} = \{0, 1, 2, 3, 4, 5, 6, 7, I_0^8, I_2^8, I_4^8, I_6^8, a + I_0^8 + I_2^8, a + I_0^8 + I_4^8, a + I_0^8 + I_6^8, \ldots, a + I_0^8 + I_2^8 + I_4^8 + I_6^8, +, \times\}$ be the natural neutrosophic semiring of finite order.

$P_1 = \{0, I_0^8\} \subseteq Z_8^I$ is a subsemiring which is an ideal.

$P_2 = \{Z_8\} \subseteq Z_8^I$ is only a subsemiring and not an ideal of Z_8^I.

$P_3 = \{0, I_0^8 + I_4^8, I_0^8 + I_4^8\}$ is a subsemiring which is also an ideal of Z_8^I.

$P_4 = \{0, I_0^8, I_4^8, I_2^8, I_6^8, I_0^8 + I_2^8, I_6^8 + I_0^8, I_2^8 + I_4^8, I_0^8 + I_4^8, I_6^8 + I_2^8, I_6^8 + I_4^8, I_0^8 + I_2^8 + I_4^8, I_0^8 + I_2^8 + I_6^8, I_0^8 + I_4^8 + I_6^8, I_2^8 + I_6^8 + I_4^8, I_0^8 + I_2^8 + I_4^8 + I_6^8\} \subseteq Z_8^I$.

Can we have other subsemiring which are not ideals of Z_8^I?

Example 1.14: Let M = $\{Z_{14}^I, +, \times\}$ be the MOD natural neutrosophic semiring.

M = {0, 1, 2, …, 13, a + I_0^{14}, a + I_2^{14}, a + I_4^{14}, a + I_6^{14}, a + I_8^{14}, a + I_{10}^{14}, a + I_{12}^{14}, a + I_7^{14}, a ∈ Z_{14}, a + $I_0^{14} + I_2^{14}$, …, a + $I_{12}^{14} + I_7^{14}$, a + I_0^{14} + I_2^{14} + I_4^{14}, …, a + I_{10}^{14} + I_{12}^{14} + I_7^{14} and so on}.

P = $\{Z_{14}\} \subseteq$ M is only subsemiring and infact a ring but not an ideal of M.

Consider {0, I_0^{14}, I_2^{14}, I_4^{14}, I_6^{14}, I_7^{14}, I_8^{14}, I_{10}^{14}, I_{12}^{14}, $I_0^{14} + I_2^{14}$, $I_0^{14} + I_4^{14}$, $I_0^{14} + I_6^{14}$, …, $I_0^{14} + I_2^{14} + I_4^{14} + I_6^{14} + I_8^{14} + I_{10}^{14} + I_{12}^{14} + I_7^{14}$} = B $\subseteq$ M is a subsemiring which is also an ideal of M.

$$(I_0^{14} + I_2^{14} + I_4^{14} + I_6^{14} + I_8^{14} + I_{10}^{14} + I_{12}^{14} + I_7^{14}) \quad I_2^{14}$$

$$(I_0^{14} + I_4^{14} + I_8^{14} + I_{12}^{14} + I_2^{14} + I_6^{14} + I_{10}^{14})$$

$$(I_0^{14} + I_2^{14} + I_4^{14} + I_6^{14} + I_8^{14} + I_{10}^{14} + I_{12}^{14} + I_7^{14}) \times I_7^{14}$$

$= I_0^{14} + I_7^{14}$ and so on.

$$I_2^{14} + I_8^{14} + I_4^{14}$$

$$= I_8^{14} + I_4^{14}$$

$$(I_2^{14} + I_4^{14}) \times I_7^{14} = I_0^{14}.$$

So is a natural neutrosophic zero divisor.

$(I_2^{14} + I_4^{14} + I_6^{14}) \times I_7^{14} = I_0^{14}$ and

$$(I_8^{14} + I_{10}^{14} + I_6^{14} + I_{12}^{14}) \times I_7^{14} = I_0^{14}$$

are all natural neutrosophic zero divisors in M.

$I_7^{14} \times I_7^{14} = I_7^{14}$ is natural neutrosophic idempotent of Z_{14}^I.

$(7 + I_7^{14}) \times (7 + I_7^{14}) = 7 + I_7^{14}$ and

$(1 + I_7^{14}) \times (7 + I_7^{14}) = 1 + I_7^{14}$ are also natural neutrosophic idempotents of Z_{14}^I.

Example 1.15: Let W = $\{Z_{25}^I, +, \times\}$ be the natural neutrosophic semiring.

W = $\{0, 1, 2, \ldots, 24, I_0^{25}, I_{20}^{25}, I_5^{25}, I_{10}^{25}, I_{15}^{25}, a + I_0^{25}, a + I_5^{25}, \ldots, a + I_0^{25} + I_5^{25} + I_{20}^{25}, I_{10}^{25} + I_{15}^{25}$ where $a \in Z_{25}\}$ be the finite MOD natural neutrosophic semiring.

$I_5^{25} \times I_5^{25} = I_0^{25}$, $\quad I_5^{25} \times I_{10}^{25} = I_0^{25}$,

$I_5^{25} \times I_{15}^{25} = I_0^{25}$, $\quad I_{10}^{25} \times I_{15}^{25} = I_0^{25}$

and so on are natural neutrosophic zero divisors and natural neutrosophic nilpotent of order two.

$I_{10}^{25} \times I_{10}^{25} = I_0^{25}$, $\quad I_{20}^{25} \times I_{20}^{25} = I_0^{25}$ and

$I_{15}^{25} \times I_{15}^{25} = I_0^{25}$ are natural neutrosophic nilpotent of order two.

P = $\{0, I_0^{25}, I_5^{25}, I_{10}^{25}, I_{15}^{25}, I_{20}^{25}, I_0^{25} + I_5^{25}, I_0^{25} + I_{10}^{25}, I_0^{25} + I_{15}^{25}, I_0^{25} + I_{20}^{25}, \ldots, I_0^{25} + I_5^{25} + I_{10}^{25} + I_{15}^{25} + I_{20}^{25}\}$.

P is such that for every $x \in P$ is such that $x \times x = I_0^{25}$.

Thus we see P is a subsemiring and infact a zero square natural neutrosophic subsemiring.

Infact x + x = x for every $x \in P$. But not a field as $I_5^{25} \times I_5^{25} = I_0^{25}$ is a natural neutrosophic zero divisor.

But P is a strict commutative finite natural neutrosophic subsemiring of W.

However W is not a strict in the usual sense natural neutrosophic semiring.

But we call them as strict for all those non strict elements have inverse under +;

5 + 20 = 0 (mod 25)
10 + 15 = 0 (mod 25)
12 + 13 = 0 (mod 25) and so on.

W is a not a semifield but has a subset which is a field.

Example 1.16: Let S = $\{ Z_9^I, +, \times\} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, a + I_0^9, a + I_3^9, a + I_6^9, a + I_0^9 + I_3^9, a + I_0^9 + I_6^9, a + I_6^9 + I_3^9, a + I_6^9 + I_0^9 + I_3^9, a \in Z_9, +, \times\}$ be the MOD natural neutrosophic semiring.

o(S) = 72. Clearly S is a strict semiring not in the usual sense. Quasi characteristic of the semiring S is 9.

$B = \{0, I_0^9, I_3^9, I_6^9, I_0^9 + I_3^9, I_0^9 + I_6^9, I_3^9 + I_6^9, I_0^9 + I_3^9 + I_6^9, +, \times\} \subseteq S$ is a subsemiring and no characteristic can be associated with it.

Infact every $x \in B$ is an idempotent with respect to both + and under × there are only some idempotents.

This helps one to define for the first time the notion of S idempotent semiring.

DEFINITION 1.2: *Let S = $\{Z_n^I, +, \times\}$ be the natural neutrosophic semiring. If P = $\{0, I_t^n$; t-appropriate values of $Z_n\} \subseteq S$ is a subsemiring then S is defined as the Smarandache idempotent semiring.*

If in addition every $x \in P$ is such that $x \times x = x$ then S is defined as the Smarandache idempotent strong semiring.

We will provide examples of them.

Example 1.17: Let S = $\{Z_6^I, +, \times\}$ be the natural neutrosophic number semiring.

$P = \{0, I_0^6, I_3^6, I_4^6, I_0^6 + I_3^6, I_0^6 + I_4^6, I_4^6 + I_3^6, I_0^6 + I_3^6 + I_4^6\} \subseteq S$ be the natural neutrosophic number semiring.

Clearly for every $x \in P$, $x + x = x$.

Now $I_0^6 \times I_0^6 = I_0^6$, $I_3^6 \times I_3^6 = I_3^6$,
$I_4^6 \times I_4^6 = I_4^6$ $(I_0^6 + I_3^6) \times (I_0^6 + I_3^6) = I_0^6 + I_3^6$,
$(I_0^6 + I_4^6) \times (I_0^6 + I_4^6) = I_0^6 + I_4^6$,
$(I_4^6 + I_3^6) \times (I_4^6 + I_3^6) = (I_4^6 + I_3^6 + I_0^6)$.
$(I_4^6 + I_0^6 + I_3^6) \times (I_0^6 + I_3^6 + I_4^6) = I_0^6 + I_3^6 + I_4^6$.

Thus baring $I_4^6 + I_3^6$ in P all elements in P are idempotents under $\times$.

Consider $P_1 = \{0, I_0^6, I_3^6, I_3^6 + I_0^6) \subseteq S$. Clearly P_1 is such that $x + x = x$ and $x \times x = x$ for all $x \in P_1$. So S is a S-strong idempotent semiring.

Let $P_2 = \{0, I_0^6, I_4^6, I_0^6 + I_4^6\} \subseteq S$.

For every $x \in P_2$ is such that $x + x = x$ and $x \times x = x$ so P_2 is an idempotent semiring with respect to + and ×; hence S is a S-strong idempotent semiring.

Example 1.18: Let S = $\{Z_{10}^{I}, +, \times\}$ be the natural neutrosophic semiring.

$P_1 = \{0, I_0^{10}, I_5^{10}, I_0^{10} + I_5^{10}\}$ and $P_2 = \{0, I_0^{10}, I_6^{10}, I_0^{10} + I_6^{10}\}$ are natural neutrosophic idempotent subsemirings of S with respect to + and ×.

So S is a S-strong idempotent semiring.

Example 1.19: Let S = $\{Z_{22}^{I}, +, \times\}$ be the natural neutrosophic semiring.

$P_1 = \{0, I_0^{22}, I_{11}^{22}, I_0^{22} + I_{11}^{22}\} \subseteq S$ and

$P_2 = \{0, I_0^{22}, I_{12}^{22}, I_0^{22} + I_{12}^{22}\} \subseteq S$ are pure natural neutrosophic subsemirings.

Both are strong idempotent subsemirings as every element in them is an idempotent with respect to + and ×.

So S is a S-strong idempotent subsemiring.

In view of this we have the following theorem.

THEOREM 1.2: *Let S = $\{Z_{2p}^{I}, +, \times\}$ (p an odd prime) be the natural neutrosophic element semiring. S is a S-strong idempotent semiring.*

Proof: Follows from the fact $P_1 = \{0, I_0^{2p}, I_p^{2p}, I_0^{2p} + I_p^{2p}\}$ and

$P_2 = \{0, I_0^{2p}, I_{p+1}^{2p}, I_0^{2p} + I_{p+1}^{2p}\}$ in S are strong idempotent subsemirings as both P_1 and P_2 are such that each element in them is an idempotent with respect to + and ×.

Hence the claim.

Example 1.20: Let S = $\{Z_{15}^I, +, \times\}$ be the natural neutrosophic semiring.

Consider $P_1 = \{0, I_0^{15}, I_{10}^{15}, I_0^{15} + I_{10}^{15}\}$ and

$P_2 = \{0, I_0^{15}, I_6^{15}, I_0^{15} + I_6^{15}\}$ are strong natural neutrosophic element idempotent subsemirings.

Hence S is a S-strong idempotent semiring.

Example 1.21: Let S = $\{Z_{16}^I, +, \times\}$ be the natural neutrosophic element semiring. S has only trivial natural neutrosophic idempotent viz., I_0^{16}.

Example 1.22: Let S = $\{Z_{27}^I, +, \times\}$ be the natural neutrosophic semiring.

S is not a S-strong natural neutrosophic semiring.

Inview of this we prove the following theorem.

THEOREM 1.3: *Let S = $\{Z_{p^n}^I, +, \times\}$ (p a prime n ≥ 1) be the natural neutrosophic semiring.*

S is not a S-strong idempotent semiring.

Follows from simple number theoretic techniques and hence left as an exercise to the reader.

Example 1.23: Let S = $\{Z_{43}^I, +, \times\}$ be the natural neutrosophic semiring.

S is not a S-strong idempotent semiring.

Next we define a strong natural neutrosophic nilpotent subsemiring of a natural neutrosophic semiring, $\{Z_n^I, +, \times\}$.

DEFINITION 1.3: *Let $S = \{Z_n^I, +, \times\}$ be the natural neutrosophic semiring. $P \subseteq S$ be the subset of pure natural neutrosophic elements. If every $x \in P$ is such that $x^m = I_0^n$, $m \geq 2$ then we define P to be a strong natural neutrosophic nilpotent subsemiring.*

If S contains a strong natural neutrosophic nilpotent subsemiring then S is defined as the S-strong natural neutrosophic nilpotent semiring.

We will provide examples of them.

Example 1.24: Let $S = \{Z_4^I, +, \times\}$ be the natural neutrosophic semiring.

$$P = \{0, 2, I_0^4, 2 + I_0^4, I_2^4, 2 + I_2^4, I_2^4 + I_0^4, 2 + I_0^4 + I_2^4\} \subseteq S.$$

is a subsemiring of S but is not a strong natural neutrosophic nilpotent subsemiring of S as $(2 + I_2^4)^2 = I_2^4 + I_0^4 \neq I_0^4$.

Let $R = \{0, I_0^4, I_2^4, I_0^4 + I_2^4\} \subseteq S$; R is a strong natural neutrosophic nilpotent subsemiring.

So S is a S-strong natural neutrosophic nilpotent semiring.

Example 1.25: Let $S = \{Z_8^I, +, \times\}$ be the natural neutrosophic semiring.

$P = \{0, I_0^8, I_4^8, I_0^8 + I_4^8\} \subseteq S$ is strong natural neutrosophic nilpotent subsemiring.

So S is a S-strong natural neutrosophic nilpotent semiring.

Example 1.26: Let S = { Z_{32}^{I} , +, ×} be the natural neutrosophic semiring.

P = {0, I_0^{32}, I_8^{32}, I_{16}^{32}, I_0^{32} + I_8^{32}, I_0^{32} + I_{16}^{32}, I_8^{32} + I_{16}^{32}, I_0^{32} + I_8^{32} + I_{16}^{32}} ⊆ S is a strong natural neutrosophic nilpotent subsemiring of S; hence S is a S-strong natural neutrosophic nilpotent semiring.

Example 1.27: Let S = { Z_{64}^{I} , +, ×} be the natural neutrosophic semiring.

P = {0, I_0^{64}, I_8^{64}, I_{16}^{64}, I_{24}^{64}, I_{32}^{64}, I_{40}^{64}, I_{48}^{64}, I_{56}^{64}, I_0^{64} + I_8^{64}, I_0^{64} + I_{16}^{64}, I_0^{64} + I_{24}^{64}, I_0^{64} + I_{32}^{64}, I_0^{64} + I_{40}^{64}, I_0^{64} + I_{48}^{64}, I_0^{64} + I_{56}^{64}, I_{48}^{64} + I_{56}^{64}, …, I_0^{64} + I_8^{64} + I_{16}^{64}, …, I_0^{64} + I_8^{64} + I_{16}^{64} + … + I_{56}^{64} } ⊆ S be the strong natural neutrosophic nilpotent subsemiring.

S is a S-strong natural neutrosophic nilpotent semiring.

In view of this we can make the following theorem,

THEOREM 1.4: *Let S = { $Z_{p^n}^{I}$, +, ×} (p a prime n ≥ 2) be the natural neutrosophic semiring. S is a S-strong natural neutrosophic nilpotent semiring.*

The proof can be got by exploiting number theoretic techniques so left as an exercise to the reader.

Next we proceed on to build natural neutrosophic matrix semirings using the natural neutrosophic semiring { Z_n^{I} , +, ×}.

Example 1.28: Let

$$V = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \\ a_{11} & a_{12} \end{bmatrix} \middle| \; a_i \in \{ Z_8^I, \times, +\}, 1 \leq i \leq 12, +, \times_n \right\}$$

be the natural neutrosophic column matrix semiring under the natural product $\times_n$.

Clearly $o(V) < \infty$.

Example 1.29: Let

$$M = \left\{ \begin{pmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \end{pmatrix} \middle| \; a_i \in \{ Z_8^I, \times, +\}, 1 \leq i \leq 12, +, \times_n \right\}$$

be the natural neutrosophic matrix semiring under natural product $\times_n$.

Let $x = \begin{pmatrix} I_0^{13} & 0 & 1 & I_0^{13}+1 \\ 0 & I_0^{13}+2 & 3 & 0 \end{pmatrix}$

and $y = \begin{pmatrix} 4 & 3+I_0^{13} & 2 & I_0^{13} \\ 5 & 0 & 6 & 7 \end{pmatrix} \in M$.

$$x \times_n y = \begin{pmatrix} I_0^{13} & 0 & 1 & I_0^{13}+1 \\ 0 & I_0^{13}+2 & 3 & 0 \end{pmatrix} \times_n \begin{pmatrix} 4 & 3+I_0^{13} & 2 & I_0^{13} \\ 5 & 0 & 6 & 7 \end{pmatrix}$$

$$= \begin{pmatrix} I_0^{13} & I_0^{13} & 2 & I_0^{13} \\ 0 & I_0^{13} & 5 & 0 \end{pmatrix} \in M.$$

This is the way natural product $\times_n$ is defined on M.

This has only one natural neutrosophic element viz I_0^{13} and $a + I_0^{13}$; $a \in Z_{13} \setminus \{0\}$.

Example 1.30: Let

$$B = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \middle| \; a_i \in \{ Z_{10}^I, \times, +\}, 1 \le i \le 16, \times_n \right\}$$

be the natural neutrosophic matrix semiring under the natural product $\times_n$.

B is a commutative semiring of finite order.

$$\text{Let } x = \begin{bmatrix} I_2^{10} + I_5^{10} & 0 & 3 & I_6^{10} + 4 \\ I_6^{10} & 4 + I_4^{10} & 0 & I_8^{10} + I_2^{10} \\ 0 & 0 & I_8^{10} + I_0^{10} & 6 \\ 7 & I_6^{10} + 3 + I_8^{10} & 6 & 2 \end{bmatrix}$$

$$\text{and } y = \begin{bmatrix} 3 + I_4^{10} & 8 & 4 & I_5^{10} + I_2^{10} \\ 0 & 2 & 3 + I_6^{10} + I_8^{10} & 0 \\ 8 & 0 & I_0^{10} & 2 \\ 2 & 3 & 4 & 5 \end{bmatrix}$$

be in B.

$$x \times_n y = \begin{bmatrix} I_2^{10} + I_8^{10} & 0 & 2 & I_5^{10} + I_2^{10} + I_0^{10} \\ I_6^{10} & 8 + I_4^{10} & I_6^{10} + I_8^{10} & I_8^{10} + I_2^{10} \\ 0 & 0 & I_0^{10} & 2 \\ 4 & I_6^{10} + I_8^{10} + 9 & 4 & 0 \end{bmatrix} \in B.$$

It is easily verified $x \times y \neq y \times_n x$.

Suppose on B we define $\times$ the usual product and not the natural product $\times_n$.

To find $x \times y$ and $y \times x$.

$$x \times y = \left[\begin{matrix} I_2^{10} + I_5^{10} + I_8^{10} + I_6^{10} + I_0^{10} + 2 & I_2^{10} + I_5^{10} + I_6^{10} + 2 \\ I_6^{10} + I_4^{10} + I_8^{10} + I_2^{10} & 8 + I_6^{10} + I_4^{10} + I_8^{10} + I_2^{10} \\ I_0^{10} + I_8^{10} + I_4^{10} + 2 & I_0^{10} + I_8^{10} + 8 \\ 3 + I_8^{10} + I_6^{10} + I_4^{10} & 8 + I_0^{10} + I_8^{10} \end{matrix}\right.$$

$$\left.\begin{matrix} I_6^{10} + I_2^{10} + I_5^{10} + I_8^{10} + I_0^{10} + 4 & 6 + I_0^{10} + I_4^{10} + I_5^{10} + I_6^{10} \\ 2 + I_0^{10} + I_2^{10} + I_4^{10} + I_6^{10} + I_8^{10} & 2 + I_0^{10} + I_2^{10} + I_6^{10} + I_8^{10} + I_4^{10} \\ 4 + I_6^{10} + I_0^{10} + I_8^{10} & I_2^{10} + I_5^{10} + I_8^{10} + I_0^{10} \\ 5 + I_0^{10} + I_6^{10} + I_8^{10} & 2 + I_5^{10} + I_2^{10} + I_6^{10} + I_8^{10} \end{matrix}\right] \in B.$$

Clearly $x \times_n y \neq x \times y$.

So both the operation $\times$ and $\times_n$ are distinct on B.

Consider $y \times x$;

$$y \times x = \begin{bmatrix} I_2^{10} + I_5^{10} + I_8^{10} + I_0^{10} + I_6^{10} & 2 + I_0^{10} + I_2^{10} + I_5^{10} + I_6^{10} + I_4^{10} & 9 + I_2^{10} + I_0^{10} + I_5^{10} + I_8^{10} + I_4^{10} & 6 + I_0^{10} + I_2^{10} + I_4^{10} + I_6^{10} + I_8^{10} \\ I_2^{10} + I_5^{10} + I_6^{10} + I_8^{10} & 8 + I_6^{10} + I_4^{10} + I_8^{10} & I_8^{10} + I_0^{10} + I_4^{10} & 8 + I_2^{10} + I_6^{10} + I_8^{10} \\ I_0^{10} + I_2^{10} + I_6^{10} + I_5^{10} + 4 & 6 + I_0^{10} + I_6^{10} + I_4^{10} + I_8^{10} & 6 + I_0^{10} & 4 + I_6^{10} + I_8^{10} + I_2^{10} + I_0^{10} \\ I_2^{10} + I_5^{10} + I_6^{10} + 5 & 7 + I_4^{10} + I_6^{10} + I_8^{10} & I_8^{10} + I_0^{10} & 2 + I_6^{10} + I_8^{10} + I_2^{10} \end{bmatrix}.$$

Clearly $x \times y \neq y \times x$.

Thus B under the operation $\times$ is not commutative.

Hence B is a non commutative natural neutrosophic semiring of finite order.

Thus if we take square natural neutrosophic matrices we can get two semirings of same order.

Working with any $m \times n$ ($m \neq n$) matrices can give only one MOD natural neutrosophic finite semiring which is always commutative.

Example 1.31: Let

$L = \{(a_1, a_2, a_3, a_4) \mid a_i \in Z_{12}^{I}, +, \times\}, 1 \leq i \leq 4, +, \times\}$ be the MOD natural neutrosophic matrix semiring of finite order.

Let $x = (3, 2, 6, 4)$ and $y = (8, 6, 4, 3) \in L$.

Clearly $x \times y = (3, 2, 6, 4) \times (8, 6, 4, 3)$

$= (0, 0, 0, 0).$

Further if $x = (I_8^{12}, I_4^{12}, I_6^{12}, I_3^{12})$ and

$y = (I_6^{12}, I_3^{12}, I_4^{12}, I_8^{12}) \in L.$

$$x \times y = (I_8^{12}, I_4^{12}, I_6^{12}, I_3^{12}) \times (I_6^{12}, I_3^{12}, I_4^{12}, I_8^{12})$$

$$= (I_0^{12}, I_0^{12}, I_0^{12}, I_0^{12})$$

is the natural neutrosophic zero divisor matrix of L.

Let $x = (3, I_4^{12}, 6, 9)$ and

$y = (8, I_6^{12}, 2, 4) \in L.$

Clearly $x \times y = (3, I_4^{12}, 6, 9) \times (8, I_6^{12}, 2, 4)$

$= (0, I_0^{12}, 0, 0) \in L.$

Thus we call $x \times y$ as the mixed natural neutrosophic zero divisor of L.

Let $a = (I_6^{12}, I_0^{12}, I_6^{12}, I_6^{12}) \in L.$

$a \times a = (I_0^{12}, I_0^{12}, I_0^{12}, I_0^{12})$ is the natural neutrosophic nilpotent matrix of L.

Let $a = (6, 6, 0, 6) \in L.$

$a \times a = (0, 0, 0, 0)$ is the real nilpotent matrix of L.

Let $b = (6, I_6^{12}, I_0^{12}, 6) \in L.$

$b \times b = (0, I_0^{12}, I_0^{12}, 0) \in L$ so b is the mixed natural neutrosophic nilpotent matrix of L.

Let $d = (4, 9, 0, 1) \in L$.

$d \times d = (4, 9, 0, 1) = d \in L$ is the real idempotent matrix of L.

Now $g = (I_9^{12}, I_4^{12}, I_4^{12}, I_0^{12}) \in L$ such that

$g \times g = (I_9^{12}, I_4^{12}, I_4^{12}, I_0^{12}) = g$ is the natural neutrosophic idempotent matrix of L.

Consider $t = (I_9^{12}, 9, 6, I_0^{12}) \in L$, we see $t \times t = t$ is the mixed natural neutrosophic matrix of L.

Thus L has real nilpotent matrices, mixed natural neutrosophic nilpotent matrices, natural neutrosophic nilpotent matrices, real idempotent matrices, mixed natural neutrosophic idempotent matrices and natural neutrosophic idempotent matrices.

Similarly L has real zero divisors, natural neutrosophic zero divisors and mixed natural neutrosophic zero divisors.

Now consider

$x = (I_9^{12}, I_8^{12} + I_4^{12}, I_6^{12} + I_3^{12}, I_9^{12} + I_6^{12})$ and

$y = (I_4^{12} + I_8^{12}, I_6^{12} + I_3^{12}, I_8^{12} + I_4^{12}, I_4^{12}) \in L.$

Clearly $x \times y = (I_0^{12}, I_0^{12}, I_0^{12}, I_0^{12})$ is the natural neutrosophic matrix zero divisors.

Let $p = (3 + I_9^{12}, 8 + I_4^{12}, 9 + I_6^{12}, 5 + I_3^{12} + I_8^{12})$ and
$q = (4 + I_8^{12}, 6 + I_6^{12}, 7 + I_4^{12}, 5 + I_4^{12}) \in L.$

We find the $p \times q = (3 + I_9^{12}, 8 + I_4^{12}, 9 + I_6^{12}, 5 + I_3^{12} + I_8^{12}) \times (4 + I_8^{12}, 6 + I_6^{12}, 7 + I_4^{12}, 5 + I_4^{12})$.

$$= (I_9^{12} + I_8^{12} + I_0^{12}, I_4^{12} + I_6^{12} + I_0^{12}, 3 + I_4^{12} + I_3^{12} + I_0^{12}, 1 + I_4^{12} + I_3^{12} + I_8^{12} + I_0^{12})$$

is a matrix in L.

This is the way product in general is performed in L.

It is an interesting and important problem to find the number of real zero divisors matrices; natural neutrosophic zero divisor matrices and mixed real natural neutrosophic zero divisor matrices of L.

The same study is to be done for idempotent matrices, nilpotent matrices.

Let $x = (5, 7, 1, 11) \in L$.

$x \times x = (1, 1, 1, 1) \in L$ is the unit.

We can have only real matrix units and it is not possible to get natural neutrosophic matrices.

This is universally true in case of MOD natural neutrosophic matrices built in general using Z_n^I.

Thus this matrix natural neutrosophic MOD semiring has units, zero divisors, idempotents and nilpotent real and otherwise, barring units.

Example 1.32: Let

$$M = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} \middle| \; a_i \in \{ Z_{23}^I, +, \times\}, 1 \leq i \leq 6, \times_n \right\}$$

be the MOD-natural neutrosophic matrix semiring under the natural product $\times_n$.

M has zero divisors only of special type.

$$\text{Let } x = \begin{bmatrix} 0 \\ 3 \\ 0 \\ 7 \\ 8 \\ 19 \end{bmatrix} \text{ and } y = \begin{bmatrix} 22 \\ 0 \\ 18 \\ 0 \\ 0 \\ 0 \end{bmatrix} \in M.$$

$$x \times_n y = \begin{bmatrix} 0 \\ 3 \\ 0 \\ 7 \\ 8 \\ 19 \end{bmatrix} \times_n \begin{bmatrix} 22 \\ 0 \\ 18 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Real zero divisors in M occurs only in this manner.

Let $a = \begin{bmatrix} I_0^{23} \\ 8 \\ I_0^{23} \\ 6 \\ 13 \\ I_0^{23} \end{bmatrix}$ and $b = \begin{bmatrix} 18 \\ I_0^{23} \\ 9 \\ I_0^{23} \\ I_0^{23} \\ 20 \end{bmatrix} \in M$.

$$a \times_n b = \begin{bmatrix} I_0^{23} \\ 8 \\ I_0^{23} \\ 6 \\ 13 \\ I_0^{23} \end{bmatrix} \times_n \begin{bmatrix} 18 \\ I_0^{23} \\ 9 \\ I_0^{23} \\ I_0^{23} \\ 20 \end{bmatrix} = \begin{bmatrix} I_0^{23} \\ I_0^{23} \\ I_0^{23} \\ I_0^{23} \\ I_0^{23} \\ I_0^{23} \end{bmatrix}$$

is the MOD natural neutrosophic matrix zero of M.

We can also have mixed MOD matrix zeros of the form

$$x_1 = \begin{bmatrix} I_0^{23} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}; x_2 = \begin{bmatrix} I_0^{23} \\ I_0^{23} \\ I_0^{23} \\ 0 \\ 0 \\ 0 \end{bmatrix}, x_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ I_0^{23} \\ I_0^{23} \\ I_0^{23} \end{bmatrix}, x_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ I_0^{23} \end{bmatrix}, \ldots,$$

$$x_t = \begin{bmatrix} I_0^{23} \\ I_0^{23} \\ I_0^{23} \\ I_0^{23} \\ I_0^{23} \\ I_0^{23} \end{bmatrix}.$$

However one can say that this M has no MOD natural neutrosophic matrix nilpotents or real matrix nilpotents.

The real idempotents one can expect in M are of the form

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \text{ and so on.}$$

The mixed idempotents of M are

$$\begin{bmatrix} I_0^{23} \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ I_0^{23} \\ I_0^{23} \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ I_0^{23} \\ 0 \\ 0 \\ 1 \\ I_0^{23} \end{bmatrix}, \begin{bmatrix} I_0^{23} \\ I_0^{23} \\ I_0^{23} \\ I_0^{23} \\ I_0^{23} \\ 0 \end{bmatrix}, \begin{bmatrix} I_0^{23} \\ I_0^{23} \\ I_0^{23} \\ I_0^{23} \\ I_0^{23} \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ I_0^{23} \\ 0 \\ I_0^{23} \\ 1 \\ 0 \end{bmatrix} \text{ and so on.}$$

These are some of the MOD mixed matrix idempotents.

Finding the number of such mixed idempotents happens to be a matter of routine.

Example 1.33: Let

$$B = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \end{bmatrix} \middle| \; a_i \in \{ Z_{19}^I, +, \times\}, 1 \leq i \leq 10, +, \times_n \right\}$$

be the MOD matrix natural neutrosophic semiring under the natural product $\times_n$.

S has real idempotents matrices as well as mixed natural neutrosophic idempotents matrices which takes entries only from the set $\{0, 1, I_0^{19}\}$.

S has real zero divisor matrices which has entries from $\{Z_{19}\}$ in a special way.

S has mixed zero divisor matrices which has entries from Z_{19}^{I} but arranged in a very special way. However it is impossible to have MOD nilpotent matrices.

The matrices of the form x = $\begin{bmatrix} 0 & 0 \\ 0 & I_0^{19} \\ 0 & 0 \\ I_0^{19} & 0 \\ 0 & I_0^{19} \end{bmatrix}$ ∈ S is such that

x^2 = x but x is not considered as MOD mixed natural neutrosophic nilpotent matrix of S as entries of x are either zero 0 or neutrosophic zero, I_0^{19}.

In view of all these we have the following theorem.

THEOREM 1.5: *Let M = {$(m_{ij})_{n\times m}$ | m_{ij} ∈ {Z_{19}^{I}, +, ×}, p a prime, 1 ≤ i ≤ n and 1 ≤ j ≤ m, +, $\times_n$} be the MOD natural neutrosophic matrix semiring under natural product $\times_n$.*

(1) The zero divisors are only of special type that is if A = (a_{ij}) and B = (b_{ij}) then A × B = (0) = ($a_{ij} \times b_{ij}$) is such that a_{ij} = 0 or b_{ij} = 0.

(ii) Similarly the natural neutrosophic zero divisors occurs in this way.

A × B = (I_0^p) = ($a_{ij} \times b_{ij}$) if and only if a_{ij} = I_0^p or b_{ij} = I_0^p.

(iii) M has no nilpotents of any form.

(iv) The only idempotents and mixed idempotents are matrices A with the entries from {0, 1, I_0^p}.

Proof is direct and hence left as an exercise to the reader.

Example 1.34: Let

$$B = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \middle| \; a_i \in \{ Z_3^I, +, \times\}, 1 \leq i \leq 3, \times_n, +\right\}$$

be the MOD natural neutrosophic matrix semiring.

$$B = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \right.$$

$$\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix},$$

$$\begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix},$$

$$\left. \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} I_0^3 \\ I_0^3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ I_0^3 \\ I_0^3 \end{bmatrix}, \begin{bmatrix} I_0^3 \\ 0 \\ I_0^3 \end{bmatrix}, \begin{bmatrix} I_0^3 \\ I_0^3 \\ I_0^3 \end{bmatrix} \text{ and so on} \right\}.$$

$$\text{Let } x = \begin{bmatrix} 2+I_0^3 \\ 1+I_0^3 \\ I_0^3 \end{bmatrix} \text{ and } y = \begin{bmatrix} 1+I_0^3 \\ I_0^3 \\ 2+I_0^3 \end{bmatrix} \in B.$$

To find $x \times_n y = \begin{bmatrix} 2+I_0^3 \\ 1+I_0^3 \\ I_0^3 \end{bmatrix} \times_n \begin{bmatrix} 1+I_0^3 \\ I_0^3 \\ 2+I_0^3 \end{bmatrix} = \begin{bmatrix} 2+I_0^3 \\ I_0^3 \\ I_0^3 \end{bmatrix}$.

This is the way product operation is performed on B.

$$x = \begin{bmatrix} I_0^3 \\ I_0^3 \\ 0 \end{bmatrix} \text{ and } y = \begin{bmatrix} I_0^3 \\ 0 \\ I_0^3 \end{bmatrix}. \ x \times_n y = \begin{bmatrix} I_0^3 \\ I_0^3 \\ I_0^3 \end{bmatrix}.$$

$$x = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{ and } y = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \in B.$$

$x \times_n y = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \times_n \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ is a trivial zero divisors.

$x \times_n y = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \times_n \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is a unit in B.

Several properties mentioned in this theorem can be verified for this B.

Example 1.35: Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \end{bmatrix} \middle| \ a_i \in Z_{10}^I ; 1 \leq i \leq 8, +, \times_n \right\}$$

be the MOD matrix natural neutrosophic semiring under + and $\times_n$.

S has several zero divisors, units and idempotents.

Further S has MOD natural neutrosophic zero divisors and MOD natural neutrosophic idempotents which will be illustrated by this example.

$$\text{Let } x = \begin{bmatrix} 2 & 4 & 6 & 8 \\ 6 & 4 & 2 & 6 \end{bmatrix} \text{ and } y = \begin{bmatrix} 0 & 5 & 5 & 5 \\ 5 & 5 & 5 & 0 \end{bmatrix} \in S.$$

$$\text{Clearly } x \times_n y = \begin{bmatrix} 2 & 4 & 6 & 8 \\ 6 & 4 & 2 & 6 \end{bmatrix} \times_n \begin{bmatrix} 0 & 5 & 5 & 5 \\ 5 & 5 & 5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ is a zero divisor.}$$

$$\text{Consider } y \times_n y = \begin{bmatrix} 0 & 5 & 5 & 5 \\ 5 & 5 & 5 & 0 \end{bmatrix} \times_n \begin{bmatrix} 0 & 5 & 5 & 5 \\ 5 & 5 & 5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 5 & 5 & 5 \\ 5 & 5 & 5 & 0 \end{bmatrix} = y.$$

Thus y is an idempotent.

$$y = \begin{bmatrix} 6 & 5 & 5 & 6 \\ 5 & 6 & 5 & 6 \end{bmatrix} \in S \text{ is such that}$$

$$y \times_n y = \begin{bmatrix} 6 & 5 & 5 & 6 \\ 5 & 6 & 5 & 6 \end{bmatrix} \times_n \begin{bmatrix} 6 & 5 & 5 & 6 \\ 5 & 6 & 5 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 5 & 5 & 6 \\ 5 & 6 & 5 & 6 \end{bmatrix} = y \text{ is also an idempotent.}$$

Now we provide MOD natural neutrosophic idempotents in S.

$$\text{Let } a = \begin{bmatrix} I_6^{10} & I_5^{10} & I_0^{10} & I_5^{10} \\ I_0^{10} & I_6^{10} & I_6^{10} & I_5^{10} \end{bmatrix} \in S$$

$$\text{Clearly } a \times_n a = \begin{bmatrix} I_6^{10} & I_5^{10} & I_0^{10} & I_5^{10} \\ I_0^{10} & I_6^{10} & I_6^{10} & I_5^{10} \end{bmatrix} = a.$$

So a is a MOD natural neutrosophic idempotent matrix.

$$\text{Let } b = \begin{bmatrix} I_5^{10} & I_2^{10} & I_6^{10} & I_0^{10} \\ I_6^{10} & I_8^{10} & I_5^{10} & I_4^{10} \end{bmatrix} \text{ and } a = \begin{bmatrix} I_4^{10} & I_5^{10} & I_5^{10} & I_7^{10} \\ I_5^{10} & I_5^{10} & I_2^{10} & I_5^{10} \end{bmatrix} \in S.$$

$$a \times_n b = \begin{bmatrix} I_0^{10} & I_0^{10} & I_0^{10} & I_0^{10} \\ I_0^{10} & I_0^{10} & I_0^{10} & I_0^{10} \end{bmatrix}$$

is a MOD natural neutrosophic zero divisor.

There are several MOD natural neutrosophic zero divisors and MOD natural neutrosophic idempotents elements of S.

We give get another example.

Example 1.36: Let

$$P = \left\{ \left. \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \end{bmatrix} \right| a_i \in Z_{16}^I ; 1 \leq i \leq 15, +, \times_n \right\}$$

be the MOD natural neutrosophic matrix semiring with product $\times_n$ and +, with entries from Z_{16}^{I}.

Let $x = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \in P$, we see $x \times_n x = x$ is an idempotent.

Apart from this type of idempotents we are not able to get any other idempotents.

All natural idempotent matrices have entries only from the set {0, 1}.

However P has nilpotents, zero divisors as well as MOD natural neutrosophic nilpotents and zero divisors.

For take $x = \begin{bmatrix} 4 & 8 & 0 & 0 \\ 8 & 0 & 4 & 8 \\ 12 & 0 & 12 & 4 \\ 12 & 4 & 8 & 0 \\ 0 & 8 & 12 & 4 \end{bmatrix} \in P$.

Clearly we see $x \times_n x = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ is a nilpotent of order two.

$$\text{Let } x = \begin{bmatrix} 8 & 2 & 4 & 6 \\ 4 & 12 & 0 & 8 \\ 2 & 4 & 10 & 6 \\ 12 & 0 & 4 & 8 \\ 6 & 10 & 2 & 6 \end{bmatrix} \text{ and}$$

$$y = \begin{bmatrix} 2 & 8 & 4 & 8 \\ 8 & 3 & 7 & 8 \\ 8 & 8 & 8 & 8 \\ 3 & 11 & 8 & 4 \\ 8 & 8 & 8 & 8 \end{bmatrix} \in P, \text{ we see}$$

$$x \times_n y = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ is a MOD zero matrix divisor of P.}$$

Now we give examples of MOD natural neutrosophic nilpotents and zero divisor in P.

$$\text{Let } x = \begin{bmatrix} 4 & 8 & 4 & 8 \\ 8 & 8 & 8 & 4 \\ 12 & 12 & 4 & 8 \\ 8 & 4 & 12 & 8 \\ 4 & 12 & 12 & 4 \end{bmatrix} \in P;$$

$$x \times_n x = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$ is a nilpotent element of order two.

Consider

$$x = \begin{bmatrix} 2 & 2 & 6 & 6 \\ 4 & 6 & 6 & 2 \\ 12 & 8 & 0 & 6 \\ 0 & 0 & 2 & 8 \\ 6 & 2 & 4 & 6 \end{bmatrix}$$ $\in$ P, clearly it is easily verified that

$$x^4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$; where $x^4 = x \times_n x \times_n x \times_n x$.

Let $$y = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$ $\in$ P is such that

$y \times_n y = y$ is a unit in P.

For $x \times_n y = y \times_n x = x$ for every $x \in P$.

Now we give nilpotents and idempotents using both MOD natural neutrosophic elements as well as elements of Z_{16}.

$$\text{Let } y = \begin{bmatrix} I_0^{16} & I_2^{16} & I_8^{16} & I_4^{16} \\ I_6^{16} & I_4^{16} & I_{12}^{16} & I_0^{16} \\ I_6^{16} & I_4^{16} & I_{12}^{16} & I_{12}^{16} \\ I_0^{16} & I_6^{16} & I_2^{16} & I_4^{16} \\ I_2^{16} & I_0^{16} & I_4^{16} & I_4^{16} \end{bmatrix} \text{ and } z = \begin{bmatrix} I_{12}^{16} & I_8^{16} & I_2^{16} & I_8^{16} \\ I_8^{16} & I_4^{16} & I_4^{16} & I_{12}^{16} \\ I_8^{16} & I_4^{16} & I_{12}^{16} & I_0^{16} \\ I_8^{16} & I_8^{16} & I_0^{16} & I_4^{16} \\ I_8^{16} & I_4^{16} & I_8^{16} & I_4^{16} \end{bmatrix} \in P.$$

$$y \times_n z = \begin{bmatrix} I_0^{16} & I_0^{16} & I_0^{16} & I_0^{16} \\ I_0^{16} & I_0^{16} & I_0^{16} & I_0^{16} \\ I_0^{16} & I_0^{16} & I_0^{16} & I_0^{16} \\ I_0^{16} & I_0^{16} & I_0^{16} & I_0^{16} \\ I_0^{16} & I_0^{16} & I_0^{16} & I_0^{16} \end{bmatrix}.$$

This type of MOD natural neutrosophic zero divisors will be known as MOD natural neutrosophic zero divisors.

$$\text{Let } x = \begin{bmatrix} I_0^{16} & I_8^{16} & 4 & I_2^{16} \\ I_8^{16} & 8 & 2 & 6 \\ I_4^{16} & 0 & I_2^{16} & I_0^{16} \\ I_8^{16} & 6 & 6 & I_4^{16} \\ I_2^{16} & 4 & 8 & I_0^{16} \end{bmatrix} \text{ and } y = \begin{bmatrix} I_8^{16} & I_8^{16} & 8 & I_8^{16} \\ I_2^{16} & 4 & 8 & 8 \\ I_8^{16} & 9 & I_8^{16} & I_8^{16} \\ I_8^{16} & 8 & 0 & I_4^{16} \\ I_8^{16} & 8 & 6 & I_8^{16} \end{bmatrix} \in P;$$

$$x \times_n y = \begin{bmatrix} I_0^{16} & I_0^{16} & 0 & I_0^{16} \\ I_0^{16} & 0 & 0 & 0 \\ I_0^{16} & 0 & I_0^{16} & I_0^{16} \\ I_0^{16} & 0 & 0 & I_0^{16} \\ I_0^{16} & 0 & 0 & I_0^{16} \end{bmatrix} \in P.$$

This type of MOD natural neutrosophic matrix zero divisors will be known as mixed MOD natural matrix zero divisors.

So we also have mixed MOD natural matrix nilpotents as well as pure MOD natural neutrosophic matrix nilpotents in P.

$$\text{Let } x = \begin{bmatrix} I_0^{16} & I_4^{16} & I_8^{16} & I_{12}^{16} \\ I_4^{16} & I_8^{16} & I_{12}^{16} & I_0^{16} \\ I_8^{16} & I_{12}^{16} & I_0^{16} & I_4^{16} \\ I_{12}^{16} & I_0^{16} & I_4^{16} & I_8^{16} \\ I_4^{16} & I_4^{16} & I_{12}^{16} & I_8^{16} \end{bmatrix} \in P$$

$$x \times_n x = \begin{bmatrix} I_0^{16} & I_0^{16} & I_0^{16} & I_0^{16} \\ I_0^{16} & I_0^{16} & I_0^{16} & I_0^{16} \\ I_0^{16} & I_0^{16} & I_0^{16} & I_0^{16} \\ I_0^{16} & I_0^{16} & I_0^{16} & I_0^{16} \\ I_0^{16} & I_0^{16} & I_0^{16} & I_0^{16} \end{bmatrix}$$

is the pure MOD natural neutrosophic nilpotent matrix of order two in P.

$$\text{Let } a = \begin{bmatrix} I_2^{16} & I_4^{16} & I_6^{16} & I_2^{16} \\ I_0^{16} & I_6^{16} & I_2^{16} & I_4^{16} \\ I_{12}^{16} & I_8^{16} & I_6^{16} & I_0^{16} \\ I_6^{16} & I_6^{16} & I_2^{16} & I_2^{16} \\ I_2^{16} & I_2^{16} & I_8^{16} & I_8^{16} \end{bmatrix} \in P.$$

$$\text{Clearly } a^4 = a \times_n a \times_n a \times_n a = \begin{bmatrix} I_0^{16} & I_0^{16} & I_0^{16} & I_0^{16} \\ I_0^{16} & I_0^{16} & I_0^{16} & I_0^{16} \\ I_0^{16} & I_0^{16} & I_0^{16} & I_0^{16} \\ I_0^{16} & I_0^{16} & I_0^{16} & I_0^{16} \\ I_0^{16} & I_0^{16} & I_0^{16} & I_0^{16} \end{bmatrix}$$

is a MOD pure natural neutrosophic nilpotent matrix of order four in P.

Thus P can have mixed MOD natural neutrosophic nilpotent matrices as well as pure MOD natural neutrosophic nilpotent matrices. Similar situation in case of zero divisor.

However it is pertinent at this stage to throw open the following conjecture.

Conjecture 1.1: Let
M = $\{(a_{ij}) \mid a_{ij} \in Z_n^I$, $1 \leq i \leq t$ and $1 \leq j \leq s$, $+, \times_n\}$ be the MOD natural neutrosophic matrix semiring under + and $\times_n$.

(i) If $n = p^m$ where p is a prime $m \geq 2$; can M have MOD natural neutrosophic idempotents or pure or mixed except the trivial ones got using I_0^n, 0 and 1.

(ii) If $n = p^m$ prove there exist MOD natural neutrosophic nilpotents matrices both pure and mixed.

(iii) if $n = p^m$, $m \geq 2$ prove there are MOD natural neutrosophic nilpotents matrices which are zero divisors.

Next we give an example of a MOD natural neutrosophic pure matrix idempotents as well as MOD natural neutrosophic mixed matrix idempotents.

Example 1.37: Let S =

$$\left\{\left[\begin{matrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \end{matrix}\right] \middle| \; a_i \in Z_{12}^I ; 1 \leq i \leq 15, +, \times_n\right\}$$

be the MOD natural neutrosophic matrix semiring under + and $\times_n$.

$$\text{Let } x = \begin{bmatrix} 4 & 9 & 0 \\ 1 & 0 & 4 \\ 9 & 9 & 4 \\ 0 & 4 & 9 \\ 4 & 9 & 4 \end{bmatrix} \in S; \text{ we see } x \times_n x = \begin{bmatrix} 4 & 9 & 0 \\ 1 & 0 & 4 \\ 9 & 9 & 4 \\ 0 & 4 & 9 \\ 4 & 9 & 4 \end{bmatrix} = x.$$

Thus $x \in S$ is an idempotent matrix.

$$\text{Let } y = \begin{bmatrix} I_4^{12} & I_9^{12} & I_0^{12} \\ I_9^{12} & I_9^{12} & I_9^{12} \\ I_4^{12} & I_4^{12} & I_4^{12} \\ I_0^{12} & I_0^{12} & I_0^{12} \\ I_4^{12} & I_9^{12} & I_0^{12} \end{bmatrix} \in S; \text{ we see } y \times_n y = \begin{bmatrix} I_4^{12} & I_9^{12} & I_0^{12} \\ I_9^{12} & I_9^{12} & I_9^{12} \\ I_4^{12} & I_4^{12} & I_4^{12} \\ I_0^{12} & I_0^{12} & I_0^{12} \\ I_4^{12} & I_9^{12} & I_0^{12} \end{bmatrix}$$

is the pure MOD natural neutrosophic idempotents matrix of S.

We will give a mixed MOD natural neutrosophic idempotents matrix of S.

$$\text{Let } z = \begin{bmatrix} 4 & I_9^{12} & 0 \\ 1 & 9 & I_4^{12} \\ I_0^{12} & 1 & I_9^{12} \\ I_9^{12} & I_4^{12} & 9 \\ 4 & 0 & 4 \end{bmatrix} \in S; \text{ clearly } z \times_n z = \begin{bmatrix} 4 & I_9^{12} & 0 \\ 1 & 9 & I_4^{12} \\ I_0^{12} & 1 & I_9^{12} \\ I_9^{12} & I_4^{12} & 9 \\ 4 & 0 & 4 \end{bmatrix}$$

is in P is a mixed MOD natural neutrosophic idempotent matrix of S.

We next give example of MOD natural neutrosophic zero divisors and MOD natural nilpotents mixed and pure.

$$\text{Let } b = \begin{bmatrix} 6 & 0 & 0 \\ 6 & I_6^{12} & I_0^{12} \\ I_6^{12} & 0 & 6 \\ 6 & I_6^{12} & 0 \\ 0 & 6 & I_6^{12} \end{bmatrix} \in S;$$

$$\text{clearly } b \times_n b = \begin{bmatrix} 0 & 0 & 0 \\ 0 & I_0^{12} & I_0^{12} \\ I_0^{12} & 0 & 0 \\ 0 & I_0^{12} & 0 \\ 0 & 0 & I_0^{12} \end{bmatrix} \in S$$

is such that b is a nilpotent of order two.

We are sure we cannot get MOD natural neutrosophic nilpotent matrices of order greater than two.

The only elements in the matrices of S which contribute to nilpotent matrices are I_6^{12}, I_0^{12}, 0, $I_6^{12} + I_0^{12}$ and 6 only.

Apart from this there is no nilpotent elements in Z_{12}^I.

We will give one or two examples of zero divisors matrices of MOD natural neutrosophic elements in S.

$$\text{Let } x = \begin{bmatrix} 4 & 3 & 6 \\ 2 & 10 & 8 \\ 9 & 7 & 4 \\ 2 & 4 & 0 \\ 5 & 6 & 2 \end{bmatrix} \text{ and } y = \begin{bmatrix} 3 & 8 & 2 \\ 6 & 5 & 3 \\ 4 & 0 & 6 \\ 6 & 3 & 11 \\ 10 & 2 & 6 \end{bmatrix} \in S.$$

Clearly it is verified $x \times_n y = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$; thus S has zero divisor matrices.

Let us give an example of a pure MOD natural neutrosophic zero divisor matrix in S.

$$x = \begin{bmatrix} I_6^{12} & I_4^{12} & I_8^{12} \\ I_3^{12} & I_0^{12} & I_9^{12} \\ I_2^{12} & I_4^{12} & I_{10}^{12} \\ I_8^{12} & I_9^{12} & I_0^{12} \\ I_4^{12} & I_6^{12} & I_2^{12} \end{bmatrix} \text{ and } y = \begin{bmatrix} I_4^{12} & I_6^{12} & I_3^{12} \\ I_8^{12} & I_2^{12} & I_4^{12} \\ I_6^{12} & I_6^{12} & I_6^{12} \\ I_3^{12} & I_4^{12} & I_9^{12} \\ I_6^{12} & I_6^{12} & I_6^{12} \end{bmatrix} \in S.$$

We see $x \times_n y = \begin{bmatrix} I_0^{12} & I_0^{12} & I_0^{12} \\ I_0^{12} & I_0^{12} & I_0^{12} \\ I_0^{12} & I_0^{12} & I_0^{12} \\ I_0^{12} & I_0^{12} & I_0^{12} \\ I_0^{12} & I_0^{12} & I_0^{12} \end{bmatrix} \in S$

is the pure MOD natural neutrosophic zero of S and x, y ∈ S contribute to pure MOD natural neutrosophic zero divisor.

Next we proceed onto give one illustration of mixed MOD natural neutrosophic zero divisor.

It is pertinent to keep on record unlike MOD pure natural neutrosophic zero $x = (I_0^{12})$ and zero matrix $y = (0)$ we have several mixed zeros.

$$A = \begin{bmatrix} I_0^{12} & 0 & 0 \\ 0 & 0 & I_0^{12} \\ 0 & I_0^{12} & 0 \\ 0 & 0 & 0 \\ I_0^{12} & I_0^{12} & I_0^{12} \end{bmatrix} \in S$$

is a MOD natural neutrosophic mixed zero of S.

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ I_0^{12} & 0 & 0 \\ I_0^{12} & 0 & 0 \\ I_0^{12} & 0 & 0 \end{bmatrix} \in S$$

is again a MOD natural neutrosophic mixed zero of S.

$$\text{Let } x = \begin{bmatrix} 4 & I_6^{12} & 0 \\ 9 & 6 & I_4^{12} \\ 2 & 8 & 4 \\ I_3^{12} & I_9^{12} & 0 \\ 10 & 6 & I_0^{12} \end{bmatrix} \text{ and } y = \begin{bmatrix} 6 & I_6^{12} & 0 \\ 4 & 6 & I_6^{12} \\ 6 & 3 & 3 \\ I_4^{12} & I_8^{12} & 6 \\ 6 & 2 & I_8^{12} \end{bmatrix} \in S.$$

$$\text{Clearly } x \times_n y = \begin{bmatrix} 0 & I_0^{12} & 0 \\ 0 & 0 & I_0^{12} \\ 0 & 0 & 0 \\ I_0^{12} & I_0^{12} & 0 \\ 0 & 0 & I_0^{12} \end{bmatrix}.$$

Thus x, y $\in$ S gives the product to be MOD natural neutrosophic mixed zero divisor.

Next we proceed onto describe MOD natural neutrosophic finite complex number semiring under + and × on $C^I(Z_n)$.

Let $C^I(Z_n) = \{\langle Z_n \cup I_t^c \rangle \mid t \in Z_n$ is a zero divisor or nilpotent or idempotent of $Z_n\}$.

$C^I(Z_n)$ is a finite semiring under + and × we call $\{C^I(Z_n), +, \times\}$ to be a MOD natural neutrosophic finite complex number semiring under usual + and ×.

We see $I_t^c + I_t^c = I_t^c$ for all appropriate $t \in C(Z_n)$ is an idempotent under +.

We will illustrate this by an example.

Example 1.38: Let S = $\{\langle C^I(Z_{10}), +, \times\rangle\}$ be a MOD natural neutrosophic finite complex number semiring. S has idempotents under + as well idempotents under ×.

$$I_5^c + I_5^c = I_5^c,\ I_{5i_F}^c + I_{5i_F}^c = I_{5i_F}^c,$$

$$I_2^c + I_2^c = I_2^c,\ I_4^c + I_4^c = I_4^c \text{ and so on.}$$

We see Z_{10} is a ring so S is a S-semiring of a special type.

Likewise Z_n^I is a MOD semiring which is a S-semiring of special type for $2 \leq n < \infty$.

We see $C^I(Z_{10})$ has zero divisor as well as idempotents which can also be MOD finite complex natural neutrosophic idempotents and MOD natural neutrosophic finite complex zero divisors.

$$I_{5i_F}^c,\ I_2^c \in C^I(Z_{10}) \text{ we see } I_{5i_F}^c \times I_2^c = I_0^c,$$

$$I_{4i_F}^c \times I_{5i_F}^c = I_0^c \text{ and so on.}$$

$I_5^c \times I_5^c = I_5^c$ is a MOD natural neutrosophic finite complex idempotent.

$I_6^c \times I_6^c = I_6^c$ is also a MOD natural neutrosophic finite complex number idempotent. $I_{6i_F}^c \times I_{6i_F}^c = I_6^c$ so $I_{6i_F}^c$ is not an idempotent.

$I_{5+5i_F}^c \times I_{5+5i_F}^c = I_0^c$,

$I_{6+6i_F}^c \times I_{5i_F}^c = I_0^c$ and so on has MOD natural neutrosophic finite complex number zero divisors.

$I_{5i_F}^c \times I_2^c = I_0^c$, $I_{5i_F}^c \times I_{2i_F}^c = I_0^c$,

$I_5^c \times I_{2i_F}^c = I_0^c$ and so on.

$I_{2+2i_F}^c \times I_5^c = I_0^c$ is also a zero divisor.

So $\langle C^I(Z_n), +, \times\rangle$ is a MOD natural neutrosophic finite complex number semiring of finite order which is a S-semiring of special type.

These MOD natural neutrosophic finite complex number semiring has MOD natural neutrosophic idempotents, nilpotents and zero divisors.

Next we describe MOD natural neutrosophic finite complex number matrix semirings under + and $\times_n$ by some examples.

Example 1.39: Let $S = \{(a_1, a_2, a_3) \mid a_i \in C^I(Z_6); 1 \leq i \leq 3, +, \times\}$ be the MOD natural neutrosophic matrix finite complex number semiring.

$x = (I_3^c \quad I_4^c \quad I_2^c)$ and $y = (I_2^c \quad I_3^c \quad I_3^c) \in S$;

$x \times y = (I_0^c \quad I_0^c \quad I_0^c)$ and

$x + y = (I_3^c + I_2^c, I_4^c + I_3^c, \; I_2^c + I_3^c) \in S.$

This is the way product and sum operation is performed on S.

Let $x = (3 \quad I_0^c \quad 4 + 2i_F)$ and

$y = (4+2i_F \quad 5+i_F \quad I_2^c) \in S.$

$x+y = (1 + 2i_F \quad I_0^c + 5+i_F \quad 4+2i_F + I_2^c) \in S.$

This is the way sum is got

$x \times y = (0 \quad I_0^c \quad I_2^c) \in S$

$x + x = (0 \quad I_0^c \quad 2+4i_F)$ and

$y + y = (2 + 4i_F \quad 4+i_F \quad I_2^c) \in S.$

$x^2 = x \times x = (3 \quad I_0^c \quad 4i_F)$ and

$y^2 = y \times y = (4i_F \quad 4i_F \quad I_4^c) \in S.$

Consider $x = (3, 4, I_4^c) \in S$, clearly $x \times x = (3, 4, I_4^c) = x$.

Thus x is a mixed MOD natural neutrosophic finite complex number matrix idempotent of S.

Let $y = (2, 3, I_3^c) \in S$.

Clearly $x \times y = (0 \; 0 \quad I_0^c) \in S$ is a mixed MOD natural neutrosophic finite complex number matrix zero of S and this pair is a called the mixed MOD natural neutrosophic matrix zero divisor.

Can this S have MOD matrix natural neutrosophic nilpotents?

It is pertinent to mention at this stage the following.

Let S = $\{C^I(Z_n), +, \times\}$ be a MOD natural neutrosophic finite complex modulo integer semiring (n fixed).

(i) Find order of S.
(ii) Find all MOD natural neutrosophic zero divisors of S.
(iii) Find all MOD natural neutrosophic finite complex number idempotents of S.
(iv) Find all MOD natural neutrosophic finite nilpotents of S.
(v) Study the situation
 (i) when n is a prime,
 (ii) when $n = p^t$ ($t \geq 2$, n a prime),
 (iii) when n is even composite number and
 (iv) when n is odd composite number.

Example 1.40: Let

$$S = \left\{\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \end{bmatrix} \middle| \; a_i \in C^I(Z_{16});\ 1 \leq i \leq 10, +, \times_n\right\}$$

be MOD natural neutrosophic finite complex number matrix semiring.

$$\text{Let } x = \begin{bmatrix} I_2^c & I_4^c \\ I_8^c & I_4^c \\ I_0^c & I_2^c \\ I_4^c & I_8^c \\ I_2^c & I_4^c \end{bmatrix} \in S.$$

Clearly $x \times_n x \times_n x \times_n x = x^4 = \begin{bmatrix} I_0^c & I_0^c \\ I_0^c & I_0^c \\ I_0^c & I_0^c \\ I_0^c & I_0^c \\ I_0^c & I_0^c \end{bmatrix}$ so x is the MOD pure natural neutrosophic finite complex nilpotent of order four and the zero is the MOD natural neutrosophic finite complex number pure zero.

We can have only one zero.

$(0) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ and only one MOD pure natural neutrosophic finite complex number matrix zero $\begin{bmatrix} I_0^c & I_0^c \\ I_0^c & I_0^c \\ I_0^c & I_0^c \\ I_0^c & I_0^c \\ I_0^c & I_0^c \end{bmatrix}$, but however we have several MOD natural neutrosophic finite complex number mixed zero matrix given by

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ I_0^c & I_0^c \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} I_0^c & 0 \\ 0 & I_0^c \\ I_0^c & 0 \\ 0 & I_0^c \\ I_0^c & 0 \end{bmatrix}, \begin{bmatrix} I_0^c & I_0^c \\ I_0^c & I_0^c \\ 0 & 0 \\ 0 & 0 \\ I_0^c & I_0^c \end{bmatrix} \text{ and so on.}$$

Let $x = \begin{bmatrix} 2 & 8 \\ 4 & 8 \\ 0 & 0 \\ 4 & 2 \\ 4 & 8 \end{bmatrix} \in S$, clearly $x \times_n x \times_n x \times_n x = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ is a

MOD natural neutrosophic nilpotent element of order four.

Let $x = \begin{bmatrix} I_6^c & I_4^c \\ I_8^c & I_2^c \\ I_{10}^c & I_0^c \\ I_{12}^c & I_4^c \\ I_6^c & I_0^c \end{bmatrix}$ and $y = \begin{bmatrix} I_8^c & I_{12}^c \\ I_8^c & I_8^c \\ I_8^c & I_6^c \\ I_4^c & I_4^c \\ I_8^c & I_{10}^c \end{bmatrix} \in S$.

Clearly $x \times_n y = \begin{bmatrix} I_0^c & I_0^c \\ I_0^c & I_0^c \\ I_0^c & I_0^c \\ I_0^c & I_0^c \\ I_0^c & I_0^c \end{bmatrix}$ is a MOD natural neutrosophic finite complex modulo integer zero divisor of S.

Let $x = \begin{bmatrix} 4 & I_2^c \\ 8 & I_4^c \\ I_6^c & I_8^c \\ I_0^c & 6 \\ 0 & 6 \end{bmatrix}$ and $y = \begin{bmatrix} 6 & I_8^c \\ 2 & I_4^c \\ I_8^c & I_2^c \\ I_6^c & 8 \\ 9 & 8 \end{bmatrix} \in S$.

Clearly $x \times_n y = \begin{bmatrix} 0 & I_0^c \\ 0 & I_0^c \\ I_0^c & I_0^c \\ I_0^c & 0 \\ 0 & 0 \end{bmatrix}$ is again a MOD natural neutrosophic matrix zero divisor of S.

Clearly we not in a position to find MOD natural neutrosophic matrix idempotents other than those matrices with entries from the set I_0^c, 1 and 0.

Some such matrix idempotents are as follows:

$$x = \begin{bmatrix} 0 & 1 \\ I_0^c & 0 \\ 1 & 1 \\ 0 & 0 \\ I_0^c & 1 \end{bmatrix} \in S \text{ is such that}$$

$$x \times_n x = \begin{bmatrix} 0 & 1 \\ I_0^c & 0 \\ 1 & 1 \\ 0 & 0 \\ I_0^c & 1 \end{bmatrix} = x$$

so x is a MOD natural neutrosophic finite complex number matrix idempotent.

We call all these types of idempotents only as trivial MOD natural neutrosophic finite complex number matrix idempotents.

$$x = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ I_0^c & I_0^c \\ 1 & 1 \end{bmatrix} \in S \text{ such that } x \times_n x = x.$$

$$\text{Let } x = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \\ 1 & 1 \\ 0 & 0 \end{bmatrix} \in S \text{ is such that } x \times_n x = x.$$

Thus we can give several such MOD natural neutrophic finite complex number trivial matrix idempotent.

In view of this we suggest the following problem.

Problem 1.1: Let $S = \{(a_{ij}) \mid a_{ij} \in C^I(Z_n)$ where $n = p^t$; $t \geq 2$, $1 \leq i \leq m$ and $1 \leq j \leq r, +, \times_n\}$ be the MOD natural neutrosophic finite complex number matrix semiring.

Can S have MOD natural neutrosophic finite complex number matrix non trivial idempotents?

Next we proceed onto supply MOD natural neutrosophic-neutrosophic matrix semirings using $\langle Z_n \cup I\rangle_I$ some examples.

Example 1.41: Let

$$B = \left\{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} \middle| a_i \in \langle Z_{10} \cup I\rangle_I;\ 1 \leq i \leq 6, +, \times_n\right\}$$

be the MOD natural neutrosophic-neutrosophic matrix semiring.

Clearly B is of finite order.

$$I = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \in B \text{ serves as the unit of B.}$$

$$(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ is the zero of B and}$$

$$(I_0^I) = \begin{bmatrix} I_0^I \\ I_0^I \\ I_0^I \\ I_0^I \\ I_0^I \\ I_0^I \end{bmatrix} \in B$$

serves as the MOD pure natural neutrosophic -neutrosophic matrix zero divisor of B.

$$m_1 = \begin{bmatrix} I_0^I \\ 0 \\ 0 \\ 0 \\ I_0^I \\ 0 \end{bmatrix}, m_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ I_0^I \\ I_0^I \end{bmatrix}, m_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ I_0^I \end{bmatrix},$$

$$m_4 = \begin{bmatrix} I_0^I \\ 0 \\ 0 \\ 0 \\ 0 \\ I_0^I \end{bmatrix}, m_5 = \begin{bmatrix} 0 \\ 0 \\ I_0^I \\ I_0^I \\ 0 \\ 0 \end{bmatrix}, m_6 = \begin{bmatrix} I_0^I \\ 0 \\ I_0^I \\ 0 \\ I_0^I \\ 0 \end{bmatrix}$$

in B are some of the MOD natural neutrosophic- neutrosophic matrix mixed zeros of B.

Let $x = \begin{bmatrix} 5 \\ I_5^I \\ 6 \\ I_6^I \\ 0 \\ 0 \end{bmatrix} \in B$ such that $x \times_n x = x$ so x is the MOD natural neutrosophic neutrosophic idempotent of B.

All these MOD natural neutrosophic-neutrosophic matrix mixed zero divisors are trivial idempotents of B.

Let $x = \begin{bmatrix} 5 \\ 5 \\ 6 \\ 6 \\ 0 \\ 5 \end{bmatrix} \in B$, $x \times_n x = x$ so x is the MOD natural neutrosophic-neutrosophic idempotent of B.

We have no MOD natural neutrosophic-neutrosophic nilpotents in B.

Now we supply a few MOD natural neutrosophic-neutrosophic matrix zero divisors of B.

$$\text{Let } x = \begin{bmatrix} I_2^I \\ I_4^I \\ I_6^I \\ I_8^I \\ I_0^I \\ I_6^I \end{bmatrix} \text{ and } y = \begin{bmatrix} I_5^I \\ I_5^I \\ I_5^I \\ I_5^I \\ I_8^I \\ I_0^I \end{bmatrix} \in B;$$

$$x \times_n y = \begin{bmatrix} I_0^I \\ I_0^I \\ I_0^I \\ I_0^I \\ I_0^I \\ I_0^I \end{bmatrix} \in B$$

is a MOD natural neutrosophic-neutrosophic matrix zero divisor.

$$\text{Let } x = \begin{bmatrix} I_5^I \\ I_2^I \\ 6 \\ 5 \\ 8 \\ 2 \end{bmatrix} \text{ and } y = \begin{bmatrix} I_2^I \\ I_5^I \\ 5 \\ 2 \\ 5 \\ 5 \end{bmatrix} \in B.$$

$$\text{Clearly } x \times_n y = \begin{bmatrix} I_0^I \\ I_0^I \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

is a MOD natural neutrosophic-neutrosophic matrix mixed zero divisor of B.

We give yet another example.

Example 1.42: Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \end{bmatrix} \middle| \; a_i \in \langle Z_9 \cup I \rangle_I; 1 \leq i \leq 8, +, \times_n \right\}$$

be the MOD natural neutrosophic-neutrosophic matrix semiring.

$$I = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \in M \text{ acts as the identity with respect to } \times_n;$$

for every $x \in M$, $x \times_n I = I \times_n x = x$.

$$\text{We see } (0) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \in M \text{ is the zero of M.}$$

$$(I_0^I) = \begin{bmatrix} I_0^I & I_0^I & I_0^I & I_0^I \\ I_0^I & I_0^I & I_0^I & I_0^I \end{bmatrix} \text{ is the pure MOD natural}$$

neutrosophic-neutrosophic zero.

$$m_1 = \begin{bmatrix} I_0^I & 0 & 0 & 0 \\ 0 & I_0^I & I_0^I & 0 \end{bmatrix},\ m_2 = \begin{bmatrix} 0 & 0 & I_0^I & 0 \\ 0 & I_0^I & 0 & 0 \end{bmatrix}$$

$$m_3 = \begin{bmatrix} 0 & 0 & 0 & I_0^I \\ I_0^I & I_0^I & 0 & 0 \end{bmatrix} \text{ and } m_4 = \begin{bmatrix} 0 & 0 & 0 & I_0^I \\ I_0^I & 0 & 0 & 0 \end{bmatrix}$$

are some of the MOD natural neutrosophic-neutrosophic zero divisors of M.

Let $x = \begin{bmatrix} 3 & 6 & 3 & 6 \\ 6 & 0 & 0 & 6 \end{bmatrix} \in M$ is such that

$$x \times_n x = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

is the MOD natural neutrosophic-neutrosophic element of M.

Let $x = \begin{bmatrix} I_{3I}^I & I_{3I+3}^I & I_{6I+3}^I & I_0^I \\ I_{6I}^I & I_{3I+6}^I & I_3^I & I_6^I \end{bmatrix} \in M$, we find $x \times_n x$;

$$x \times_n x = \begin{bmatrix} I_0^I & I_0^I & I_0^I & I_0^I \\ I_0^I & I_0^I & I_0^I & I_0^I \end{bmatrix}$$

$$= (I_0^I)$$

So x is a MOD natural neutrosophic nilpotent matrix of order two.

We have also MOD natural matrix zero divisors of M.

$$\text{Let } x = \begin{bmatrix} 3 & 6 & I_3^I & I_6^I \\ I_0^I & I_6^I & 0 & 6 \end{bmatrix} \text{ and } y = \begin{bmatrix} 6 & 3 & I_6^I & I_3^I \\ I_6^I & I_0^I & 5 & 6 \end{bmatrix} \in M.$$

$$x \times_n y = \begin{bmatrix} 0 & 0 & I_0^I & I_0^I \\ I_0^I & I_0^I & 0 & 0 \end{bmatrix}$$

is a MOD natural neutrosophic-neutrosophic matrix mixed zero divisor.

Thus we can work with MOD natural neutrosophic-neutrosophic matrix zero divisors in this way.

Interested reader can work with many more examples.

Next we proceed on to develop the notion of MOD natural neutrosophic dual number matrix semirings of finite order by some examples.

Example 1.43: Let

$$S = \left\{ \begin{bmatrix} a_1 & a_6 & a_{11} \\ a_2 & a_7 & a_{12} \\ a_3 & a_8 & a_{13} \\ a_4 & a_9 & a_{14} \\ a_5 & a_{10} & a_{15} \end{bmatrix} \middle| \; a_i \in \langle Z_{12} \cup g\rangle_I;\ 1 \leq i \leq 15, +, \times_n \right\}$$

be the MOD natural neutrosophic dual number matrix semiring.

$$(0) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ is the natural zero.}$$

$$(I_0^g) = \begin{bmatrix} I_0^g & I_0^g & I_0^g \\ I_0^g & I_0^g & I_0^g \\ I_0^g & I_0^g & I_0^g \\ I_0^g & I_0^g & I_0^g \\ I_0^g & I_0^g & I_0^g \end{bmatrix}$$

is the MOD pure natural neutrosophic dual number matrix zero.

$$p_1 = \begin{bmatrix} 0 & 0 & 0 \\ I_0^g & I_0^g & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I_0^g \end{bmatrix}, \; p_2 = \begin{bmatrix} I_0^g & I_0^g & I_0^g \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ I_0^g & I_0^g & I_0^g \end{bmatrix} \text{ and } p_3 = \begin{bmatrix} I_0^g & 0 & 0 \\ 0 & I_0^g & 0 \\ 0 & 0 & I_0^g \\ 0 & 0 & 0 \\ I_0^g & 0 & I_0^g \end{bmatrix}$$

are some of the mixed zeros of S.

$$\text{Let } a = \begin{bmatrix} I_{2g}^g + I_{5g}^g & 1 & 6 \\ 0 & I_g^g & 7 \\ I_g^g & I_2^g + I_{5g}^g & 0 \\ 0 & I_4^g & I_6^g \\ 5 & 6 & I_0^g \end{bmatrix}$$

$$\text{and } b = \begin{bmatrix} I_g^g & I_0^g & 2g \\ 6g & I_g^g & 0 \\ I_{5g}^g & I_{6g}^g & 11g \\ 10+5g & I_3^g & I_{2g}^g \\ 0 & 2 & I_4^g \end{bmatrix} \in S.$$

$$\text{Clearly } a \times_n b = \begin{bmatrix} I_0^g & I_0^g & 0 \\ 0 & I_0^g & 0 \\ I_0^g & I_0^g & 0 \\ 0 & I_0^g & I_0^g \\ 0 & 0 & I_0^g \end{bmatrix} \in S.$$

Thus a and b gives the MOD natural neutrosophic dual number mixed zero divisor.

$$a = \begin{bmatrix} 4 & 9 & 4 \\ 0 & 1 & 4 \\ 9 & 4 & 1 \\ 0 & 1 & 4 \\ 9 & 4 & 4 \end{bmatrix} \in S \text{ is such that } a \times_n a = \begin{bmatrix} 4 & 9 & 4 \\ 0 & 1 & 4 \\ 9 & 4 & 1 \\ 0 & 1 & 4 \\ 9 & 4 & 4 \end{bmatrix} = a.$$

Thus a is an idempotent of S.

$$\text{Let } b = \begin{bmatrix} I_0^g & I_4^g & I_9^g \\ I_4^g & I_0^g & I_4^g \\ I_0^g & I_9^g & I_0^g \\ I_9^g & I_4^g & I_4^g \\ I_0^g & I_0^g & I_0^g \end{bmatrix} \in S.$$

Clearly b $\times_n$ b = b so b is a MOD natural neutrosophic dual number matrix idempotent of S.

We have also mixed MOD natural neutrosophic dual number idempotents given by

$$x = \begin{bmatrix} 4 & 9 & I_0^g \\ 0 & I_4^g & 0 \\ 9 & 4 & 0 \\ 1 & I_9^g & 0 \\ 0 & 1 & I_9^g \end{bmatrix} \in S \text{ is such that } x \times_n x = x \text{ so x is a MOD}$$

natural neutrosophic dual number mixed matrix idempotent of S.

$$\text{Let } y = \begin{bmatrix} 0 & 1 & I_0^g \\ 0 & I_0^g & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \\ I_0^g & 0 & 1 \end{bmatrix} \in S \text{ is such that it is a MOD natural}$$

neutrosophic mixed idempotent matrix of x. But we call it as trivial.

We have several trivial idempotent in S. But S has also MOD natural neutrosophic mixed idempotent matrices which are not trivial.

Example 1.44: Let
$W = \{(a_1, a_2, a_3, a_4) \mid a_i \in \langle Z_7 \cup g\rangle_I, 1 \leq i \leq 4, +, \times\}$ be the MOD natural neutrosophic dual number matrix semiring.

(1) = (1 1 1 1) serves as the multiplicative identity.

(0) = (0 0 0 0) is the natural zero.

$(I_0^g) = (I_0^g\ \ I_0^g\ \ I_0^g\ \ I_0^g) \in S$ is the MOD natural neutrosophic dual number matrix zero.

$P_1 = (I_0^g\ 0\ 0\ 0)$, $P_2 = (I_0^g\ \ I_0^g\ 0\ 0)$, $P_3 = (I_0^g\ 0\ I_0^g\ 0)$,

$P_4 = (0\ 0\ 0\ I_0^g)$, $P_5 = (0\ I_0^g\ I_0^g\ 0)$ are $P_6 = (I_0^g\ I_0^g\ I_0^g\ 0)$ are some of the MOD natural neutrosophic matrix mixed zeros of W.

Let x = (g, g, g, 3g) $\in$ W.

Clearly x $\times$ x = (0 0 0 0) so x is a nilpotent element of W.

Let y = $(I_g^g, I_{3g}^g, I_{7g}^g, I_{8g}^g) \in$ W, clearly

y $\times$ y = $(I_0^g, I_0^g\ I_0^g\ I_0^g)$ is a MOD natural neutrosophic nilpotent matrix of order two.

Let x = $(5g\ I_{7g}^g\ 6g\ I_g^g) \in$ W, we have x $\times$ x = $(0\ I_0^g\ 0\ I_0^g)$ is a MOD natural neutrosophic mixed nilpotent matrix of order two in W.

W has several MOD natural neutrosophic matrix nilpotent and usual nilpotents and MOD natural neutrosophic mixed matrix nilpotents for the reader to analyse about MOD natural neutrosophic matrix nilpotents of order greater than two.

Let x = $(7\ I_{3g}^g\ 4\ I_{2g}^g)$ and y = $(0\ I_g^g\ 0\ I_{5g}^g) \in$ W.

We see x $\times$ y = $(0\ I_0^g\ 0\ I_0^g)$ is a mixed zero divisors of MOD natural neutrosophic matrix in W.

We have several such MOD natural neutrosophic dual number matrix zero divisors of all the three types.

We now proceed onto describe MOD natural neutrosophic special dual like number semiring.

Just before we wish to keep on record only in case MOD natural neutrosophic dual number we get subsemirings which are MOD natural neutrosophic dual number zero square subsemirings of all the three types.

If A = $\{(a_1g,\ a_2g,\ a_3g,\ a_4g) \mid a_i \in Z_9, 1 \leq i \leq 4, +, \times\}$ then A is a zero square matrix subsemiring of W.

If B = $\{(I^g_{ag}\ \ I^g_{bg}\ \ I^g_{cg}\ \ I^g_{dg}) \mid a, b, c, d \in Z_9, +, \times\} \subseteq W$ is a MOD natural neutrosophic dual number matrix zero square subsemiring of W.

If S = $(a, b, c, d) \mid a, b, c, d \in Z_9g$ or
$a, b, c, d \in \{I^g_0, I^g_g, I^g_{2g}, \ldots, I^g_{8g}\} +, \times\} \subseteq W$ is also a special mixed MOD natural neutrosophic dual number matrix subsemiring which is a mixed zero square subsemiring of W.

Now this property is very different from other MOD natural neutrosophic matrix semirings.

Example 1.45: Let

$$B = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \end{bmatrix} \middle| a_i \in \langle Z_9 \cup h\rangle_I; 1 \leq i \leq 7, +, \times_n \right\}$$

be the MOD natural neutrosophic special dual like number matrix semiring.

B is a finite semiring.

B has $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ = (1) to be the unit for x (1) = (1)x for all x $\in$ B.

$$(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ is the zero of B.}$$

$(I_0^h) = \begin{bmatrix} I_0^h \\ I_0^h \\ I_0^h \\ I_0^h \\ I_0^h \\ I_0^h \\ I_0^h \end{bmatrix}$ is the MOD natural neutrosophic special dual like number matrix zero.

Apart from this we have MOD natural neutrosophic special dual like number matrix mixed zeros given by

$$\begin{bmatrix} I_0^h \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ I_0^h \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ I_0^h \\ I_0^h \\ I_0^h \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ I_0^h \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} I_0^h \\ I_0^h \\ I_0^h \\ 0 \\ 0 \\ 0 \\ I_0^h \end{bmatrix}$$

and so on.

This has MOD natural neutrosophic special dual like number matrix idempotents, nilpotents and zero divisors.

We will illustrate these situations by some examples.

$$\text{Let } x = \begin{bmatrix} 3 \\ 6 \\ I_0^h \\ 3+6h \\ 6+3h \\ I_3^h \\ I_{3h}^h \end{bmatrix} \in B.$$

$$\text{We see } x \times_n x = \begin{bmatrix} 0 \\ 0 \\ I_0^h \\ 0 \\ 0 \\ I_0^h \\ I_0^h \end{bmatrix}$$

is the MOD natural neutrosophic special dual like number matrix mixed nilpotent element of order two in B.

Let x = $\begin{bmatrix} I_0^h \\ I_h^h \\ h \\ I_h^h + h \\ 1 \\ I_h^h + 1 \\ 0 \end{bmatrix}$ $\in$ B is such that

x $\times_n$ x = $\begin{bmatrix} I_0^h \\ I_h^h \\ h \\ I_h^h + h \\ 1 \\ I_h^h + 1 \\ 0 \end{bmatrix}$ = x so x is a mixed MOD natural neutrosophic special dual like number idempotent matrix of B.

We have several idempotents of this type.

Let x = $\begin{bmatrix} I_{6h}^h \\ 4 \\ 6 \\ 5 \\ I_3^h \\ I_0^h \\ I_{3+3h}^h \end{bmatrix}$ and y = $\begin{bmatrix} I_6^h \\ 0 \\ 6 \\ 0 \\ I_{3h}^h \\ I_3^h \\ I_3^h \end{bmatrix}$ be in B.

$$x \times_n y = \begin{bmatrix} I_0^h \\ 0 \\ 0 \\ 0 \\ I_0^h \\ I_0^h \\ I_0^h \end{bmatrix}$$ is a MOD natural neutrosophic special dual like number matrix mixed zero divisor of B.

Infact we have several such zero divisors.

Example 1.46: Let

$$G = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \middle| \; a_i \in \langle Z_4 \cup h \rangle_I;\ 1 \leq i \leq 4, +, \times \right\}$$

be the MOD natural neutrosophic special dual like number matrix semiring.

Clearly G is a non commutative ring with $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_{2\times 2}$ the identity element of G.

We see G has MOD right zero divisors which are not MOD left zero divisors.

Consider $(0) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is the usual zero and

$(I_0^h) = \begin{bmatrix} I_0^h & I_0^h \\ I_0^h & I_0^h \end{bmatrix}$ is the MOD natural neutrosophic special dual like number matrix zero of G.

$$x = \begin{bmatrix} I_{2h}^{h} & I_{2}^{h} \\ 2 & 3 \end{bmatrix} \text{ and } (I_0^h) = \begin{bmatrix} I_0^h & I_0^h \\ I_0^h & I_0^h \end{bmatrix} \in G.$$

$$x \times (I_0^h) = \begin{bmatrix} I_{2h}^{h} & I_{2}^{h} \\ 2 & 3 \end{bmatrix} \times \begin{bmatrix} I_0^h & I_0^h \\ I_0^h & I_0^h \end{bmatrix}$$

$$= \begin{bmatrix} I_0^h & I_0^h \\ I_0^h & I_0^h \end{bmatrix} = (I_0^h).$$

$$(I_0^h) \times x = \begin{bmatrix} I_0^h & I_0^h \\ I_0^h & I_0^h \end{bmatrix} \times \begin{bmatrix} I_{2h}^{h} & I_{2}^{h} \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} I_0^h & I_0^h \\ I_0^h & I_0^h \end{bmatrix}.$$

Let $x = \begin{bmatrix} 3+I_2^h & 2 \\ 1 & I_{2h}^h \end{bmatrix}$ and $y = \begin{bmatrix} 2+I_2^h & I_0^h \\ 2 & I_2^h \end{bmatrix} \in G.$

$$x \times y = \begin{bmatrix} 3+I_2^h & 2 \\ 1 & I_{2h}^h \end{bmatrix} \times \begin{bmatrix} 2+I_2^h & I_0^h \\ 2 & I_2^h \end{bmatrix}$$

$$= \begin{bmatrix} 2+I_2^h+I_0^h & I_0^h+I_2^h \\ 2+I_{2h}^h+I_2^h & I_0^h \end{bmatrix} \quad \text{---} \quad \text{I}$$

Now we find

$$y \times x = \begin{bmatrix} 2+I_2^h & I_0^h \\ 2 & I_2^h \end{bmatrix} \times \begin{bmatrix} 3+I_2^h & 2 \\ 1 & I_{2h}^h \end{bmatrix}$$

$$= \begin{bmatrix} 2+I_2^h+I_0^h & I_0^h+I_2^h \\ 2+I_2^h & I_0^h \end{bmatrix} \quad \text{---} \quad \text{II}$$

Clearly I and II are distinct so we see G is a non commutative semiring of finite order.

Finding MOD natural neutrosophic special dual like number matrix zero divisors, idempotents and nilpotents happens to be a difficult problem.

For we may have left zero divisors which are not right zero divisors and vice versa.

Let x = $\begin{bmatrix} 2 & 2+I_0^h \\ 2+I_0^h & 2 \end{bmatrix} \in G$.

$$x \times x = \begin{bmatrix} 2 & 2+I_0^h \\ 2+I_0^h & 2 \end{bmatrix} \times \begin{bmatrix} 2 & 2+I_0^h \\ 2+I_0^h & 2 \end{bmatrix}$$

$= \begin{bmatrix} I_0^h & I_0^h \\ I_0^h & I_0^h \end{bmatrix}$ is a MOD natural neutrosophic

special dual like number matrix nilpotent of order two.

Now we show if in the above example '×' usual product is replaced by natural product $\times_n$ then G is commutative and

$$x \times_n x = \begin{bmatrix} 2 & 2+I_0^h \\ 2+I_0^h & 2 \end{bmatrix} \times_n \begin{bmatrix} 2 & 2+I_0^h \\ 2+I_0^h & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & I_0^h \\ I_0^h & 0 \end{bmatrix}$$

is again a mixed zero so a nilpotent of matrix of order two.

However we see $x \times_n x \neq x \times x$.

Let x = $\begin{bmatrix} 3+I_2^h & 2 \\ 1+I_0^h & I_2^h+I_h^h \end{bmatrix}$ and y = $\begin{bmatrix} I_h^h & 2+I_2^h \\ I_0^h+I_h^h & I_2^h+2 \end{bmatrix} \in G$.

We find x × y and x $\times_n$ y and show the product × and $\times_n$ are distinct.

$$x \times y = \begin{bmatrix} 3+I_2^h & 2 \\ 1+I_0^h & I_2^h+I_h^h \end{bmatrix} \times \begin{bmatrix} I_h^h & 2+I_2^h \\ I_0^h+I_h^h & I_2^h+2 \end{bmatrix}$$

$$= \begin{bmatrix} I_{2h}^h+I_h^h+I_0^h & 2+I_2^h+I_0^h \\ I_h^h+I_0^h+I_{2h}^h & 2+I_0^h+I_2^h+I_h^h+I_{2h}^h \end{bmatrix} \quad \text{--- I}$$

Consider

$$x \times_n y = \begin{bmatrix} 3+I_2^h & 2 \\ 1+I_0^h & I_2^h+I_h^h \end{bmatrix} \times_n \begin{bmatrix} I_h^h & 2+I_2^h \\ I_0^h+I_h^h & I_2^h+2 \end{bmatrix}$$

$$= \begin{bmatrix} I_h^h+I_2^h & I_2^h \\ I_0^h+I_h^h & I_2^h+I_0^h+I_h^h \end{bmatrix} \quad \text{--- II}$$

Clearly I and II are distinct so x × y ≠ x $\times_n$ y in general.

Further y × x ≠ x × y in general we find

$$y \times x = \begin{bmatrix} I_h^h & 2+I_2^h \\ I_0^h+I_h^h & I_2^h+2 \end{bmatrix} \times \begin{bmatrix} 3+I_2^h & 2 \\ 1+I_0^h & I_2^h+I_h^h \end{bmatrix}$$

$$= \begin{bmatrix} I_h^h+I_{2h}^h+2+I_0^h+I_2^h & I_h^h+I_0^h+I_{2h}^h \\ I_0^h+I_{2h}^h+I_h^h+2+I_2^h & I_0^h+I_h^h+I_2^h \end{bmatrix}.$$

x × y ≠ y × x in general for x, y ∈ G.

So if x = $\begin{bmatrix} 2 & I_0^h \\ I_2^h & I_{2h}^h \end{bmatrix}$ $\in$ G we see

$$x \times_n x = \begin{bmatrix} 2 & I_0^h \\ I_2^h & I_{2h}^h \end{bmatrix} \times_n \begin{bmatrix} 2 & I_0^h \\ I_2^h & I_{2h}^h \end{bmatrix}$$

$$= \begin{bmatrix} 0 & I_0^h \\ I_0^h & I_0^h \end{bmatrix}$$ is a MOD natural neutrosophic

special dual like number nilpotent of order two.

$$x \times x = \begin{bmatrix} 2 & I_0^h \\ I_2^h & I_{2h}^h \end{bmatrix} \times \begin{bmatrix} 2 & I_0^h \\ I_2^h & I_{2h}^h \end{bmatrix}$$

$$= \begin{bmatrix} I_0^h & I_0^h \\ I_2^h + I_0^h & I_0^h \end{bmatrix}$$; clearly x is not a

natural neutrosophic special dual like number which is nilpotent.

Hence we find the two operations $\times$ and $\times_n$ act in a different way.

So in case of MOD natural neutrosophic special dual like number square matrices G we can either define $\times$ the usual product or the $\times_n$ natural product and under natural product G will be a commutative semiring of finite order where as in case of usual product G be a non commutative semiring of finite order.

So this concept will be used in case of finding MOD natural neutrosophic linear algebras which are non commutative.

This sort of working is true in case when Z_n^I or $C^I(Z_n)$ or $\langle Z_n \cup I\rangle_I$ or $\langle Z_n \cup g\rangle_I$ or $\langle Z_n \cup k\rangle_I$ is used.

We now give one or two examples of MOD natural neutrosophic special quasi dual number matrix semiring and discuss their associated properties.

Example 1.47: Let

$$H = \left\{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} \middle|\, a_i \in \langle Z_6 \cup k\rangle_I;\ 1 \le i \le 5, +, \times_n\right\}$$

be the MOD natural neutrosophic special quasi dual number matrix semiring.

$$(1) = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \text{ is the unit matrix of H as}$$

$x \times_n (1) = (1) \times_n x = x$; for all $x \in H$.

$$(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \in \text{H is the zero matrix of H.}$$

$$(I_0^k) = \begin{bmatrix} I_0^k \\ I_0^k \\ I_0^k \\ I_0^k \\ I_0^k \end{bmatrix} \in H \text{ is the pure MOD natural neutrosophic}$$

special quasi dual number zero matrix of H.

$$p_1 = \begin{bmatrix} I_0^k \\ 0 \\ I_0^k \\ 0 \\ 0 \end{bmatrix}, p_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ I_0^k \end{bmatrix}, p_3 = \begin{bmatrix} 0 \\ 0 \\ I_0^k \\ 0 \\ 0 \end{bmatrix}, p_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ I_0^k \end{bmatrix}, p_5 = \begin{bmatrix} 0 \\ I_0^k \\ I_0^k \\ 0 \\ I_0^k \end{bmatrix} \text{ and}$$

so on are the MOD natural neutrosophic special quasi dual number matrix mixed zeros of H.

Clearly $o(H) < \infty$ and H is a commutative semiring.

This MOD natural neutrosphic special quasi dual number matrix semirings has zero divisors, idempotents but finding nilpotents happens to be a difficult task.

$$\text{Let } x = \begin{bmatrix} I_4^k \\ 0 \\ 3 \\ 4 \\ I_{3k}^k \end{bmatrix} \in H \text{ is such that}$$

$x \times_n x = x$, so x is a MOD natural neutrosophic special quasi dual number idempotent matrix of H.

Consider y = $\begin{bmatrix} 4 \\ 3 \\ 0 \\ 1 \\ 4 \end{bmatrix}$ ∈ H is such that y $\times_n$ y = y so y is again a idempotent matrix of H.

We have several idempotents.

$$\text{Let } x = \begin{bmatrix} I_{2k}^k \\ I_{3k}^k \\ I_{4k+4}^k \\ I_{2k+4}^k \\ I_{4k}^k \end{bmatrix} \text{ and } y = \begin{bmatrix} I_{3k}^k \\ I_4^k \\ I_3^k \\ I_{3k}^k \\ I_3^k \end{bmatrix} \in H.$$

It is easily verified x $\times_n$ y = $\begin{bmatrix} I_0^k \\ I_0^k \\ I_0^k \\ I_0^k \\ I_0^k \end{bmatrix}$ so x, y is a MOD pure natural neutrosophic special quasi dual number matrix zero divisor of H.

$$\text{Let } x = \begin{bmatrix} 2 \\ 2 \\ 3 \\ 3 \\ 4 \end{bmatrix} \text{ and } y = \begin{bmatrix} 0 \\ 3 \\ 4 \\ 2 \\ 3 \end{bmatrix} \in H.$$

We see $x \times_n y = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, thus x, y is a MOD natural neutrosophic special quasi dual number real matrix zero divisor of H.

Let $x = \begin{bmatrix} I_{3k}^k \\ 3k+3 \\ 4+I_4^k \\ I_2^k \\ I_3^k \end{bmatrix}$ and $y = \begin{bmatrix} I_2^k \\ 2 \\ I_0^k \\ I_3^k \\ I_{4k}^k \end{bmatrix} \in H$, it is easily verified.

$x \times_n y = \begin{bmatrix} I_0^k \\ 0 \\ I_0^k \\ I_0^k \\ I_0^k \end{bmatrix}$, so x, y is the MOD natural neutrosophic special quasi dual number matrix mixed zero divisor of H.

Let $x = \begin{bmatrix} I_4^k \\ 3 \\ 4 \\ 2 \\ I_2^k \end{bmatrix}$ and $y = \begin{bmatrix} I_{3k}^k \\ 2 \\ 3 \\ 3 \\ I_3^k \end{bmatrix} \in H$, we see $x \times_n y = \begin{bmatrix} I_0^k \\ 0 \\ 0 \\ 0 \\ I_0^k \end{bmatrix}$; thus x, y is a MOD natural neutrosophic special quasi dual number matrix mixed zero divisor of H.

Thus H has all types of zero divisor matrices.

We give get another example.

Example 1.48: Let

$$P = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \middle| \ a_i \in \langle Z_8 \cup k\rangle_I;\ 1 \leq i \leq 9, +, \times \right\}$$

be the MOD natural neutrosophic special quasi dual number matrix semiring of finite order.

Clearly P is a non commutative semiring.

$$(0) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \in P \text{ is the zero of P.}$$

$$I_{3\times3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ is the unit element of P.}$$

$$(I_0^k) = \begin{bmatrix} I_0^k & I_0^k & I_0^k \\ I_0^k & I_0^k & I_0^k \\ I_0^k & I_0^k & I_0^k \end{bmatrix} \in P \text{ is the pure MOD natural}$$

neutrosophic special quasi dual number zero divisor matrix of P.

$$x_1 = \begin{bmatrix} 0 & I_0^k & 0 \\ I_0^k & 0 & I_0^k \\ 0 & I_0^k & I_0^k \end{bmatrix}$$ of P is a MOD natural neutrosophic

special quasi dual number mixed zero divisor of P.

Let $a = \begin{bmatrix} 4+2k & 5+3k & 2k \\ I_2^k & 4+2k & I_{4+6k}^k \\ I_2^k + 3k & 4+I_{2k+6}^k & I_{4k}^k \end{bmatrix} \in P$.

Consider

$$x_1 \times a = \begin{bmatrix} 0 & I_0^k & 0 \\ I_0^k & 0 & I_0^k \\ 0 & I_0^k & I_0^k \end{bmatrix} \times \begin{bmatrix} 4+2k & 5+3k & 2k \\ I_2^k & 4+2k & I_{4+6k}^k \\ I_2^k + 3k & 4+I_{2k+6}^k & I_{4k}^k \end{bmatrix}$$

$$= \begin{bmatrix} I_0^k & I_0^k & I_0^k \\ I_0^k & I_0^k & I_0^k \\ I_0^k & I_0^k & I_0^k \end{bmatrix}$$

$$= (I_0^k).$$

Thus a MOD mixed zero divisor can lead to give a pure MOD natural neutrosophic special quasi dual like number matrix with $a \in P$ which is not a mixed zero and a x_1 which is a mixed MOD natural neutrosophic special quasi dual like number matrix zero.

This is a very strange property.

Further if $\times_n$ is used in the place of $\times$ in P.

We see

$$a \times_n x = \begin{bmatrix} 4+2k & 5+3k & 2k \\ I_2^k & 4+2k & I_{4+6k}^k \\ I_2^k+3k & 4+I_{2k+6}^k & I_{4k}^k \end{bmatrix} \times_n \begin{bmatrix} 0 & I_0^k & 0 \\ I_0^k & 0 & I_0^k \\ 0 & I_0^k & I_0^k \end{bmatrix}$$

$$= \begin{bmatrix} 0 & I_0^k & 0 \\ I_0^k & 0 & I_0^k \\ I_2^k & I_0^k & I_0^k \end{bmatrix} \neq x.$$

So even in this case we cannot in general claim if $x \in P$ is a mixed MOD natural neutrosophic special quasi dual number matrix of P then for every $a \in P$.

$a \times_n x \neq x$ in general.

Thus mixed MOD natural neutrosophic special quasi dual number matrix in general does not behave like zeros or pure MOD natural neutrosophic zeros.

So if x is a MOD mixed natural neutrosophic matrix zero that for all $a \in P$.

$a \times_n x \neq x$ and $a \times x \neq x$.

Inview of this we have the following theorem.

THEOREM 1.6: *Let $B = \{(a_{ij})_{m\times m} = A \mid a_{ij} \in Z_n^I$ or $\langle Z_n \cup g\rangle_I$ or $\langle Z_n \cup h\rangle_I$ or $\langle Z_n \cup I\rangle_I$ or $\langle Z_n \cup k\rangle_I$ or $C^I(Z_n)$; $1 \leq i, j \leq m$ are square matrices $+, \times_n$ (or $\times$)$\}$ be the MOD natural neutrosophic matrix semiring commutative (or otherwise);*

(i) For $(0) = (0)_{m\times m}$, $(I_0^t) = (I_0^t)_{m\times m}$ in B the real zero matrix and pure MOD natural neutrosophic matrix zero respectively then we have
$x \times (0) = (0) = (0) \times (x)$

$x \times_n (0) = (0)$ and

$x \times (I_0^t) = (I_0^t) = (I_0^t) \times x$ and $x \times_n (I_0^t) = (I_0^t)$ for all $x \in B$; ($t = n$ or g or k or h or I or c).

(ii) If a is a mixed MOD natural neutrosophic matrix of B then in general $a \times x \neq a$ and $x \times a \neq a \times a$ and $x \times_n a \neq a$ for $x \in B$.

The proof is direct and hence left as an exercise to the reader.

Now having seen MOD natural neutrosphic matrix semirings we proceed on to describe MOD natural neutrosphic polynomial semiring both of finite order as well as of infinite order.

Example 1.49: Let

$$Z_{10}^I[x] = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in Z_{10}^I; +, \times\right\}$$

be the MOD natural neutrosophic polynomial semiring.

Clearly $o(Z_{10}^I[x]) = \infty$.

Example 1.50: Let

$$S = \langle Z_7 \cup I\rangle_I[x] = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in \langle Z_n \cup I\rangle_I; +, \times\right\}$$

be the MOD natural neutrosophic-neutrosophic coefficient polynomial semiring.

Clearly $o(S) = \infty$.
S is commutative.

Example 1.51: Let

$$P = C^I(Z_{12})[x] = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in C^I(Z_{12}); +, \times \right\}$$

be the MOD natural neutrosophic finite complex coefficient polynomial semiring. P is of infinite order.

Example 1.52: Let

$$W = \langle Z_n \cup g\rangle_I [x]\} = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in \langle Z_n \cup g\rangle_I; +, \times \right\}$$

be the MOD natural neutrosophic dual number coefficient polynomial semiring of infinite order.

W is a commutative semiring.

Example 1.53: Let

$$B = \{\langle Z_n \cup h\rangle_I [x]\} = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in \langle Z_n \cup h\rangle_I; +, \times \right\}$$

be the MOD natural neutrosophic special dual like number coefficient polynomial semiring of infinite order.

Finally we give an example of a MOD natural neutrosophic special quasi dual number coefficient polynomial semiring.

Example 1.54: Let

$$G = \{\langle Z_{12} \cup k\rangle_I [x]\} = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in \langle Z_{12} \cup k\rangle_I; +, \times \right\}$$

be the MOD natural neutrosophic special quasi dual number coefficient polynomial semiring of infinite order.

Before we start to analyse the properties of these semirings of infinite order we give examples of MOD natural neutrosophic polynomial coefficient semiring of finite order in the following.

Example 1.55: Let

$$S = Z_{16}^{I}[x]_9 = \left\{ \sum_{i=0}^{9} a_i x^i \,\middle|\, a_i \in Z_{16}^{I};\ 0 \le i \le 9,\ x^{10} = 1, +, \times \right\}$$

be the MOD natural neutrosophic coefficient polynomial semiring of finite order.

Example 1.56: Let

$$A = \{\langle Z_{18} \cup I\rangle_I [x]_7\} = \left\{ \sum_{i=0}^{7} a_i x^i \,\middle|\, a_i \in \langle Z_{18} \cup I\rangle_I;\ x^8 = 1, +, \times \right\}$$

be the MOD natural neutrosophic-neutrosophic coefficient polynomial semiring of finite order.

Example 1.57: Let

$$B = \{\langle Z_{11} \cup g\rangle_I [x]_5\} = \left\{ \sum_{i=0}^{5} a_i x^i \,\middle|\, a_i \in \langle Z_{11} \cup g\rangle_I;\ x^6 = 1, +, \times \right\}$$

be the MOD natural neutrosophic dual number polynomial coefficient semiring of finite order.

Example 1.58: Let

$$D = C^{I}(Z_6)[x]_{10} = \left\{ \sum_{i=0}^{10} a_i x^i \,\middle|\, a_i \in C^{I}(Z_6);\ x^{11} = 1, +, \times \right\}$$

be the MOD natural neutrosophic finite complex coefficient polynomial semiring of finite order.

Example 1.59: Let

$$B = \{\langle Z_{19} \cup k\rangle_I [x]_6\} = \left\{\sum_{i=0}^{6} a_i x^i \,\middle|\, a_i \in \langle Z_{19} \cup g\rangle_I;\ x^7 = 1, +, \times\right\}$$

be the MOD natural neutrosophic special quasi dual number coefficient polynomial semirng of finite order.

Now we will define zero and MOD natural neutrosophic zero in semirings of both finite and infinite order.

I_0^n, I_0^C, I_0^h, I_0^k, I_0^I and I_0^g are the MOD natural neutrosophic zeros of polynomials semirings both of finite and infinite order.

We will describe a few of the properties associated with them by some examples.

Example 1.60: Let

$$S = Z_{10}^I [x] = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in Z_{10}^I, +, \times\right\}$$

be the MOD natural neutrosophic polynomial semiring.

S has zero divisors but has no MOD natural neutrosophic zero divisors.

Let $p(x) = 5x^8 + 5x^3 + 5$ and
$q(x) = 2x^7 + 4x^5 + 6x^3 + 8x + 4 \in S$.

Clearly $p(x) \times q(x) = 0$, so this S has zero divisors.

Let $p(x) = I_0^{10} x^3 + I_2^{10} + I_4^{10}$ and
$q(x) = I_5^{10} x + I_0^{10} \in S$.

Clearly $p(x) \times q(x) = (I_0^{10} x^3 + I_2^{10} x + I_4^{10}) \times (I_5^{10} x + I_0^{10})$

$$= I_0^{10} x^4 + I_0^{10} x^2 + I_0^{10} x + I_0^{10} x^3 + I_0^{10} x + I_0^{10}$$
$$= I_0^{10} (x^4 + x^3 + x^2 + x + 1)$$

$\neq I_0^{10}$ as $I_0^{10} x \neq I_0^{10}$ it is a neutrosophic element so $I_0^{10} x^n$ is never a MOD natural neturosophic zero though the coefficient of x^n is a MOD natural neutrosophic zero.

Hence finding MOD natural neutrosophic zero divisors is not possible.

Next we cannot have MOD natural neutrosophic idempotents or usual idempotents.

Since Z_{10}^I has no nilpotents we cannot study about it in this example.

We call $5 \times 5 = 5 \pmod{10}$ and $6 \times 6 = 6 \pmod{10}$ as trivial idempotents of S.

Next we present another example.

Example 1.61: Let

$$B = Z_{16}^I [x] = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in Z_{16}^I ; +, \times \right\}$$

be the MOD natural neutrosophic coefficient polynomial semiring of infinite order.

Let $p(x) = 4x^3 + 8x + 4$ and $q(x) = 4x + 8 \in B$.

Clearly $p(x) . q(x) = 0$.

Let $r(x) = 2x^3 + 4x^2 + 8x + 4 \in B$.

$$r(x) \times r(x) = 4x^6$$

$$r(x) \times r(x) \times r(x) \times r(x) = 0.$$

Thus r(x) is a nilpotent polynomial of order four.

However we do not have MOD natural neutrosophic nilpotent or zero divisor in B.

Infact B has polynomial subsemirings which are not MOD natural neutrosophic given by $Z_{16}[x]$.

P[x] where P = {0, 2, 4, 6, 8, 10, 12, 14}.

Both these polynomial subsemirings are infact rings of infinite order of B. Both of them are not ideals of B.

Example 1.62: Let

$$S = \{C^I(Z_7) = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in C^I(Z_7); +, \times\right\}$$

be the MOD natural neutrosophic finite complex number coefficient polynomial semiring. S is of infinite order.

Finding zero divisors and nilpotents is a difficult task.

Example 1.63: Let

$$P = \{C^I(Z_{12})[x]\} = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in C^I(Z_{12}); +, \times\right\}$$

be the MOD natural neutrosophic finite complex number coefficient polynomial semiring.

This has nilpotents and zero divisors, however P has no MOD natural neutrosophic zero divisors or nilpotents.

$p(x) = 6x^3 + 6 \in S$ is such that
$p(x) \times p(x) = 0$ so it p(x) is a nilpotent in order two.

Let $p(x) = 6x^3 + 3x^2 + 9$ and

$q(x) = 8x^2 + 4 \in S$; clearly $p(x) \times q(x) = 0$ is the usual zero divisor and not a MOD natural neutrosophic zero divisor.

Let $p(x) = (6 + 6i_F)x^3 + 6 \in S$, $p(x) \times p(x) = 0$.

Let $p(x) = 6i_Fx^2 + 6x + (6 + 6i_F)$ and
$q(x) = 2x_8 + (4 + 8i_F)x^4 + (2 + 4i_F) \in S$.

We have $p(x) \times q(x) = 0$ is the usual zero divisor of S and not the MOD natural neutrosophic zero divisor.

Let $p(x) = (6 + 6i_F)x^3 + 6 \in S$, $p(x) \times p(x) = 0$.
Let $p(x) = 6i_Fx^2 + 6x + (6 + 6i_F)$ and
$q(x) = 2x^8 + (4 + 8i_F)x^4 + (2+4i_F) \in S$.

We have $p(x) \times q(x) = 0$ is the usual zero divisor of S and not the MOD natural neutrosophic zero divisor.

Consider the MOD natural neutrosophic - neutrosophic coefficient polynomial semiring given by the following examples.

Example 1.64: Let

$$B = \{\langle Z_{15} \cup I\rangle_I [x]\} = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in \langle Z_{15} \cup I\rangle_I; +, \times \right\}$$

be the MOD natural neutrosophic - neutrosophic coefficient polynomial semiring of infinite order.

Clearly B has zero divisors finding nilpotents happens to be a challenging one.

However B has no MOD natural neutrosophic-neutrosophic zero divisors or nilpotents.

We have the following theorem in view of all these properties.

THEOREM 1.7: *Let*

$$B = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in Z_n^I \text{ or } C^I(Z_n) \text{ or } \langle Z_n \cup I\rangle_I; +, \times \right\}$$

be the MOD natural neutrosophic coefficient polynomial semiring.

(i) B is a polynomial semiring of infinite order.

(ii) B has no MOD natural neutrosophic nilpotents or zero divisors or idempotents.

(iii) If n is a composite number, B has zero divisors and nilpotents (nilpotents depending on n).

Proof is direct and hence left as an exercise to the reader.

Example 1.65: Let

$$M = \{\langle Z_9 \cup g\rangle_I [x]\} = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in \langle Z_9 \cup I\rangle_I; +, \times \right\}$$

be the MOD natural neutrosophic dual number coefficient polynomial semiring.

This semiring M has subsemirings which are zero square rings as substructures.

This has lots of zero divisors and nilpotents of order two. However finding MOD nilpotents or zero divisors or idempotents of natural neutrosophic dual numbers is not possible.

Example 1.66: Let

$$V = \{\langle Z_{18} \cup k\rangle_I [x]\} = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in \langle Z_{18} \cup k\rangle_I; +, \times \right\}$$

be the MOD natural neutrosophic special quasi dual number coefficient polynomial semiring.

V has zero divisors and nilpotents, however V has no MOD natural neutrosophic special quasi dual number zero divisors or nilpotents or idempotents.

Example 1.67: Let

$$W = \{\langle Z_{19} \cup h\rangle_I [x]\} = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in \langle Z_{19} \cup h\rangle_I; +, \times \right\}$$

be the MOD natural neutrosophic special dual like number coefficient polynomial semiring.

W is of infinite order.

Finding even zero divisors happens to be a difficult problem as Z_{19} is used.

On similar lines we can describe MOD natural neutrosophic special quasi dual like number coefficient polynomial semirings using $\langle Z_n \cup k\rangle_I$ and construct $\langle Z_n \cup k\rangle_I[x]$ as polynomial semiring of infinite order.

THEOREM 1.7: *Let*

$$G = \left\{ \sum_{i=0}^{\infty} a_i x^i \;\middle|\; a_i \in Z_n^I \text{ or } \langle Z_n \cup g\rangle_I \text{ or } \langle Z_n \cup h\rangle_I \text{ or } \langle Z_n \cup k\rangle_I \text{ or } C^I(Z_n) \text{ or } \langle Z_n \cup I\rangle_I; +, \times \right\}$$

be the MOD *natural neutrosophic coefficient polynomial semiring.*

G has subrings of infinite order.

Proof is direct and hence left as an exercise to the reader.

Finding ideals of G happens to be a challenging problem.

Next we derive and describe some properties associated with MOD natural neutrosophic coefficient polynomial semirings of infinite order.

Example 1.68: Let

$$S = \{ Z_{12}^I [x]_7\} = \left\{ \sum_{i=0}^{6} a_i x^i \;\middle|\; a_i \in Z_{12}^I;\ x^7 = 1,\ +, \times \right\}$$

be the MOD natural neutrosophic coefficient polynomial semiring of finite order.

Finding order of S is a challenging job.

This S has zero divisors and nilpotents of order two.

$Z_{12}[x]_6 \subseteq Z_{12}^I [x]_6$ is a subsemiring of S which is a ring of finite order.

The reader is left with the task of finding ideals.

Let P = $\left\{\sum_{i=0}^{6} a_i x^i \,\middle|\, a_i \in \langle 0, 6, I_0^{12}, I_2^{12}, I_4^{12}, I_6^{12}, I_8^{12}, I_{10}^{12}, I_3^{12}, I_9^{12}\rangle \text{ under } +, \times\right\} \subseteq Z_{12}^I[x]_6$ is a Mod natural neutrosophic coefficient polynomial subsemiring which is also an ideal of finite order.

$p(x) = 6 + 6x^2 \in S$ is such that $p(x) \times p(x) = 0$.

0 is a nilpotent zero divisor.

Take $p(x) = 4x^3 + 8x^2 + 2x + 6$ and
$q(x) = 6x^3 + 6x + 6$ in S; clearly $p(x) \times q(x) = 0$.

S has no idempotents other than 4, $I_0^{12}, I_4^{12}, I_9^{12}$ and their sums, these will be only known as trivial idempotents of S.

Example 1.69: Let

$$S = \{\langle Z_{10} \cup g\rangle_I [x]_4\} = \left\{\sum_{i=0}^{4} a_i x^i \,\middle|\, a_i \in \langle Z_{10} \cup g\rangle_I, +, \times\right\}$$

be the MOD natural neutrosophic dual number coefficient polynomial semiring of finite order.

$Z_{10}[x]_4$ is a subsemiring which is a subring of finite order.

$Z_{10}^I[x]_4$ is a subsemiring which is not a subring of finite order.

$\langle Z_{10} \cup g\rangle[x]$ is again a subsemiring which is a subring of finite order and so on.

None of them are ideals of $\langle Z_{10} \cup g\rangle_I[x]_4$.

Consider $\langle Z_{10}g\rangle_I[x]_4$ is an ideal of $\langle Z_{10} \cup g\rangle_I[x]_4$.

In view of this we have the following theorem.

THEOREM 1.9: *Let $S = \{\langle Z_n \cup g\rangle_I[x]_t;\ 2 \leq t < \infty\} =$*

$$\left\{\sum_{i=0}^{t} a_i x^i \,\middle|\, a_i \in \langle Z_n \cup g\rangle_I,\ x^{t+1}=1,\ +,\ \times\right\}$$

be the MOD natural neutrosophic dual number coefficient polynomial semiring.

(i) S is a finite commutative semiring.
(ii) S contains an ideal I such that $I^2 = 0$.

Proof is direct and hence left as an exercise to the reader.

However it is pertinent to keep on record that the special feature is enjoyed only by MOD natural neutrosophic dual number coefficient polynomial semiring.

Next we provide examples of MOD natural neutrosophic special dual like number coefficient polynomial semirings of finite order.

Example 1.70: Let

$$G = \{\langle Z_{14} \cup h\rangle_I\,[x]_5\} = \left\{\sum_{i=0}^{5} a_i x^i \,\middle|\, a_i \in \langle Z_{14} \cup h\rangle_I,\ x^6 = 1,\ +,\ \times\right\}$$

be the MOD natural neutrosophic special dual like number coefficient semiring of finite order.

This has zero divisors but no MOD natural neutrosophic zero divisors.

$p(x) = 7x^3 + 7x + 7$ and $q(x) = 4x^2 + 2x + 8 \in G$

Clearly $p(x) \times q(x) = 0$.

Consider $Z_{14}[x]_5 \subseteq G$ is a subsemirings of G which is not an ideal of G but a subring.

$\langle Z_{14}h \cup \{ I_t^h, t = 2, 0, 4, 6, 8, 10, 12, 7\}\rangle[x]_5 \subseteq G$ is a subsemiring which is also an ideal of G.

Example 1.71: Let

$$H = \{\langle Z_{12} \cup h\rangle_I [x]_{11}\} = \left\{ \sum_{i=0}^{11} a_i x^i \,\middle|\, a_i \in \langle Z_{12} \cup h\rangle_I,\ x^{12} = 1, +, \times \right\}$$

be the MOD natural neutrosophic special dual like number coefficient polynomial semiring of finite order.

$o(H) < \infty$ and H is a commutative semiring.

We see H is only zero divisors and nilpotents but has no MOD natural neutrosophic special dual like number zero divisors or nilpotents.

Always idempotents are ruled out only trivial idempotents exists even in case of MOD natural neutrosophic ones.

$Z_{12}[x]_{11} \subseteq H$ is a subsemiring which is also a subring of H.

$\langle Z_{12} \cup h\rangle [x]_{11}$ is again a subsemiring which is also subring of H.

$\langle Z_{12}h \cup \{ I_t^h, t = \{0, 2, 4, 6, 8, 10, 3, 9\}\rangle[x]_{11}$ is a subsemiring of H which is also an ideal of H.

Inview of all these we state the following theorem.

THEOREM 1.10: *Let*

$$B = \{\langle Z_n \cup h\rangle_I [x]_m\} = \left\{ \sum_{i=0}^{t} a_i x^i \,\middle|\, a_i \in \langle Z_n \cup h\rangle_I,\ x^{m+1} = 1,\ 2 \le m < \infty, +, \times \right\}$$

be the MOD natural neutrosophic special dual like number coefficient polynomial semiring.

(i) $o(B) < \infty$ and B is a commutative semiring.
(ii) B has subsemirings which are subrings and not ideals of B.
(iii) B has $S = \{\langle Z_n\, h \cup I_t^h \rangle$ where t = 0 or an idempotent or nilpotent or zero divisor of $Z_n\}[x]_m\} \subseteq B$ is a subsemiring of B which is also an ideal of B.

We leave the proof of this theorem as an exercise to the reader.

Next we provide a few examples and related properties of MOD natural neutrosophic special quasi dual like number coefficient polynomial semirings of finite order.

Example 1.72: Let

$$M = \{\langle Z_{10} \cup k\rangle_I\, [x]_8\} = \left\{\sum_{i=0}^{8} a_i x^i \;\middle|\; a_i \in \langle Z_{10} \cup k\rangle_I,\ x^9 = 1, +, \times\right\}$$

be the MOD natural neutrosophic special quasi dual number coefficient polynomial semiring of finite order.

M has only zero divisors but has no MOD natural neutrosophic special quasi dual number zero divisors or nilpotents or idempotents.

However M has trivial natural neutrosophic special quasi dual number zero divisors and idempotents but no nilpotents.

We see M has subsemirings which are not ideals but subrings.

$$P = \left\{\sum_{i=0}^{8} a_i x^i \,\middle|\, a_i \in Z_{10},\ x^9 = 1, +, \times\right\} \subseteq M$$

is a subsemiring of M which is not an ideal of M but a subring of M.

$P_1 = \langle Z_{10} \cup k\rangle_I [x]_8 \subseteq M$ is again a subsemiring of M which is not an ideal but only a subring.

$$\text{Let } P_3 = \left\{\sum_{i=0}^{8} a_i x^i \,\middle|\, a_i \in \{0, 2, 4, 6, 8\},\ x^9 = 1, +, \times\right\}$$

$\subseteq$ M is a subsemiring of M which is not an ideal but only a subring.

***Example 1.73**:* Let

$$W = \{\langle Z_9 \cup k\rangle_I [x]_3\} = \{\left\{\sum_{i=0}^{3} a_i x^i \,\middle|\, a_i \in \langle Z_9 \cup k\rangle_I, x^4 = 1, +, \times\right\}$$

be the MOD natural neutrosophic special quasi dual number coefficient polynomial semiring of finite order.

$$S_1 = \left\{\sum_{i=0}^{3} a_i x^i \,\middle|\, a_i \in Z_9, +, \times\right\} \subseteq W$$

is a subsemirings of W which is not an ideal but a subring.

$$S_2 = \left\{\sum_{i=0}^{3} a_i x^i \,\middle|\, a_i \in \langle Z_9 \cup k\rangle; +, \times\right\} \subseteq W$$

is a subsemiring and not an ideal of W but a subring of W.

Let

$$P_2 = \left\{ \sum_{i=0}^{3} a_i x^i \right| a_i \in \langle\{0, 3, 6\} \cup k\rangle; +, \times\} \subseteq W$$

is once again only a subsemiring and not an ideal of W but is also subring of W.

Let

$$S_3 = \left\{ \sum_{i=0}^{3} a_i x^i \right| a_i \in \langle Z_9 k \cup \{ I_0^k \ I_3^k \ I_6^k \}\rangle; +, \times\} \subseteq W$$

is a subsemiring as well as ideal of W but is not a subring of W.

$p(x) = 3x^3 + 6$ and $q(x) = 3x^2 + 6x + 3 \in W$.

Clearly $p(x) \times q(x) = 0$. W has only classical zero divisors and nilotents of order two as $p(x) \times p(x) = 0$ and $q(x) \times q(x) = 0$, however W has no MOD natural neutrosophic nilpotents are zero divisors W has no usual idempotents or MOD natural neutrosophic idempotents.

In view of all these we have the following theorem.

THEOREM 1.11: *Let*

$$G = \{\langle Z_n \cup k\rangle_I [x]_s\} = \left\{ \sum_{i=0}^{s} a_i x^i \right| a_i \in \langle Z_n \cup k\rangle_I, +, \times, x^{s+1} = 1\}$$

be the MOD natural neutrosophic special quasi dual number coefficient polynomial semiring.

i) $o(G) < \infty$ and is commutative.
ii) G has several subsemirings which are subrings and not ideals.

iii) If n is a composite number, G has more subsemirings which are subrings than when n is a prime.

iv) $P = \left\{ \sum_{i=0}^{s} a_i x^i \,\middle|\, a_i \in \langle Z_n \cup k \rangle \cup \{ I_t^k \,/\, t \text{ is a nilpotent or 0 or zero divisor or an idempotents}\}; +, \times \right\} \subseteq G$ *is a subsemiring of G which is also an ideal of G.*

Proof is direct and hence left as an exercise to the reader. In all cases of MOD natural neutrosophic semirings we do not indulge in taking MOD natural neutrosophic decimal zero divisors and so on.

Next we proceed onto describe pseudo interval semirings of infinite order constructed using I([0, n)) or C^I([0, n)) or $\langle [0, n) \cup I \rangle_I$ and so on by some examples.

For more refer [66].

***Example 1.74**:* Let

$$S = \{{}^I[0, 20), +, \times\}$$

be a MOD natural neutrosophic interval pseudo semiring o(S) = ∞.

For more refer [66].

Once again we keep in record that we do not include MOD natural neutrosophic decimal zero divisors.

Similarly we have the following types of MOD natural neutrosophic interval pseudo semirings.

***Example 1.75**:* Let W = $\{\langle [0, n) \cup I \rangle_I, +, \times\}$ be the MOD natural neutrosophic neutrosophic interval pseudo semiring of infinite order.

W has subsemirings which are not pseudo.

Z_n is a subsemiring which is a ring in W.

$\langle Z_n \cup I\rangle = P \subseteq W$ is also subsemiring which is a subring of finite order.

$\langle Z_n \cup I\rangle_I$ is a MOD natural neutrosophic- neutrosophic subsemiring of W which is not pseudo but of finite order.

$\{[0, n), \times, +\} \subseteq W$ is a pseudo subsemiring which is only a pseudo subring of infinite order.

$P_1 = \{\langle [0, n) \cup I\rangle, +, \times\}$ is again a pseudo subsemiring which is a pseudo subring of W.

Example 1.76: Let $S = \{\langle [0, 9) \cup g\rangle_I, +, \times\}$ be the MOD natural neutrosophic dual number pseudo interval semiring.

S has MOD natural neutrosophic zero divisors as well as zero divisors. S has also MOD natural neutrosophic nilpotents and nilpotents.

Z_9 and $\langle Z_9 \cup g\rangle$ are subsemirings are not pseudo and are of finite order.

$\{\langle [0, 9), +, \times\} \subseteq S$ is a pseudo subsemiring of infinite order which is also a pseudo subring.

None of them are ideals only subsemirings.

$3g \times 6g = 0$, $g \times 3g = 0$ $4g \times g = 0$ $3 \times 3 = 0$ some are nilpotents of order two and some are zero divisors.

$(3 + 5g)\ 3g = 0$ is a zero divisor. $1.007g \times 4.0002g = 0$ is again a zero divisor and so on.

Example 1.77: Let $P = \{\langle [0, 15) \cup h\rangle_I, +, \times\}$ be the MOD natural neutrosophic special dual like number pseudo semiring of infinite order.

This has zero divisors and idempotent however finding nilpotents in this case happens to be challenging problem. P has subsemirings which are subrings of finite order. $Z_{15} \subseteq P$ is a subsemiring which is a subring. $\langle Z_{15} \cup h\rangle$ is a subsemiring which is a subring of finite order.

$L = \langle Z_{15} \cup h\rangle_I$ is a subsemiring of finite order which is only MOD natural neutrosophic subsemiring and is not a subring. However L is not pseudo.

$T = \{[0, 15), +, \times\} \subseteq P$ is a pseudo subsemiring which is also a pseudo subring of infinite order.

This is also not ideal of P_1.

***Example 1.78**:* Let $M = \{\langle[0, 12) \cup k\rangle_I, +, \times\}$ be the MOD natural neutrosophic special quasi dual number pseudo semiring of infinite order.

Clearly $P_1 = \{\langle[0, 12)\ k\rangle_I, +, \times\}$ be the MOD natural neutrosophic special quasi dual number pseudo subsemiring of infinite order which is also a pseudo ideal of M.

Consider $P_2 = \{\langle[0, 12)\rangle_I, +, \times\}$ to be the MOD natural neutrosophic special quasi dual number pseudo subsemiring of M which is also a MOD natural neutrosophic special quasi dual number pseudo subring of M, which is not an ideal of M.

$P_3 = \langle\{[0, 12), +, \times\} \subseteq M$ is again a pseudo subsemiring of M which is also a pseudo subring of M; this is also not an ideal of M.

$Z_{12} \subseteq M$ is a subsemiring and it is also a subring of M of finite order $\langle Z_{12} \cup k\rangle \subseteq M$ is a subsemiring of M which is also subring of M which is of finite order $\langle Z_{12} \cup k\rangle_I \subseteq M$ is a MOD natural neutrosophic special quasi dual number subsemiring which is not a subring.

$Z_{12}^{I} \subseteq$ M is a MOD natural neutrosophic dual number subsemiring which is not a subring of finite order.

We see M hs zero divisors and idempotents.

Example 1.79: Let M = $\{C^{I}([0, 7)), +, \times\}$ be the MOD natural neutrosophic finite complex number pseudo semiring of infinite order.

$Z_7 \subseteq$ M is a finite subsemiring which is also a subring $C(Z_7) \subseteq$ M is a finite subsemiring which is also a finite subring $Z_7^{I} \subseteq$ M is a finite pseudo subsemiring which is not a subring $C^{I}(Z_7)$ is a finite pseudo subsemiring which is not a subring.

P = $\{[0, 7), \times, +\} \subseteq$ M is a pseudo subsemiring which is also a pseudo subring of infinite order.

In view of all these examples.

We have the following theorems.

THEOREM 1.12: *Let S = $\{^{I}[0, n), + \times\}$ be the MOD natural neutrosophic pseudo interval semiring of infinite order.*

i) S has MOD natural neutrosophic interval subsemirings of finite order which are not pseudo but just subrings.
ii) S has MOD natural neutrosophic pseudo subsemirings which are not pseudo and not subrings of finite order.
iii) If n is a non prime S has MOD natural neutrosophic pseudo subsemirings of infinite order which are ideals.

Proof is direct and hence is left as an exercise to reader.

THEOREM 1.13: *Let $S = \{\langle[0, n) \cup I\rangle_I, +, \times\}$ be the MOD natural neutrosophic-neutrosophic pseudo interval semiring of infinite order.*

i) S has finite subsemirings which are subrings and not pseudo.
ii) S has finite subsemiring which are not subrings and which are pseudo
iii) S has infinite order pseudo subsemirings which are also pseudo subrings.
iv) S has an infinite pseudo subsemiring which is a pseudo ideal of infinite order.

Proof is direct and hence left as an exercise to the reader.

THEOREM 1.14: *Let $M = \{\langle[0, n) \cup g\rangle_I, +, \times\}$ be the MOD natural neutrosophic dual number interval pseudo semiring of infinite order.*

i) M has subsemirings of finite order which are subrings and not pseudo.
ii) M has subsemirings of finite order which are not subrings and pseudo.
iii) M has subsemiring of infinite order which are not pseudo as well as ideals.
iv) M has zero square subsemirings of finite order and infinite order.

Proof is left as an exercise to the reader.

However it is pertinent to keep on record that only this MOD natural neutrosophic dual number interval pseudo subsemiring which is a zero square subsemiring of infinite order.

THEOREM 1.15: *Let $S = \{\langle[0, n) \cup h\rangle_I, +, \times\}$ be the MOD natural neutrosophic special dual like number pseudo interval semiring of infinite order.*

i) S has subsemirings which are subrings and not pseudo of finite order.
ii) S has subsemirings which are not subrings of finite order which are pseudo.
iii) S has pseudo subsemirings which are pseudo ideals and is of infinite order.

Proof is direct and hence left as an exercise to the reader.

The above theorem is valid in case of MOD natural neutrosophic special quasi dual number interval pseudo semiring B = $\{\langle [0, n) \cup k \rangle_I, +, \times\}$.

However in case of C = $\{C^I ([0, n))\ +, \times\}$ the same results may not be true as $i_F^2 = (n - 1)$.

Study in this direction is little different however can be derived with appropriate modifications.

Next we proceed onto give examples of MOD natural neutrosophic matrix interval pseudo semiring in the following.

Example 1.80: Let M = $\{(a_1\ a_2\ a_3\ a_4) / a_i \in {}^I[0, 10); 1 \leq i \leq 4, +, \times\}$ be the MOD natural neutrosophic interval matrix pseudo semiring M has zero divisors (1 1 1 1) is the identity element of M. (0 0 0 0) is the zero of M and ($I_0^{[0,10)}, I_0^{[0,10)}, I_0^{[0,10)}, I_0^{[0,10)}$) is the pure MOD natural neutrosophic zero of M.

Reader is left with the task of finding finite subsemirings which are subrings and not pseudo $\{(a, 0, 0, 0) / a \in {}^I[0, 10), + \times\}$ is a MOD natural neutrosophic pseudo interval matrix subsemiring which is also a pseudo ideal of M.

S = $\{(a, 0\ 0\ 0) / a \in [0, 10), + \times\}$ is a pseudo interval matrix subsemiring of M which is not an ideal of M.

Study in this direction is left as an exercise to the reader.

Example 1.81: Let

$$P = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} \middle| \; a_i \in \{\langle [0, 16) \cup g\rangle_I, +, \times_n\} \right\}$$

be the MOD natural neutrosophic dual number pseudo interval semiring, $0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ is the zero of the pseudo semiring P.

$$I_0^{\langle [0,16) \cup g\rangle} = \begin{bmatrix} I_0^{\langle [0,16) \cup g\rangle} \\ I_0^{\langle [0,16) \cup g\rangle} \\ I_0^{\langle [0,16) \cup g\rangle} \\ I_0^{\langle [0,16) \cup g\rangle} \\ I_0^{\langle [0,16) \cup g\rangle} \\ I_0^{\langle [0,16) \cup g\rangle} \end{bmatrix}$$

is the MOD natural neutrosophic dual number zero of the semiring.

$$(1) = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \text{ is the unit element of P.}$$

There are several zero divisors both MOD natural neutrosophic zero divisors and usual zero divisors. For consider

$$x = \begin{bmatrix} I_{4g}^{g} \\ I_{2g}^{g} \\ I_{0}^{g} \\ I_{8g}^{g} \\ I_{6}^{g} \\ I_{2+2g}^{g} \end{bmatrix} \text{ and } y = \begin{bmatrix} I_{g}^{g} \\ I_{5g}^{g} \\ I_{9g}^{g} \\ I_{g}^{g} \\ I_{8}^{g} \\ I_{8g}^{g} \end{bmatrix}.$$

Clearly

$$x \times_n y = \begin{bmatrix} I_{0}^{g} \\ I_{0}^{g} \\ I_{0}^{g} \\ I_{0}^{g} \\ I_{0}^{g} \\ I_{0}^{g} \end{bmatrix}$$

is the MOD natural neutrosophic zero divisor matrix of dual numbers.

$$\text{Consider } x = \begin{bmatrix} I_4^g \\ I_{6g}^g \\ I_{2g}^g \\ I_8^g \\ I_g^g \\ I_0^g \end{bmatrix} \times P,$$

$$\text{Clearly } x \times_n y = \begin{bmatrix} I_0^g \\ I_0^g \\ I_0^g \\ I_0^g \\ I_0^g \\ I_0^g \end{bmatrix}$$

is the MOD natural neutrosophic nilpotent elements of order two.

Thus this P has infinite number of MOD natural neutrosophic dual number matrices of order two.

Find non trivial idempotents or MOD natural neutrosophic dual number idempotents happens to be an impossibility.

We have only trivial idempotents of the form

$$x_1 = \begin{bmatrix} I_0^g \\ 1 \\ 0 \\ I_0^g \\ 0 \\ 1 \end{bmatrix}, x_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ I_0^g \end{bmatrix}, x_3 = \begin{bmatrix} 1 \\ I_0^g \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, x_4 = \begin{bmatrix} I_0^g \\ I_0^g \\ I_0^g \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ and so on.}$$

All these idempotents are built using only the set $\{0, 1, I_0^g\}$.

Next we describe by examples the MOD natural neutrosophic special dual like number matrix pseudo semirings in the following.

Example 1.82: Let

$$W = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \end{bmatrix} \middle| a_i \in \langle [0, 12) \cup h \rangle_I; 1 \leq i \leq 6, +, \times_n \right\}$$

be the MOD natural neutrosophic special dual like number matrix pseudo semiring of infinite order.

This semiring has MOD natural neutrosophic zero divisor, idempotent and nilpotent matrices.

$$\text{Let } x = \begin{bmatrix} I_6^h & I_0^h & I_{6h}^6 \\ I_{6+6h}^h & I_{6h}^h & I_6^h \end{bmatrix} \in W; \; x \times_n x = \begin{bmatrix} I_0^h & I_0^h & I_0^h \\ I_0^h & I_0^h & I_0^h \end{bmatrix}$$

is the MOD natural neutrosophic special dual like number nilpotent matrix of order two in W.

$$\text{Let } x = \begin{bmatrix} I_4^h & I_{3+3h}^h & 6 \\ 2 & 4 & I_{6h}^h \end{bmatrix} \text{ and } y = \begin{bmatrix} I_{3h}^h & I_4^h & 2 \\ 6 & 3 & I_2^h \end{bmatrix} \in W$$

$$x \times_n y = \begin{bmatrix} I_0^h & I_0^h & 0 \\ 0 & 0 & I_0^h \end{bmatrix}$$

is the MOD natural neutrosophic mixed zero divisor of W.

Infact W has several mixed zero.

It is pertinent to record that if (O_m) is mixed MOD natural neutrosophic zero of W then for any x ∈ W; $x \times (O_m) \neq O_m$ in general.

We shall prove this by some examples.

Let $p = \begin{bmatrix} I_0^h & 0 & I_0^h \\ 0 & I_0^h & I_0^h \end{bmatrix}$ and $y = \begin{bmatrix} I_4^h & 4 & I_0^h \\ I_8^h & I_9^h & 3 \end{bmatrix} \in W$

$$y \times_n p = \begin{bmatrix} I_4^h & 4 & I_0^h \\ I_8^h & I_9^h & 3 \end{bmatrix} \times_n \begin{bmatrix} I_0^h & 0 & I_0^h \\ 0 & I_0^h & I_0^h \end{bmatrix} = \begin{bmatrix} I_0^h & 0 & I_0^h \\ I_8^h & I_0^h & I_0^h \end{bmatrix} \neq p.$$

This is not even a MOD natural neutrosophic mixed zero divisor of W.

Let $m = \begin{bmatrix} I_0^h & 0 & 0 \\ 0 & I_0^h & 0 \end{bmatrix}$ and $y = \begin{bmatrix} 4 & I_8^h & I_6^h \\ 8 & I_9^h & I_{3+3h}^h \end{bmatrix} \times W$

$$m \times_n y = \begin{bmatrix} I_0^h & 0 & 0 \\ 0 & I_0^h & 0 \end{bmatrix} \times_n \begin{bmatrix} 4 & I_8^h & I_6^h \\ 8 & I_9^h & I_{3+3h}^h \end{bmatrix}$$

$$= \begin{bmatrix} I_0^h & I_8^h & I_6^h \\ 0 & I_0^h & I_{3+3h}^h \end{bmatrix} \neq (O_m);$$

this is the product of a MOD natural neutrosohic mixed zero divisor with some matrix in W does not result in a mixed MOD natural neutrosophic zero divisor.

However it is pertinent to keep on record that we have MOD natural neutrosophic matrices. x, y ∈ W such that $x \times_n y$ gives a MOD natural neutrosophic mixed zero that is x is a MOD natural neutrosophic mixed zero divisor of y.

$$\text{Let } x = \begin{bmatrix} 9 & I_8^h & I_2^h \\ I_3^h & I_0^h & 3 \end{bmatrix} \quad \text{and } y = \begin{bmatrix} 4 & I_{6h}^h & I_6^h \\ I_{4h+4}^h & I_9^h & 4 \end{bmatrix} \in W.$$

$$\text{Clearly } x \times_n y \quad = \begin{bmatrix} 9 & I_8^h & I_2^h \\ I_3^h & I_0^h & 3 \end{bmatrix} \times_n \begin{bmatrix} 4 & I_{6h}^h & I_6^h \\ I_{4+4h}^h & I_9^h & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & I_0^h & I_0^h \\ I_0^h & I_0^h & 0 \end{bmatrix}$$

is a MOD natural neutrosophic mixed matrix zero divisor of W. Hence the claim.

$$\text{Let } x = \begin{bmatrix} 4 & I_4^h & I_9^h \\ 9 & I_0^h & 1 \end{bmatrix} \in W, \text{ we see } x \times_n x = \begin{bmatrix} 4 & I_4^h & I_9^h \\ 9 & I_0^h & 1 \end{bmatrix} = x$$

so $x \in W$ is a MOD natural neutrosophic mixed special dual like number matrix mixed idempotent of W.

Infact W has several such mixed idempotents.

$$\text{Let } m = \begin{bmatrix} h & I_h^h & I_{4h}^h \\ 4 & 0 & 1 \end{bmatrix} \in W \text{ is such that } m \times_n m = m \text{ is a}$$

MOD natural neutrosophic special dual like number matrix mixed idempotent of W.

Infact we have several such MOD natural neutrosophic mixed matrix idempotents in W.

We have also MOD natural neutrosophic mixed matrix nilpotents in W.

$$\text{Let } x = \begin{bmatrix} 6 & 6h & I_6^h \\ I_{6h}^h & 0 & I_0^h \end{bmatrix} \in W,$$

$$\text{clearly } x \times_n x = \begin{bmatrix} 6 & 6h & I_6^h \\ I_{6h}^h & 0 & I_0^h \end{bmatrix} \times \begin{bmatrix} 6 & 6h & I_6^h \\ I_{6h}^h & 0 & I_0^h \end{bmatrix} = \begin{bmatrix} 0 & 0 & I_0^h \\ I_0^h & 0 & I_0^h \end{bmatrix}$$

so x is a MOD natural neutrosophic special dual like number mixed nilpotent matrix of W.

Infact W has several such mixed MOD nilpotents.

Next we proceed onto describe MOD natural neutrosophic interval special quasi dual number matrix pseudo semiring.

***Example 1.83**:* Let

$$S = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \end{bmatrix} \middle|\, a_i \in \langle [0, 4) \cup k \rangle_I ; 1 \leq i \leq 15, +, \times_n \right\}$$

be the MOD natural neutrosophic dual number semiring of special quasi dual number interval pseudo semiring.

$$P = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \end{bmatrix} \middle|\, a_i \in \langle [0, 4)k \rangle_I ; 1 \leq i \leq 15, +, \times_n \right\} \subseteq S$$

is a MOD natural neutrosophic quasi dual number subsemiring of S.

Clearly P is an semi ideal of S.

S has several numbers of zero divisors as well mixed zero divisors and MOD natural neutrosophic zero divisors and MOD natural neutrosophic mixed zero divisors.

These will be described by some examples.

$$\text{Let } x = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix} \in S \text{ clearly } x \times_n x = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

is nilpotent of order two.

$$\text{Let } x = \begin{bmatrix} I_0^4 & I_2^4 & I_0^4 \\ I_2^4 & I_2^4 & I_2^4 + 2 \\ I_0^4 & I_2^4 & I_0^4 \\ I_0^4 & I_0^4 & I_2^4 \\ I_0^4 & I_2^4 & I_2^4 \end{bmatrix} \in S.$$

$$\text{Clearly } x \times_n x = \begin{bmatrix} I_0^4 & I_0^4 & I_0^4 \\ I_0^4 & I_0^4 & I_0^4 \\ I_0^4 & I_0^4 & I_0^4 \\ I_0^4 & I_0^4 & I_0^4 \\ I_0^4 & I_2^4 & I_0^4 \end{bmatrix}$$

so x is a pure MOD natural neutrosophic special quasi dual number zero divisor or in particular x is the MOD natural neutrosophic special quasi dual nilpotent or order two.

$$\text{Let } x = \begin{bmatrix} 2 & 2+I_0^4 & I_2^4 \\ 0 & 0 & 2 \\ 3 & 1 & I_2^4 + I_0^4 \\ 1 & 0 & 3 \\ 2 & I_2^4 + I_0^4 + 1 & I_0^4 + 1 \end{bmatrix} \text{ and } y = \begin{bmatrix} 2 & I_0^4 & I_2^4 \\ 3 & 1 & 2 \\ I_0^4 & I_0^4 & I_0^4 \\ 0 & I_0^4 & I_0^4 \\ 2 & I_0^4 & I_0^4 \end{bmatrix}.$$

$$\text{Clearly } x \times_n y = \begin{bmatrix} 0 & I_0^4 & I_0^4 \\ 0 & 0 & 0 \\ I_0^4 & I_0^4 & I_0^4 \\ 0 & I_0^4 & I_0^4 \\ 0 & I_0^4 & I_0^4 \end{bmatrix}$$

is a mixed MOD natural neutrosophic special quasi dual number zero.

Thus x is a MOD natural neutrosophic special quasi dual number mixed zero divisor of S.

It is pertinent to keep on record that if $x \in S$ is a mixed MOD natural neutrosophic special quasi dual number zero then $x \times_n y = x$ for all $y \in S$.

This is just evident from the following.

$$\text{Let } x = \begin{bmatrix} 0 & I_0^4 & I_0^4 \\ I_0^4 & 0 & I_0^4 \\ 0 & I_0^4 & I_0^4 \\ 0 & I_0^4 & 0 \\ 0 & 0 & I_0^4 \end{bmatrix} \in S.$$

$$\text{Take } y = \begin{bmatrix} I_2^4 & I_0^4 & I_0^4 \\ I_2^4 & I_2^4 & 2+I_2^4 \\ I_2^4 & 0 & I_2^4+I_0^4 \\ I_2^4+I_0^4 & 0 & I_2^4 \\ 0 & I_2^4+I_0^4 & 0 \end{bmatrix} \in S$$

$$\text{Consider } x \times_n y = \begin{bmatrix} 0 & I_0^4 & I_0^4 \\ I_0^4 & 0 & I_0^4 \\ 0 & I_0^4 & I_0^4 \\ 0 & I_0^4 & 0 \\ 0 & 0 & I_0^4 \end{bmatrix} \times_n \begin{bmatrix} I_2^4 & I_0^4 & I_0^4 \\ I_2^4 & I_2^4 & I_2^4+2 \\ I_2^4 & 0 & I_2^4+I_0^4 \\ I_2^4+I_0^4 & 0 & I_2^4 \\ 0 & I_2^4+I_0^4 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} I_2^4 & I_0^4 & I_0^4 \\ I_0^4 & I_2^4 & I_0^4 \\ I_2^4 & I_0^4 & I_0^4 \\ I_2^4+I_0^4 & I_0^4 & I_2^4 \\ 0 & I_2^4+I_0^4 & I_0^4 \end{bmatrix} \neq x.$$

Hence the claim.

But S has element x and y such that it can yield mixed MOD natural neutrosophic special quasi dual number zero.

Further S has finite order subsemirings which are just rings.

$$B = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \end{bmatrix} \middle| a_i \in Z_4; 1 \leq i \leq 15, +, \times_n \right\} \subseteq S$$

is a MOD natural neutrosophic subsemiring which is a subring not an ideal of S.

$$C = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \end{bmatrix} \middle| \; a_i \in \langle Z_4 \cup k \rangle; 1 \le i \le 15 \right\} \subseteq S$$

is a MOD natural neutrosohic interval subsemiring which is a subring of S and not ideal of S.

$$\text{Let } W = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \end{bmatrix} \middle| \; a_i \in [0,4)k; 1 \le i \le 15, +, \times_n \right\} \subseteq S;$$

W is a MOD natural neutrosophic interval pseudo subsemiring which is also a pseudo ideal.

Study in this direction is both innovative and interesting but is considered as a matter of routine and left as an exercise to the reader.

Now having seen examples of MOD natural neutrosophic special quasi dual number interval matrix semiring of infinite order.

We next describe MOD natural neutrosophic interval coefficient polynomial pseudo semiring of varying types of some examples.

***Example 1.84.**:* Let

$$V = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in [0,10); +, \times \right\}$$

be the MOD natural neutrosophic interval coefficient polynomial pseudo semiring of infinite order.

$$M_1 = \langle Z_{10}[x] = \sum_{i=0}^{\infty} a_i x^i \} \subseteq V$$

is a MOD natural neutrosophic polynomial subsemiring which is not pseudo but is a subring and not an ideal of V.

$$\text{Let } M_2 = \{ Z_{10}^{I}[x] = \sum_{i=0}^{\infty} a_i x^i \,\Big|\, a_i \in Z_{10}^{I}, +, \times \} \subseteq V$$

be MOD natural neutrosophic polynomial subsemiring which is not pseudo but is only a subring or an ideal.

This MOD polynomial semiring has zero divisors but has no idempotents. These zero divisors includes nilpotents also.

***Example 1.85**:* Let

$$W = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in \langle [0,16) \cup g \rangle_I, +, \times \right\}$$

be the MOD natural neutrosophic interval pseudo polynomial semiring of infinite order. W has zero divisors as well as nilpotents.

Let $p(x) = 4x^8 + 8x^2 + 12 \in W$, $p(x) \times p(x) = 0$ is a MOD natural neutrosophic nilpotent polynomial of order two.

Let $q(x) = 2 + 10x^9 + 8x^{23} + 4x^{42}$ and $p(x) = 8 + 8x^2 \in W$;

$p(x) \times q(x) = 0$ is a MOD natural neutrosophic zero divisor polynomial.

Let $w(x) = 5 + 10x^3 + 8x^7$ and $v(x) = 12 + 12x^9 \in W$; $w(x) \times v(x) = 0$ is a MOD polynomial natural neutrosophic zero divisor of W.

Finding ideals of W happens to be a challenging one. Finding finite subsemirings is not possible.

Example 1.86: Let

$$M = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in \langle [0,12) \cup I \rangle_I \, Z_{10}^I, +, \times \right\}$$

be the MOD natural neutrosophic-neutrosophic polynomial coefficient pseudo semiring. M has zero divisors and nilpotent.

Let $p(x) = 6x^3 + 8x + 4$ and $q(x) = 6x^7 + 6 \in M$.

$p(x) \times q(x) = 0$ is the MOD natural neutrosophic zero divisor polynomial.

$q(x) \times q(x) = 0$ so $q(x)$ is MOD natural neutrosophic nilpotent polynomial of order two.

$o(M) = \infty$.

M has only infinite subsemirings and subrings of infinite order.

Take $Z_{12}[x] \subseteq M$ is a subsemiring which is a ring of infinite order but is not an ideal of M.

$Z_{12}^I[x] \subseteq M$ is a MOD natural neutrosophic subsemiring of infinite order but is not a ring.

$\langle Z_{12} \cup I\rangle[x]$ is a subsemring of infinite order which is not an ideal but a ring $\langle Z_{12} \cup I\rangle_I$ [x] is a subsemiring of infinite order which is not an ideal or subring [0, 12)[x] is a pseudo subsemiring which is not an ideal of M but infact a pseudo ring.

L = [0, 12)I[x] is a pseudo subsemiring which is not an ideal which is also pseudo of M. Infact L is a pseudo ring.

$^I[0, 12)I[x]$ is a pseudo subsemiring which is also a pseudo ideal of M and not a pseudo ring.

In view of all these we have the following theorem.

THEOREM 1.16: *Let $M = \{\langle[0\ n) \cup I\rangle_I, +, \times\}$ be a MOD natural neutrosophic neutrosophic interval coefficient polynomial pseudo semiring.*

i) M is commutative and is of infinite order.
ii) M has subsemirings of infinite order which are not pseudo but rings.
iii) M has pseudo subsemirings which are pseudo subrings.
iv) All polynomial subsemirings of M are infinite order.
v) M has pseudo ideals none of them are subring.

Proof is direct and hence left as an exercise to the reader.

THEOREM 1.17: Let

$$B = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in \langle[0, n) \cup g\rangle, +, \times\right\}$$

be the MOD natural neutrosophic dual number interval coefficient polynomial pseudo semiring.

i) $o(B) = \infty$ and B is commutative.

ii) B has subsemirings all of which are of infinite order and some of them are rings and some are not pseudo.
iii) B has subsemirings of infinite order which are pseudo and are not rings.'
iv) B has subsemirings of infinite order which is not pseudo but is an ideal of B.

Proof is direct and hence left as an exercise to the reader.

It is pertinent to keep on record that only MOD natural neutrosophic dual number coefficient polynomial semirings has ideals which are not pseudo and are not rings.

We give examples of other types of MOD natural neutrosophic interval polynomial pseudo semiring.

Example 1.87: Let

$$N = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in \langle [0, 18) \cup h \rangle_I, +, \times \right\}$$

be the MOD natural neutrosophic special dual like number coefficient polynomial interval pseudo semiring.

Clearly N is a commutative pseudo semiring and is of infinite order.

All polynomial subsemirings of N are of infinite order need not necessarily be pseudo

$$P_1 = Z_{18}[x] = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in Z_{18}, +, \times \right\} \subseteq N$$

is a MOD natural neutrosophic special dual like number semiring o f polynomials of infinite order. P_1 is a ring and P_1 is not an ideal and is not a pseudo subsemiring of N.

$$P_2 = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in \langle Z_{18} \cup h\rangle, +, \times\right\}$$

be the MOD natural neutrosophic special dual like number subsemiring of polynomial. P_2 is a subring not an ideal and is not pseudo but of infinite order.

$$P_3 = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in Z_{18}^I, +, \times\right\} \subseteq N$$

be the MOD natural neutrosophic subsemiring which is not a subring or ideal of infinite order, but is not possible.

$$P_4 = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in \langle Z_{18} \cup h\rangle_I, +, \times\right\} \subseteq N$$

be the MOD natural neutrosophic special dual like number coefficient polynomial subsemiring.

P_4 is of infinite order and is not a subring which is not an ideal.

$$P_5 = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in [0,18) +, \times\right\}$$

be the MOD natural neutrosophic interval coefficient polynomial pseudo subsemiring of N, which is not an ideal but is a pseudo ring.

$$P_6 = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in {}^I[0,18), +, \times\right\} \subseteq N$$

be the MOD natural neutrosophic interval coefficient polynomial pseudo subsemiring.

Clearly P_6 is of infinite order P_6 is not an ideal or ring of N.

$$P_7 = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in {}^{I}[0,18)h, +, \times\right\} \subseteq N$$

be the MOD natural neutrosophic special dual like number coefficient interval polynomial pseudo subsemiring.

P_7 is not a subring but is an pseudo ideal of P_7.

In view of all these we have the following result.

THEOREM 1.18: *Let S =* $\left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in \langle[0, n) \cup h\rangle_I, +, \times\right\}$ *be the MOD natural neutrosophic special dual like number coefficient polynomial interval semiring.*

i) o(S) = ∞ and is commutative.
ii) S has pseudo ideals which are not subrings.
iii) S has subsemirings of infinite order which are not pseudo and enjoy ring structure.
iv) S has subsemiring which are not pseudo and are not rings.

Proof is direct and hence left as an exercise to the reader.

Example 1.88: Let H = $\left\{\sum_{i=0}^{\infty} a_i x^i \in C^I([0,9)), +, \times\right\}$ be the MOD natural neutrosophic finite complex coefficient interval polynomial semiring o(H) = ∞.

H has subsemirings, subrings which are not pseudo but all of them are of infinite order.

Let $p(x) = 6x^3 + 3$ and $q(x) = 6x^2 + 6x + 3 \in H$.

Clearly $p(x) \times q(x) = 0$ is a zero divisor.

$(p(x))^2 = p(x) \times p(x) = 0$ so p(x) is a nilpotent element of order two.

Let $p(x) = 4.5\,x^3$ and $q(x) = 2x^4 + 4x^2 + 8 \in H$.

Clearly $p(x) \times q(x) = 0$.

Once again a zero divisor.

But p(x) and q(x) are both not nilpotent elements of H.

$$\text{Let } W_1 = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in Z_9; +, \times\right\} \subseteq H$$

is a subsemiring which is a ring and W_1 is not pseudo.

$$W_2 = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in C(Z_9); +, \times\right\} \subseteq H$$

is again a subsemiring of H which is not pseudo also a ring.

$$W_3 = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in [0, 9), +, \times\right\} \subseteq H$$

is a pseudo subsemiring which is also a pseudo ring of infinite order.

W_3 is not an ideal of H.

$$W_4 = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in C^I(Z_9); +, \times\right\} \subseteq H$$

is a subsemiring which is not a subring and is also not an ideal of H.

Finding pseudo ideals using the complex interval $C^I([0, n))$ happens to be a challenging problem.

***Example 1.89**:* Let

$$S = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in \langle [0, 10) \cup k\rangle_I, +, \times\right\}$$

be the MOD natural neutrosophic interval special quasi dual number coefficient polynomial pseudo semiring. o(S) = ∞.

$$P_1 = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in Z_{10}, +, \times\right\}$$

is a subsemiring of infinite order and is a ring and is not an ideal.

$$P_2 = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in Z_{10}^I, +, \times\right\}$$

is a subsemiring which is not an ideal and which is not a subring.

$$P_3 = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in \langle Z_{10} \cup k\rangle, +, \times\right\}$$

is a subsemiring of infinite order which is not an ideal but a subring.

$$P_4 = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in \langle Z_{10} k\rangle_I, +, \times\right\}$$

is a subsemiring which is not an ideal and not a subring.

$$P_5 = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in \langle Z_{10} \cup k\rangle_I, +, \times\right\}$$

is a subsemiring which is not an ideal and not a subring.

$$P_6 = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in [0, 10), +, \times\right\}$$

is a pseudo subsemiring which is also a pseudo subring but not an ideal.

$$P_7 = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in [0, 10)\, k, +, \times\right\}$$

is a pseudo subsemiring which is also a pseudo subring and not an ideal.

$$P_8 = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in {}^{I}[0,10)\, k, +, \times\right\}$$

is a pseudo subsemiring which is not a pseudo subring but an ideal of S.

This pseudo semiring h as zero divisors, no idempotents or nilpotents. In view of all these we give the following theorem.

THEOREM 1.19: *Let* $S = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in \langle [0, 10) \cup k\rangle_I, +, \times\right\}$ *be the MOD natural neutrosophic interval special quasi dual number coefficients polynomial pseudo semiring.*

i) S is commutative and o(S) = ∞.
ii) S has subsemirings which are rings and not pseudo and not ideals.

iii) S has subsemirings which are not rings or ideals and are not pseudo.
iv) S has a pseudo ideal.
v) All subsemirings of S are of infinite oroder.
vi) S has zero divisors and nilpotents and no nontrivial idempotents.

Proof of the theorem is left as an exercise to the reader.

Next we proceed onto describe the notion of MOD natural neutrosohic interval coefficient polynomial pseudo semirings which has only polynomials of degree less than or equal to m; $1 \leq m < \infty$, by some examples.

Example 1.90.: Let

$$P = \left\{\sum_{i=0}^{9} a_i x^i \,\middle|\, a_i \in \langle[0, 16)\rangle_I = {}^I[0, 16);\ x^{10} = 1, +, \times\right\}$$

be the MOD natural neutrosophic interval coefficient polynomial pseudo semiring with polynomials of degree less than or equal to 9.

$o(P) = \infty$, but P has subsemirings and subrings of finite order. This P has nilpotents and zero divisors.

$p(x) = 2 + 6x + 4x^3$ and $q(x) = 8 + 8x^2 \in P$, $p(x) \times q(x) = 0$ so P has nontrivial zero divisor polynomials.

Also $q(x) \times q(x) = 0$ so $q(x)$ is a nontrivial nilpotent polynomial of order two.

Let $B_1 = \left\{\sum_{i=0}^{9} a_i x^i \,\middle|\, a_i \in Z_{16}, +, \times\right\}$ be the subsemiring which is also a subring of P of finite order and not pseudo.

$B_2 = \left\{\sum_{i=0}^{9} a_i x^i \,\middle|\, a_i \in \{0, 2, 4, 6, 8, 10, 12, 14\}; +, \times\right\}$ is also a subsemiring of P which is a subring and not an ideal of P and is not pseudo.

$B_3 = \left\{\sum_{i=0}^{9} a_i x^i \,\middle|\, a_i \in \{0, 4, 8, 12\}, +, \times\right\} \subseteq P$ is again a subsemirin of finite order which is a subring and B_3 is not pseudo.

$B_4 = \left\{\sum_{i=0}^{9} a_i x^i \,\middle|\, a_i \in Z_{16}^{I}, +, \times\right\}$ is a subsemiring of P of finite order and is not an ideal or subring of P.

$B_5 = \left\{\sum_{i=0}^{9} a_i x^i \,\middle|\, a_i \in [0,16), +, \times\right\}$ is a pseudo subsemiring of P which is a pseudo subring but not an ideal of P.

The task of finding ideals in P is left as an exercise to the reader.

Example 1.91: Let

$$W = \left\{\sum_{i=0}^{6} a_i x^i \,\middle|\, a_i \in \langle[0,12) \cup I\rangle_I;\ x^7 = 1, +, \times\right\}$$

be the MOD natural neutrosophic - neutrosophic interval coefficient polynomial pseudo semiring of infinite order.

This has zero divisors and nilpotents but has no idempotent which are nontrivial.

Let $V_1 = \left\{\sum_{i=0}^{6} a_i x^i \,\middle|\, a_i \in Z_{12}, +, \times\right\}$ is a finite subsemiring which is a subring but not an ideal of W.

$V_2 = \left\{\sum_{i=0}^{6} a_i x^i \,\middle|\, a_i \in Z_{12}^{I},\ x^7 = 1, +, \times\right\} \subseteq W$ is the subsemiring which is not an ideal or subring and not pseudo.

$V_3 = \left\{\sum_{i=0}^{6} a_i x^i \,\middle|\, a_i \in \langle Z_{12} \cup I\rangle;\ x^7=1, +, \times\right\} \subseteq W$ is aa subsemiring which is not pseudo and is not an ideal but a subring of finite order.

$V_4 = \left\{\sum_{i=0}^{6} a_i x^i \,\middle|\, a_i \in \langle Z_{12} \cup I\rangle_I\ x^7 = 1, +, \times\right\} \subseteq W$ is a subsemiring which is not pseudo but is not a subring V_4 is not an ideal but V_4 is a finite order MOD natural neutrosophic - neutrosophic subsemiring.

$V_5 = \left\{\sum_{i=0}^{6} a_i x^i \,\middle|\, a_i \in [0,12),\ x^7 = 1, +, \times\right\} \subseteq W$ is a pseudo subsemirin of infinite order which is also a pseudo subring but not a pseudo ideal.

$V_6 = \left\{\sum_{i=0}^{6} a_i x^i \,\middle|\, a_i \in [0,12)I;\ x^7 = 1, +, \times\right\} \subseteq W$ is a pseudo subsemiring which is also a pseudo subring but not an ideal.

$V_7 = \left\{\sum_{i=0}^{6} a_i x^i \,\middle|\, a_i \in {}^{I}[0,12)\ I,\ x^7 = 1, +, \times\right\}$ is a pseudo MOD natural neutrosophic-neutrosophic subsemiring which is not a subring but an ideal of W.

In view of all these we have the following result.

THEOREM 1.20.: *Let*

$$S = \left\{ \sum_{i=0}^{m} a_i x^i \,\middle|\, a_i \in \langle [0, n) \cup I \rangle_I;\ x^{m+1} = 1, +, \times \right\}$$

be the MOD natural neutrosophic-neutrosophic interval coefficient pseudo semiring of polynomials of finite degree.

i) S is commutative and o(S) = ∞.
ii) S has subsemirings which are subrings and that not pseudo. But they are not ideals.
iii) All ideals of S are of infinite order.
iv) S has subsemirings of finite order which are not subrings and ideals.
v) S has zero divisors and nilpotents but has no nontrivial idempotents.

Proof is direct and hence left as exercise to the reader.

Example 1.92: Let

$$W = \left\{ \sum_{i=0}^{4} a_i x^i \,\middle|\, x^5 = 1, a_i \in \langle [0,6) \cup g \rangle_I, +, \times \right\}$$

be the MOD natural neutrosophic dual number coefficient interval polynomial pseudo semiring.

W has zero divisors but no nilpotents or idempotent which are nontrivial.

$B_1 = \left\{ \sum_{i=0}^{4} a_i x^i \,\middle|\, x^5 = 1, a_i \in Z_6, +, \times \right\} \subseteq W$ is a subsemiring which is not pseudo but is a ring of finite order which is not an ideal.

$B_2 = \left\{\sum_{i=0}^{4} a_i x^i \,\middle|\, x^5 = 1, a_i \in Z_6^I, +, \times\right\} \subseteq W$ is a MOD natural neutrosophic subsemiring of finite order which is not pseudo and is not a subring or ideal.

$B_3 = \left\{\sum_{i=0}^{4} a_i x^i \,\middle|\, x^5 = 1, a_i \in \langle Z_6 \cup g\rangle_I, +, \times\right\} \subseteq W$ is a MOD natural neutrosophic subsemiring of finite order which is a subring and it is not pseudo and is not an ideal.

$B_4 = \left\{\sum_{i=0}^{4} a_i x^i \,\middle|\, x^5 = 1, a_i \in \langle Z_6 \cup g\rangle_I, +, \times\right\} \subseteq W$ is a MOD natural neutrosophic subsemiring of finite order which is not pseudo which is not a subring or ideal of W.

$B_5 = \left\{\sum_{i=0}^{4} a_i x^i \,\middle|\, x^5 = 1, a_i \in [0, 6), +, \times, x^5 = 1\right\} \subseteq W$ is a pseudo subsemiring of W of infinite order which is also a pseudo subring but not an ideal of W.

$B_6 = \left\{\sum_{i=0}^{4} a_i x^i \,\middle|\, a_i \in {}^I[0,6), +, \times, x^5 = 1\right\}$ is a pseudo MOD natural neutrosophic interval subsemiring of infinite order which is not a subring or pseudo ideal of W.

$B_7 = \left\{\sum_{i=0}^{4} a_i x^i \,\middle|\, a_i \in [0,6)g, +, \times, x^5 = 1\right\} \subseteq W$ is a MOD natural neutrosophic interval dual number subsemiring polynomial of infinite order which is a subring but is not a pseudo subsemiring.

Infact a zero square subsemiring of infinite order. It is not an ideal.

$$B_8 = \left\{\sum_{i=0}^{4} a_i x^i \,\middle|\, x^5 = 1, a_i \in {}^{I}[0,6)_g; +, \times\right\} \subseteq W \text{ be the MOD}$$

natural neutrosophic interval dual number coefficient polynomial subsemiring, which is not pseudo but is an ideal of W.

In view of all these we have the following theorem.

THEOREM 1.21: *Let*

$$S = \left\{\sum_{i=0}^{m} a_i x^i \,\middle|\, a_i \in \langle[0,n) \cup g\rangle_I, x^{m+1} = 1, +, \times\right\}$$

be the MOD natural neutrosophic dual number coeffinite polynomiral pseudo semring of degree less than or equal to m; $2 \leq m < \infty$.

i) $o(S) = \infty$ and S is a commutative semiring.

ii) S has subsemirings of finite order which are rings and not ideals.

iii) S has subsemirings of finite order which are not subrings or ideals.

iv) S has subsemirings of infinite order which are pseudo and are pseudo subrings and not ideals of S.

v) S has subsemirings of infinite order which are not pseudo but ideals. They are not subrings of S.

Proof is direct and hence left as an exercise to the reader.

Property (iv) of this theorem is very unique and it pertains only to those MOD natural neutrosophic interval pseudo polynomial semirings whose coefficients are MOD natural neutrosophic interval dual numbers from $\langle[0,n) \cup g\rangle_I$.

This is not true in case of all other types of MOD natural neutrosophic semiring of polynomials.

Next we proceed onto describe the notion of MOD natural neutrosophic interval special dual like number coefficient polynomial semirings by examples.

***Example 1.93**:* Let

$$M = \left\{\sum_{i=0}^{8} a_i x^i \,\middle|\, a_i \in \langle [0,19) \cup h\rangle_I;\ x^9 = 1, +, \times\right\}$$

be the MOD natural neutrosophic special dual like number coefficient polynomial interval pseudo semiring o(M) = ∞.

M has only zero divisors and no nilpotents or nontrivial idempotents.

Let $p(x) = 9.5\,x^8 + 4.75\,x^6 + 9.5$ and

$q(x) = 8x^3 + 4x + 12 \in M$.

Clearly $p(x) \times q(x) = 0$.

So is a nontrivial zero divisor in M.

M has no nontrivial idempotents or nilpotents x.

We see $P_1 = \left\{\sum_{i=0}^{8} a_i x^i \,\middle|\, a_i \in Z_{19}, +, \times\right\} \subseteq M$ is a subsemiring of M which is not an ideal but a proper subring of M.

Let $P_2 = \left\{\sum_{i=0}^{8} a_i x^i \,\middle|\, x^9 = 1,\ a_i \in Z_{19}^I, +, \times\right\} \subseteq M$ is a subsemiring of M of finite order and not an ideal or subring of M.

Let $P_3 = \left\{\sum_{i=0}^{8} a_i x^i \right| a_i \in \langle Z_{19} \cup h\rangle;\ x^9 = 1, +, \times\} \subseteq M$ is a subsemiring which is not pseudo and not an ideal only a subring of finite order.

$P_4 = \left\{\sum_{i=0}^{8} a_i x^i \right| a_i \in \langle Z_{19} \cup h\rangle_I;\ x^9 = 1, +, \times\} \subseteq M$ is a subsemiring of finite order and not a subring or ideal or it is not pseudo substructure of M.

$P_5 = \left\{\sum_{i=0}^{8} a_i x^i \right| x^9 = 1, a_i \in [0,19), +, \times\} \subseteq M$ is a pseudo subsemiring of infinite order P_5 is a pseudo subring but not an ideal.

$P_6 = \left\{\sum_{i=0}^{8} a_i x^i \right| a_i \in {}^I[0,19);\ x^9 = 1, +, \times\} \subseteq M$ is a pseudo MOD natural neutrosophic special dual like number coefficient interval subsemiring which is not a subring ideal of M. $o(P_6) = \infty$.

Let $P_7 = \left\{\sum_{i=0}^{8} a_i x^i \right| x^9 = 1, a_i \in [0,19)h, +, \times\} \subseteq M$ is again a pseudo subsemiring of M which is not an ideal.

But P_7 is a pseudo subring of M of infinite order.

Let $P_8 = \left\{\sum_{i=0}^{8} a_i x^i \right| a_i \in {}^I[0,19)h;\ +, \times\} \subseteq M$ is a MOD natural neutrosophic interval pseudo special dual like number coefficient polynomial semiring of infinite order.

Clearly P_8 is not a subring but an ideal of M.

In view of all these we prove the following result.

THEOREM 1.21: *Let*

$$S = \left\{ \sum_{i=0}^{m} a_i x^i \;\middle|\; a_i \in \langle [0,n) \cup h \rangle_I;\ x^{m+1} = 1, \times, + \right\}$$

be the MOD natural neutrosophic special dual like number interval coefficient polynomial pseudo semiring of finite degree polynomials; $2 \le m < \infty$.

Then the following are true

i) *$o(S) = \infty$ and S is a commutative semiring.*
ii) *S has subsemiriings of finite order which are subrings and not ideals and are not pseudo*
iii) *S has subsemirings of finite o rder which is not subrings or ideals and are not pseudo*
iv) *S has pseudo subsemirings of infinite order which are pseudo subrings but not ideals.*
v) *S has pseudo MOD natural neutrosophic subsemirings of infinite or ideals.*
vi) *S has pseudo MOD natural neutrosophic subsemirings of infinite order which are not subrings but are ideals.*

Proof of all these notions are direct and hence left as an exercise to the reader.

Next we provide an example of MOD natural neutrosophic special quasi dual number interval coefficient polynomials pseudo semirings.

Example 1.94: Let $S = \left\{ \sum_{i=0}^{12} a_i x^{12} \;\middle|\; x^{13} = 1,\ a_i \in \langle [0,15) \cup k \rangle_I;\ +, \times \right\} \subseteq S$ be the MOD natural neutrosophic interval special quasi dual number coefficient polynomial pseudo semiring of degree less than or equal to 12.

We see $P_1 = \left\{\sum_{i=0}^{12} a_i x^i \middle| a_i \in Z_{15}, \quad x^{13} = 1, +, \times\right\} \subseteq S$ is a subsemiring which is also a subring of S.

$P_2 = \left\{\sum_{i=0}^{12} a_i x^i \middle| a_i \in Z_{15}^I, x^{13} = 1, +, \times\right\} \subseteq S$ is a MOD natural neutrosophic special quasi dual number polynomial subsemiring of finite order and is not a subring.

$P_3 = \left\{\sum_{i=0}^{12} a_i x^i \middle| a_i \in \langle Z_{15} \cup k\rangle, x^{13} = 1, +, \times\right\} \subseteq S$ is a subsemiring of polynomials of finite order, which is also a subring.

Clearly P_1, P_2 and P_3 are not ideals of S.

$P_4 = \left\{\sum_{i=0}^{12} a_i x^i \middle| a_i \in \langle Z_{15} \cup k\rangle_I, x^{13} = 1, +, \times\right\} \subseteq S$ be the MOD natural neutrosophic special quasi dual number semiring of polynomials and is not a subring or ideal or pseudo and P_4 is of finite order.

$P_5 = \left\{\sum_{i=0}^{12} a_i x^i \middle| a_i \in Z_{15}k, x^{13} = 1, +, \times\right\} \subseteq S$ be the MOD natural neutrosophic special pure quasi dual number subsemiring of finite order. P_5 is a subring and not an ideal P_5 is not pseudo.

$P_6 = \left\{\sum_{i=0}^{12} a_i x^i \middle| x^{13} = 1, +, \times, a_i \in [0,15)\right\} \subseteq S$ is a MOD natural neutrosophic interval pseudo subsemiring and also a pseudo subring but is not an ideal.

$P_7 = \left\{\sum_{i=0}^{12} a_i x^i \,\middle|\, a_i \in [0,15)\ k,\ x^{13} = 1, +, \times\right\} \subseteq S$ is a MOD natural neutrosophic pseudo subsemiring also a interval pseudo subring but not an ideal of S.

$P_8 = \left\{\sum_{i=0}^{12} a_i x^i \,\middle|\, x^{13} = 1,\ a_i \in {}^I[0,15), +, \times\right\} \subseteq S$ be the MOD natural neutrosophic pseudo interval polynomial subsemirin of S. P_8 is not a pseudo subring. P_8 is not an ideal of S.

$P_9 = \left\{\sum_{i=0}^{12} a_i x^i \,\middle|\, x^{13} = 1,\ a_i \in {}^I[0,15)\ k, +, \times\right\} \subseteq S$ be the pseudo MOD natural neutrosophic interval quasi dual number coefficient polynomial subring of S.

Clearly P_9 is an ideal but is not a subring.

In view of all these we have the following result.

Before we mention this S has nontrivial zero divisors but only trivial idempotents and nilpotents.

THEOREM 1.23: *Let*

$$S = \left\{\sum_{i=0}^{m} a_i x^i \,\middle|\, x^{m+1} = 1,\ a_i \in \langle [0,n) \cup k\rangle_I; +, \times\right\}$$

be the MOD natural neutrosophic special quasi dual number coefficient polynomial pseudo interval semiring which has polynomials of degree less than or equal to m.

Then the following facts are true.

i) $o(S) = \infty$ and S is a commutataive structure.

ii) S has nontrivial zero divisors but has only trivial idempotents and nontrivial nilpotents if and only if Z_n has nontrivial nilpotents.

iii) S has subsemirings of finite order which are subrings but not ideals and they are not pseudo.

iv) S has subsemirings of finite order which are not ideals, not subrings and not pseudo.

v) S has subsemirings of infinite order which are pseudo and subrings but not ideals.

vi) S has pseudo subsemirings of infinite order which are not subrings or ideals of S.

vii) S has pseudo subsemirings of infinite order which are not pseudo subrings but are ideals of S.

Proof follows from direct calculations, so left as an exercise to the reader.

Now it is pertinent to keep on record that we see $\langle[0, n)\rangle_I$ contains only MOD natural neutrosophic elements of Z_n^I.

Clearly $I_t^{[0,n)}$, $t \in [0, n) \setminus Z_n$ is not included in this book.

For in many cases they may not be closed with respect special factors which will be described and developed in the forth coming books.

However $I_t^{[0,n)}$ will be a pseudo MOD natural neutrosophic zero divisor but we do not include them in $\langle[0,n)\rangle_I$.

Similarly all MOD natural neutrosophic dual numbers in $\langle[0,n) \cup g\rangle_I$ are from $[0, n)g$ and Z_n^I and not from $[0, n) \setminus Z_n$ when included we will have no problem for closure.

Otherwise the set $\langle[0,n)\rangle_I$ is not in general closed order $\times$ which we brought out in the forthcoming books.

This has been also discussed in several places in the books [ ]. Thus we always put a condition the collection of all pseudo natural neutrosophic numbers arising from $\langle[0,n)\rangle_I \setminus Z_n^I$ are not actually natural neutrosophic numbers.

This has been already addressed but a systematic study exploiting their properties are not carried out.

That is why while defining $\langle[0,n) \cup I\rangle_I$ or $\langle[0,n)\rangle_I$ or $C^I([0, n))$ and so on we have only used the term MOD natural neutrosophic numbers and not MOD pseudo natural neutrosophic numbers or pseudo MOD natural neutrosophic numbers whatever be the situation [0, n) or $\langle[0,n)\rangle_I$ or $\langle[0,n) \cup I\rangle$ or $\langle[0,n) \cup I\rangle_I$ or $\langle[0,n) \cup g\rangle$ or $\langle[0,n) \cup g\rangle_I$ and so on will not be rings semirings they will only be pseudo rings or pseudo semirings as in all these sets we will not have the distributive laws to be true, for

$a \times (b + c) \neq a \times b + a \times c$ for a, b c $\in [0, n) \setminus Z_n$ or $\langle[0,n) \cup g\rangle_I \setminus \langle Z_n \cup g\rangle_I$ and so on.

So this study is little different for these algebraic structures do not behave like the existing structures.

We see at all stages we are forced to marvel at the behavior of $\langle[0,n)\rangle_I$ under + and $\times$.

We now suggest a few problems in the following.

Problems:

1. Let S = $\{ Z_{22}^I, +, \times\}$ be the natural neutrosophic semring.

    i) Find all ideals of S.
    ii) Find all subsemirings which are not ideals of S.
    iii) Find all zero divisors, nilpotents and idempotents of Z_{22}.
    iv) Find all natural neutrosophic zero divisors, nilpotents and idempotents of Z_{22}^I.
    v) What is o(S)?
    vi) How many elements of S are natural neutrosophic?

2. Let B = { Z^I_{19} , +, ×} be the natural neutrosophic semiring.

   i) Study questions (i) to (vi) of problem (1) to this B.
   ii) Compare the natural neutrosophic elements of B and S.

3. Let P = { $Z^I_{2^{10}}$, + ×} be the natural neutrosophic semiring.

   i) Study questions (i) to (vi) of problem (1) for thisP.
   ii) Compare P with S and B of problems (1) and (2) respectively.

4. Obtain any other specil properties enjoyed by these finite natural neutrosophic semirings { Z^I_n , +, ×}.

5. Prove { Z^I_n , +, ×} can never be a semifield.

6. Let M ={ $Z^I_{2^2.3^4.5^7.13^6.19^2.23.43^2}$, +, ×} be the natural neutrosophic semiring.

   i) Study questions (i) to (vi) of problem (1) for this M.
   ii) Compare M with S, B and P of problems (1), (2) and (3) respectively.

7. Let S_1 = { Z^I_{120} , +, ×} be the MOD natural neutrosophic semiring.

   i) Study questions (i) to (vi) of problem (1) for this S,
   ii) Compare this S_1 with S of problem 1, B of problem 2 and P of problem 3.
   iii) Is it true only S_1 has more number of MOD natural neutrosophic elements?
   iv) Can we prove S_1 has more number of ideals and subsemiring?
   v) Is it true S_1 has more number of subrings?

8. Characterize those M = $\{ Z_n^I, +, \times\}$ the MOD natural neutrosophic semiring to have more number subring.

   i) For what value of n M will have least number of subrings.
   ii) Find all MOD natural neutrosophic elements of M.
   iii) Under what condition on n, M will have maximum number of MOD natural neutrosophic elements.
   iv) Will the collection of all MOD natural neutrosophic elements of M form a subsemiring?

   Justify your claim.

9. Study problem 8 in case of N = $\{ Z_{19}^I, +, \times\}$ the MOD natural neutrosophic semiring.

10. Study problem 8 in case of P = $\{ Z_{64}^I, +, \times\}$, the MOD natural neutrosophic semiring.

11. Study problem 8 in case of V = $\{ Z_{180}^I, +, \times\}$, the MOD natural neutrosophic semiring.

12. Study problem 8 for W = $\{ Z_{12}^I, +, \times\}$, the semiring of MOD natural neutrosophic numbers.

13. Study problem 8 for Z = $\{ Z_{35}^I, +, \times\}$ the semiring of MOD natural neutrosophic numbers.

14. Compare the properties of semirings in problems 9, 10, 11, 12, and 13.

    Can we say the properties completely depend on the n that is on 19, 64, 180, 12 and 35 respectively?

15. Enumerate all special and interesting features associated with MOD natural neutrosophic semiring $\langle Z_n^I, +, \times\rangle$, $2 \leq n < \infty$.

16. Let B = $\langle Z_n \cup I\rangle_I$, +, ×} be MOD natural neutrosophic neutrosophic semiring.

    i) Study questions (i) to (vi) of problem (1) for this B.
    ii) Study questions (i) to (iv) of problem 8 for this B.

17. Let M = {$\langle Z_{125} \cup I\rangle_I$, +, ×} be the MOD natural neutrosophic neutrosophic semiring.

    i) Study questions (i) to (vi) of problem (1) for this M.
    ii) Study questions (i) to (iv) of problem (8) for this M.

18. Let N = {$\langle Z_{29} \cup I\rangle_I$, + ×} be the MOD natural neutrosophic neutrosophic semiring.

    i) Study questions (i) to (vi) of problem (1) for this N.
    ii) Study questions (i) to (iv) of problem (8) for this N.

19. Let S = {$\langle Z_{72} \cup I\rangle_I$, +, ×} be the MOD natural neutrosophic neutrosophic semiring.

    i) Study questions (i) to (vi) of problem (1) for this S.
    ii) Study questions (i) to (iv) of problem (8) for this S.

20. Compare M, N and S of problems 17, 18 and 19 respectively with each other.

21. Compare $\langle Z_n^I$, +, ×$\rangle$ = P_1 with {$\langle Z_n \cup I\rangle_I$, +, ×} = P_2 as MOD natural neutrosophic semirings.

    Which of the semirings P_1 or P_2 has more number of MOD natural neutrosophic elements?

22. Let V = {$C^I(Z_{42})$, +, ×} be MOD finite complex number natural neutrosophic semiring.

    i) Study questions (i) to (vi) of problem (1) for this V.
    ii) Study questions (i) to (iv) of problem (8) for this v.

23. Let L = $\{C^I(Z_{43}), +, \times\}$ be the MOD finite complex number natural neutrosophic semiring.

    i) Study questions (i) to (vi) of problem (1) for this L.
    ii) Study questions (i) to (iv) of problem (8) for this L.

24. Compare L and V in problem (23) and (22) respectively.

25. Obtain any other special feature associated with $\{C^I(Z_n), +, \times\}$ the MOD natural neutrosophic finite complex number semiring.

26. Compare the semirings $P_1 = \{C^I(Z_{12}), +, \times\}$, $\{Z_{12}^I, +, \times\} = P_2$ and $P_3 = \{\langle Z_{12} \cup I\rangle_I, +, \times\}$.

    i) Which has maximum number of MOD natural neutrosophic elements?
    ii) Which of the semiring has maximum number of subrings?

27. Let T = $\{\langle Z_{10} \cup g\rangle_I, +, \times\}$ be the MOD natural neutrosophic dual number semiring.

    i) Study questions (i) to (vi) of problem (1) for this T.
    ii) Study questions (i) to (iv) of problem (8) for this T.
    iii) Prove T has more number of subrings.
    iv) Prove T has subrings which are zero square rings.

28. Let P = $\{\langle Z_{28} \cup g\rangle_I +, \times\}$ be the MOD natural neutrosophic dual number semiring.

    i) Study questions (i) to (vi) of problem (1) for this P.
    ii) Study questions (i) to (iv) of problem (8) for this P.

29. Compare MOD natural neutrosophic dual number semirings with MOD natural neutrosophic-neutrosophic semirings and MOD natural finite complex integer natural neutrosophic semirings all built using Z_n, the same n.

30. Let S = $\{\langle Z_{14} \cup h\rangle_I, +, \times\}$ be the MOD natural neutrosophic special dual like number semiring.

    i) Study questions (i) to (vi) of problem (1) for this S.
    ii) Study questions (i) to (iv) of probelom (8) for this S.

31. Study problem 30 if $\langle Z_{14} \cup h\rangle_I$ is replaed by $\langle Z_{43} \cup h\rangle_I$ and $\langle Z_{243} \cup h\rangle_I$. Compare all the three spaces.

32. Let W = $\{\langle Z_{48} \cup k\rangle_I, +, \times\}$ be the MOD natural neutrosophic special quasi dual number semiring.

    i) Study questions (i) to (vi) of problem (1) for this W.
    ii) Study questions (i) to (iv) of problem (8) for this W.

33. Compare W in problem 33 with MOD natural neutrosophic dual number semiring $\{\langle Z_{48} \cup g\rangle_I, +, \times\}$, $\{ Z_{48}^I, + \times\}$ and $C^I(Z_{498})$.

34. Let S = $\{(a_1\ a_2\ a_3\ a_4\ a_5) / a_i \in Z_{24}^I; 1 \leq i \leq 5, +, \times\}$ be the MOD natural neutrosophic matrix semiring.

    i) Find o(S) and prove S is commutative.
    ii) Find all subrings of S.
    iii) Find all subsemirings which are not ideals or subrings of S.
    iv) Prove subrings of S cannot be ideal of S.
    v) Find all zero divisors of S.
    vi) Find all idempotents of S.
    vii) Can S have nilpotents?
    viii) Obtain any other interesting or special feature associated with S.

35. Let M = $\left\{\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \end{bmatrix} \middle| a_i \in Z_{15}^I; 1 \leq i \leq 10, +, \times_n\right\}$ be the

MOD natural neutrosophic matrix semiring.

Study questions (i) to (viii) of problem 34 for this M.

36. Let P = $\left\{\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \middle| a_i \in Z_{32}^I; 1 \leq i \leq 9, +, \times_n\right\}$ be the

MOD natural neutrosophic matrix semiring.

i) Study questions (i) to (viii) of problem 34 for this P.
ii) In P '$\times_n$' is replaced by usual matrix product $\times$ prove P is a non commutative semiring.
iii) Obtain the special feature enjoyed by this noncommutative matrix MOD natural neutrosophic semiring.

37. Let W = $\left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \end{bmatrix} \middle| a_i \in \langle Z_{12} \cup I\rangle_I;\right.$

$1 \leq i \leq 15, +, \times_n\}$ be the MOD natural neutrosophic matrix semiring.

i) Study questions (i) to (viii) of problem (34) for this W.
ii) Derive any other special feature associated with this W.

38. Enumerate all special features associated with

$V = \{ (a_{ij})_{p \times q} / a_i \in \langle Z_n \cup I \rangle_I, (p \neq q), 1 \leq i \leq p, 1 \leq j \leq q, +, \times_n\}$ the MOD natural neutrosophic - neutrosophic matrix semirings $2 \leq n < \infty$.

39. Let $T = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} \end{bmatrix} \middle| \; a_i \in \langle Z_{16} \cup g \rangle_I; \right.$

$1 \leq i \leq 18, +, \times_n\}$

be the MOD natural neutrosophic dual number matrix semiring.

i) Study questions (i) to (viii) of problem (34) for this T.
ii) Compare T with S_1 and S where S_1 is in T; $\langle Z_{16} \cup g \rangle_I$ is replaced by $\langle Z_{16} \cup I \rangle_I$ and S_2 is in T $\langle Z_{16} \cup g \rangle_I$ is replaced by $C^I(Z_{16})$.

40. Let $M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 & a_{10} & a_{11} & a_{12} \end{bmatrix} \middle| \; a_i \in \langle Z_{45} \cup h \rangle_I; \right.$

$1 \leq i \leq 12, +, \times_n\}$

be the MOD natural neutrosophic special dual like number matrix semiring.

i) Study questions (i) to (vi) of problem (1) for this M.
ii) Study questions (i) to (iv) of problem (8) for this M.
iii) Obtain any other special and distinct features enjoyed by M.

iv) Compare M with N where N is the same order matrix semiring but $\langle Z_{45} \cup h\rangle$ is replaced by $C^I(Z_{45})$.

41. Let W = $\left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \middle| \; a_i \in \langle Z_{42} \cup k\rangle_I;\ 1 \leq i \leq 16, \right.$

$+, \times_n\}$

be the MOD natural neutrosophic special quasi number matrix semiring.

i) Study questions (i) to (vi) of problem 1 for this W.
ii) Study questions (i) to (iv) of problem 8 for this W.
iii) If the product $\times_n$ in W is replaced by $\times$ find the distinct features enjoyed by it.

42. Let W = $\{{}^I[0, 9), +, \times\}$ be the MOD interval natural neutrosophic semiring.

i) Prove W is of infinite order.
ii) Prove W has subsemirings which are subrings of finite order.
iii) Prove W has pseudo subsemirings of infinite order which can be pseudo subring.
iv) Can W have pseudo ideals?
v) Can pseudo subsemirings which are pseudo subrings be ideals? Justify your claim.
vi) Find all zero divisors and nilpotents of W.
vii) Can W have idempotents which are non trivial?

43. Let S = $\{{}^I[0, 19), +, \times\}$ be the MOD interval natural neutrosophic pseudo semiring.

Study questions (i) to (vii) of problem (42) for this S.

44. Let $L = \{{}^{I}[0,24), +, \times\}$ be the MOD interval natural neutrosophic pseudo semiring.

    Study questions (i) to (vii) of problem (42) for this L.

45. Let $N = \{\langle[0,13) \cup I\rangle_I, +, \times\}$ be the MOD interval natural neutrosophic-neutrosophic pseudo semiring.

    Study questions (i) to (vii) of problem (42) for this W.

46. Let $B = \{\langle[0,48) \cup I\rangle_I, +, \times\}$ be the MOD interval natural neutrosophic-neutrosophic pseudo semiring.

    Study questions (i) to (vii) of problem (42) for this B.

47. Let $A = \{\langle[0,10) \cup g\rangle_I, +, \times\}$ be the MOD interval natural neutrosophic dual number pseudo semiring.

    Study questions (i) to (vii) of problem (42) for this A.

48. Let $D = \{\langle[0,16) \cup I\rangle_I, +, \times\}$ be the MOD natural neutrosophic dual number interval pseudo semiring

    Study questions (i) to (vii) of problem (42) for this D.

49. Let $E = \{\langle[0,45) \cup g\rangle_I, +, \times\}$ be the MOD natural neutrosophic dual number interval pseudo semiring.

    Study questions (i) to (vii) of problem (42) for this E.

50. Let $F = \{\langle[0,16) \cup h\rangle_I, +, \times\}$ be the MOD natural neutrosophic interval special dual like number pseudo semiring.

    i) Study questions (i) to (vii) of problem (42) for this F.
    ii) Compare F with E in problem 42.

51. Let G = $\{\langle[0,49) \cup k\rangle_I, +, \times\}$ be the MOD natural neutrosophic special quasi dual number interval pseudo semiring.

    i) Study questions (i) to (vii) of problem (42) for this G.
    ii) Make a comparative study when $\langle[0, n) \cup k\rangle_I$, with $\langle[0, n) \cup g\rangle$.

52. Let P = $\{\langle[0,27) \cup I\rangle_I, +, \times\}$ be the MOD natural neutrosophic-neutrosophic interval pseudo semiring. Prove this has nontrivial nilpotents.

    Study questions (i) to (vi) of problem (42) for this P.

53. Let B = $\{C^I[0,26)), +, \times\}$ be the MOD natural neutrosophic finite complex number interval pseudo semiring.

    i) Study questions (i) to (vii) of problem (42) for this B.

54. Let M = $\{(a_1, a_2, \ldots, a_9) / a_i \in {}^I[0,14), +, \times\}$ be the MOD interval nautral neutrosophic row matrix pseudo semiring.

    i) Prove o(M) = ∞ and M is commutative.
    ii) Prove M has zero divisors.
    iii) Prove M has ideals and none of the ideals are pseudo subrings but one pseudo subsemirings.
    iv) Can M have nontrivial idempotents?
    v) Prove M has subsemirings of finite order which are subrings nut not ideals or pseudo semirings.
    vi) Prove M has subsemirings of finite order wich are not subrings but are also not ideals.
    vii) Prove M has pseudo subsemirings subrings or ideals of M.
    viii) Prove all ideals of M are pseudo and only subsemirings and not subrings.

55. Let P = $\left\{\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \\ a_{11} & a_{12} \end{bmatrix} \middle|\ a_i \in {}^{I}[0,48), 1 \leq i \leq 12, +, \times_n\right\}$

be the MOD interval natural neutrosophic pseudo semiring.

Study questions (i) to (viii) of problem (54) for this P.

56. Let S = $\left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \\ a_{17} & a_{18} & a_{19} & a_{20} \end{bmatrix} \middle|\ a_i \in \langle [0,29) \cup I \rangle_I;\right.$

$\left. 1 \leq i \leq 20, +, \times_n \right\}$

be the MOD natural neutrosophic interval matrix pseudo semiring.

Study questions (i) to (viii) of problem (54) for this S.

57. Let S_1 = $\left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \end{bmatrix} \middle|\ a_i \in \{\langle [0,15) \cup I \rangle_I;\right.$

$\left. 1 \leq i \leq 10, +, \times_n \right\}$

be the MOD interval natural neutrosophic - neutrosophic matrix pseudo semiring.

Study questions (i) to (viii) of problem (54) for this S_1.

58. Let $B = \left\{\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \end{bmatrix} \middle| \; a_i \in \langle [0,24) \cup g\rangle_I;\right.$

$1 \leq i \leq 15, +, \times_n\}$

be the MOD pseudo natural neutrosophic dual number interval matrix semiring.

Study questions (i) to (viii) of problem (54) for this B.

59. Let $M = \left\{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} \middle| \; a_i \in \langle [0,28) \cup k\rangle_I, 1 \leq i \leq 6, +, \times_n\right\}$

be the MOD natural neutrosophic interval special dual like number matrix pseudo semiring.

Study questions (i) to (viii) of problem (54) for this M.

60. Let $J = \left\{\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \end{bmatrix} \middle| \; a_i \in \langle [0,15) \cup h\rangle_I, 1 \leq i \leq 10, +, \times_n\right\}$

be the MOD pseudo natural neutrosophic special dual like interval semiring.

Study questions (i) to (viii) of problem (54) for this J.

61. Let P = $\left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in Z_{27}^{I}, +, \times_n\right\}$ be the MOD natural neutrosophic coefficient polynomial semring.

62. Study questions (i) to (viii) of problem (54) for this semiring.

    i) Prove it is impossible to have finite order subring or subsemirings.
    ii) Prove this semiring is not pseudo.

63. Let B = $\left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in Z_{48}^{I}, +, \times_n\right\}$ be the MOD natural neutrosophic coefficient polynomial semiring.

    i) Prove B is commutative and is of infinite order.
    ii) Prove B has no subsemirings of finite order.
    iii) Prove B has no subrings of finite order.
    iv) Prove all ideals of B are of infinite order and they are not subrings.
    v) Prove B can h ave zero divisors but cannot have nilpotents or zero divisors.
    vi) Prove Z_{48}, Z_{48}^{I} and subrings of Z_{48} are subrings which are subsemirings of finite order but they are not polynomial subsemiring.

    [Note by subsemirings in polynomial semirings we mean that they must be also polynomial semirings].

64. Let W = $\left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in \langle Z_{12} \cup g\rangle_I, +, \times\right\}$ be the MOD natural neutrosophic dual number coefficient polynomial semiring.

i) Study questions (i) to (v) of problem 64 for this W.
ii) Prove W can have zero square subrings of finite order.
iii) Prove also W can have subsemirings of infinite order which are subrings.
iv) Show this W or in general MOD natural neutrosophic dual coefficient polynomial semirings behave differently from other MOD natural neutrosophic polynomial coefficient semirings.

65. Let S = $\left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in \langle Z_{24} \cup I\rangle_I, +, \times_n\right\}$ be the MOD natural neutrosophic-neutrosophic polynomial coefficient semiring.

Study questions (i) to (iv) of problem (64) for this S.

i) Compare S with W of problem (64).

66. Let V = $\left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in \langle Z_{19} \cup h\rangle_I, +, \times_n\right\}$ be the MOD natural neutrosophic special dual like number coefficient polynomial semiring.

Study questions (i) to (iv) of problem (64) for this V.

67. Let B = $\left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in \langle Z_{48} \cup k\rangle_I, +, \times_n\right\}$ be the MOD natural neutrosophic special quasi dual number coefficient polynomial semiring.

Study questions (i) to (iv) of problem (64) for this B.

68. Let T = $\left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in \langle Z_{493} \cup k\rangle_I, +, \times\right\}$ be the MOD natural neutrosophic special quasi dual number coefficient polynomial semiring.

Study questions (i) to (iv) of problem (64) for this T.

69. Let M = $\left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in C^I(Z_{48}), +, \times_n\right\}$ be the MOD natural neutrosophic finite complex number coefficient polynomial semiring.

Study questions (i) to (iv) of problem (64) for this M.

70. Let S = $\left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in {}^I[0,8), +, \times_n\right\}$ be the MOD natural neutrosophic coefficient polynomial interval pseudo semiring.

i) Study questions (i) to (iv) of problem (64) for this S.
ii) Prove there are subsemirings of infinite order which are not pseudo.

71. Let T = $\left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in {}^I[0,23), +, \times_n\right\}$ be the MOD natural neutrosophic interval coefficient polynomial pseudo semiring.

Study questions (i) to (iv) of problem (64) for this T.

72. Let V = $\left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in C^I([0,42)); +, \times_n\right\}$ be the MOD interval neutrosophic finite complex number pseudo semiring.

Study questions (i) to (iv) of problem (64) for this V.

73. Let W = $\left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in \langle [0,40) \cup I\rangle_I, +, \times\right\}$ be the MOD interval natural neutrosophic-neutrosophic polynomial coefficient pseudo semiring.

Study questions (i) to (iv) of problem (64) for this W.

74. Let x = $\left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in \langle [0,24) \cup g\rangle_I, +, \times\right\}$ be the MOD natural neutrosophic dual number coefficient polynomial interval pseudo semiring.

Study questions (i) to (iv) of problem (64) for this ×.

75. Let M = $\left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in \langle [0,15) \cup h\rangle_I, +, \times\right\}$ be the MOD natural neutrosophic interval special dual like number coefficient polynomial semiring.

Study questions (i) to (iv) of problem (64) for this M.

76. Let S = $\left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in \langle [0,24) \cup k\rangle_I, +, \times\right\}$ be the MOD natural neutrosophic special quasi dual like number coefficient polynomial interval pseudo semiring.

Study questions (i) to (vi) of problem (63) for this S.

77. Let B = $\left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in Z_{42}^{I}, +, \times, x^{25} = 1\right\}$ be the MOD natural neutrosophic coefficient polynomial semiring of polynomials of degree less than or equal to 24.

i) Find o(B).
ii) Find all subsemirings which are subrings of B.
iii) Find all subsemirings which are not subrings.
iv) Can ideals of B be a subring?
v) Can B have zero divisors?
vi) Can B have idempotents?
vii) Can B have nilpotents?
viii) Find all MOD neutrosophic coefficient subsemirings.

78. Let S = $\left\{\sum_{i=0}^{9} a_i x^i \,\middle|\, a_i \in Z_{23}^{I},\ x^{10} = 1, +, \times\right\}$ be the MOD natural neutrosophic coefficient polynomial semiring of degree less than or equal to 9.

    Study questions (i) to (viii) of problem (77) for this S.

79. Let M = $\left\{\sum_{i=0}^{18} a_i x^i \,\middle|\, a_i \in \langle Z_{14} \cup I\rangle_I,\ x^{19} = 1, +, \times\right\}$ be the MOD natural neutrosophic neutrosophic coefficient polynomial semiring of degree less than or equal to 19.

    i) Study questions (i) to (viii) of problem (77) for this M.
    ii) Obtain all the special and distinct features enjoyed by this M.

80. Let V = $\left\{\sum_{i=0}^{5} a_i x^i \,\middle|\, a_i \in C^{I}(Z_{24}),\ x^{6} = 1, +, \times\right\}$ be the MOD natural neutrosophic finite complex number coefficient polynomial semiring.

    i) Study questions (i) to (viii) of problem (77) for this V.
    ii) Enumerate all special features associated with V.

81. Let W = $\left\{\sum_{i=0}^{10} a_i x^i \,\middle|\, a_i \in \langle Z_{16} \cup g\rangle_I,\ x^{11} = 1, +, \times\right\}$ be the MOD natural neutrosophic dual number coefficient polynomial semiring.

    i) Study questions (i) to (viii) of problem (77) for this W.
    ii) Determine all the special features associated with this W.

82. Let T = $\left\{\sum_{i=0}^{20} a_i x^i \,\middle|\, a_i \in \langle Z_{21} \cup h\rangle_I,\ x^{21} = 1, +, \times\right\}$ be the MOD natural neutrosophic special dual like number coefficient polynomial semirng.

    Study questions (i) to (viii) of problem (77) for this T.

83. Let R = $\left\{\sum_{i=0}^{11} a_i x^i \,\middle|\, a_i \in \langle Z_4 \cup k\rangle_I,\ x^{12} = 1, +, \times\right\}$ be the MOD natural neutrosophic special quasi dual number coefficient polynomial semiring.

    Study questions (i) to (viii) of problem (77) for this R.

84. Let B = $\left\{\sum_{i=0}^{8} a_i x^i \,\middle|\, a_i \in {}^{I}[0,42), +, \times, x^9 = 1\right\}$ be the MOD natural neutrosophic interval coefficient polynomial pseudo semiring, which has polynomials to be of degree less than or equal to 8.

    i) Prove o(B) = ∞ and B is a commutative semiring.
    ii) Prove B has subsemirings of finite order wich are subrings.
    iii) Prove B has subsemirings of infinite order which are not subrings.
    iv) Prove B has subsemirings of infinite order which are pseudo subrings.
    v) Prove B has subsemirings of infinite order which are not subrings but are pseudo subsemirings.
    vi) Prove all ideals of B are only pseudo subsemirings of infinite order.

85. Let D = $\left\{\sum_{i=0}^{6} a_i x^i \,\middle|\, a_i \in \langle [0,7) \cup g\rangle_I,\ x^7 = 1, +, \times\right\}$ be the MOD natural neutrosophic interval dual number coefficient polynomial pseudo semiring.

Study questions (i) to (vi) of problem (84) for this D.

86. Let M = $\left\{\sum_{i=0}^{11} a_i x^i \,\middle|\, a_i \in \langle [0, 27) \cup h\rangle_I, x^{12} = 1, +, \times\right\}$ be the MOD natural neutrosophic interval special dual like number coefficient polynomial pseudo semiring.

Study questions (i) to (vi) of problem (84) for this M.

87. Let N = $\{\sum_{i=0}^{4} a_i x^i$ / $a_i \in \langle [0,9 \cup k\rangle_I, x^8 = 1, +, \times\}$ be the MOD natural neutrosophic interval special dual like number coefficient polynomial pseudo semiring.

Study questions (i) to (vi) of problem (84) for this N.

88. Let W = $\left\{\sum_{i=0}^{23} a_i x^{23} \,\middle|\, a_i \in C^I ([0,42)), x^{24} = 1, +, \times\right\}$ be the MOD natural neutrosophic finite complex number coefficient polynomial interval pseudo to semiring.

i) Study questions (i) to (vi) of problem (84) for this W.
ii) Enumerate all special and distinct features associated in W and compare them with other semirings.

89. Let V = $\left\{\sum_{i=0}^{12} a_i x^i \,\middle|\, a_i \in \langle [0, 24), x^{13} = 1, +, \times\right\}$ be the MOD natural neutrosophic neutrosophic interval coefficient polynomial pseudo semiring.

i) Study questions (i) to (vi) of problem (84) for this V.
ii) Distinguish and differential this V with other semirings built using $C^I([0,n))$, $\langle [0,n) \cup g\rangle_I$ $\langle [0,n) \cup h\rangle_I$ and so on.

Chapter Two

MOD SUBSET PSEUDO SEMIRINGS AND MOD NATURAL NEUTROSOPHIC SUBSET PSEUDO SEMIRINGS

The notion of MOD subset semigroups and MOD natural neutrosophic subset semigroups have been introduced in [66]. The authors have dealt with these two types of MOD subset semigroups as well as their related properties.

Infact they have dealt with two types of products. In this chapter we for the first time define MOD subset pseudo semirings and MOD subset natural neutrosophic pseudo semirings.

It is important to record at this juncture that we call them pseudo as $A \times B + A \times C \neq A \times (B + C)$ in general where A, B, C are subsets of Z_n or $C(Z_n)$ or so on Z_n^I or $C^I(Z_n)$ or $(\langle Z_n \cup g\rangle_I$ and so on.

We will illustrate first these situations by some examples.

Example 2.1: Let $S = \{S(Z_6), +, \times\}$ be the MOD subset pseudo semiring built on $S(Z_6)$.

Let $A = \{3, 4, 0, 2\}$ and $B = \{2\}$ and $C = \{4\} \in S$.

$$A \times (B + C) = \{3, 4, 0, 2\} \times (\{2\} + \{4\})$$
$$= \{3, 4, 0, 2\} \times \{0\} = \{0\} \qquad \text{I}$$

Now

$$A \times B + A \times C = \{3, 4, 0, 2\} \times \{2\} + \{3, 4, 0, 2\} \times \{4\}$$

$$= \{0, 2, 4\} + \{0, 4, 2\} = \{0, 4, 2\} \qquad \text{II}$$

Clearly I and II are distinct so the distributive law is not true on S.

That is why we choose to call these MOD subset semirings as pseudo MOD subset semirings.

Clearly P = {collection of all subsets from {0, 2, 4}, +, ×} ⊆ S is a MOD subset subsemiring; which is also an ideal.

Let A = {0, 2, 5, 3} and B = {1, 2, 4} ∈ S,

$$A + B = \{0, 2, 5, 3\} + \{1, 2, 4\}$$
$$= \{1, 3, 0, 4, 2, 5\} = Z_6.$$

So A and B are MOD universal subsets of S.

$$A \times B = \{0, 2, 5, 3\} \times \{1, 2, 4\} = \{0, 2, 5, 3, 4\} \neq Z_6.$$

Suppose we have MOD subsets A, B ∈ S such that A + B = $\{Z_6\}$ and A × B = $\{Z_6\}$ then we call A, B the MOD subset pair to be a MOD universal subset pair of S.

If only A + B = $\{Z_6\}$ or A × B = $\{Z_6\}$ we call A, B the MOD subset semi universal pair of S.

Let A = {0, 4, 2} ∈ S,

$$A \times A = \{0, 2, 4\} \times \{0, 2, 4\} = \{0, 2, 4\} = A.$$

Thus A is an MOD subset idempotent of S.

Let A = {0, 3} $\in$ S,

$A \times B = \{0, 3\} \times \{0, 3\} = \{0, 3\} = A \in S$ is also a MOD subset idempotent of S.

Let A = {0, 4, 2} and B = {0, 3} $\in$ S, $A \times B = \{0\}$ is the MOD subset zero of S.

Example 2.2: Let M = {collection of all subsets from Z_7, +, $\times$} = {$S(Z_7)$, +, $\times$} be the MOD subset pseudo semiring.

Let A = {3, 5, 2}, B = {3} and C = {4} $\in$ M.

Is $A \times (B + C) = A \times B + A \times C$?

$$\begin{aligned} A \times B + A \times C &= \{3, 5, 2\} \times \{3\} + \{5, 3, 2\} \times \{4\} \\ &= \{2, 1, 6\} + \{1, 6, 5\} \\ &= \{3, 2, 0, 1, 5, 6\}. \qquad \text{I} \end{aligned}$$

$$\begin{aligned} A \times (B + C) &= \{3, 5, 2\} \times (\{3\} + \{4\}) \\ &= \{3, 5, 2\} + 0 = 0. \qquad \text{II} \end{aligned}$$

I and II are not equal hence we claim M is only a MOD subset pseudo semiring.

This M has no MOD zero divisor pair. M has only trivial MOD idempotents. M has no nontrivial MOD nilpotents.

This has P = {{0}, {0, 1, 2, 3, 4, 5, 6}) is a MOD subset subsemiring as well as ideal of M.

In view of all these we prove the following result.

THEOREM 2.1: *Let S = {collection of all subsets from Z_n,$\times$, +} = {$S(Z_n)$, +, $\times$} be the MOD subset pseudo semiring.*

i) $o(S) < \infty$
ii) S has also MOD subset ideals and MOD subset pseudo subsemirings.
iii) S has zero divisors, idempotents and nilpotents only if n is a non prime and must be special type of the form $n = p^t q$, p a prime $t \geq 2$ $q \neq p$ but can be a prime.

Proof is direct and hence left as a exercise to the reader.

Example 2.3: Let W = {collection of all subsets from Z_{12}, +, ×} = {$S(Z_{12})$, +, ×} be the MOD pseudo subset semiring.

This W has MOD subset zero divisor, MOD subset idempotents and MOD subset nilpotents.

W has MOD subset pseudo ideals and MOD subset pseudo subsemirings.

Next we consider using $S(C(Z_n))$, which will be illustrate by examples.

Example 2.4: Let P = {$S(C(Z_{10})$, +, ×} be the MOD subset finite complex number pseudo semiring.

Let A = {0, 5, 6, 2}, B = {3} and C = {7} ∈ P.

$$\begin{aligned} A \times (B + C) &= \{0, 5, 6, 2\} \times (\{3\} + \{7\}) \\ &= \{0, 5, 2, 6\} \times \{0\} = \{0\}. \qquad \text{I} \end{aligned}$$

$$\begin{aligned} A \times B + A \times C &= \{0, 2, 5, 6\} \times \{3\} + \{0, 2, 5, 6\} \times \{7\} \\ &= \{0, 6, 5, 8\} + \{0, 4, 5, 2\} \\ &= \{0, 6, 5, 8, 4, 9, 2, 1, 3, 7\} \\ &= \{Z_{10}\}. \qquad \text{II} \end{aligned}$$

We see I and II are distinct so P is only a MOD subset pseudo semiring.

Let T = {5, 0, 1} ∈ P.

We see $T \times T = \{0, 1, 5\} = T$.

Thus T is an idempotent MOD subset of P.

$L = \{0, 1, 6\} \in P$ is such that
$L \times L = L$ is a MOD idempotent subset.

$B = \{0, 1, 6, 5\} \in P$ is such that
$B \times B = B$ is a MOD idempotent subset of P.

Let $W = \{5, 0, 5i_F + 5, 5i_F\}$ and
$V = \{2, 2 + 2i_F, 8i_F + 4, 4i_F, 4i_F + 2, 8 + 2i_F\} \in P$,
we see $W \times V = \{0\}$ is the MOD subset zero divisor of P.

Thus this P the MOD subset pseudo finite complex number semiring has zero divisors and idempotents.

However P does not contain nilpotents.

Example 2.5: Let $D = \{S(C(Z_{16}), \times, +\}$ be the MOD subset finite complex number pseudo semiring.

This has MOD subset idempotents, nilpotents and zero divisors. D has MOD subset subsemirings as well as ideals.

Example 2.6: Let $E = \{S(C(Z_{17})), +, \times\}$ be the MOD subset pseudo finite complex number semiring.

This has no zero divisors or nilpotents or idempotents.

Infact the task of proving this is left as an exercise to the reader. But E has ideals as well as subsemirings.

Thus tasks is considered as a matter of routine so left as an exercise to the reader.

Now we see the result of theorem 2.1 is true if $S(Z_n)$ is replaced by $S(C(Z_n))$.

Next we proceed onto describe MOD neutrosophic subset pseudo semirings by examples.

Example 2.7: Let M = $\{S(\langle Z_{13} \cup I\rangle) +, \times\}$ be the MOD subset neutrosophic pseudo semiring.

Clearly $\{S(Z_{13}), + \times\} \subseteq M$ is a MOD subset pseudo subsemiring of M which is not an ideal of M.

$W = \{S(Z_{13}I), +, \times\} \subseteq M$ is a MOD subset pseudo subsemiring which is an ideal of M.

Thus has zero divisors of the form $P_1 = \{10 + 3h, 12 + h, 10h + 3, 7 + 6h\}$ and $P_2 = \{h\}$ is such that $P_1 \times P_2 = \{0\}$.

However finding idempotents and nilpotents of M is a challenging task.

Example 2.8: Let N = $\{S(\langle Z_{12} \cup I\rangle), +, \times\}$ be the MOD subset pseudo neutrosophic semiring.

This N has zero divisors, nilpotents and idempotents. Further N has both MOD pseudo subset subsemirings which are not ideals as well as ideals.

Those task is left as an exercise to the reader.

This can be proved for $\{S(\langle Z_n \cup I\rangle), +, \times\}$ by replacing $S(Z_n)$ by $S(\langle Z_n \cup I\rangle)$ with appropriate modifications in case of zero divisors.

Next we describe MOD dual number subset pseudo semirings by some examples.

Example 2.9: Let A = $\{S(\langle Z_7 \cup g\rangle), +, \times\}$ be the MOD dual number subset pseudo semiring. A has zero divisors and nilpotents.

However as Z_7 is used it is difficult to find idempotents.

A has MOD subset subsemirings which are not pseudo.

For take B = {S(Z_7g), + ×} ∈ A is a MOD subset subsemiring which is not pseudo as a × b = {0} for all a, b ∈ B.

This special feature is enjoyed only by MOD subset dual number pseudo semiring.

Further S(⟨Z_p ∪ g⟩), p a prime has zero divisors and nilpotents. This is yet another striking feature of MOD dual number subset pseudo semiring.

Example 2.10: Let W = {S(⟨Z_{48} ∪ g⟩) +, ×} be the MOD subset pseudo semiring. W has zero divisors and nilpotents of order two.

This has MOD subset idempotents also.

W has a MOD subset zero square subsemiring I which is also an ideal such that I × I = {0}.

Working with this W happens to be interesting.

In view of this we have the following result.

THEOREM 2.2: *Let B = {S(⟨Z_n ∪ g⟩), +, ×} be the MOD subset dual number pseudo semiring.*

i) o(B) < ∞.
ii) B has MOD nilpotents subsets of order two whatever be n.
iii) B has MOD zero divisor subsets whatever be n.
iv) B has MOD subset subsemiring which is an ideal and which is not pseudo but which is a zero square MOD subsemiring whatever be n.
v) B has MOD subset subsemirings as well as ideals which are pseudo.

Existence of ideals is dependent on n.

Proof is direct and hence left as an exercise to the reader.

Next we proceed onto describe MOD subset special dual like number pseudo semirings by some examples.

Example 2.11: Let T = $\{S(\langle Z_{15} \cup h\rangle), +, \times\}$ be the MOD subset special dual like number pseudo semiring of finite order. T has MOD subset zero divisors.

$P_1 = \{S(Z_{15}), \times, +\} \subseteq T$ is a MOD subset special dual like number pseudo subsemiring which is not an ideal.

$P_2 = \{S(Z_{15}h), +, \times\} \subseteq T$ is a MOD subset special dual like number pseudo subsemiring which is an ideal.

$P_3 = \{S(\langle\{0, 3, 6, 9, 12\} \cup h\rangle), +, \times\} \subseteq T$ is a MOD subset pseudo special dual like number subsemiring which is also an ideal of T.

However T has MOD subset idempotents but has no non trivial MOD subset ideals.

Example 2.12: Let W = $\{S(\langle Z_{11} \cup h\rangle), +, \times\}$ be the MOD subset special dual like number pseudo semiring. W has no nontrivial MOD subset nilpotents but has MOD non nontrivial subset idempotents.

P = $\{0, 1, h\} \in W$ is such that $P \times P = P$ which we choose to call as trivial MOD subset idempotent.

M = $\{0, h\} \times W$ is such that $M \times M = M$ and $V \times V = V$ where $V = \{1, h\}$ and so on. All these are considered as MOD subset trivial idempotents of W.

Let $B_1 = \{S(Z_{11}), +, \times\}$ be the MOD special dual like number subset pseudo subsemiring.

Clearly B_1 is not an ideal.

$B_2 = \{S(Z_{11}h), +, \times\} \subseteq W$ is a MOD special dual like number subset pseudo subsemiring which is also an ideal of W.

However single element set zero divisors of this form are possible.

Let B = {10 + h, 10h + 1, 8h + 3, 3 + 8h, 6 + 5h, 5 + 6 h} and C = {h}. Clearly B × C = {0}, is a nontrivial zero divisors.

We can call these MOD subset zero divisors also as MOD subset h induced zero divisors.

Example 2.13: Let $W = \{S(\langle Z_{32} \cup h\rangle), +, \times\}$ be the MOD subset special dual like number pseudo semiring.

Clearly W has nontrivial idempotents. P = {8h, 8 + 8h, 8, 16, 16 + 8h 16h, 16h + 8} ∈ W is such that P × P = {0} is a MOD subset special dual like number nilpotent of order two.

Let V = {2, 2h, 4, 4h, 2 + 2h} ∈ W.

$$\begin{aligned} V \times V &= \{2, 2h, 4, 4h, 2 + 2h\} \times \{2, 2h, 4, 4h, 2 + 2h\} \\ &= \{4, 4h, 8, 8h, 4 + 4h, 16h, 8 + 8h, 4 + 12h\} = V^2. \end{aligned}$$

$$\begin{aligned} V^2 \times V &= \{4, 4h, 8, 8h, 4 + 4h, 16h, 8 + 8h, 4 + 12h\} \times \\ &\qquad \{2, 2h, 4, 4h, 2 + 2h\} \\ &= \{8, 8h, 8 + 8h, 16, 16h, 16 + 16h, 0, 8 + 24h, \\ &\qquad 16 + 16h\} \\ &= V^3. \end{aligned}$$

$$\begin{aligned} V^3 \times V &= \{8, 8h, 8 + 8h, 16, 16h, 16 + 16h, 8 + 24h, \\ &\qquad 16 + 16h\} \times \{2, 2h, 4, 4h, 2 + 2h\} \\ &= \{16, 16h, 16 + 16h, 0\} = V^4. \end{aligned}$$

$$\begin{aligned} V^4 \times V &= \{16, 16h, 16 + 16h, 0\} \times \{2, 2h, 4, 4h, 2 + 2h\} \\ &= \{0\} = V^5. \end{aligned}$$

Thus V is a MOD special dual like number subset nilpotent of order 5.

Clearly $\{S(Z_{32}h), +, \times\} = D$ is a MOD subset special dual like number pseudo ideal of D.

Now $E = \{S(Z_{32}), +, \times\}$ is a MOD subset special dual like number pseudo subsemirings of W which is not an ideal of W.

However W has MOD subset zero divisors which are not nilpotents of W.

For $P_1 = \{16, 16h, 16 + 16h, 8 + 8h, 8 + 16h, 16 + 8h\}$ and $P_2 = \{4, 4 + 4h, 8h + 8, 16\} \in W$ is such that $P_1 \times P_2 = \{0\}$ is a MOD subset zero divisor of W.

In view of all these we have the following result.

THEOREM 2.3: *Let $V = \{S(\langle Z_n \cup h\rangle, +, \times\}$ be the MOD subset special dual like number subset pseudo semiring.*

i) $o(V) < \infty$.
ii) V has MOD subset special dual like number pseudo subsemirings which are not ideals for any n, $2 \leq n < \infty$.
iii) V has MOD subset special dual like number pseudo subsemirings which are ideals for any n, $2 \leq n < \infty$.
iv) V has MOD subset zero divisors for any n; $2 \leq n < \infty$.
v) V has only trivial MOD subset idempotents if n is a prime,
vi) V has only trivial MOD subset nilpotents if n is a prime.
vii) If $n = p^{\alpha}q$, $\alpha \geq 2$; p and q primes or $p \times q$, q any number then V has both MOD nontrivial idempotents and nilpotents.

Proof is direct and hence left as an exercise to the reader.

Next we proceed onto describe MOD subset special quasi dual number pseudo semiring by examples and derive some special properties about them.

Example 2.14: Let S = $\{S(\langle Z_7 \cup k\rangle) \times, +\}$ be the MOD subset special quasi dual number pseudo semiring. S has MOD zero divisor subsets.

For Y = $\{2 + 2k, 3 + 3k, k + 1, 0, 6 + 6k, 5 + 5k\}$ and T = $\{k\} \in$ S.

$$\begin{aligned} T \times Y &= \{k\} \times \{2 + 2k, 3 + 3k, 1 + k, 6 + 6k, 5 + 5k\} \\ &= \{2k + 2k^2, 3k + 3k^2, k + k^2, 6k + 6k^2, 5k + 5k^2\} \\ &\quad \text{(using } k^2 = 6k) = \{0\}. \end{aligned}$$

Thus we can have MOD subset zero divisors of this type.

However we are not able to find non trivial MOD subset idempotents.

We can have MOD subset ideals and subsemirings both of them are only pseudo.

Finding nontrivial MOD nilpotents subsets is a challenging job.

Example 2.15: Let B = $\{S(\langle Z_{12} \cup k\rangle), +, \times\}$ be the MOD subset special quasi dual number pseudo semiring.

B has MOD subset zero divisors, MOD subset idempotents and MOD subset nilpotents.

B also has MOD subset special quasi dual numbers pseudo subsemirings which are not ideals as well as subsemirings which are ideals.

In view of all these we have the following result.

THEOREM 2.4: *Let M = {S(⟨$Z_n \cup k$⟩), +, ×} be the MOD subset special quasi dual number pseudo semiring.*

i) o(M) < ∞.
ii) M has MOD subset zero divisors of special type if n = p, a prime otherwise has MOD subset zero divisors.
iii) Whatever be n M has MOD subset special quasi dual number subset subsemirings which are not ideals as well as subsemirings which are ideals.
iv) M has MOD subset idempotents and nilpotents only if n = p^tq; (p, q) = 1, t ≥ 2; p is a prime q any number.

Proof is direct and hence left as an exercise to the reader.

Next we proceed onto describe MOD subset semirings built using the MOD natural neutrosophic elements.

We will describe them by examples.

Example 2.16: Let B = {S(Z_{18}^I), +, ×} be the MOD natural neutrosophic subset pseudo semiring.

This has two types of products defined on B, we can have product in which zero is dominant that is 0 × I_0^t = 0, but however we use only in this book use MOD natural neutrosophic dominant product only.

Clearly this MOD natural neutrosophic subset semiring has idempotents with respect to + also.

We see B has MOD subset natural neutrosophic nilpotents as well as MOD subset natural neutrosophic idempotents.

This has MOD subset natural neutrosophic subsemirings.

$A_1 = \{ I_9^{18}, I_3^{18}, I_{12}^{18}, I_0^{18} \}$ and $A_2 = \{ I_6^{18}, I_0^{18} \} \in$ B.

$A_1 \times A_2 = \{0\}$ is a MOD subset natural neutrosophic zero divisor.

$Y = \{ I_9^{18}, I_0^{18} \} \in B$, $Y^2 = Y \times Y = Y$.

Thus Y is a MOD subset natural neutrosophic idempotent.

For MOD subset natural neutrosophic nilpotents for take D = $\{ I_6^{18}, I_0^{18} \} \in B$, $D \times D = \{0\}$ is a MOD subset natural neutrosophic nilpotent of order two.

M = {S({0, 3, 6, 9, 12}), +, ×} is a MOD subset natural neutrosophic subsemiring which is also a MOD subset natural neutrosophic ideal.

Example 2.17: Let W = $\{S(Z_{13}^{I}); +, \times\}$ be the MOD natural neutrosophic subset pseudo semiring.

Finding if W has no nontrivial MOD natural neutrosophic subset idempotent, nilpotents and zero divisors is a difficult task.

Has MOD natural neutrosophic subset pseudo subsemirings.

In view of all these we have the following theorem.

THEOREM 2.5: *Let W = $\{S(Z_n^{I}), +, \times\}$ be the MOD natural neutrosophic subset pseudo semiring.*

i) W has nontrivial MOD subset natural neutrosophic zero divisors, idempotents and nilpotents only when n is a non prime and n is in the appropriate form.
ii) W has MOD natural neutrosophic subset subsemirings which are not ideals as well as subsemirings which are ideals.
iii) $o(W) < \infty$.

Proof is direct and hence left as an exercise to the reader.

Next we proceed onto describe briefly the notion of MOD natural neutrosophic finite complex number subset semirings by some examples.

Example 2.18: Let M = $\{S(C^I(Z_{12})), +, \times\}$ be the MOD subset natural neutrosophic finite complex number pseudo semiring.

It is easily verified M has MOD subset natural neutrosophic finite complex number idempotents, nilpotents and zero divisors which are nontrivial. M also has additive MOD subset idempotents.

We see M has MOD subset natural neutrosophic finite complex number pseudo subsemirings which are not ideals as well as subsemirings which are ideals.

Take $A = \{I_0^C + I_2^C + I_4^C + I_6^C + I_8^C + I_{10}^C\} \in M$ is such that

$A + A = A.$

$$\begin{aligned} A \times A &= \{I_0^C + I_2^C + I_4^C + I_6^C + I_8^C + I_{10}^C\} \times \\ &\qquad \{I_0^C + I_2^C + I_4^C + I_6^C + I_8^C + I_{10}^C\} \\ &= \{I_0^C + I_4^C + I_8^C + I_6^C\} \neq A. \end{aligned}$$

So A is not a MOD subset idempotent with respect to product.

Let $B = \{I_0^C + I_4^C + I_8^C\} \in M;$

$B \times B = \{I_0^C + I_4^C + I_8^C\} = B.$

So B is a MOD subset idempotent with respect to product.

Consider B + B = B so B is a MOD subset idempotent with respect to +.

Such type of MOD subset idempotents will be called as double MOD subset idempotents or strong MOD subset idempotents of M.

Let P = $\{Z_{12}\} \in$ M, clearly P + P = P and P × P = P so P is a MOD subset strong idempotent of M.

Infact B = $\{ I_0^C + I_6^C + I_4^C + I_8^C + I_2^C + I_{10}^C + I_3^C + I_9^C \} \in$ M.

B + B = B so B is a MOD subset idempotent of M under +.

However B × B = $\{ I_0^C + I_2^C + I_4^C + I_6^C + I_8^C + I_9^C + I_{10}^C + I_3^C \} \times \{ I_0^C + I_2^C + I_4^C + I_6^C + I_8^C + I_9^C + I_{10}^C + I_3^C \} = \{ I_0^C + I_4^C + I_8^C + I_6^C + I_2^C + I_9^C + I_3^C \} \neq$ B so is not a MOD subset idempotent under ×.

Let us consider W = $\{S(\langle I_0^C, I_2^C, I_4^C, I_6^C, I_{10}^C, I_9^C, I_3^C \rangle), +, \times\}$ be the MOD subset finite complex number subsemiring which is also an ideal of M.

Consider L = $\{S(Z_{12}), +, \times\} \subseteq$ M, clearly L is a MOD subset finite complex number pseudo subsemiring of M and is not an ideal of M.

N = $\{(Z_{12}^I), +, \times\} \subseteq$ M is also a MOD subset finite complex number pseudo subsemiring which is not an ideal of M.

Consider A = $\{S(\langle 0, 3, 6, 9, I_0^C, I_3^C, I_9^C, I_6^C \rangle), +, \times\}$ be MOD subset finite complex number pseudo subsemiring. This is also not an ideal of M.

$B = \{S(\langle 0, 3, 7, 9, I_0^C, I_3^C, I_9^C, I_9^C, I_{6i_F}^C, I_{3+3i_F}^C, I_{6+6i_F}^C, I_{9+9i_F}^C, I_{9+3i_F}^C, \ldots, I_{6+9i_F}^C \rangle) +, \times\} \subseteq M$ is a MOD subset finite complex number pseudo subsemiring which is also an ideal of M.

Thus the reader is left with the task of finding MOD finite complex number pseudo ideals, MOD subset pseudo subsemirings, MOD subset idempotents MOD subset nilopotents and MOD subset zero divisors.

Example 2.19: Let $M = \{S(C^I(Z_2)), \times, +\}$ be the MOD subset finite complex number pseudo semiring. $C^I(Z_2) = \{\langle 0, 1, i_F, 1 + i_F, I_0^C, I_{1+i_F}^C \rangle\}$.

$(C^I(Z_2)) = \{0, 1, i_F, 1 + i_F, 1 + I_0^C, I_0^C, I_{1+i_F}^C, i_F + I_0^C, 1 + i_F + I_0^C, i_F + I_0^C + I_{1+i_F}^C, i_F + 1 + I_0^C + I_{1+i_F}^C, 1 + I_{1+i_F}^C, 1 + i_F + I_{1+i_F}^C, I_0^C + I_{1+i_F}^C, 1 + I_0^C + I_{i_F}^C, i_F + I_{1+i_F}^C\}$.

We can find $S(C^I(Z_2))$.

$A = \{1 + i_F, I_{1+i_F}^C, 1 + i_F + I_{1+i_F}^C, 0, I_0^C\} \in M,$

$$\begin{aligned} A \times A &= \{0, I_0^C, 1 + i_F, I_{1+i_F}^C, 1 + i_F, I_{1+i_F}^C\} \times \{0, I_0^C, 1 + i_F, \\ &\qquad I_{1+i_F}^C, 1 + i_F, + I_{1+i_F}^C\} \\ &= \{0, I_0^C, I_{1+i_F}^C, I_{1+i_F}^C + I_0^C\} \neq A. \end{aligned}$$

$A + A = \{0, I_0^C, 1 + i_F + I_0^C, 1 + i_F + I_{1+i_F}^C + I_{1+i_F}^C\} = A.$

However we have both strong MOD idempotents as well as MOD idempotents with respect to + and $\times$.

For $P = \{Z_2\} \in M$ is such that $P \times P = P$ and $P + P = P$. Thus P is a MOD subset strong idempotent of M.

We see P can have MOD subset idempotents as well as MOD strong subset idempotents. So study in this direction is interesting and innovative.

In view of all these we have the following theorem.

THEOREM 2.6: *Let $W = \{S(C^I(Z_n)), +, \times\}$ be the MOD subset finite complex number pseudo semiring.*

i) $o(W) < \infty$.
ii) W has both MOD subset finite complex number pseudo subsemirings which are ideals as well as subsemirings which are not ideals.
iii) W has both MOD subset idempotents as well as strong MOD subset idempotents.
iv) W has MOD subsets nilpotents and zero divisors mainly depending on the n.

Proof is direct and hence left as an exercise to the reading.

Next we proceed onto describe MOD subset natural neutrosophic - neutrosophic pseudo semirings by examples.

Example 2.20: Let $S = \{S(\langle Z_{18} \cup I\rangle_I), +, \times\}$ be the MOD natural neutrosophic-neutrosophic pseudo semiring.

Clearly S is a commutative semiring of finite order. S has MOD subset zero divisor and nilpotents. S has also MOD subset idempotents.

For $A = \{I, 0, 9, 9I\} \in S$ is such that $A \times A = A$.

For $W_1 = \{S(Z_{18}I), +, \times\}$ is a MOD subset natural neutrosophic-neutrosophic pseudo semiring which is also an ideal of S.

$W_2 = \{S(Z_{18}), +, \times\}$ is a MOD subset natural neutrosophic - neutrosophic pseudo subsemiring of S which is an ideal of S.

Interested reader is left with the task of finding the special features of S.

Example 2.21: Let P = $\{S(\langle Z_5 \cup I\rangle_I, +, \times\}$ be the MOD subset natural neutrosophic - neutrosophic pseudo semiring P has MOD subset natural neutrosophic - neutrosophic pseudo subsemiring which is not an ideal as well as MOD subsemiring which ideals.

Finding nontrivial zero divisors, idempotents and nilpotents in P happens to be a difficult job.

In view of all these we have the following result.

THEOREM 2.7: *Let S = $\{S(\langle Z_n \cup I\rangle_I, +, \times\}$ be the MOD natural neutrosophic - neutrosophic subset pseudo semiring.*

- *i) $o(S) < \infty$.*
- *ii) S has nontrivial MOD subset nilpotents, MOD subset idempotents and MOD subset zero divisors only for n = $p^\alpha q$; $\alpha \geq 2$, p a prime (p, q) = 1.*
- *iii) For all n S has MOD natural neutrosophic - neutrosophic subset pseudo subsemirings which are not ideals as well as pseudo subsemirings which are ideals.*
- *iv) S has MOD strong subset idempotents as well as MOD subset idempotents.*

Proof is direct and hence left as an exercise to the reader.

Next we proceed onto describe MOD natural neutrosophic dual number subset pseudo semirings by examples.

Example 2.22: Let B = $\{S(\langle Z_{10} \cup g\rangle), +, \times\}$ be the MOD natural neutrosophic dual number subset pseudo semiring.

B has several MOD natural neutrosophic subset zero divisors and nilpotents.

Let A = {5g, 6g, 3g, 2g} and

$D = \{0, 5g, 7g, 9g, g, 8g\} \in B$.

Clearly $A \times D = \{0\}$.
$A \times A = \{0\}$ and $D \times D = \{0\}$.

However $A + A \neq \{0\}$.

Let $S = \{ I_g^g, I_0^g, I_{4g}^g, I_{7g}^g, 0\}$ and
$R = \{ I_{2g}^g, I_{9g}^g, I_{8g}^g, I_g, 0, I_{4g}^g \} \in S$.

$S \times R = \{ I_0^g, 0\}$ $R \times R = \{ I_0^g, 0\}$ and $S \times S = \{ I_0^g, 0\}$. Thus all these subsets yield under product the MOD mixed zero set.

We can get MOD mixed subset nilpotents in fact R and S are MOD mixed subset nilpotents A and D are MOD subset nilpotents.

$V = \{ I_0^g, I_{2g}^g, I_{4g}^g, I_{8g}^g \}$ and
$W = \{ I_0^g, I_{5g}^g, I_{8g}^g, I_{7g}^g, I_{9g}^g \} \in B$ are such that
$V \times W = \{ I_0^g \}$, $V \times V = \{ I_0^g \}$ and $\{ I_0^g \}$
$= W \times W$

so V and W are MOD subset natural neutrosophic dual number nilpotents. (V, W) is the MOD natural neutrosophic dual number subset zero divisors.

It is easily verified that $M = \{S(Z_{10}), +, \times\} \subseteq B$ is a MOD natural neutrosophic dual number subset pseudo subsemiring which is not an ideal.

Let $N = \{S(Z_{10}g), +, \times\} \subseteq B$ is a MOD natural neutrosophic dual number subset subsemiring which is not pseudo and N is also an ideal of B.

Thus only MOD natural neutrosophic dual number subset pseudo semirings always has MOD natural neutrosophic subset pseudo dual number semirings which has MOD natural neutrosophic subset dual number subsemirings which are ideals and are not pseudo.

Always these MOD natural neutrosophic dual number subset pseudo semirings has MOD subset zero divisors and MOD subset nilpotents of usual form yielding {0} or mixed form $\{0, I_0^g\}$ or MOD natural neutrosophic zero $\{I_0^g\}$.

This is the marked difference between the other MOD natural neutrosophic subset pseudo semiring.

In view of all these we have the following theorem.

THEOREM 2.8: *Let S ={S(⟨$Z_n \cup g$⟩$_I$), +, ×} be the MOD natural neutrosophic dual number subset pseudo semiring.*

i) o(S) < ∞.
ii) S has MOD natural neutrosophic dual number subset nilpotents and zero divisors for all n; 2 ≤ n < ∞.
iii) S has MOD subset natural neutrosophic dual number pseudo subsemiring which is not an ideal for all n, 2 ≤ n < ∞.
iv) S has MOD subset natural neutrosophic dual number subsemiring which is not pseudo and which is also an ideal for all n, 2 ≤ n < ∞.
v) S has MOD subset natural neutrosophic dual number idempotents for all n, 2 ≤ n < ∞.

Proof is direct and hence left as an exercise to the reader.

Next we proceed onto describe MOD natural neutrosophic special dual like number subset pseudo semirings by some examples.

Example 2.23: Let S = {S(⟨Z_{12} ∪ h⟩), +, ×} be the MOD natural neutrosophic special dual like number subset pseudo semiring.

We see S has MOD natural neutrosophic special dual like number subset subsemiring which are not ideals as well as MOD subset special dual like number subset subsemirings which are ideals.

Infact S has MOD subset idempotents, nilpotents and zero divisors. This task of finding them is left as an exercise to the reader.

Example 2.24: Let M = {S(⟨Z_{17} ∪ h⟩), +, ×} be the MOD subset natural neutrosophic special dual like number pseudo semiring.

Finding MOD zero divisors in M happens to be a challenging job. However this has MOD subset idempotents. Also this M has no MOD subset nilpotents.

However P ={⟨Z_{17} ∪ h⟩$_I$), +, ×} ⊆ M is a MOD subset natural neutrosophic special dual like number pseudo subsemiring which is also an ideal of M.

In view of all these we can prove the following result.

THEOREM 2.9: *Let W = {S(⟨Z_n ∪ h⟩$_I$), +, ×} be the MOD subset natural neutrosophic pseudo special dual like number semiring.*

- *i)* *o(W) < ∞.*
- *ii)* *W has MOD natural neutrosophic special dual like number subset pseudo subsemiring which are ideals as well as subsemirings which are not ideals what ever be n; 2 ≤ n < ∞.*
- *iii)* *W has MOD natural neutrosophic special dual like number subset idempotents whatever be n, 2 ≤ n < ∞.*
- *iv)* *W has MOD natural neutrosophic special dual like number subset nilpotents as well as zero divisors*

for only special values of n; $n = p^{\alpha}q$; $\alpha \geq 2$, $(p, q) = 1$, p a prime.

Proof is direct and hence left as an exercise to the reader.

Next we proceed onto briefly describe MOD subset natural neutrosophic special quasi dual number semiring by some examples and give a few of its related properties.

Example 2.25: Let $B = \{S(\langle Z_{48} \cup k\rangle), +, \times\}$ be the MOD natural neutrosophic special quasi dual number subset semiring.

$A = \{12 + 6k, 12k, 12, 6, 6 + k, 24k, 24, 24 + 12 k\}$ and
$C = \{8k, 8 + 8k, 16, 16 + 8, 16k + 8\} \in B$.

$A \times C = \{0\}$ thus A, C in B is a MOD natural neutrosophic special quasi dual number subset zero divisor pair.

$V = \{24, 24k, 12, 12k, 12 + 12k, 24 + 24k, 12 + 24k\} \in B$ is such that $V \times V = \{0\}$ is the MOD natural neutrosophic special quasi dual number subset nilpotent of order two.

Let $P = \{I^k_{12}, I^k_{24k}, I^k_0, I^k_{24+12k}, I^k_{12k}, I^k_{24+24k}, I^k_{12+12k}, I^k_{24k}\} \in B$; clearly $P \times P = \{I^k_0\}$ thus P is a MOD natural neutrosophic special quasi dual number subset, MOD natural neutrosophic nilpotent of order two which is clearly different from V.

Let $R = \{I^k_{12}, I^k_{24k}, 0\ I^k_{36}, I^k_{36k+36}, I^k_0\} \in B$ we see $R \times R = \{0, I^k_0\}$; that is R is a MOD natural neutrosophic special quasi dual number subset mixed MOD natural neutrosophic nilpotent of order two.

We just recall in B, $\{0\}$ is the real zero subset $\{I^k_0\}$ is the MOD natural neutrosophic special quasi dual number zero subset, $\{I^k_0, 0\}$ is the MOD natural neutrosophic special quasi dual number mixed zero subset.

Thus we have three types of zeros.

Further A × {0} ≠ {0} for all A ∈ B.

A × { I_0^k } = { I_0^k } for all A ∈ B. A × { I_0^k, 0} ≠ { I_0^k, 0} for all A ∈B.

Now we proceed onto show W_1 = {S(Z_{48}^I); + ×} ⊆ B is a MOD natural neutrosophic subset special quasi dual number subset pseudo subsemiring which is not an ideal of B.

W_2 = {S(⟨Z_{48} ∪ k⟩), +, ×} ⊆ B is again a MOD natural neutrosophic special quasi dual number pseudo subset subsemiring which is not an ideal.

Let W_3 = {S(⟨Z_{48} ∪ k⟩), +, ×} be the MOD natural neutrosophic special quasi dual number pseudo subset subsemiring and is not an ideal.

We see we have several interesting and special properties associated with B.

We give yet another example.

Example 2.26: Let S = {S(⟨Z_{19} ∪ k⟩$_I$), ×, +} be the MOD natural neutrosophic special quasi dual number subset pseudo semiring.

We do not find MOD natural neutrosophic special quasi dual number subset zero divisors or nilpotents in S.

However if P = {Z_{19}} then P + P = P and P × P = P is a MOD natural neutrosophic subset strong idempotent of S.

Let M = {Z_{19}k} ∈ S, M + M = M and M × M = M is again a MOD natural neutrosophic subset strong idempotents of S.

{ I_0^k, 0, 1} = W such that W × W = W is only a MOD natural neutrosophic subset idempotent not a strong subset idempotent.

One can study the structure of MOD natural neutrosophic subset pseudo semiring properties when n is a prime and n is a non prime.

This task is left as an exercise to the reader.

THEOREM 2.10: *Let S = {S($\langle Z_n \cup k\rangle_I$), +, ×} be the MOD natural neutrosophic special quasi dual number pseudo semiring.*

i) $o(S) < \infty$.
ii) S has MOD natural neutrosophic special quasi dual number subset idempotents as well as strong idempotents $2 \leq n < \infty$.
iii) S has MOD natural neutrosophic special quasi dual number subset pseudo subsemirings which are not ideals as well as has ideals; $2 \leq n < \infty$.
iv) S has nontrivial MOD natural neutrosophic special quasi dual number subset nilpotents and zero divisors only for n a non prime and $n = p^{\alpha}q$, $\alpha \geq 2$, p a prime (p, q) = 1.

Proof is direct hence left as an exercise to the reader.

Thus we see all the six distinct MOD natural neutrosophic subset pseudo semirings behave differently. The MOD natural neutrosophic dual number subset pseudo semiring only can have MOD subset subsemirings which are not pseudo and which are zero square subsemirings and not ideals.

Thus the special feature enjoyed by these MOD natural neutrosophic dual number subset pseudo semirings S can find applications in near future.

For there are several elements in S which are MOD nilpotents subsets under all the three types of zeros of S.

Further MOD natural neutrosophic - neutrosophic subset pseudo semirings enjoy a specialty of their own as they involve the classical neutrosophic element I.

Finally the MOD natural neutrosophic finite complex number subset pseudo semiring is very different as $i_F^2 = (n - 1)$ and I_t^C has very many features.

Problems.

1. Find all the special features associated with S = {S(Z_n^I), +, ×} the MOD natural neutrosophic subset pseudo semiring.

2. Let S = {S(Z_{48}^I), +, ×} be the MOD natural neutrosophic subset pseudo semiring.

   i) Find o(S).
   ii) Find all MOD natural neutrosophic strong subset idempotent of S.
   iii) Find all MOD natural neutrosophic subset idempotents with respect to +.
   iv) Find all MOD natural neutrosophic subset idempotents with respect to ×.
   v) Find all MOD natural neutrosophic subset pseudo subsemirings which are not ideals of S.
   vi) Find all MOD natural neutrosophic subset pseudo ideals of S.
   vii) Show S has MOD natural neutrosophic subset zero divisors, mixed zero divisors and MOD natural neutrosophic zero divisors.
   viii) Find all MOD natural neutrosophic nilpotent subsets P of S with $P^2 = \{0\}$ or $P^2 = \{I_0^n\}$ or $P^2 = \{0, I_0^n\}$.
   ix) Discuss any other special feature associated with S.

3. Let M ={S(Z_{128}^I), +, ×} be the MOD subset natural neutrosophic pseudo semiring.

Study questions (i) to (ix) of problem (2) for this M.

4. Let W = $\{S(Z_{43}^{I}), +, \times\}$ be the MOD subset natural neutrosophic pseudo semiring.

   Study questions (i) to (ix) of problem (2) for this W.

5. Let B = $\{S(\langle Z_{24} \cup I\rangle_I), +, \times\}$ be the MOD natural neutrosophic neutrosophic subset pseudo semiring.

   i) Study (i) to (ix) of problem (2) for this B.
   ii) Compare B with W in problem 4.

6. Let E = $\{S(\langle Z_{47} \cup I\rangle); +, \times\}$ be the MOD subset natural neutrosophic - neutrosophic pseudo semiring.

   Study questions (i) to (ix) of problem (2) for this E.

7. Let G = $\{S(\langle Z_{3^{10}} \cup I\rangle_I), +, \times\}$ be the MOD natural neutrosophic - neutrosophic subset pseudo semiring.

   Study questions (i) to (ix) of problem (2) for this G.

8. Let H = $\{S(\langle Z_{12} \cup g\rangle_I); +, \times\}$ be the MOD natural neutrosophic dual number subset pseudo semiring.

   i) Study questions (i) to (ix) of problem (2) for this H.
   ii) Show only MOD natural neutrosophic dual number subset pseudo semiring has zero square subsemirings which are not pseudo.
   iii) Compare H of this problem with E and B of problem 6 and 5 respectively.

9. Let Z = $\{S(\langle Z_{53} \cup g\rangle_I); +, \times\}$ be the MOD natural neutrosophic dual number subset semiring.

   i) Study questions (i) to (ix) of problem (2) for this Z.
   ii) Compare Z with H of problem 8.

10. Let T = $\{S(\langle Z_{5^6} \cup g\rangle_I); +, \times\}$ be the MOD natural neutrosophic dual number subset pseudo semiring.

    Study questions (i) to (ix) of problem (2) for this T.

11. Let V = $\{S(C^I(Z_{15})), +, \times\}$ be the MOD natural neutrosophic finite complex number pseudo subset semiring.

    i) Study questions (i) to (ix) of problem (2) for this V.
    ii) Compare V with this problem (10)

12. Let W = $\{S(C^I(Z_{193})), +, \times\}$ be the MOD natural neutrosophic finite complex number subset pseudo semiring.

    i) Study questions (i) to (ix) of problem (2) for this W.
    ii) Compare W with T and V of problems (10) and (11) respectively.

13. Let F = $\{S(\langle Z_9 \cup h\rangle_I), +, \times\}$ be the MOD natural neutrosophic special dual like number subset pseudo semiring.

    Study questions (i) to (ix) of problem (2) for this F.

14. Let K = $\{S(\langle Z_{144} \cup h\rangle_I), +, \times\}$ be the MOD natural neutrosophic special dual like number subset pseudo semiring.

    i) Study questions (i) to (ix) of problem (2) for this K.
    ii) Compare this K with F of problem (13).

15. Let V = $\{S(\langle Z_{160} \cup h\rangle_I), +, \times\}$ be the MOD natural neutrosophic special dual like number subset pseudo semiring.

    Study questions (i) to (ix) of problem (2) for this V.

16. Let $T = \{S(\langle Z_{124} \cup k\rangle_I), +, \times\}$ be the MOD natural neutrosophic special quasi dual number subset pseudo semiring.

    i) Study questions (i) to (ix) of problem (2) for this T.
    ii) Compare this T with problem (15) for this V.

17. Let $W = \{S(\langle Z_{47} \cup k\rangle_I), +, \times\}$ be the MOD natural neutrosophic special quasi dual number subset pseudo semiring.

    i) Study questions (i) to (ix) of problem (2) for this W.
    ii) Compare W with T of problem (16).

18. Let $Z = \{S(\langle Z_{16} \cup k\rangle_I), +, \times\}$ be the MOD subset special quasi number natural neutrosophic pseudo semiring.

    Study questions (i) to (ix) of problem (2) for this Z.

19. Let $H_1 = \{S(\langle Z_{48} \cup I\rangle_I), +, \times\}$, $H_2 = \{S(\langle Z_{48} \cup h\rangle_I), +, \times\}$, $H_3 = \{S(Z_{48}^{I}), +, \times\}$, $H_4 = \{S(\langle Z_{48} \cup k\rangle_I), +, \times\}$, $H_5 = \{S(C^I(Z_{48})), +, \times\}$ and $H_6 = \{S(\langle Z_{48} \cup g\rangle_I), +, \times\}$ be the MOD natural neutrosophic-neutrosophic subset pseudo semiring MOD natural neutrosophic special dual like number subset pseudo semiring, MOD natural neutrosophic subset pseudo semiring, MOD natural neutrosophic special quasi dual number subset pseudo semiring, MOD natural neutrosophic finite complex number pseudo subset semiring and MOD natural neutrosophic dual number subset pseudo semiring respectively.

    Compare the six MOD pseudo semirings with every other MOD pseudo semirings.

Chapter Three

MOD SUBSET MATRIX PSEUDO SEMIRINGS AND MOD SUBSET POLYNOMIAL PSEUDO SEMIRINGS

In this chapter we for the first time introduce the notion of MOD subset matrix pseudo semirings and MOD matrix subset pseudo semirings and discuss their properties.

Likewise MOD subset polynomial pseudo semirings and MOD polynomial subset pseudo semirings are introduced in this chapter. We first describe this situation by some examples.

Example 3.1: Let G = {S(M), the collection of all matrices from the set M = $\{(a_1, a_2, a_3) / a_i \in Z_{12}^{I}; 1 \leq i \leq 3\}$, +, ×} be the MOD natural neutrosophic matrix subset pseudo semiring.

Clearly $o(G) < \infty$.

Let A = $\{(3, 0, 5), (I_4^{12}, 4, I_0^{12}), (I_6^{12}, 6, 3)\}$ and

B = $\{(4, 3, I_{10}^{12}), (I_3^{12}, 0, 1)\} \in G$.

$$A + B = \{(3, 0, 5), (I_4^{12}, 4, I_0^{12}), (I_6^{12}, 6, 3)\} + \{(4, 3, I_{10}^{12}), (I_3^{12}, 0, 1)\}$$

$= \{(3, 0, 5) + (4, 3, I_{10}^{12}), (3, 0, 5) + (I_3^{13}, 0, 1), (I_4^{12}, 4, I_0^{12}) + (4, 3, I_0^{12}), (I_4^{12}, 4, I_0^{12}) + (I_3^{12}, 0, 1), (I_6^{12}, 6, 3) + (4, 3, I_{10}^{12}), (I_6^{12}, 6, 3) + (I_3^{12}, 0, 1)\} = \{ I_4^{12} + 4, 7, I_0^{12}), (I_4^{12} + I_3^{12}, 4\ 1 + I_0^{12}) (4 + I_6^{12}, 9, 3, I_{10}^{12}), (I_6^{12} + I_3^{12}, 6, 4)\} \in G.$

This is the way + operation is performed on G.

$A \times B = \{(3, 0, 5), (I_4^{12}, 4, I_0^{12}), (I_6^{12}, 6, 3)\} \times \{(4, 3, I_0^{12}), (I_3^{12}, 0, 1)\} \{(0, 0, I_0^{12}), (I_4^{12}, 0, I_0^{12}) (I_6^{12}, 6, I_{10}^{12}), (I_0^{12}, 0, 5), (I_0^{12}, 0, I_0^{12}), (I_6^{12}, 0, 3)\} \in G.$

This is the way product operation is performed on G.

However G is not a MOD subset semiring on the other hand G is only pseudo for if $A = \{(1, 0, 4) (I_0^{12}, I_6^{12}, 0), (0, 1, I_8^{12})\}$, $B = \{(6, 4, 8)\}$ and $C = \{(6, 8, 4)\} \in G$.

A (B + C) = A({6, 4, 8} + {6, 8, 4}) = A × {0} = {0} I
I

$A \times B + A \times C = \{(1, 0, 4), (I_0^{12}, I_6^{12}, 0), (0, 1, I_8^{12})\} \times \{(6, 4, 8)\} + \{(1, 0, 4), (I_0^{12}, I_6^{12}, 0), (0, 1, I_8^{12})\} \times \{(6, 8, 4)\}$

$= \{(6, 0, 8), (I_0^{12}, I_6^{12}, 0), (0, 4, I_8^{12})\} + \{(6, 0, 4), (I_0^{12}, I_6^{12}, 0), (0, 8, I_8^{12})\}$

$$= \{(0, 0, 0), (6 + I_0^{12}, I_6^{12}, 4), (6, 4, 4 + I_8^{12}), (6 + I_0^{12}, I_6^{12}, 8), (I_0^{12}, I_6^{12}, 0), (I_0^{12}, 4 + I_6^{12}, I_8^{12}), (6, 8, 8 + I_{12}^{8}), (I_0^{12}, 8 + I_6^{12}, I_8^{12}), (0, 0, I_8^{12})\} \quad \text{II}$$

I and II are distinct so the MOD subset semiring is only pseudo.

G has MOD subset zero divisors given by A = {4, 8} and B = {6, 0} $\in$ G, A $\times$ B = {4, 8} $\times$ (6, 0} = {0} is a MOD subset zero divisor.

Let D = {4, 0, 1, 9} $\in$ G is such that D $\times$ D = D is a MOD subset idempotent of G.

$A = \{I_0^{12}, I_6^{12}, I_4^{12}, I_8^{12}\}$ and $B = \{I_0^{12}, I_6^{12}\} \in G$ is such $A \times B = \{I_0^{12}\}$ is the MOD subset natural neutrosophic zero divisor.

Let $Z = \{I_0^{12}, I_6^{12}\} \in G$ is such that $Z \times Z = \{I_0^{12}\}$ is a MOD natural neutrosophic nilpotent zero of G.

Thus using $M = \{(I_0^{12}, I_0^{12}, I_0^{12}), (I_6^{12}, I_6^{12}, I_6^{12}), (I_4^{12}, I_4^{12}, I_4^{12}), (I_8^{12}, I_8^{12}, I_8^{12})\}$ and

$N = \{(I_0^{12}, I_0^{12}, I_0^{12}), (I_6^{12}, I_6^{12}, I_6^{12})\} \in G$ is such that

$M \times N = \{(I_0^{12}, I_0^{12}, I_0^{12})\}$.

Let $W = \{I_0^{12}, I_0^{12}, I_0^{12}) (I_6^{12}, I_6^{12}, I_6^{12})\} \in G$ is such that $W \times W = \{(I_0^{12}, I_0^{12}, I_0^{12})\}$.

Example 3.2: Let L = {S(P) = {collection of all matrix subsets from

$$P = \{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} / a_i \in Z_7^I, 1 \leq i \leq 4\}, +, \times_n\}, +, \times_n\}$$

be the MOD natural neutrosophic matrix subsets pseudo semiring.

$$\text{Let } A = \{\begin{bmatrix} 0 \\ 3 \\ I_0^7 \\ 6 \end{bmatrix} \begin{bmatrix} 6 \\ 0 \\ 4 \\ 1+I_0^7 \end{bmatrix}\} \text{ and } B = \{\begin{bmatrix} I_0^7+5 \\ 0 \\ 2 \\ 6 \end{bmatrix} \begin{bmatrix} 0 \\ I_0^7+1 \\ 0 \\ 3 \end{bmatrix} \in L$$

$$A \times_n B = \{\begin{bmatrix} 0 \\ 3 \\ I_0^7 \\ 6 \end{bmatrix}, \begin{bmatrix} 6 \\ 0 \\ 4 \\ 1+I_0^7 \end{bmatrix}\} \times_n \{\begin{bmatrix} I_0^7+5 \\ 0 \\ 2 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ I_0^7+1 \\ 0 \\ 3 \end{bmatrix}\}$$

$$= \{\begin{bmatrix} I_0^7 \\ 0 \\ I_0^7 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 3+I_0^7 \\ I_0^7 \\ 4 \end{bmatrix}, \begin{bmatrix} I_0^7+2 \\ 0 \\ 1 \\ 6+I_0^7 \end{bmatrix}, \begin{bmatrix} 0 \\ I_0^7 \\ 0 \\ 3+I_0^7 \end{bmatrix}\}.$$

This is the way the product operation is performed on L.

$$A + B = \{\begin{bmatrix} 0 \\ 3 \\ I_0^7 \\ 6 \end{bmatrix}, \begin{bmatrix} 6 \\ 0 \\ 4 \\ 1+I_0^7 \end{bmatrix}\} + \{\begin{bmatrix} I_0^7+5 \\ 0 \\ 2 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ I_0^7+1 \\ 0 \\ 3 \end{bmatrix}\}$$

$$= \left\{ \begin{bmatrix} 5+I_0^7 \\ 3 \\ 2+I_0^7 \\ 5 \end{bmatrix}, \begin{bmatrix} 4+I_0^7 \\ 0 \\ 6 \\ I_0^7 \end{bmatrix}, \begin{bmatrix} 0 \\ 4+I_0^7 \\ I_0^7 \\ 2 \end{bmatrix} \begin{bmatrix} 6 \\ 1+I_0^7 \\ 4 \\ 4+I_0^7 \end{bmatrix} \right\} \in L.$$

This is the way the sum operation is performed on A and B.

$$A = \left\{ \begin{bmatrix} 0 \\ I_0^7 \\ 0 \\ I_0^7 \end{bmatrix} \right\} \text{ and } B = \left\{ \begin{bmatrix} 6 \\ 4 \\ 5 \\ 2 \end{bmatrix} \right\} \in L$$

$$A \times_n B = \left\{ \begin{bmatrix} 0 \\ I_0^7 \\ 0 \\ I_0^7 \end{bmatrix} \right\} \times_n \left\{ \begin{bmatrix} 6 \\ 4 \\ 5 \\ 2 \end{bmatrix} \right\} = \left\{ \begin{bmatrix} 0 \\ I_0^7 \\ 0 \\ I_0^7 \end{bmatrix} \right\} \in L.$$

Thus we say L has only trivial zero divisors, trivial MOD natural neutrosophic zero divisors and MOD mixed zero divisors.

Example 3.3: Let W = {S(M) where

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \middle| \; a_i \in Z_{18}^I ; 1 \leq i \leq 9, +, \times \right\}; +, \times\}$$

be the MOD natural neutrosophic subset matrix pseudo semiring.

Clearly W is a non commutative pseudo semiring.

$$\text{Let } P = \left\{\begin{bmatrix} 3 & 6 & I_6^{18} \\ 0 & I_0^{18} & 9 \\ 0 & 0 & I_2^{18} \end{bmatrix}, \begin{bmatrix} 6 & 3 & I_0^{18} \\ I_2^{18} & 0 & 1 \\ 5 & 0 & 9 \end{bmatrix}\right\} \text{ and}$$

$$R = \left\{\begin{bmatrix} 4 & 8 & I_3^{18} \\ I_2^{18} & 0 & I_2^{18} \\ 6 & I_0^{18} & 0 \end{bmatrix}\right\} \in W;$$

$$P \times R = \left\{\begin{bmatrix} 3 & 6 & I_6^{18} \\ 0 & I_0^{18} & 9 \\ 0 & 0 & I_2^{18} \end{bmatrix}, \begin{bmatrix} 6 & 3 & I_0^{18} \\ I_2^{18} & 0 & 1 \\ 5 & 0 & 9 \end{bmatrix}\right\} \times \left\{\begin{bmatrix} 4 & 8 & I_3^{18} \\ I_2^{18} & 0 & I_2^{18} \\ 6 & I_0^{18} & 0 \end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix} 3 & 6 & I_6^{18} \\ 0 & I_0^{18} & 9 \\ 0 & 0 & I_2^{18} \end{bmatrix} \times \begin{bmatrix} 4 & 8 & I_3^{18} \\ I_2^{18} & 0 & I_2^{18} \\ 6 & I_0^{18} & 0 \end{bmatrix}, \begin{bmatrix} 6 & 3 & I_0^{18} \\ I_2^{18} & 0 & 1 \\ 5 & 0 & 9 \end{bmatrix} \times \begin{bmatrix} 4 & 8 & I_3^{18} \\ I_2^{18} & 0 & I_2^{18} \\ 6 & I_0^{18} & 0 \end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix} 12 + I_2^{18} + I_6^{18} & 6 + I_0^{18} & I_3^{18} + I_2^{18} + I_6^{18} \\ I_0^{18} & I_0^{18} & I_0^{18} \\ I_2^{18} & I_0^{18} & I_3^{18} + I_2^{18} \end{bmatrix}\right.$$

This is the way product operation is performed on W.

This W is a MOD natural neutrosophic matrix subset pseudo non commutative semiring. If $\times$, the usual operation is replaced by the natural product $\times_n$ then W is a commutative pseudo semiring.

In view of all these we have the following theorem.

THEOREM 3.1: *Let W = {collection of all subsets from S = {Collection of all m × t matrices with entries from* Z_n^I *; +, $\times_n$} +, $\times_n$} be the MOD natural neutrosophic subset matrix pseudo semiring.*

- *i) o(W) < ∞.*
- *ii) W has MOD natural neutrosophic subset matrix pseudo subsemirings which are ideals as well as pseudo subsemirings for all 2 ≤ n < ∞.*
- *iii) W has no nontrivial MOD natural neutrosophic subset matrix zero divisors and nilpotents when n is a prime.*
- *iv) Only for special values of n we have W to have nontrivial MOD natural neutrosophic subset zero divisors and nilpotents.*

Proof is left as exercise to the reader.

Next we describe MOD natural neutrosophic subset matrix finite complex number pseudo semirings by examples.

Example 3.4: Let S(T) = {collection of matrix subsets from

$$T = \left\{\left.\begin{pmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \end{pmatrix}\right|\ a_i \in C^I(Z_4);\ 1 \le i \le 8, +, \times_n\right\}, +, \times_n\}$$

be the MOD natural neutrosophic finite complex number matrix subset pseudo semiring.

$$\text{Let } A = \{\begin{pmatrix} 0 & 2 & I_2^C & I_0^C \\ 1 & 0 & 1 & 1+i_F \end{pmatrix},$$
$$\begin{pmatrix} 3+i_F+I_2^C & 0 & 0 & 1 \\ 0 & 2+i_F & 0 & i_F \end{pmatrix}\} \in S(T).$$

We can as a matter of routine find A + A and A $\times_n$ A; this task is left as an exercise to the reader.

$$\text{Let } P = \{\begin{pmatrix} I_0^C & I_2^C & 3 & 2 \\ 1+i_F & 2i_F & 0 & 1 \end{pmatrix}\} \text{ and}$$

$$Q = \{\begin{pmatrix} 1 & 2 & 3 & i_F + I_0^C \\ 0 & 1 & I_0^C & 1+I_2^C \end{pmatrix}\} \in S(T).$$

$$P + Q = \{\begin{pmatrix} I_0^C & I_2^C & 3 & 2 \\ 1+i_F & 2i_F & 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 2 & 3 & i_F + I_0^C \\ 0 & 1 & I_0^C & 1+I_2^C \end{pmatrix}\}$$

$$= \begin{pmatrix} 1+I_0^C & 2+I_2^C & 2 & 2+i_F+I_0^C \\ 1+i_F & 2i_F+1 & I_0^C & 2+I_2^C \end{pmatrix}\} \in S(T).$$

This is the way + operation is performed on S(T).

$$P \times Q = \{\begin{pmatrix} I_0^C & I_2^C & 3 & 2 \\ 1+i_F & 2i_F & 0 & 1 \end{pmatrix}\} \times_n \{\begin{pmatrix} 1 & 2 & 3 & i_F + I_0^C \\ 0 & 1 & I_0^C & 1+I_2^C \end{pmatrix}\}$$

$$= \{\begin{pmatrix} I_0^C & I_2^C & 1 & 2i_F+I_0^C \\ 0 & 2i_F & I_0^C & 1+I_2^C \end{pmatrix}\} \in S(T).$$

This is the way the natural product operation $\times_n$ is performed on S(T). S(T) is a MOD matrix subset commutative.

Certainly S(T) has MOD subset zero divisors, MOD matrix subset nilpotents.

Also S(T) has MOD subset matrix idempotents.

This task is also left as an exercise to the reader.

Next we give one more example.

Example 3.5: Let S(W) = {collection of all matrix subsets from

$$W = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \end{bmatrix} \middle| \ a_i \in C^I(Z_{10}),\ 1 \leq i \leq 10, +, \times_n\}, +, \times_n\right\}$$

be the MOD natural neutrosophic subset matrix finite complex number pseudo semiring.

W has MOD matrix subset zero divisors, idempotents and nilpotents W also has MOD matrix subset subsemirings which are ideals as well as not ideals.

In view of all these we have the following theorem.

THEOREM 3.2: *Let S(B) = {collection of all matrix subsets from B = {m × t matrices witih entries from $C^I(Z_n)$; +, $\times_n$}; +, $\times_n$} be the collection of all MOD natural neutrosophic finite complex number subset matrix pseudo semiring.*

i) $o(S(B)) < \infty$.
ii) S(B) has MOD subset matrix strong idempotents as well as idempotents.
iii) S(B) has MOD subset matrix pseudo ideals as well as pseudo subsemirings which are not ideals.
iv) S(B) has MOD subset matrix nontrivial zero divisors and nilpotents only if $n = p^\alpha q$, p a prime, $\alpha \geq 2$, (p, q) = 1.

Proof is direct and hence left as an exercise to the reader.

Now we proceed onto describe by an example the MOD natural neutrosophic - neutrosophic matrix subset pseudo semiring.

Example 3.6: Let S(W) = {collection of all matrix subsets from

$$W = \left\{ \left. \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \end{bmatrix} \right| a_i \in \langle Z_{15} \cup I\rangle_I;\ 1 \le i \le 10, +, \times_n\}, +, \times_n\right\}$$

be the MOD natural neutrosophic-neutrosophic matrix subset pseudo semiring.

Clearly S(W) is a finite commutative pseudo semiring.

S(W) has MOD subset matrix zero divisors and has no nontrivial MOD nilpotents. Working with S(W) is a matter of routine and hence left as an exercise to the reader.

Let S(P) = {collection of all subsets from

$$P = \left\{ \left. \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \end{bmatrix} \right| a_i \in \langle Z_{15} \cup I_t^I \rangle \text{ where } t \in \langle Z_{15} \cup I\rangle_I \right.$$

is an idempotent or a zero divisor or nilpotent; +, $\times_n$ } $\subseteq$ S(W) is a MOD matrix subset natural neutrosophic-neutrosophic pseudo subsemiring of S(W) which is also an ideal.

Interested reader is expected to find MOD matrix subset pseudo subsemirings and ideals of S(W) using the fact $0 \times I_t^I = I_t^I$ and $0 \times a = 0$ for $a \in \langle Z_{15} \cup I\rangle$.

Next we describe by an example MOD natural neutrosophic dual number matrix subsets pseudo semiring.

Example 3.7: Let S(B) = {collection of subset matrices with entries from

$$B = \left\{\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \end{bmatrix} \middle| a_i \in \langle Z_{12} \cup g\rangle_I; 1 \leq i \leq 6, +, \times_n\right\}, +, \times_n\}$$

be the MOD natural neutrosophic dual number matrix subset pseudo semiring.

$$\text{Let } A_1 = \{\begin{bmatrix} 3 & I_g^g & 1 \\ I_2^g & 0 & 5 \end{bmatrix}, \begin{bmatrix} I_{6+6g}^g & 0 & 1 \\ I_{2+8g}^g & 5 & 3 \end{bmatrix}\} \text{ and}$$

$$B_1 = \{\begin{bmatrix} I_4^g & 6 & 2 \\ I_{6g}^g & 1 & 6 \end{bmatrix}, \begin{bmatrix} 1 & 2 & I_8^g \\ 0 & 3 & I_g^g \end{bmatrix}\} \in S(B).$$

$$A_1 + B_1 = \{\begin{bmatrix} 3 & I_g^g & 1 \\ I_2^g & 0 & 5 \end{bmatrix}, \begin{bmatrix} I_{6+6g}^g & 0 & 1 \\ I_{2+8g}^g & 5 & 3 \end{bmatrix}\} +$$

$$\{\begin{bmatrix} I_4^g & 6 & 2 \\ I_{6g}^g & 1 & 6 \end{bmatrix}, \begin{bmatrix} 1 & 2 & I_8^g \\ 0 & 3 & I_g^g \end{bmatrix}\}$$

$$= \{\begin{bmatrix} 3+I_4^g & 6+I_g^g & 3 \\ I_2^g+I_{6g}^g & 1 & 11 \end{bmatrix} \begin{bmatrix} 4 & 2+I_g^g & 1+I_8^g \\ I_2^g & 3 & 5+I_g^g \end{bmatrix},$$

$$\begin{bmatrix} I^g_{6+6g} + I^g_4 & 6 & 3 \\ I^g_{6g} + I^g_{2+8g} & 6 & 9 \end{bmatrix}, \begin{bmatrix} 1 + I^g_{6+6g} & 2 & 1 + I^g_8 \\ I^g_{2+8g} & 8 & 3 + I^g_g \end{bmatrix}\} \in S(B).$$

This is the way '+' operation on S(B) is performed.

$$A_1 \times_n B_1 = \{\begin{bmatrix} 3 & I^g_g & 1 \\ I^g_2 & 0 & 5 \end{bmatrix}, \begin{bmatrix} I^g_{6+6g} & 0 & 1 \\ I^g_{2+8g} & 5 & 3 \end{bmatrix}\} \times_n \{\begin{bmatrix} I^g_4 & 6 & 2 \\ I^g_{6g} & 1 & 6 \end{bmatrix}, \begin{bmatrix} 1 & 2 & I^g_8 \\ 0 & 3 & I^g_g \end{bmatrix}\}$$

$$= \{\begin{bmatrix} I^g_4 & I^g_g & 2 \\ I^g_0 & 0 & 6 \end{bmatrix}, \begin{bmatrix} 3 & I^g_g & I^g_8 \\ I^g_2 & 0 & I^g_g \end{bmatrix}, \begin{bmatrix} I^g_0 & 0 & 2 \\ I^g_0 & 5 & 6 \end{bmatrix}, \begin{bmatrix} I^g_{6+6g} & 0 & I^g_8 \\ I^g_{2+8g} & 3 & I^g_g \end{bmatrix}\} \in S(B).$$

This is the way $\times_n$ operation is performed on S(B). It is to be noted that if P, Q, R $\in$ S(B) then $P \times_n (Q + R) \neq P \times_n Q + P \times_n R$ is true in general that is why we are forced to call S(B) a pseudo ring. Another fact is if

$$C = \{\begin{bmatrix} g & 3g & 4g \\ 5g & 6g & 7g \end{bmatrix}, \begin{bmatrix} 0 & 8g & 9g \\ 11g & 2g & g \end{bmatrix}, \begin{bmatrix} 2g & g & 6g \\ 8g & g & 0 \end{bmatrix}\} \text{ and}$$

$$D = \{\begin{bmatrix} g & g & g \\ g & 2g & 3g \end{bmatrix}, \begin{bmatrix} 4g & 5g & 6g \\ 7g & 8g & 9g \end{bmatrix}\} \in S(B);$$

we see $C \times_n D = \{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\}$. $D \times_n D = \{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\}$ and

$$C \times_n C = \{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\}.$$

Thus this MOD natural neutrosophic subset matrix dual number pseudo semirings always has MOD matrix subset nilpotents and MOD subset matrix zero divisors.

This is a special and distinct feature enjoyed by S(B).

Further S(T) = {collection of matrix subsets from

$$T = \left\{\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \end{bmatrix} \middle|\ a_i \in Z_{12}g,\ 1 \leq i \leq 6,\ +,\ \times_n\right\},\ +,\ \times\} \subseteq S(B)$$

is a MOD natural neutrosophic dual number matrix subset pseudo subsemiring which is not pseudo and not an ideal of S(B).

We leave it as an exercise to the reader to find ideals of S(B).

In view of this we have the following theorem.

THEOREM 3.3: *Let S(B) = {collection of all matrix subsets from B = {m × t matrices with entries from $\langle Z_n \cup g\rangle_I$; +, $\times_n$}, +, $\times_n$} be the MOD natural neutrosophic matrix subset dual number pseudo semiring.*

- *i) $o(S(B)) < \infty$.*
- *ii) S(B) has MOD matrix subsets which are nontrivial zero divisors, $2 \leq n \leq \infty$.*
- *iii) S(B) has nontrivial MOD matrix subsets which are nilpotents of order two, $2 \leq n < \infty$.*
- *iv) S(B) has nontrivial MOD matrix subset subsemirings which are zero square subsemirings and not pseudo.*
- *v) S(B) has MOD natural neutrosophic matrix subset idempotents only for special n.*

Proof is direct and hence left as an exercise to the reader.

Next we describe MOD natural neutrosophic special dual like number subset matrix pseudo semirings and MOD natural neutrosophic special quasi dual number subset matrix pseudo semirings by an example each.

All the properties related with them can be derived as a matter of routine so is left as an exercise to the reader.

Example 3.8: Let S(W) = {collection of all matrix subsets from

$$W = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \end{bmatrix} \middle| \; a_i \in \langle Z_{18} \cup h\rangle_I,\ 1 \leq i \leq 12, +, \times_n\right\},$$

$$+, \times_n\}$$

be the MOD natural neutrosophic special dual like number matrix subset pseudo semiring.

This has nontrivial MOD matrix subset zero divisors, nilpotents and idempotents. This S(W) has MOD matrix subset pseudo subsemirings as well as pseudo ideals.

Example 3.9: Let S(V) = {collection of all matrix subsets from

$$V = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} \middle| \; a_i \in \langle Z_6 \cup h\rangle_I,\ 1 \leq i \leq 5, +, \times_n\right\}, +, \times_n\}$$

be the MOD matural neutrosophic special quasi dual number subset matrix pseudo semiring.

$$\text{Let } A = \{\begin{bmatrix} 0 \\ 3 \\ 3k \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 3+3k \\ 0 \\ 0 \\ 3k \end{bmatrix}, \begin{bmatrix} 3k+3 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\} \text{ and}$$

$$B = \{\begin{bmatrix} 2k \\ 4 \\ 2 \\ 5 \\ 4k \end{bmatrix}, \begin{bmatrix} 4k \\ 2 \\ 2k \\ 1 \\ 4k \end{bmatrix}, \begin{bmatrix} 2 \\ 2+2k \\ 4k \\ 4 \\ 4+4k \end{bmatrix}\} \in S(V).$$

$$A \times_n B = \{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\}$$

is a MOD natural neutrosophic special quasi dual number subset matrix zero divisor.

The reader is left with the task of finding nontrivial MOD natural neutrosophic special quasi dual number subset matrix strong idempotents, idempotents and nilpotents.

Next we proceed onto describe MOD natural neutrosophic matrix pseudo semirings whose entries are subsets from Z_n^I or $\langle Z_n \cup g\rangle_I$ or $C^I(Z_n)$ or $\langle Z_n \cup I\rangle_I$ or $\langle Z_n \cup h\rangle_I$ or $\langle Z_n \cup k\rangle_I$ by some examples.

Example 3.10: Let

$$W = \left\{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \middle| \; a_i \in S(Z_{12}^I); 1 \leq i \leq 3, +, \times_n\right\}$$

be the MOD natural neutrosophic matrix with subset entries pseudo semiring.

W has MOD natural neutrosophic matrix zero divisors given by

$$A = \{\begin{bmatrix} \{4,0\} \\ \{2,4\} \\ \{8,2\} \end{bmatrix}, \begin{bmatrix} \{0\} \\ \{4,8\} \\ \{0\} \end{bmatrix}\}, B = \{\begin{bmatrix} \{3,6,0\} \\ \{6,0\} \\ \{6\} \end{bmatrix}\} \in W$$

$$A \times_n B = \begin{bmatrix} \{0\} \\ \{0\} \\ \{0\} \end{bmatrix}$$

is a MOD natural neutrosophic subset matrix zero divisors.

$$\text{Let } A = \begin{bmatrix} \{4,9,0\} \\ \{9\} \\ \{4,0\} \end{bmatrix} \in W \text{ clearly}$$

$$A \times A = \begin{bmatrix} \{4,9,0\} \\ \{9\} \\ \{4,0\} \end{bmatrix} = A$$

is an MOD natural neutrosophic idempotent element of W.

We see P = $\begin{bmatrix} \{6\} \\ \{0,6\} \\ \{0\} \end{bmatrix}$ $\in$ W is such that

$$P \times P = \begin{bmatrix} \{0\} \\ \{0\} \\ \{0\} \end{bmatrix} \in W.$$

P is a MOD natural neutrosophic nilpotent element of order two.

This has both MOD natural neutrosophic subset matrix pseudo subsemirings which are not ideals as well as ideals.

$$B = \left\{ \begin{bmatrix} \{a\} \\ \{0\} \\ \{0\} \end{bmatrix} \middle| \; a_i \in S(Z_{12}^I), +, \times_n \right\} \subseteq W,$$

B is a MOD natural neutrosophic subset matrix pseudo sub semiring which is not a pseudo ideal for if,

$$A = \begin{bmatrix} \{a\} \\ \{0\} \\ \{0\} \end{bmatrix} \in B \text{ and } T = \begin{bmatrix} \{b\} \\ \{I_0^{12}\} \\ \{I_0^{12}\} \end{bmatrix} \in W \text{ then}$$

$$A \times_n T = \begin{bmatrix} \{c\} \\ \{I_0^{12}\} \\ \{I_6^{12}\} \end{bmatrix} \notin B.$$

The reader is expected to find MOD natural neutrosophic matrix with subset entries which are pseudo ideals of W.

Example 3.11: Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \\ a_{11} & a_{12} \end{bmatrix} \middle| \; a_i \in S(Z_{13}^I), 1 \leq i \leq 12, +, \times_n \right\}$$

be the MOD natural neutrosophic subset matrix pseudo semiring.

M has several trivial MOD subset zero divisors. However finding nontrivial zero divisors and nilpotents happens to be a challenging problem.

In view of all these we have the following result.

THEOREM 3.4: *Let S = {m × t matrices with entries from $S(Z_n^I)$; +, $\times_n$} be the MOD natural neutrosophic subset matrix pseudo semiring.*

i) $o(S) < \infty$.
ii) S has MOD natural subset matrix idempotents and strong idempotents.
iii) S has nontrivial MOD natural neutrosophic zero divisors and nilpotents only for appropriate n.
iv) S has several MOD natural neutrosophic subset matrix pseudo subsemirings which are not ideals.
v) S has MOD natural neutrosophic subset matrix pseudo ideals.

Proof is direct hence left as an exercise to the reader.

Next we describe MOD natural neutrosophic matrix subset finite complex number pseudo semiring by examples.

Example 3.12: Let

$$S = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \end{bmatrix} \middle| \; a_i \in S(C^I(Z_{10})); 1 \leq i \leq 8, +, \times_n \right\}$$

be the MOD natural neutrosophic finite number complex subset matrix pseudo semiring.

S has MOD subset idempotents strong and other wise.

S has MOD subset pseudo subsemirings as well as ideals. S has nontrivial MOD matrix subset zero divisors and nilpotents which are nontrivial does not exist.

Example 3.13: Let

$$V = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \middle| \; a_i \in S(C^I(Z_{16})), 1 \leq i \leq 4, +, \times_n \text{ or } \times \right\};$$

$+ \times$ (or $\times_n$)} be the MOD natural neutrosophic finite complex number subset matrix pseudo semiring.

V is non commutative under usual product and commutative under $\times_n$.

Let us show by a simple subsets.

$$A = \begin{bmatrix} \{I_0^C, 2+I_4^C, 4+i_F\} & \{I_2^C, I_0^C+5, 2+4i_F\} \\ \{0\} & \{3+i_F+I_0^C+I_8^C, I_{12}^C\} \end{bmatrix} \text{ and}$$

$$B = \begin{bmatrix} \{8+I_4^C, 2, 4+I_2^C\} & \{6, 2+I_2^C\} \\ \{5+I_4^C, I_{4+8i_F}^C, I_{2+4i_F}^C\} & \{2, 3, I_2^C\} \end{bmatrix} \in V.$$

We find

$$A \times B;\ A \times B = \begin{bmatrix} \{I_0^C, 2+I_4^C, 4+i_F\} & \{I_2^C, I_0^C+5, 2+4i_F\} \\ \{0\} & \{3+i_F+I_0^C+I_8^C, I_{12}^C\} \end{bmatrix} \times$$

$$\begin{bmatrix} \{8+I_4^C, 2, 4+I_2^C\} & \{6, 2+I_2^C\} \\ \{5+I_4^C, I_{4+8i_F}^C, I_{2+4i_F}^C\} & \{2,3,I_2^C\} \end{bmatrix}$$

$$= \begin{bmatrix} \begin{matrix} \{I_0^C, I_4^C+I_0^C, 4+I_4^C, 8+I_2^C \\ +I_4^C+I_8^C, 8i_F+I_4^C \\ 8+2i_F, I_2^C+4i_F\} + \\ \{I_2^C+I_8^C, I_8^C, I_{4+8i_F}^C, I_0^C + \\ I_4^C+9, I_0^C+I_{4+8i_F}^C, I_0^C + \\ I_{2+4i_F}^C, 10+4i_F+I_4^C, \\ I_{4+8i_F}^C, I_{2+4i_F}^C\} \end{matrix} & \begin{matrix} \{I_0^C, 12+I_4^C, 6i_F, I_0^C, 4+I_4^C+I_2^C \\ +I_8^C, 8+2i_F+I_2^C\} + \{I_2^C, I_{0+10}^C, 4+8i_F, \\ 15+I_0^C,\ 6+12i_F, I_4^C, I_0^C+I_4^C, I_2^C\} \end{matrix} \\ \begin{matrix} \{I_4^C, I_2^C\} + \{7+5i_F+I_4^C+I_0^C \\ +I_8^C, I_{2+4i_F}^C, I_{12}^C+I_0^C, I_{4+8i_F}^C \\ +I_0^C+I_0^C, I_0^C\} \end{matrix} & \begin{matrix} \{I_2^C, 0, 6+2i_F+I_0^C+I_8^C, \\ I_{12}^C, 9+3i_F+I_0^C+I_8^C, \\ I_3^C+I_0^C, I_0^C\} \end{matrix} \end{bmatrix}$$

This is the way product operation '×' is performed. It is left as an exercise to the reader to prove $A \times B \neq B \times A$.

However $A \times_n B = B \times_n A$. We give the following result.

THEOREM 3.5: *Let S = {m × t matrices with entries from $S(C^I(Z_n))$, + $\times_n$} the MOD natural neutrosophic finite complex number subset matrix pseudo semiring.*

i) $o(S) < \infty$.
ii) S has MOD natural neutrosophic strong matrix subset idempotents as well as MOD natural neutrosophic matrix subset idempotents.

iii) S has MOD *natural neutrosophic finite complex number subset matrix zero divisors and nilpotents only for certain value of n.*

iv) S has MOD *natural neutrosophic subset matrix pseudo subsemirings.*

v) S has MOD *natural neutrosophic subset matrix pseudo subsemirings which are ideals.*

Proof is direct and hence left as an exercise to the reader.

Next we proceed onto describe MOD natural neutrosophic - neutrosophic subset matrix pseudo semiring by some examples.

Example 3.14: Let

$$V = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \end{bmatrix} \middle| \, a_i \in S\langle Z_5 \cup h\rangle_I);\ 1 \leq i \leq 8, +, \times_n \right\}$$

be the MOD natural neutrosophic-neutrosophic subset matrix pseudo semiring.

$$\text{Let } A = \begin{bmatrix} \{I, 3I\} & \{4I, 0\} \\ \{1, I\} & \{2, I_0^I\} \\ \{I_{2I,1}^I\} & \{1 + I_0^I + I_{2I}^I\} \\ \{0\} & \{I_0^I\} \end{bmatrix} \text{ and}$$

$$B = \begin{bmatrix} \{0\} & \{4\} \\ \{1 + I_0^I\} & \{2 + 3I\} \\ \{0, 1, 2\} & \{2I, I_0^I\} \\ \{I_{4I}^I, 2I\} & \{3I + 2, I_{2I}^I\} \end{bmatrix} \in V$$

$$A \times_n B = \begin{bmatrix} \{I, 3I\} & \{4I, 0\} \\ \{1, I\} & \{2, I_0^I\} \\ \{I_{2I,1}^I\} & \{1 + I_0^I + I_{2I}^I\} \\ \{0\} & \{I_0^I\} \end{bmatrix}.$$

This is the way $\times_n$ operation is performed on V.

$$A + B = \begin{bmatrix} \{I, 3I\} & \{4 + 4I, 4\} \\ \{2 + I_0^I, 1 + I + I_0^I\} & \{4 + 3I, 2 + 3I + I_0^I\} \\ \{I_{2I,1}^I 1, 2, 1 + I_{2I}^I\} & \{2I + 1 + I_0^I + I_{2I}^I, \\ 3, 2 + I_{2I}^I\} & 1 + I_0^I, I_{2I}^I\} \\ \{I_{4I}^I, 2I\} & \{I_0^I + 3I + 2, I_0^I + I_{2I}^I\} \end{bmatrix} \in V.$$

This is the way + operation on V is performed.

$$[\{0\}] = \begin{bmatrix} \{0\} & \{0\} \\ \{0\} & \{0\} \\ \{0\} & \{0\} \\ \{0\} & \{0\} \end{bmatrix}$$

acts as the additive identity for A + [{0}] = A for all $A \in A$.

However $A \times [\{0\}] \neq [\{0\}]$ in general for A in V.

All properties can be derived in case of MOD natural neutrosophic - neutrosophic subset matrix pseudo semiring.

This task is left as an exercise to the reader as it is a matter of routine.

Next we illustrate by an example the MOD natural neutrosophic dual number subset matrix pseudo semiring.

Example 3.15: Let

$$D = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} \middle| \; a_i \in S(\langle Z_{12} \cup g \rangle_I);\ 1 \leq i \leq 5, +, \times_n \right\}$$

be the MOD natural neutrosophic dual number subset matrix pseudo semiring.

Clearly D has nontriial MOD natural neutrosophic dual subset matrix zero divisors and nilpotents of order two.

$$\text{Let } A = \begin{bmatrix} \{2g,3g,5g,g\} \\ \{g,0,9g\} \\ \{11g,3g,8g\} \\ \{g,4g,6g\} \\ \{10g,9g,7g,g\} \end{bmatrix} \text{ and } B = \begin{bmatrix} \{0,g,3g,5g\} \\ \{2g,5g,9g,g\} \\ \{3g,g,11g,5g\} \\ \{4g,0,2g\} \\ \{8g\} \end{bmatrix} \in D,$$

$$\text{clearly } A \times_n B = \begin{bmatrix} \{0\} \\ \{0\} \\ \{0\} \\ \{0\} \\ \{0\} \end{bmatrix},$$

$$A \times_n A = \begin{bmatrix} \{0\} \\ \{0\} \\ \{0\} \\ \{0\} \\ \{0\} \end{bmatrix} \text{ and } B \times_n B = \begin{bmatrix} \{0\} \\ \{0\} \\ \{0\} \\ \{0\} \\ \{0\} \end{bmatrix}.$$

Thus D has several nontrivial MOD natural neutrosophic subset matrix zero divisors and nilpotents of order two.

$$P = \begin{bmatrix} \{I_g^g, I_{2g}^g, I_0^g\} \\ \{I_0^g, I_{5g}^g, I_g^g\} \\ \{I_{10g}^g, I_{11g}^g, I_{7g}^g\} \\ \{I_{5g}^g, I_{6g}^g\} \\ \{I_{2g}^g, I_{6g}^g\} \end{bmatrix} \in D. \text{ We see } P \times_n P = \begin{bmatrix} \{I_0^g\} \\ \{I_0^g\} \\ \{I_0^g\} \\ \{I_0^g\} \\ \{I_0^g\} \end{bmatrix}$$

so P is a MOD natural neutrosophic zero associated subset nilpotent matrix. Clearly one can see P is different from A and B.

The reader is left with the task of finding MOD natural neutrosophic dual number subset matrix pairs which result in mixed zeros.

Also we see D has MOD subset pseudo subsemirings which are zero square subsemirings so does not result in pseudo nature. This task is also left as an exercise to the reader. However we record the following result.

THEOREM 3.6: *Let S = {m × t matrices with entries from S(⟨Z_n ∪ g⟩$_I$) = {collection of all subsets from ⟨Z_{48} ∪ g⟩$_I$}, +, $\times_n$} be the MOD natural neutrosophic dual number subset matrix pseudo semiring.*

i) o(S) < ∞.

ii) S has MOD natural neutrosophic subset matrix nilpotents P of order two such that P × P = ({0}) or P × P = [{ I_0^g }] or P × P = [{ I_0^g,0}], 2 ≤ n < ∞.

iii) S has MOD natural neutrosophic subset matrix zero divisors of all the three types; 2 ≤ n < ∞.

iv) S has MOD natural neutrosophic subset matrix pseudo subsemirings which is not an ideal and it is a zero square subsemiring so is not pseudo, 2 ≤ n < ∞.

v) *S has MOD natural neutrosophic subset matrix pseudo subsemirings which are ideals for all* $2 \leq n < \infty$.

vi) *S has nontrivial MOD natural neutrosophic subset matrix strong idempotents as well as idempotents only for special values of* n.

Proof is direct and hence left as an exercise to the reader.

It is important to keep on record that only MOD natural neutrosophic subset matrix dual numbers behave in a very different way from other pseudo semirings.

However all MOD pseudo semirings enjoy some or other special feature associated with.

Another observation we see is $S(Z_n^I) \subseteq S(C^I(Z_n))$, $S(Z_n^I) \subseteq S(\langle Z_n \cup I\rangle_I)$; $S(Z_n^I) \subseteq S(\langle Z_n \cup g\rangle_I)$; $S(Z_n^I) \subseteq S(\langle Z_n \cup h\rangle_I)$ and $S(Z_n^I) \subseteq S(\langle Z_n \cup k\rangle_I)$.

This all MOD pseudo semirings will contain a nontrivial MOD pseudo subsemiring which is not an ideal.

Next we describe by examples MOD natural neutrosophic subset matrix special dual like number pseudo semiring and MOD natural neutrosophic subset matrix special quasi dual number pseudo semiring.

Example 3.16: Let

$$B = \left\{ \begin{bmatrix} a_1 & a_2 & a_{11} \\ a_3 & a_4 & a_{12} \\ a_5 & a_6 & a_{13} \\ a_7 & a_8 & a_{14} \\ a_9 & a_{10} & a_{15} \end{bmatrix} \middle| \; a_i \in S((\langle Z_{42} \cup h\rangle_I); 1 \leq i \leq 15, +, \times_n\} \right.$$

be the MOD natural neutrosophic subset matrix pseudo semiring.

$$P_1 = \left\{ \begin{bmatrix} a_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \middle| \; a_i \in S((\langle Z_{42} \cup h\rangle_I); +, \times_n\} \subseteq B$$

is a MOD natural neutrosophic subset special dual like number matrix pseudo subsemiring which is not an ideal.

$$P_2 = \left\{ \begin{bmatrix} 0 & a_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \middle| \; a_2 \in S(\langle Z_{42} \cup h\rangle_I); +, \times_n \right\}$$

is again a MOD natural neutrosophic special dual like number subset matrix subsemiring which is not an ideal.

Here we wish to state if the product is changed as usual zero dominated product that is $\{0\} \times x = \{0\}$ for all $x \in S(\langle Z_n \cup h\rangle_I)$ then both P_1 and P_2 will be pseudo ideals but we have not used this product throughout this book.

We use only $I_t^h \times a = I_t^h$ a not a natural neutrosophic number.

Study of all properties associated with B happens to be a matter of routine so left as an exercise to the reader.

Example 3.17: Let

$$V = \{(a_1\ a_2\ a_3\ a_4) / a_i \in S(\langle Z_{10} \cup k\rangle_I); 1 \leq i \leq 4, +, \times_n\}$$

be the MOD natural neutrosophic special quasi dual number subset matrix pseudo semiring. We have MOD natural neutrosophic special quasi dual number subset matrix idempotents as well as strong idempotents.

We see V has no nontrivial MOD natural neutrosophic special quasi dual number subset matrix nilpotents.

V has MOD natural neutrosophic special quasi dual number subset matrix pseudo subsemirings.

Let ({5, 6, 0}, {0, 6, 5k}, {0, 5}, {5 + 5k, 5, 1}) = A $\in$ A.

Clearly A $\times$ A = A but

A + A = ({5,6, 0}, {0, 6, 5k}, {0, 5}, {5 + 5k, 5, 1}) + ({5, 6, 0}, {0, 6, 5k}, {0, 5}, {1, 5, 5 + 5k})

= ({0, 2, 1, 5, 6}, {0, 6, 5k, 2, 6 + 4k}, {0, 5}, {2, 0, 6, 5k, 6 + 5k} $\in$ V.

We see A is only a MOD natural neutrosophic idempotent but is not a MOD natural neutrosophic strong idempotent. Study in this direction is considered as a matter of routine.

Let E = ({5, 5 + 5k, 0}, {5k, 0}, {5k, 5 + 5k, 5}, {5 + 5k, 0})

B = ({2, 4, 8}, {0, 4, 6k}, {6, 2 , 8k, 8 + 8k}, {2, 2 + 2k, 4 + 8k, 8 + 6k}) $\in$ V;

E $\times$ B = ({0}, {0}, {0}, {0}) is a MOD natural neutrosophic subset matrix zero divisor.

Let D = ({ I_8^k , I_0^k , I_0^k }, { I_{6+4k}^k , I_{8k+8}^k , I_{4+4k}^k }, { I_{8k}^k , I_{2k}^k , I_6^k }) and

$E = (\{I_0^k, I_5^k\}, \{I_{5+5k}^k, I_5^k\}, \{I_{5+5k}^k, I_5^k, I_5^k\}, \{I_0^k, I_5^k, I_5^k\}) \in V.$

$D \times E = (\{I_0^k\}, \{I_0^k\}, \{I_0^k\}, \{I_0^k\})$ is again a MOD natural neutrosophic special quasi dual number subset matrix zero divisor, different from usual zero divisor.

However V has no nontrivial MOD natural neutrosophic nilpotents. As in case of other MOD natural neutrosophic subset matrix special quasi dual number pseudo semirings all related properties can be derived.

This is left as an exercise to the reader.

Next we describe MOD natural neutrosophic polynomial subsets pseudo semirings by some examples.

Example 3.18: Let S(M[x]) = {collection of all MOD natural neutrosophic polynomial subsets from

$$M[x] = \left\{\sum_{i=0}^{\infty} a_i x^i \;\middle|\; a_i \in Z_8^I; +, \times\right\}, +, \times\}$$

be the MOD natural neutrosophic polynomial subsets pseudo semiring.

Let $A = \{5x^3 + 2x + I_0^8, 2x + 3, I_2^8 + I_4^8 x^2\}$ and

$B = \{4x^2 + 1, I_2^8 x^3 + I_0^8, I_0^8 x + 4\} \in S(M[x])$

$A + B = \{5x^3 + 2x + I_0^8, 2x+3, I_4^8 x^2 + I_2^8\} + \{4x^2 + 1, I_2^8 x^3 + I_0^8, I_0^8 x + 4\}$

$= \{5x^3 + 4x^2 + 2x + 1 + I_0^8, 4x^2 + 2x + 4, (4 + I_4^8)x^2 + 1 + I_2^8, (5 + I_2^8) x^3 + 2x + I_0^8, I_2^8 x^3 + 2x + 3 + I_0^8 +$

$I_2^8 x^3 + I_4^8 x^2 + I_0^8 + I_2^8, 5x^3 + (2 + I_0^8) x + 4 + I_0^8 (I_0^8 + 2) x + 7, I_0^8 x + I_4^8 x^2 + 4 + I_2^8\} \in S(M[x])$.

This is the way + operation is performed on S(M[x]).

Consider $A \times B = \{5x^3 + 2x + I_0^8, 2x + 3, I_2^8 + I_4^8 x^2\} \times \{4x^2 + 1, I_2^8 x^3 + I_0^8, I_0^8 x + 4\} = \{4x^5 + I_0^8 x^2 + 5x^3 + 2x + I_0^8, 4x^2 + 3 + 2x, I_4^8 x^4 + I_2^8 x^2 + I_2^8 + I_4^8 x^2, I_2^8 x^6 + I_2^8 x^4 + I_0^8 x^3 + I_0^8 x + I_0^8, I_2^8 I_2^8 x^4 + I_0^8 x + I_2^8 x^3 + I_0^8, I_4^8 x^3 + I_0^8 x^5 + I_0^8 + I_0^8 x^2, I_0^8 x^4 + I_0^8 x^2 + I_0^8 x + 4x^3 + I_0^8, I_0^8 x^2 + I_0^8 x + 4, I_0^8 x + I_0^8 x^3 + I_2^8 + I_4^8 x^2\} \in S(M[x])$.

This has nilpotents of order two which are infinite in number.

$A = \{4x^3 + 4, 4x^2 + 4\} \in S(M[x])$ is such that $A \times A = \{0\}$.

Interested reader can find the MOD zero divisors and nilpotent subset polynomials.

Example 3.19: Let S[P(x)] = {collection of all subsets from $P[x] = \{\sum_{i=0}^{\infty} a_i x^i \,/\, a_i \in Z_{19}^I; +, \times\}, +, \times\}$ be the MOD natural neutrosophic polynomial subset pseudo semiring. S(P[x]) has no nontrivial MOD natural neutrosophic nilpotents or zero divisors.

S(M[x]) = {collection of all subsets from

$$M[x] = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in Z_{19}; +, \times\right\}, +, \times\} \subseteq S(P[x])$$

is a MOD natural neutrosophic polynomial subset pseudo subsemiring of S(P[x]) which is not an ideal of S(P[x]).

Study of other properties are a matter of routine and S(M[x]) is a MOD natual neutrosophic polynomial subset idempotent which is strong as S(M[x]) + S(M[x]) = S(M[x]) and S(M[x]) × S(M[x]) = S(M[x]), however apart from this we cannot find nontrivial strong idempotents of S(P[x]).

In view of all these we have the following theorem.

THEOREM 3.7: *Let S(V[x]) = {collection of all subsets from*

$$V[x] = \left\{ \sum_{i=0}^{\infty} a_i x^i \;\middle|\; a_i \in Z_n^I \,;\, +, \times \right\}, +, \times\}$$

be the MOD natural neutrosophic polynomial subset pseudo semiring.

- *i) S(V[x]) has MOD natural neutrosophic strong idempotents which are only MOD natural neutrosophic polynomial pseudo subsemirings of S(V[x]) for $2 \leq n < \infty$.*
- *ii) S(V[x]) has nontrivial MD natural neutrosophic strong nilpotents and zero divisors only if $n = p^{\alpha}q$, $\alpha \geq 2$, $(p, q) = 1$ and p a prime.*
- *iii) S(V[x]) has MOD natural neutrosophic polynomial subset pseudo subsemirings which are ideals as well as not ideals dependent only on n.*

Proof is direct and hence left as an exercise to the reader.

We proceed to give examples using $C^I(Z_n)$.

Example 3.20: Let S(W[x]) = {collection of all polynomial subsets from

$$W[x] = \left\{ \sum_{i=0}^{\infty} a_i x^i \;\middle|\; a_i \in C^I(Z_{12}); +, \times \right\}, +, \times\}$$

be the MOD natural neutrosophic polynomial subset finite complex number pseudo semiring.

This has nontrivial MOD natural neutrosophic zero divisors and nilpotents however idempotents are only of the type mentioned earlier as in case of Z_n^I.

Study of this is a matter of routine so left as an exercise to the reader.

Further the theorem 3 is also true in case of MOD natural neutrosophic subset polynomial finite complex number pseudo semiring.

This work is also left to the reader.

Next we briefly describe MOD natural neutrosophic-neutrosophic subset polynomial pseudo semirings by some examples.

Example 3.21: Let S(B[x]) = {collection of all subset polynomials from

$$B[x] = \left\{\sum_{i=0}^{\infty} a_i x^i \;\middle|\; a_i \in \langle Z_{15} \cup I\rangle_I +, \times\}, +, \times\right\}$$

be the MOD natural neutrosophic-neutrosophic subset polynomial pseudo semiring.

This has nontrivial zero divisors however no nilpotents or idempotents. Idempotents are only contributed by MOD natural neutrosophic-neutrosophic subset polynomial pseudo subsemirings.

Example 3.22: Let S(D[x]) = {collection of all subsets from

$$D[x] = \left\{\sum_{i=0}^{\infty} a_i x^i \;\middle|\; a_i \in \langle Z_{23} \cup I\rangle_I +, \times\}, +, \times\right\}$$

be the MOD natural neutrosophic-neutrosophic subset polynomial pseudo semiring.

S(D[x]) has no nontrivial MOD subset polynomial zero divisors or nilpotents.

The result / Theorem 3.7 is also true in case of MOD natural neutrosophic - neutrosophic subset polynomial, pseudo semirings with simple modifications.

Next we briefly describe MOD natural neutrosophic dual number subset polynomial pseudo semirings by some examples.

Example 3.23: Let S(N[x]) = {collection of all polynomial subsets from

$$N[x] = \left\{ \sum_{i=0}^{\infty} a_i x^i \;\middle|\; a_i \in \langle Z_{13} \cup g\rangle_I +, \times \right\}, +, \times\}$$

be the MOD natural neutrosophic dual number polynomial subset pseudo semiring.

S(N[x]) has MOD natural neutrosophic nilpotents of order two as well as infinite number of zero divisors.

However finding idempotents which are non trivial happens to be a challenging problem.

Another special feature enjoyed by these class of pseudo semirings is that they have MOD subset pseudo subsemirings which are zero square subsemirings. Further these also have finite order MOD natural neutrosophic pseudo subsemirings.

This are the special and distinct features which can be associated only with MOD natural neutrosophic dual number subset polynomial pseudo semirings.

The reader is left with the task of studying the special features associated with this structure. Further these MOD pseudo semirings are not affected by taking n a prime. When n is composite the specialty is one can get MOD pseudo sub semirings which are ideals.

Next we give an example of each MOD natural neutrosophic subset polynomial special dual like number pseudo semirings and MOD natural neutrosophic special quasi dual number subset polynomial pseudo semirings.

Example 3.24: Let S(Z[x]) = {collection of all subset polynomials from

$$Z[x] = \left\{\sum_{i=0}^{\infty} a_i x^i \;\middle|\; a_i \in \langle Z_{14} \cup h\rangle_I +, \times\right\}, +, \times\}$$

be the MOD natural neutrosophic special dual like number subset polynomial pseudo semiring.

We can as in case of other MOD natural neutrosophic subset polynomial pseudo semirings baring MOD natural neutrosophic dual number subset polynomial pseudo semirings obtain all properties.

This work is considered as a matter of routine and hence left as an exercise to the reader.

Example 3.25: Let S(F[x]) = {collection of all subset polynomials from

$$F[x] = \left\{\sum_{i=0}^{\infty} a_i x^i \;\middle|\; a_i \in \langle Z_{24} \cup k\rangle_I +, \times\right\}, +, \times\}$$

be the MOD natural neutrosophic special quasi dual number subset polynomial pseudo semiring.

S(F[x]) has non trivial MOD natural neutrosophic special quasi dual number subset polynomial nilpotents and zero divisors.

This has MOD natural neutrosophic special quasi dual number subset polynomial pseudo ideals as well as pseudo subsemirings which are not ideals.

The reader is left with the task of finding them.

However if Z_{24} is replaced by Z_p, p a prime certainly several of the claims are not true.

Next we describe MOD natural neutrosophic subset polynomials from polynomial semirings of finite order by some examples.

Example 3.26: Let $S(G[x]_9)$ = {collection of all MOD natural neutrosophic special quasi dual number subset polynomials from

$$G[x]_9 = \left\{\sum_{i=0}^{9} a_i x^i \;\middle|\; a_i \in \langle Z_{17} \cup k\rangle_I,\ x^9 = 1, +, \times\right\}, +, \times\}$$

be the MOD natural neutrosophic special quasi dual number subset polynomial pseudo semiring $o(S(G[x]_9)) < \infty$.

Finding nontrivial zero divisors, nilpotents and idempotents in $S(G[x]_9)$ happens to be challenging S(P[x]) = {collection of all subset polynomials from

$$P[x] = \left\{\sum_{i=0}^{9} a_i x^i \;\middle|\; a_i \in Z_{17},\ x^9 = 1, +, \times\right\}, +, \times\} \subseteq S(G[x])$$

is a MOD natural neutrosophic special quasi dual number subset polynomial pseudo subsemiring of finite order which is not an ideal of S(G[x]).

Example 3.27: Let $S(T[x]_3)$ = {collection of all subset polynomials from

$$T[x]_3 = \left\{ \sum_{i=0}^{3} a_i x^i \;\middle|\; a_i \in \langle Z_{48} \cup I\rangle_I,\ x^4 = 1, +, \times\right\}, +, \times\}$$

be the MOD natural neutrosophic-neutrosophic subset polynomial pseudo semiring.

S(T[x]) has nontrivial MOD natural neutrosophic subset polynomial nilpotents, zero divisors and idempotents.

This task is considered as a matter of routine so left as an exercise to the reader.

Example 3.28: Let $S(M[x]_{10})$ = {collection of all polynomial subsets from

$$M[x]_{10} = \left\{ \sum_{i=0}^{10} a_i x^i \;\middle|\; a_i \in \langle Z_{11} \cup g\rangle_I,\ x^{11} = 1, +, \times\right\}, +, \times\}$$

be the MOD natural neutrosophic dual number subset polynomial pseudo semiring.

This has finite number of nontrivial nilpotents as well as zero divisors.

Finding idempotents other than substructures is a challenging problem.

This has MOD subset pseudo subsemirings as well as ideals. The study in this direction is a matter of routine so left as an exercise to the reader. However it is pertinent to keep on record that the presence of nontrivial nilpotents and zero divisors happens to be different from other types of MOD pseudo semirings.

Another deviation is that these MOD pseudo semirings also have subsemirings which are not pseudo as well as which are zero square subsemirings.

Example 3.29: Let $S(V[x]_7)$ = {collection of all subset polynomials from

$$V[x]_7 = \left\{ \sum_{i=0}^{7} a_i x^i \;\middle|\; a_i \in C^I(Z_{12});\ x^8 = 1, +, \times \right\}, +, \times\}$$

be the MOD natural neutrosophic finite complex number subset polynomial pseudo semiring. It is a matter of routine to find MOD subset zero divisors, nilpotents and substructures of $S(V[x]_7)$.

Next we proceed onto describe MOD natural neutrosophic subset coefficient polynomial pseudo semirings by examples.

Example 3.30: Let

$$S[x] = \left\{ \sum_{i=0}^{\infty} a_i x^i \;\middle|\; a_i \in S(Z_{24}^I);\ +, \times \right\}$$

be the MOD natural neutrosophic subset coefficient polynomial pseudo semiring. This S[x] has nontrivial zero divisors and nilpotents.

However has no MOD idempotents of finite order. Infinite order MOD idempotents can arise from substructures.

$p(x) = \{6, 12, 0\} x^4 + \{0, 18\}$ and
$q(x) = \{4, 8, 0\}x^3 + \{4, 12\} \in S[x]$.
Clearly $p(x) \times q(x) = \{0\}$.

Let $m(x) = \{12, 0\}x^2 + \{12\} \in S[x]$; $m(x) \times m(x) = \{0\}$. Thus S[x] has nontrivial MOD zero divisors and nilpotents.

Let $p(x) = \{2, 4, I_0^{24}, 1\} x^3 + \{3,2, 8 + I_6^{24}\}$ and $q(x) = \{8, I_6^{24} + 6\} x^3 \{ 4+ I_8^{24}, I_6^{24}\} \in S[x]$. We find $p(x) + q(x)$ and $p(x) \times q(x)$ in the following.

p(x) + q(x) = $\{2, 4, 1, I_0^{24}\}$ x^3 + $\{3, 2, 8, I_6^{24}\}$ + $\{8, 6 + I_6^{24}\}x^3$ + $\{4, I_8^{24}, I_6^{24}\}$ = $\{10, 12, 9, 8 + I_0^{24}, 8 + I_6^{24}, 10 + I_6^{24}, 7 + I_6^{24}, I_0^{24} + I_6^{24} + 6\}$ x^3 + $\{7 + I_8^{24}, 6 + I_8^{24}, 12 + I_6^{24} + I_8^{24}, 3 + I_6^{24}, 2 + I_6^{24}, 8 + I_6^{24}\}$.

This is the way the operation of addition is performed on S[x].

p(x) × q(x) = $(\{2, 4, I_0^{24}, 1\}$ x^3 + $(3, 2, 8 + I_6^{24}\}) \times (\{8, I_6^{24} + 6\}$ x^3 + $\{4 + I_8^{24}, I_6^{24}\})$

= $\{16, 8, I_0^{24}, 12 + I_6^{24}, I_6^{24}, I_6^{24} + 6, I_0^{24}\}$ x^6 + $\{0, 6 + I_6^{24}, 16, I_6^{24} + 12, 16 + I_6^{24}, I_6^{24} + I_{12}^{24}\}x^3$ + $\{8 + I_8^{24}, 16 + I_8^{24}, I_6^{24}, 4 + I_8^{24}, I_0^{24}\}x^3$ + $\{12 + I_8^{24}, 8 + I_8^{24}, 8 + I_6^{24} + I_8^{24}, I_0^{24}, I_6^{24}, I_6^{24} + I_{12}^{24}\}$ ∈ S[x].

This is the way product operation is performed on S[x].

This is the way infact operations are performed on all other MOD natural neutrosophic subset coefficient polynomial pseudo semirings.

We will a few examples in this direction.

Example 3.31: Let

$$S[x] = \left\{\sum_{i=0}^{\infty} a_i x^i \;\middle|\; a_i \in S(\langle Z_{11} \cup k\rangle); +, \times\right\}$$

be the MOD natural neutrosophic special quasi dual number subset coefficient polynomial semiring.

Finding nontrivial MOD natural neutrosophic subset coefficient polynomial zero divisors and nilpotents does not arise as S[x] does not contain such special elements.

However S[x] has substructure S[x] cannot have nontrivial idempotents.

Example 3.32: Let

$$S[x] = \left\{ \sum_{i=0}^{\infty} a_i x^i \;\middle|\; a_i \in S\,(C^I(Z_6)), +, \times \right\}$$

be the MOD natural neutrosophic finite complex number subset coefficient polynomial pseudo semiring.

S[x] has nontrivial MOD natural neutrosophic zero divisors and a few nilpotents. S[x] has substructures.

$p(x) = \{3 + 3i_F, 0\}\, x^6 + \{3 + 3i_F\} \in S[x]$ is such that

$p(x) \times p(x) = (\{0\})$.

Example 3.33: Let

$$S[x] = \left\{ \sum_{i=0}^{\infty} a_i x^i \;\middle|\; a_i \in S\,(\langle Z_{10} \cup g\rangle_I); +, \times \right\}$$

be the MOD natural neutrosophic dual number subset coefficient polynomial pseudo semiring.

S[x] has nontrivial MOD natural neutrosophic dual number subset zero divisors and nilpotents.

This has also MOD subset zero square subsemirings.

Example 3.34: Let

$$V[x] = \left\{ \sum_{i=0}^{\infty} a_i x^i \;\middle|\; a_i \in S\,(\langle Z_{18} \cup h\rangle_I); +, \times \right\}$$

be the MOD natural neutrosophic special dual like number subset coefficient polynomial semiring.

Finding nontrivial MOD natural neutrosophic nilpotents, zero divisors and substructures are left as an exercise at it is considered as a matter of routine.

Next we give one more example.

Example 3.35: Let

$$B[x] = \left\{\sum_{i=0}^{\infty} a_i x^i \;\middle|\; a_i \in S\,(\langle Z_9 \cup I\rangle_I); +, \times\right\}$$

be the MOD natural neutrosophic-neutrosophic subset coefficient polynomial pseudo semiring.

All related properties can be derived for B[x] so left as an exercise to the reader.

Next we proceed onto describe MOD natural neutrosophic subset coefficient polynomial semirings of finite order by examples.

Example 3.36: Let

$$W[x]_7 = \left\{\sum_{i=0}^{7} a_i x^i \;\middle|\; a_i \in S\,(Z_{48}^{I}); x^8 = 1, +, \times\right\}$$

be the MOD natural neutrosophic subset coefficient polynomial pseudo semiring.

$o(W[x]_7) < \infty$ and $W[x]_7$ has nontrivial MOD subset zero divisors and nilpotents. $W[x]_7$ has also MOD substructures.

Example 3.37: Let

$$B[x]_3 = \left\{ \sum_{i=0}^{7} a_i x^i \;\middle|\; a_i \in S(\langle Z_5 \cup I\rangle_I);\ x^4 = 1, +, \times \right\}$$

be the MOD natural neutrosophic-neutrosophic subset coefficient polynomial pseudo semiring.

$o(B[x]_3) < \infty$.

Finding in $B[x]_3$ nontrivial nilpotents and zero divisors happens to be a challenging problem.

However it has pseudo subsemirings which are not ideals.

Example 3.38: Let

$$V[x]_{10} = \left\{ \sum_{i=0}^{10} a_i x^i \;\middle|\; a_i \in S(C^I(Z_{42}));\ x^{11} = 1, +, \times \right\}$$

be the MOD natural neutrosophic finite complex number subset coefficient pseudo semiring.

$V[x]_{10}$ has MOD substructure as well as nontrivial MOD subset zero divisors and nilpotents.

Example 3.39: Let

$$S[x]_5 = \left\{ \sum_{i=0}^{5} a_i x^i \;\middle|\; a_i \in S(\langle Z_{12} \cup h\rangle_I);\ x^6 = 1, +, \times \right\}$$

be the MOD natural neutrosophic special dual like number subset coefficient polynomial pseudo semiring.

All properties related to $S[x]_5$ can be studied by any interested reader as it is considered as a matter of routine.

Study of MOD natural neutrosophic subset coefficient polynomial pseudo semirings of both infinite and finite order can be analysed.

Almost all properties in this direction can be derived as a matter of routine.

However finding special MOD elements happens to be challenging one.

For the MOD natural neutrosophic dual number subset coefficient polynomial pseudo semirings alone behave in a very different way from others.

We suggest a set of problems some of which are easy and at research level.

Problems

1. Let S = {S(G) collection of all subsets from G = { m × t matrices with entries from Z^I_{48}, + ×}, +, ×} be the MOD natural neutrosophic m × t matrix subsets.

   Obtain all special features associated with S.

2. Let P = {S(G) collection of all subsets from

$$G = \left\{ \left. \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} \end{bmatrix} \right| a_i \in Z^I_{29},\ 1 \leq i \leq 18, +, \times_n \right\}, +, \times_n\}$$

   be the MOD natural neutrosophic matrix subset pseudo semring.

i) Find o(P).
ii) Find all nontrivial MOD natural neutrosophic matrix subset zero divisors and nilpotents
iii) Can P have nontrivial MOD subset natural neutrosophic idempotents?
iv) Find all MOD natural neutrosophic subset matrix pseudo subsemirings which are not ideals.
v) Find all MOD natural neutrosophic matrix subset pseudo subsemirings which are ideals.
vi) Study this problem when Z_{19}^{I} is replaced by Z_{48}^{I} and $Z_{5^7}^{I}$.

3. Let W = {Collection of all matrix subsets from

$$M = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \end{bmatrix} \middle| \; a_i \in \langle Z_{18} \cup I \rangle_I,\ 1 \leq i \leq 8, +, \times_n \right\}, +, \times_n \}$$

be the MOD natural neutrosophic neutrosophic matrix subset pseudo semiring.

i) Study questions (i) to (v) of problem (2) for this W.
ii) Study this problem when $\langle Z_{18} \cup I \rangle_I$ is replaced by $\langle Z_{96} \cup I \rangle_I$ and $\langle Z_{3^9}^{I} \cup I \rangle_I$.
iii) Compare W with P in problem (2).

4. Let V = {collection of all matrix subsets from

$$J = \left\{\left.\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \end{bmatrix}\right| a_i \in \langle Z_{12} \cup g\rangle_I,\ 1 \leq i \leq 8, +, \times_n\},\right.$$

$$+, \times_n \}$$

be the MOD natural neutrosophic dual number matrix subset pseudo semiring.

i) Study questions (i) to (v) of problem (2) for this V.
ii) Compare V with W of the problem (3)
iii) Study V when $\langle Z_{12} \cup I\rangle_I$ is replaced by $\langle Z_{19} \cup I\rangle_I$ and $\langle Z_{49} \cup g\rangle_I$.
iv) Show what ever be Z_n the MOD natural neutrosophic dual number matrix subsets has several zero divisors and nilpotents.
v) Show only these MOD natural neutrosophic pseudo semiring alone has subsemirings which are zero square subsemirings and are not pseudo.
vi) Enumerate any other special feature enjoyed by V.

5. Let M = {collection of all matrix subsets from

$$F = \left\{\left.\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \\ a_{11} & a_{12} \\ a_{13} & a_{14} \end{bmatrix}\right| a_i \in C^I(Z_{45});\ 1 \leq i \leq 14, \times_n, +\}, +, \times_n\right\}$$

be the MOD natural neutrosophic finite complex number matrix subsets pseudo semiring.

i) Study questions (i) to (v) of problem (2) for this M.
ii) Compare M with V of problem 4.
iii) Enumerate all the special features enjoyed by M.
iv) Compare M with W of problem 3.

6. Let G = {collection of all matrix subsets from

$$H = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \middle| \; a_i \in \langle Z_{92} \cup k \rangle_I,\ 1 \leq i \leq 9,\ +,\ \times_n \text{ (or } \times)\right\},$$

$+, \times_n$ (or $\times$)}

be the MOD natural neutrosophic special quasi dual number matrix subset pseudo semiring.

i) Study questions (i) to (v) of problem (2) for this G.
ii) Study G when $\langle Z_{92} \cup k\rangle_I$ is replaced by $\langle Z_{41} \cup k\rangle_I$ and $\langle Z_{7^4} \cup k\rangle_I$.
iii) Enumerate any special feature associated with G.
iv) Compare G with M and V of problem (5) and (4) respectively.

7. Let L = {collection of all matrix subsets from

$$Z = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{26} & a_{27} & a_{28} & a_{29} & a_{30} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \end{bmatrix} \middle| \; a_i \in \langle Z_{43} \cup h \rangle_I,\ 1 \leq i \leq 35,\ +, \right.$$

$\times_n$ (or $\times$)}, $+, \times_n$}

be the MOD natural neutrosophic special dual like number matrix subset pseudo semiring.

i) Study questions (i) to (v) of problem (2) for this L.
ii) Study L by replacing the set $\langle Z_{43} \cup h\rangle_I$ by $\langle Z_{3^6} \cup h\rangle_I$.
iii) Compare L with G, M and V of problems 4, 6 and 5

respectively.

8. Let S = $\left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} \middle| a_i \in S(Z_{16}^{I}), 1 \leq i \leq 5, +, \times_n \right\}$

be the MOD natural neutrosophic matrices whose entries are subset (subset matrices) from Z_{16}^{I} be the pseudo semiring.

i) Find o(S).
ii) Find all MOD natural neutrosophic subset matrix zero divisors.
iii) Find all MOD natural neutrosophic subset matrix zero divisors.
iv) Can S have MOD natural neutrosophic idempotents?
v) Find all MOD natural neutrosophic subset matrix pseudo subsemirings which are not ideals.
vi) Find all MOD natural neutrosophic subset matrix pseudo subsemirings which are ideals.
vii) Find all special features associated with S.
viii) Study S when $S(Z_{16}^{I})$ is replaced by $S(Z_{17}^{I})$ and $S(Z_{48}^{I})$.

9. Let W = $\left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \middle| \; a_i \in S(C^I(Z_{44})); 1 \leq i \leq 16, \right.$

$+, \times_n$ (or $\times$)} be the MOD natural neutrosophic subset matrix finite complex number pseudo semiring.

i) Study questions (i) to (vii) of problem 8 for this W.
ii) Study W as a MOD pseudo non commutative semiring.
iii) In case of (W, $\times$, +) find all MOD right pseudo ideals which are not MOD left pseudo ideals.
iv) Find all MOD natural neutrosophic subset right zero divisors which are not left zero divisors.

10. Let B = $\left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \end{bmatrix} \middle| \; a_i \in S(\langle Z_{47} \cup I\rangle_I, \right.$

$1 \leq i \leq 10, +, \times_n\}$

be the MOD natural neutrosophic-neutrosophic subset matrix pseudo semiring.

i) Study questions (i) to (vii) of problem 8 for this B.
ii) Enumerate all special features enjoyed by B.
iii) Study B if S $(\langle Z_{48} \cup I\rangle_I)$ or $(\langle Z_{64} \cup I\rangle_I)$.

11. Let Z = $\left\{\begin{bmatrix} a_1 & a_5 \\ a_2 & a_6 \\ a_3 & a_7 \\ a_4 & a_8 \end{bmatrix} \middle| a_i \in S(\langle Z_{10} \cup g\rangle_I, 1 \leq i \leq 8, +, \times_n\right\}$

be the MOD natural neutrosophic dual number subset matrix pseudo semiring.

i) Study questions (i) to (vii) of problem 8 for this Z.
ii) Enumerate all special features enjoyed by Z.
iii) Study this if $S(\langle Z_{10} \cup g\rangle_I)$ is replaced by $S(\langle Z_{47} \cup g\rangle_I)$ or by $S(\langle Z_{128} \cup g\rangle_I)$.

12. Let G = $\left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} & a_{17} & a_{18} \end{bmatrix} \middle| a_i \in S(\langle Z_{25} \cup h\rangle_I,\right.$

$\left. 1 \leq i \leq 18, +, \times_n\right\}$

be the MOD natural neutrosophic special dual like number subset matrix pseudo semiring.

i) Study questions (i) to (vii) of problem 8 for this G.
ii) Enumerate all special features enjoyed by this G.
iii) Study G if $S(\langle Z_{25} \cup h\rangle_I)$ is replaced by $S(\langle Z_{53} \cup h\rangle_I)$ or $S(\langle Z_{484} \cup h\rangle_I)$.

13. Let H = $\left\{\left.\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} \\ a_{19} & a_{20} & a_{21} \end{bmatrix} \right| a_i \in S(\langle Z_{48} \cup k\rangle_I, +, \times_n\right\}$ be the

MOD natural neutrosophic special quasi dual number subset matrix pseudo semiring.

i) Study questions (i) to (vii) of problem 8 for this H.
ii) Enumerate all special features enjoyed by H.
iii) Study H if $S(\langle Z_{48} \cup k\rangle_I)$ is replaced by $S(\langle Z_{23} \cup k\rangle_I)$ and (or) $S(\langle Z_{16} \cup k\rangle_I)$.
iv) Compare H if $S(\langle Z_{48} \cup k\rangle_I)$ is replaced by $S(\langle Z_{48} \cup g\rangle_I)$

14. Let P[x] = $\left\{\left.\sum_{i=0}^{\infty} a_i x^i \right| a_i \in S(Z_{12}^I); +, \times\right\}$

be the MOD natural neutrosophic subset coefficient polynomial pseudo semiring.

i) Prove P[x] is of infinite order.
ii) Show P[x] has MOD natural neutrosophic zero divisors.
iii) Show P[x] has MOD natural neutrosophic nilpotents.
iv) Prove P[x] has no nontrivial MOD natural neutrosophic idempotents.
v) Find all MOD natural neutrosophic subset coefficient polynomial pseudo subsemirings which are not ideals.
vi) Find all MOD natural neutrosophic subset coefficient polynomial pseudo subsemiring which are ideals.
vii) Enumerate all special features associated with P[x].

15. Let $M[x] = \left\{\sum_{i=0}^{\infty} a_i x^i \;\middle|\; a_i \in S\ (C^I(Z_{15})); +, \times\right\}$ be the MOD natural neutrosophic finite complex number subset coefficient polynomial pseudo semiring.

    i) Study questions (i) to (vi) of problem (14) for this M[x].
    ii) Study M[x] if $S(C^I(Z_{15}))$ is replaced by $S(C^I(Z_{43}))$ (and) or $S(C^I(Z_{48}))$ or by $S(C^I(Z_{128}))$.

16. Let $W[x] = \left\{\sum_{i=0}^{\infty} a_i x^i \;\middle|\; a_i \in S\ (\langle Z_{12} \cup I\rangle_I); +, \times\right\}$ be the MOD natural neutrosophic-neutrosophic subset coefficient polynomial pseudo semiring.

    i) Study questions (i) to (vi) of problem (14) for this W[x].
    ii) Compare M[x] of problem (15) with this W[x].

17. Let $B[x] = \left\{\sum_{i=0}^{\infty} a_i x^i \;\middle|\; a_i \in S\ (\langle Z_{12} \cup g\rangle_I); +, \times\right\}$

    be the MOD natural neutrosophic dual number subset coefficient polynomial pseudo semiring.

    i) Study questions (i) to (vi) of problem (14) for this B[x].
    ii) Show B[x] has infinite number of nontrivial MOD natural neutrosophic dual number zero divisors and nilpotents.
    iii) Show B[x] cannot have nontrivial MOD idempotents.
    iv) What are special features enjoyed by B[x]?
    v) Show B[x] has MOD natural neutrosophic dual number subset pseudo subsemirings which are zero square subsemirings are not pseudo.
    vi) Obtain all special features enjoyed by B[x].

18. Let $V[x] = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in S(\langle Z_{18} \cup h\rangle_I); +, \times\right\}$

be the MOD natural neutrosophic special dual like number subset coefficient polynomial pseudo semiring.

i) Study questions (i) to (vi) of problem (14) for this V[x].
ii) Enumerate all special features enjoyed by V[x].
iii) Study V[x] if $S(\langle Z_{18} \cup h\rangle_I)$ is replaced by $S(\langle Z_{11} \cup h\rangle_I)$ or by $S(\langle Z_{48} \cup h\rangle_I)$.

19. Let $W[x] = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in S(\langle Z_{25} \cup k\rangle_I); +, \times\right\}$ be the

MOD natural neutrosophic special quasi dual number subset coefficient polynomial pseudo semiring.

i) Study questions (i) to (vi) of problem (14) for this W[x].
ii) Enumerate all special features associated with W[x].
iii) Study W[x] if $S(\langle Z_{25} \cup k\rangle_I)$ is replaced by $S(\langle Z_{29} \cup k\rangle_I)$ (and) or by $S(\langle Z_{148} \cup k\rangle_I)$

20. Let S(P[x]) = {collection of all subsets from

$P[x] = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in Z_{90}^I; +, \times\right\}, +, \times\}$ be the MOD natural neutrosophic polynomial subset pseudo semiring.

i) Prove $o(S(P[x])) = \infty$.
ii) Find all nontrivial MOD natural neutrosophic subset zero divisors and nilpotents.
iii) Find all MOD natural neutrosophic subset pseudo subsemirings which are not ideals.
iv) Find all MOD natural neutrosophic pseudo ideals.
v) Can S(P[x]) have MOD subset polynomial idempotents?

vi) Study S(P[x]) if Z_{90}^{I} is replaced by Z_{24}^{I} and also by Z_{47}^{I}.

21. Obtain all special features special features enjoyed by MOD natural neutrosophic dual number subset polynomial pseudo semirings.

22. Let S(M[x]) = {collection of all subsets from

$$M[x] = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in C^I(Z_{42}), +, \times\right\}, +, \times\}$$ be the MOD natural neutrosophic finite complex number subset polynomial pseudo semiring.

i) Study questions (i) to (v) of problem for this S(M[x]).
ii) Compare S(P[x]) of problem (20) for S(M[x]).
iii) Study S(M[x]) if $C^I(Z_{42})$ is replaced by $C^I(Z_{53})$ or $C^I(Z_{3^7})$.

23. Let S(N[x]) = {collection of all subsets from

$$N[x] = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in \langle Z_{12} \cup I\rangle I, +, \times\right\}, +, \times\}$$ be the MOD natural neutrosophic-neutrosophic subset polynomial pseudo semiring.

i) Study questions (i) to (v) of problem (20) for this S(N[x]).
ii) Compare this S(N[x]) with S(M[x]) of problem 22.

24. Let S(Z[x]) = {collection of all subsets from

$$Z[x] = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in \langle Z_{17} \cup g\rangle_I, +, \times\right\}, +, \times\}$$ be the MOD natural neutrosophic dual number subset polynomial pseudo semiring.

i) Study questions (i) to (v) of problem (2) for this S (Z[x]).
ii) Show S(Z[x]) has infinite number of MOD subset zero divisors and nilpotents.
iii) Show S(Z[x]) has MOD subset subsemirings which are not pseudo as well as which are zero square subsemirings.

25. Let S(T[x]) = {collection of subsets from

$$T[x] = \left\{\sum_{i=0}^{\infty} a_i x^i \;\middle|\; a_i \in \langle Z_{15} \cup h\rangle_I +, \times\}, +, \times\}$$

be the MOD subset polynomial natural neutrosophic special dual like number pseudo semiring.

i) Study questions (i) to (v) of problem (20) for this S(T[x]).
ii) Compare S(T[x]) with S(Z[x]) of problem 25.

26. Let S(D[x]) = {collection of all subsets from $D[x] = \left\{\sum_{i=0}^{\infty} a_i x^i \;\middle|\; a_i \in (\langle Z_{42} \cup k\rangle_I), +, \times\}, +, \times\}$ be the MOD natural neutrosophic special quasi dual number polynomial subset pseudo semiring.

i) Study questions (i) to (v) of problem (20) for this S(D[x]).
ii) Study S(D[x]) if $\langle Z_{42} \cup k\rangle_I$ is replaced by $\langle Z_{13} \cup k\rangle_I$ or $\langle Z_{64} \cup k\rangle_I$.

27. Let S(P[x]) = {collection of all subsets from $P[x] = \left\{\sum_{i=0}^{12} a_i x^i \;\middle|\; a_i \in Z_{49}^I,\ x^{13} = 1, +, \times\}, +, \times\}$ be the MOD natural neutrosophic polynomial subset pseudo semiring.

i) Prove $o(S(P[x])) < \infty$.
ii) Can S(P[x]) have MOD natural neutrosophic zero divisors and nilpotents.
iii) Can S(P[x]) have MOD natural neutrosophic subset idempotents? Justify!
iv) Obtain all MOD subset pseudo subsemirings which are not ideals.
v) Obtaini all MOD subset pseudo ideals.
vi) Enumerate all special features associated with S(P[x]).
vii) Study S(P[x]) if Z_{49}^{I} is replaced by Z_{53}^{I} or Z_{48}^{I}.

28. Let S(W[x]) = {collection of all subsets from

$$W[x] = \left\{\sum_{i=0}^{8} a_i x^i \;\middle|\; x^9 = 1, a_i \in C^I(Z_{48}), +, \times\right\}, +, \times\}$$

be the MOD natural neutrosophic finite complex number polynomial subset pseudo semiring.

Study questions (i) to (vi) of problem (27) for this S(W[x]).

29. Let $S(V[x]_9)$ = {collection of all subsets from

$$V[x] = \left\{\sum_{i=0}^{9} a_i x^i \;\middle|\; a_i \in \langle Z_{10} \cup g\rangle_I), x^{10} = 1, +, \times\right\}, +, \times\}$$ be the

MOD natural neutrosophic dual number coefficient subset polynomial pseudo semiring.

i) Study questions (i) to (vi) of problem (27) for this $S(V[x]_9)$
ii) Prove $S(V[x]_9)$ has several MOD subset zero divisors and nilpotents.
iii) Prove $S(V[x]_9)$ has MOD subset subsemiring which are zero square subsemiring and not pseudo
iv) Obtain any other special feature associated with $S(V[x]_9)$.

30. Let $S(B[x]_3)$ = {collection of all subsets from $B[x]_3 = \left\{ \sum_{i=0}^{3} a_i x^i \;\middle|\; x^4 = 1, a_i \in \langle Z_{11} \cup h \rangle_I, +, \times \right\}, +, \times\}$ be the MOD natural neutrosophic special dual like number polynomial subset pseudo semiring.

    i) Study questions (i) to (vi) of problem (27) for this $S(B[x]_3)$.
    ii) Compare $S(B[x]_3)$ with $S(V[x]_9)$ in problem 30.
    iii) Study $S(B[x]_3)$ when $\langle Z_{11} \cup h \rangle_I$ is replaced by $\langle Z_{48} \cup h \rangle_I$ and $\langle Z_{5^6} \cup h \rangle_I$,

31. Let $S(Z[x])_5$ = {collection of all polynomial subset from

$$Z[x]_5 = \left\{ \sum_{i=0}^{5} a_i x^i \;\middle|\; x^6 = 1, ai \in \langle Z_{12} \cup k \rangle_I), +, \times \right\}, +, \times\}$$ be the

MOD natural neutrosophic special quasi dual number coefficient polynomial subset pseudo semiring.

Study questions (i) to (vi) of problem (27) for this $S(Z[x]_5)$.

32. Let $S([x])_6 = \left\{ \sum_{i=0}^{6} a_i x^i \;\middle|\; x^7 = 1, a_i \in S(Z_{48}^I), +, \times \right\}$

be the MOD natural neutrosophic subset coefficient polynomial pseudo semiring.

i) Find $o(S([x]_6))$.
ii) Can $S([x]_6)$ have nontrivial MOD subset zero divisors and nilpotents?
iii) Can $S([x]_6)$ have nontrivial MOD subset idempotents?
iv) Find all MOD subset pseudo subsemirings which are not ideals of $S([x]_6)$.
v) Find all MOD subset pseudo ideals of $S([x]_6)$

vi) Study this $S([x]_6)$ if $S(Z^I_{48})$ is replaced by $S(Z^I_{59})$ and $S(Z^I_{128})$.

33. Let $S([x]_{10}) = \left\{\sum_{i=0}^{10} a_i x^i \;\middle|\; x^{11} = 1, a_i \in S(C^I(Z_{40})), +, \times\right\}$

be the MOD natural neutrosophic finite complex number subset polynomial pseudo semiring.

Study questions (i) to (v) of problem (32) for this $S([x]_{10})$.

34. Let $S([x])_{40}) = \left\{\sum_{i=0}^{40} a_i x^i \;\middle|\; a_i \in S(\langle Z_{10} \cup I\rangle_I), x^{41} = 1, +, \times\right\}$

be the MOD natural neutrosophic-neutrosophic subset polynomial pseudo semiring.

Study questions (i) to (v) of problem 32 for this $S([x]_{40})$.

35. Let $S([x]_4) = \left\{\sum_{i=0}^{4} a_i x^i \;\middle|\; x^5 = 1, a_i \in S(\langle Z_{14} \cup g\rangle_I), +, \times\right\}$

be the MOD natural neutrosophic dual number subset polynomial pseudo semiring.

i) Study questions (i) to (v) of problem (32) for this $S([x]_4)$.
ii) Find all MOD zero square subsemirings.
iii) Find all MOD nilpotents and zero divisors.
iv) Compare $S([x]_4)$ with $S([x]_{40})$ of problem 34.

36. Let $S([x]_{12}) = \left\{\sum_{i=0}^{12} a_i x^i \;\middle|\; x^{13} = 1, a_i \in S(\langle Z_{12} \cup h\rangle_I), +, \times\right\}$ be

the MOD natural neutrosophic special dual like number subset polynomial pseudo semiring.

i) Study questions (i) to (v) of problem (32) for this $S([x]_{12})$.
ii) Compare $S([x]_{12})$ with $S([x]_4)$ of problem 35.

37. Let $S([x]_9) = \left\{\sum_{i=0}^{9} a_i x^i \,\middle|\, x^{10} = 1, ai \in S(\langle Z_{27} \cup k\rangle_I), +, \times\right\}$

be the MOD natural neutrosophic special quasi dual number subset polynomial pseudo semiring.

i) Study questions (i) to (v) of problem (32) for this $S([x]_9)$.
ii) Compare $S([x]_9)$ with $S([x]_{12})$ and $S([x]_{14})$ in problem (36) and (35) respectively.
iii) Enumerate all special features enjoyed by $S([x]_9)$.

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INDEX

N

Q

S

Z

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On India's 60th Independence Day, Dr.Vasantha was conferred the Kalpana Chawla Award for Courage and Daring Enterprise by the State Government of Tamil Nadu in recognition of her sustained fight for social justice in the Indian Institute of Technology (IIT) Madras and for her contribution to mathematics. The award, instituted in the memory of Indian-American astronaut Kalpana Chawla who died aboard Space Shuttle Columbia, carried a cash prize of five lakh rupees (the highest prize-money for any Indian award) and a gold medal.

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In this book authors introduce the notion of MOD subset semirings and MOD subset natural neutrosophic semirings. Further MOD subset interval subset pseudo semirings and its natural neutrosophic analogue is introduced. We see these MOD subset semirings when built on MOD intervals are both non associative and does not satisfy the distributive law. This book will attract the researchers working on semirings

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