Bell's theorem refuted: EPR rule OK

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Abstract: EPR (1935) famously argue that additional variables will bring locality and causality to QM's completion; we show that they are right. More famously, Bell (1964) cried 'impossible' against such variables; we give the shortest possible refutation of his theorem. With EPR-based variables – and no QM – a thought-experiment delivers common-sense locally-causal accounts of EPRB and GHZ in 3-space. We then find the flaw in Bell's theorem: Bell's 1964:(14a) $\neq$ Bell's 1964:(14b). Thus, at odds with EPR (and us), Bell's unrealistic theorem and its many variants (eg, Mermin, Peres) miss their mark. In short, mixing common-sense with undergrad math and physics in the classical way so favored by Einstein, we interpret QM locally and realistically. Long may EPR rule OK we say.

Keywords: Bell's error, causality, CLR, completeness, EPR, EPRB, equivalence, locality, realism

Notes to the Reader: (i) Paragraphs and equations are numbered to facilitate discussion, improvement, correction. (ii) Texts freely available online – see References – are taken as read. (iii) In accord with convention, the term *particle* here includes objects with a *wavicle* nature. (iv) Taking math to be the best logic, we let it flow. (v) All results here accord with quantum theory and experiment.

1 Introduction

> #1.0. "Einstein argued that the EPR correlations can be made intelligible only by completing the quantum mechanical account in a classical way," Bell (2004:86). "In a complete physical theory of the type envisioned by Einstein, the hidden variables would have dynamical significance and laws of motion; our λ can be thought of as initial values of these variables at some suitable instant," Bell (1964:196). We agree.

#1.1. Following Bell's (1964) example – merging his valid formalisms with principles consistent with EPR – we'll be working with λ to account for EPRB correlations in a classical way. Our postulates match Einstein's passions (causality, completeness, locality, reality, separability; with no exceptions) and EPR's belief (see #3.0-3.1 below).

#1.2. Taking realism to be the view that external reality exists and has definite properties, our core principle will be common-sense local realism (CLR), the union of local-causality (no causal influence propagates superluminally) and physical-realism (some physical properties change interactively).

#1.3. Then, since CLR precludes nonlocal mechanisms, we'll find erroneous parameters and unphysical assumptions associated with Bellian conclusions like these:

> *Bell's theorem*: "In a theory in which parameters are added to [QM] to determine the results of individual measurements, without changing the statistical predictions, there must be a mechanism whereby the setting of one measuring device can influence the reading of another instrument, however remote. Moreover, the signal involved must propagate instantaneously, so that such a theory could not be Lorentz invariant," Bell (1964:199). "Detailed analysis [Bell's theorem] shows that any classical account of these correlations has to contain just such a 'spooky action at a distance' as Einstein could not believe in ... [rendering] Einstein's conception of the world untenable," Bell (2004:86). We prove the contrary. [#7.1.]

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#1.4. Such errors – at odds with EPR/EPRB, and of his own making – void Bell's impossibility theorem and its kin. Against this, and at one with EPR/EPRB, we deliver a common-sense locally-causal understanding of EPRB and GHZ in 3-space; all done in the classical way so favored by Einstein.

#1.5. Our theory, WM – dubbed wholistic mechanics in 1989, after a discussion with David Mermin re his (1988) – seeks to unify QM and relativity. Accepting Einstein-separability (eg, Laudisa 1995), our particles are presumed separable; rejecting ideas like *inseparable entanglement* (Feingold & Peres 1985), each pair being a *single nonlocal indivisible entity* (Mermin 1985), etc.

#1.6. The key to our analysis is a thought-experiment; developed in 1989, reading Mermin (1988). But here we reverse the order. With Fig.1 capturing technical aspects of our study, Fig.2 presents the thought-experiment that helps make sense of it all; our debt to Mermin (1988) clear.

2 Analysis

#2.0. In WM, with its precise math, the big and the small merge smoothly; after Bell (2004:190). You learn by doing it; for though you think you know it, you have no certainty until you succeed; after Sophocles (c. 496-406 BCE). Importantly: only the impossible is impossible.

$$A^{\pm} \equiv \pm 1 = [\hat{\mathbf{a}}\cdot\hat{\mathbf{a}}^{\pm}] \leftarrow q(\hat{\mathbf{a}}^{\pm}) \leftarrow \Delta^{\pm}_{\hat{\mathbf{a}}} \leftarrow q(\lambda_i) \leftarrow \mathrm{S}_E \rightarrow q(\mu_i) \rightarrow \Delta^{\pm}_{\hat{\mathbf{b}}} \rightarrow q(\hat{\mathbf{b}}^{\pm}) \rightarrow [\hat{\mathbf{b}}\cdot\hat{\mathbf{b}}^{\pm}] = \pm 1 \equiv B^{\pm}$$

⟦ Alice's locale ⟧ ⟦Source⟧ ⟦ Bob's locale ⟧

Figure 1: Experiment E, based on the idealized EPRB experiment in Bell (1964).

#F1.1. In 3-space, and under Einstein-completeness, every relevant element of the subject reality is shown. Since A and B are discrete, we employ discrete variables λ_i, μ_i. This rigor accords with our take on EPRB and Bell's (1964:195) indifference.

#F1.2. With pristine spin-related properties λ_i and μ_i, spin-$\frac{1}{2}$ particles $q(\lambda_i)$ and $q(\mu_i)$ emerge from S_E via a spin-conserving decay such that $\lambda_i + \mu_i = 0$. The particles interact with dichotomic linear-polarizer-analyzers ($\Delta^{\pm}_{\hat{\mathbf{x}}}$, $\hat{\mathbf{x}}$ any direction-vector in 3-space) – freely and independently operated by Alice and Bob – built from polarizers $\delta^{\pm}_{\hat{\mathbf{x}}}$ and analyzers $[\hat{\mathbf{x}}\cdot\hat{\mathbf{x}}^{\pm}]$. These interactions/events are locally-causal and spacelike-separated. Thus, under Einstein-causality – ie, elements in spacelike-separated locales commute – the respective elements (though correlated) are physically independent.

#F1.3. If the output of the interaction $\delta^{\pm}_{\hat{\mathbf{a}}} q(\lambda_i)$ is $q(\hat{\mathbf{a}}^{+})$, then $q(\lambda_i) \sim q(\hat{\mathbf{a}}^{+})$ where $\sim$ denotes *has the same output under* $\delta^{\pm}_{\hat{\mathbf{a}}}$ (a dichotomic operator that dyadically partitions its domain); etc. The polarized particle $q(\hat{\mathbf{a}}^{+})$ goes to an analyzer which reports – via its inner-product function $[\mathbf{a}\cdot\hat{\mathbf{a}}^{+}]$ – the $\hat{\mathbf{a}}$-related spin-projection $A^{+}_i =$ $+1$ in units of $s\hbar$; with intrinsic spin $s = \frac{1}{2}$ here. Elements in Bob's locale similarly.

#F1.4. Under E, to confirm related probabilities – including our generalization of Malus' Law at #2.8 – we allow experiments like $\delta^{\pm}_{\hat{\mathbf{b}}} q(\hat{\mathbf{a}}^{+}) \rightarrow q(\hat{\mathbf{a}}^{+}) \oplus q(\hat{\mathbf{b}}^{-})$. We also control enough dichotomic-polarizers (and single-channel variants $\delta^{+}_{\hat{\mathbf{x}}}, \delta^{-}_{\hat{\mathbf{x}}}$), to conduct any combination of experiments. Moreover, having good internet connections, these experiments may be organized by any subset of the team {us, Alice, Bob}.

#2.1. From Fig.1: two spin-$\frac{1}{2}$ particles $q(\lambda_i)$ and $q(\mu_i)$ with pristine spin-related properties λ_i and μ_i – multivectors in 3-space, each incorporating spin s – emerge from a spin-conserving decay such that

$$\lambda_i + \mu_i = 0;\ \text{ie, } \mu_i = -\lambda_i \equiv \lambda^{-}_i \text{ for notational convenience.} \tag{1}$$

#2.2. (1) shows the correlation of λ_i and μ_i, the EPR-based variables we use to form a more complete locally-causal specification of E. Since one pristine property may be pairwise represented in terms of the other, let's first focus on λ_i and call it the primary random variable for now (for the choice and the name matter not). Then μ_i becomes the secondary variable (for now) with $\mu_i = \lambda_i^-$. [#7.2.]

$$[G|R] \leftarrow q(\hat{\mathbf{a}}^{\pm}) \leftarrow \Delta^{\pm}_{\hat{\mathbf{a}}} \leftarrow q(\lambda_i) \leftarrow \mathrm{S}_F \rightarrow q(\mu_i) \rightarrow \Delta^{\pm}_{\hat{\mathbf{b}}} \rightarrow q(\hat{\mathbf{b}}^{\pm}) \rightarrow [G|R]$$

$$[G|R] \leftarrow q(\hat{\mathbf{a}}^{\pm}) \leftarrow \Delta^{\pm}_{\hat{\mathbf{a}}} \leftarrow q(\hat{\mathbf{a}}^{\pm}) \leftarrow \delta^{\pm}_{\hat{\mathbf{a}}} \leftarrow q(\lambda_i) \leftarrow \mathrm{S}_F \rightarrow q(\mu_i) \rightarrow \Delta^{\pm}_{\hat{\mathbf{b}}} \rightarrow q(\hat{\mathbf{b}}^{\pm}) \rightarrow [G|R]$$

$$[G|R] \leftarrow q(\hat{\mathbf{a}}^{\pm}) \leftarrow \Delta^{\pm}_{\hat{\mathbf{a}}} \leftarrow q(\hat{\mathbf{a}}^{\pm}) \leftarrow \delta^{\pm}_{\hat{\mathbf{a}}} \leftarrow q(\lambda_i) \leftarrow \mathrm{S}_F \rightarrow q(\mu_i) \rightarrow \delta^{\pm}_{\hat{\mathbf{a}}} \rightarrow q(\hat{\mathbf{a}}^{\pm}) \rightarrow \Delta^{\pm}_{\hat{\mathbf{b}}} \rightarrow q(\hat{\mathbf{b}}^{\pm}) \rightarrow [G|R]$$

⟦ Alice's locale ⟧ ⟦Source⟧ ⟦ Bob's locale ⟧

Figure 2: Experiment F – F for *Fantasy* – the essence of experiment E in Fig.1.

#F2.1. See Fig.1 for operating protocols, etc. Apart from new analyzers, the main change to E is the new source S_F; F being a thought-experiment.

#F2.2. To resolve alleged mysteries in Fig.1, we build three equivalent experiments (shown above) around a fantasy-source S_F. S_F delivers identical particle-pairs on demand; each pair identically tagged $q(\lambda_i), q(\mu_i)$. Differing from E in Fig.1, each analyzer now reports its results via green ($G = +1$) or red ($R = -1$) lights. The following paired-results are observed over any fair run of the experiment:

$$\text{If the detectors have the same setting, the lights flash different colors.} \qquad (2)$$

$$\text{In all runs of the experiment, same and different colors flash equally.} \qquad (3)$$

#F2.3. Recalling that Alice's and Bob's direction-vectors $(\hat{\mathbf{a}}, \hat{\mathbf{b}})$ can take any orientation in 3-space whereas, under F, $q(\lambda_i)$ and $q(\mu_i)$ are always the same, the following relation captures the observations in (2)-(3):

$$\langle AB\,|\,E\rangle = \langle AB\,|\,F\rangle = \tfrac{1}{2}(GG+RR)\sin^2\tfrac{1}{2}(\hat{\mathbf{a}}, \hat{\mathbf{b}}) + \tfrac{1}{2}(GR+RG)\cos^2\tfrac{1}{2}(\hat{\mathbf{a}}, \hat{\mathbf{b}}) \qquad (4)$$

$$= \sin^2\tfrac{1}{2}(\hat{\mathbf{a}}, \hat{\mathbf{b}}) - \cos^2\tfrac{1}{2}(\hat{\mathbf{a}}, \hat{\mathbf{b}}) = -\hat{\mathbf{a}}\cdot\hat{\mathbf{b}} \because G = +1, R = -1. \, QED. \blacksquare \qquad (5)$$

#F2.4. The clue to (4), from highschool, is that the required probability relations come thus: convert Malus' classical $\cos^2$ Law for the relative intensity of beams of polarized photons (spin $s = 1$) to a law for spin-$\frac{1}{2}$ particles (spin $s = \frac{1}{2}$). We can then derive an exact solution for two of the above experiments: at the same time satisfying the results for the topmost experiment, which is essentially E in Fig.1.

#F2.5. We then see that $q(\mu_i) \neq q(\hat{\mathbf{a}}^+)$ or $q(\mathbf{b}^+)$ – not even by magic – because, in breach of (2), the lights might not flash different colors when the detectors have the same settings. Reason: per #F2.2, $q(\mu_i) = q(\mu_j) = q(\mu_k)$ and so on, but $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ are arbitrary; within a run or from run to run. (Bell and d'Espagnat reason contrariwise at #7.3, and into trouble as we'll see.)

#F2.6. However, weaker than an equality relation (=), an equivalence relation ($\sim$) meets all our needs: if the output of $\delta^{\pm}_{\hat{\mathbf{a}}} q(\lambda_i)$ is $q(\hat{\mathbf{a}}^+)$, then $q(\lambda_i) \sim q(\hat{\mathbf{a}}^+)$ where $\sim$ denotes *has the same output under* $\delta^{\pm}_{\hat{\mathbf{a}}}$; etc. So we return to Fig.1 with new insights and work to do. [#7.4.]

#2.3. Under E, Fig.1, from the well-known action of a linear-polarizer $\delta^{\pm}_{\hat{\mathbf{a}}}$ on polarized particles $q(\hat{\mathbf{a}}^+)$, we can match a laboratory operation $\delta^{\pm}_{\hat{\mathbf{a}}} q(\hat{\mathbf{a}}^+) \rightarrow q(\hat{\mathbf{a}}^+)$ with the interaction $\delta^{\pm}_{\hat{\mathbf{a}}} q(\lambda_i) \rightarrow q(\hat{\mathbf{a}}^+)$; etc. We also know that $\delta^{\pm}_{\hat{\mathbf{a}}}$ is a dichotomic operator that dyadically partitions its domain. So let $\sim$ be the equivalence relation *has the same output under* $\delta^{\pm}_{\hat{\mathbf{a}}}$. Then, for the i-th and j-th particles:

$$\text{If } \delta^{\pm}_{\hat{\mathbf{a}}} q(\lambda_i) \to q(\hat{\mathbf{a}}^+) \text{ then } q(\lambda_i) \sim q(\hat{\mathbf{a}}^+) \because \delta^{\pm}_{\hat{\mathbf{a}}} q(\hat{\mathbf{a}}^+) \to q(\hat{\mathbf{a}}^+) \text{ exclusively.} \tag{6}$$

$$\text{If } \delta^{\pm}_{\hat{\mathbf{a}}} q(\lambda_j) \to q(\hat{\mathbf{a}}^-) \text{ then } q(\lambda_j) \sim q(\hat{\mathbf{a}}^-) \because \delta^{\pm}_{\hat{\mathbf{a}}} q(\hat{\mathbf{a}}^-) \to q(\hat{\mathbf{a}}^-) \text{ exclusively.} \tag{7}$$

#2.4. That is, from (6): polarizing-operator $\delta^{\pm}_{\hat{\mathbf{a}}}$ delivers $q(\lambda_i)$ and $q(\hat{\mathbf{a}}^+)$ to the same codomain, and it is impossible for $\delta^{\pm}_{\hat{\mathbf{a}}}$ to deliver $q(\lambda_i)$ and $q(\hat{\mathbf{a}}^+)$ to two different codomains. An equivalence relation $\sim$ thus holds between $q(\lambda_i)$ and $q(\hat{\mathbf{a}}^+)$ under $\delta^{\pm}_{\hat{\mathbf{a}}}$; etc. So, consistent with our acceptance of Bell's 1964:(1), the analyzer-functions and outputs in Fig.1 (and their expectations) can be written:

$$A(\hat{\mathbf{a}}, \lambda) = A^{\pm} = \cos(\hat{\mathbf{a}}, \lambda \,|\, \lambda \sim \hat{\mathbf{a}}^{\pm}) = \pm 1; \langle A \,|\, E\rangle = 0 \because P(\lambda \sim \hat{\mathbf{a}}^+ \,|\, E) = P(\lambda \sim \hat{\mathbf{a}}^- \,|\, E) = \tfrac{1}{2}. \tag{8}$$

$$B(\hat{\mathbf{b}}, \mu) = B^{\pm} = \cos(\hat{\mathbf{b}}, \mu \,|\, \mu \sim \hat{\mathbf{b}}^{\pm}) = \pm 1; \langle B \,|\, E\rangle = 0 \because P(\mu \sim \hat{\mathbf{b}}^+ \,|\, E) = P(\mu \sim \hat{\mathbf{b}}^- \,|\, E) = \tfrac{1}{2}. \tag{9}$$

#2.5. In words, LHS (8) reads: given $q(\lambda)$ equivalent to $q(\hat{\mathbf{a}}^+)$, $\cos(\hat{\mathbf{a}}, \lambda \,|\, \lambda \sim \hat{\mathbf{a}}^+)$ denotes the cosine of the angle $(\hat{\mathbf{a}}, \hat{\mathbf{a}}^+)$: so the outcome is $A^+ = +1$; etc. Thus, from (8)-(9) and Fig.1, WM is locally-causal: $A^{\pm}$ and $B^{\pm}$ are locally-caused by precedent local events $\delta^{\pm}_{\hat{\mathbf{a}}} q(\lambda_i)$ and $\delta^{\pm}_{\hat{\mathbf{b}}} q(\mu_i)$ which are spacelike-separated; etc.

#2.6. Now, given hidden random variables λ and μ, the expectations in (8) and (9) are zero. But (1) invokes $P(XY) = P(X)P(Y \,|\, X) = P(Y)P(X \,|\, Y)$; the probability relation that cannot fail. Under E, A^+_i and B^+_i are pairwise correlated via the pairwise correlation of λ_i and μ_i in (1); etc.

#2.7. So we now move to derive $\langle AB \,|\, E\rangle$, the expectation for experiment E, via the probabilities for the conjunction of the outcomes in (8) and (9). [See discussion at #7.5-7.6.] Since primacy is arbitrary, (#2.2), and given the correlation in (1), the following string of probability relations holds:

$$P(\lambda \sim \hat{\mathbf{a}}^+ \,|\, E, \mu \sim \hat{\mathbf{b}}^+) = P(\mu \sim \hat{\mathbf{b}}^+ \,|\, E, \lambda \sim \hat{\mathbf{a}}^+) = P(\lambda^- \sim \hat{\mathbf{b}}^+ \,|\, E, \lambda \sim \hat{\mathbf{a}}^+) = P(\lambda \sim \hat{\mathbf{b}}^- \,|\, E, \lambda \sim \hat{\mathbf{a}}^+) \tag{10}$$

$$= P(\delta^{\pm}_{\hat{\mathbf{b}}} q(\lambda \sim \hat{\mathbf{a}}^+) \to q(\lambda \sim \hat{\mathbf{b}}^-) \,|\, E) = P(\delta^{\pm}_{\hat{\mathbf{b}}} q(\hat{\mathbf{a}}^+) \to q(\hat{\mathbf{b}}^-) \,|\, E) = \cos^2 s\, (\hat{\mathbf{a}}^+, \hat{\mathbf{b}}^-) = \sin^2 \tfrac{1}{2}(\hat{\mathbf{a}}, \hat{\mathbf{b}}). \tag{11}$$

#2.8. The probability relation LHS (11) is, we see, equivalent to a classical test on spin-$\frac{1}{2}$ particles of known polarization. So, per RHS (11), this probability relation is given by Malus' $\cos^2 s\, (\hat{\mathbf{a}}^+, \hat{\mathbf{b}}^-)$ Law for the relative intensity of beams of polarized spin-s particles. Then, since our equivalence relations hold under probability functions P, we say that P is well-defined under $\sim$ and is thus a law. That is, P under $\sim$ is Malus' Law generalized to entangled particles as follows:

$$P(\delta^{\pm}_{\hat{\mathbf{b}}} q(\lambda \sim \hat{\mathbf{a}}^+) \to q(\hat{\mathbf{b}}^+) \,|\, E) = P(\delta^{\pm}_{\hat{\mathbf{b}}} q(\hat{\mathbf{a}}^+) \to q(\hat{\mathbf{b}}^+) \,|\, E) = \cos^2 \tfrac{1}{2}(\hat{\mathbf{a}}^+, \hat{\mathbf{b}}^+) = \cos^2 \tfrac{1}{2}(\hat{\mathbf{a}}, \hat{\mathbf{b}}); \tag{12}$$

$$P(\delta^{\pm}_{\hat{\mathbf{b}}} q(\lambda \sim \hat{\mathbf{a}}^-) \to q(\hat{\mathbf{b}}^+) \,|\, E) = P(\delta^{\pm}_{\hat{\mathbf{b}}} q(\hat{\mathbf{a}}^-) \to q(\hat{\mathbf{b}}^+) \,|\, E) = \cos^2 \tfrac{1}{2}(\hat{\mathbf{a}}^-, \hat{\mathbf{b}}^+) = \sin^2 \tfrac{1}{2}(\hat{\mathbf{a}}, \hat{\mathbf{b}}); \textbf{ etc.} \tag{13}$$

#2.9. Given the experimentally validated (and QM-supported) relations in (12)-(13) – [see #7.7 for their direct relevance to QM via WM] – we now derive $\langle AB \,|\, E\rangle$ using (8)-(13):

$$\langle A^+B^+ \,|\, E\rangle = P(\lambda \sim \hat{\mathbf{a}}^+ \,|\, E) \cos(\hat{\mathbf{a}}, \lambda \,|\, \lambda \sim \hat{\mathbf{a}}^+) P(\mu \sim \hat{\mathbf{b}}^+ \,|\, E, \lambda \sim \hat{\mathbf{a}}^+) \cos(\hat{\mathbf{b}}, \mu \,|\, \mu \sim \hat{\mathbf{b}}^+) \tag{14}$$

$$= \tfrac{1}{2} P(\mu \sim \hat{\mathbf{b}}^+ \,|\, E, \lambda \sim \hat{\mathbf{a}}^+) = \tfrac{1}{2} \sin^2 \tfrac{1}{2}(\hat{\mathbf{a}}, \hat{\mathbf{b}}); \text{ from(10)-(11).} \tag{15}$$

$$\text{Similarly: } \langle A^+B^- \,|\, E\rangle = \langle A^-B^+ \,|\, E\rangle = -\tfrac{1}{2}\cos^2 \tfrac{1}{2}(\hat{\mathbf{a}}, \hat{\mathbf{b}}); \langle A^-B^- \,|\, E\rangle = \tfrac{1}{2}\sin^2 \tfrac{1}{2}(\hat{\mathbf{a}}, \hat{\mathbf{b}}). \tag{16}$$

$$\therefore \langle AB \,|\, E\rangle = \langle A^+B^+ \,|\, E\rangle + \langle A^+B^- \,|\, E\rangle + \langle A^-B^+ \,|\, E\rangle + \langle A^-B^- \,|\, E\rangle = -\hat{\mathbf{a}} \cdot \hat{\mathbf{b}}.\ QED. \blacksquare \tag{17}$$

$$\text{Finally: } P(AB = +1 \,|\, E) = \sin^2 \tfrac{1}{2}(\hat{\mathbf{a}}, \hat{\mathbf{b}}).\ P(AB = -1 \,|\, E) = \cos^2 \tfrac{1}{2}(\hat{\mathbf{a}}, \hat{\mathbf{b}}). \tag{18}$$

#2.10. (17) thus reproduces the results of quantum theory in full accord with EPR and EPRB (a 2-particle setting). So we next show the validity and utility of (1)-(18) in a 3-particle setting.

#2.11. To that end – beginning at #3.1 with a small refinement of EPR to fully accord with (1)-(18) – we reproduce the results of quantum theory in the context of a 3-particle experiment, Mermin (1990; 1990a). With its supposed 'always-vs-never refutation' of EPR, Mermin's Bell-based analysis is (for us) a crucial all-or-nothing test of our perception of EPR against that of Bell and Mermin.

3 Mermin's "always-vs-never refutation of EPR" refuted

> #3.0. *EPR's belief:* "We shall be satisfied with the following criterion, which we regard as reasonable. If, without any way disturbing a system, we can predict with certainty (ie, with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity," EPR (1935:777). "While we have thus shown that the wave function does not provide a complete description of the physical reality, we left open the question of whether or not such a description exists. *We believe, however, that such a theory is possible*," EPR (1935:780); *emphasis added.*

#3.1. *EPR's belief delivered*: "If, without in any way disturbing $q(\mu_i)$, Alice can predict with certainty that Bob's result will be $B_i = -1$ when he tests $q(\mu_i)$ with $\Delta^{\pm}_{\hat{\mathbf{a}}}$ (which may be a disturbance), then elements of reality $\Delta^{\pm}_{\hat{\mathbf{a}}}$ and $q(\mu_i \sim \hat{\mathbf{a}}^-)$ mediate Bob's result. Thus the element of reality corresponding to Bob's $B_i = -1$ result will be $q(\hat{\mathbf{a}}^-)$," after Watson (1998:417; 1999). So here's how Alice predicts Bob's result after observing the outcome of her test $\Delta^{\pm}_{\hat{\mathbf{a}}} q(\lambda_i) \Rightarrow A_i = +1$:

$$A_i = +1 \because \delta^{\pm}_{\hat{\mathbf{a}}} q(\lambda_i) \to q(\hat{\mathbf{a}}^+) \to [\hat{\mathbf{a}} \cdot \hat{\mathbf{a}}^+] = +1. \therefore q(\lambda_i) = q(\lambda_i \sim \hat{\mathbf{a}}^+). \therefore q(\mu_i) = q(\mu_i \sim \hat{\mathbf{a}}^-). \tag{19}$$

$$\therefore \delta^{\pm}_{\hat{\mathbf{a}}} q(\mu_i) = \delta^{\pm}_{\hat{\mathbf{a}}} q(\mu_i \sim \hat{\mathbf{a}}^-) \to q(\hat{\mathbf{a}}^-) \to [\hat{\mathbf{a}} \cdot \hat{\mathbf{a}}^-] = -1 = B_i. \; QED. \blacksquare \text{ And vice-versa.} \tag{20}$$

#3.2. Thus showing that EPR is valid in the context of (1)-(18), we now consider experiment M, Mermin's (1990; 1990a) 3-particle variant of GHZ (1989). Respectively, consistent with our notation for the polarizer orientations: Three spin-$\frac{1}{2}$ particles with spin-related properties λ, μ, ν emerge from a spin-conserving decay such that (taking ν to be the tertiary variable; see #2.2),

$$\lambda + \mu + \nu = \pi. \therefore \nu = \pi - \lambda - \mu. \tag{21}$$

#3.3. The particles separate in the y-z plane and interact with spin-$\frac{1}{2}$ polarizers that are orthogonal to the related line of flight. Let a, b, c denote the angle of each polarizer's principal-axis relative to the positive x-axis; let the test results be A, B, C. Then, as in (8)-(9) with two particles, let

$$A(a, \lambda) = A^{\pm} = \cos(a - \lambda \,|\, \lambda \sim a^{\pm}) = \pm 1, \tag{22}$$

$$B(b, \mu) = B^{\pm} = \cos(b - \mu \,|\, \mu \sim b^{\pm}) = \pm 1, \tag{23}$$

$$C(c, \nu) = C^{\pm} = \cos(c - \nu \,|\, \nu \sim c^{\pm}) = \pm 1. \tag{24}$$

#3.4. Via the principles in (1)-(18) – and nothing more – we now derive $\langle ABC \,|\, M\rangle$, the expectation for experiment M. (The qualifier M is missing from (21) to limit its length; explanatory notes follow.)

$$\big\langle A^+B^+C^+ \,|\, M \big\rangle$$

$$= P(\lambda \sim a^+)\cos(a-\lambda \,|\, \lambda \sim a^+) P(\mu \sim b^+)\cos(b-\mu \,|\, \mu \sim b^+) P(\nu \sim c^+ \,|\, \lambda \sim a^+, \mu \sim b^+)\cos(c-\nu \,|\, \nu \sim c^+) \tag{25}$$

$$= \tfrac{1}{4} P(\nu \sim c^+ \,|\, M, \lambda \sim a^+, \mu \sim b^+) = \tfrac{1}{4} P((\pi - \lambda - \mu) \sim c^+ \,|\, M, \lambda \sim a^+, \mu \sim b^+) \tag{26}$$

$$= \tfrac{1}{4} P((\pi - a^+ - b^+) \sim c^+ \,|\, M) = \tfrac{1}{4}\cos^2 \tfrac{1}{2}(\pi - a^+ - b^+ - c^+) = \tfrac{1}{4}\sin^2 \tfrac{1}{2}(a+b+c). \tag{27}$$

$$\text{Similarly: } \big\langle A^+B^-C^- \,|\, M \big\rangle = \big\langle A^-B^+C^- \,|\, M \big\rangle = \big\langle A^-B^-C^+ \,|\, M \big\rangle = \tfrac{1}{4}\sin^2 \tfrac{1}{2}(a+b+c), \text{ and} \tag{28}$$

$$\big\langle A^+B^+C^- \,|\, M \big\rangle = \big\langle A^+B^-C^+ \,|\, M \big\rangle = \big\langle A^-B^+C^+ \,|\, M \big\rangle = \big\langle A^-B^-C^- \,|\, M \big\rangle = -\tfrac{1}{4}\cos^2 \tfrac{1}{2}(a+b+c). \tag{29}$$

$$\therefore \langle ABC \,|\, M \rangle \equiv \Sigma \big\langle A^{\pm}B^{\pm}C^{\pm} \,|\, M \big\rangle \tag{30}$$

$$= \sin^2 \tfrac{1}{2}(a+b+c) - \cos^2 \tfrac{1}{2}(a+b+c) = -\cos(a+b+c). \; QED. \blacksquare \tag{31}$$

$$\text{Finally: } P(ABC = +1 \,|\, M) = \sin^2 \tfrac{1}{2}(a+b+c). \; P(ABC = -1 \,|\, M) = \cos^2 \tfrac{1}{2}(a+b+c). \tag{32}$$

#3.5. (27) follows from (26) via the allocation of the equivalence relations in the conditioning space to the related variables. Thus, in words, LHS (27) is one-quarter the probability that ν — ie, $\nu \sim (\pi - a^+ - b^+)$ — will be equivalent to c^+ under $\delta^{\pm}_c$.

#3.6. In other words: LHS (27) $= \frac{1}{4}P(\delta_c^{\pm}q(\nu \sim \pi - a^+ - b^+) \to q(c^+))$ = RHS (27) via Malus' (now generalized) Law; as in (15) in the two-particle EPRB example above.

#3.7. (31) is the correct result for experiment M: delivering Mermin's (1990a:733) *crucial minus sign*. That is, from (32): $\langle ABC\,|\,M\rangle = -1$ when $(a+b+c)=0$. Thus, consistent with CLR and classical rules for operators and functions in 3-space, we again deliver classically-intelligible EPR correlations.

#3.8. With (17), (20), and now (31) – as one with EPR but so clearly in conflict with the Bellian conclusions cited at #1.3 above – we turn to Bell's work to find our differences.

4 Bell's theorem misses its target

> #4.0. *Bell's dictum*: "We are not at all concerned with sequences of measurements on a given particle, or of pairs of measurements on a given particle. We are concerned with experiments in which for each pair the 'spin' of each particle is measured once only," Bell (2004:65); from 1975. "It is a matter of indifference . . . whether λ denotes a single variable or a set, or even a set of functions, and whether the variables are discrete or continuous. However, [Bell writes] as if λ were a single continuous parameter," Bell (1964:195). λ may denote "any number of hypothetical additional complementary variables needed to complete QM in the way envisaged by EPR," Bell (2004:242). NB: we work with discrete variables.

#4.1. To establish the famed inequality in Bell 1964:(3), Bell (1964:197) takes us to his proof via 'Contradiction – The main result will now be proved.' That is, we are taken to Bell 1964:(15) via Bell 1964:(14), vector **c**, and three unnumbered equations.

#4.2. Numbering them (14a)-(14c), and using the principles that rightly deliver (17), (20), (31), we now audit Bell's derivation of his Bell 1964:(15). In our terms, $\stackrel{?}{=}$ denotes a questionable Bellian relation; each one false, as we'll see next.

#4.3. In E (ie, EPRB per Fig.1 above), let $3n$ random particle-pairs be equally distributed over three randomized polarizer-pairings $(\hat{\mathbf{a}}, \mathbf{b})$, $(\hat{\mathbf{a}}, \mathbf{c})$, $(\mathbf{b}, \mathbf{c})$. With each particle-pair uniquely indexed, let n be such that (for convenience in presentation and to an adequate accuracy hereafter; but see #7.8):

$$\text{Bell 1964:(14a)} = \langle AB\,|\,E\rangle - \langle AC\,|\,E\rangle = -\frac{1}{n}\sum_{i=1}^{n}[A(\hat{\mathbf{a}},\lambda_i)A(\hat{\mathbf{b}},\lambda_i) - A(\hat{\mathbf{a}},\lambda_{n+i})A(\hat{\mathbf{c}},\lambda_{n+i})] \tag{33}$$

$$= -\frac{1}{n}\sum_{i=1}^{n} A(\hat{\mathbf{a}},\lambda_i)A(\hat{\mathbf{b}},\lambda_i)[A(\hat{\mathbf{a}},\lambda_i)A(\hat{\mathbf{b}},\lambda_i)A(\hat{\mathbf{a}},\lambda_{n+i})A(\hat{\mathbf{c}},\lambda_{n+i}) - 1] \stackrel{?}{=} \text{Bell 1964:(14b)}. \tag{34}$$

$$\therefore |\,\langle AB\,|\,E\rangle - \langle AC\,|\,E\rangle\,| \leq 1 - \frac{1}{n}\sum_{i=1}^{n} A(\hat{\mathbf{a}},\lambda_i)A(\hat{\mathbf{b}},\lambda_i)A(\hat{\mathbf{a}},\lambda_{n+i})A(\hat{\mathbf{c}},\lambda_{n+i}) \stackrel{?}{=} \text{Bell 1964:(14c)}. \blacksquare \tag{35}$$

$$\therefore |\,\langle AB\,|\,E\rangle - \langle AC\,|\,E\rangle\,| \leq 1 - \langle AB\,|\,E\rangle\langle AC\,|\,E\rangle \stackrel{?}{=} \text{LHS Bell 1964:(15)} \leq 1 + \langle BC\,|\,E\rangle\,. \tag{36}$$

#4.4. In (34)-(35), we use $A(\hat{\mathbf{a}},\lambda_i)A(\mathbf{b},\lambda_i) = \pm 1$ from (8)-(9). The central term in (35) is the correct discrete form of LHS (35), which equals LHS (36) as follows: the random variables (RVs) generated by λ_i and λ_{n+i} are independent, and the expectation over the product of two independent RVs is the product of their individual expectations.

#4.5. So we now study the physical significance of Bell's mathematical reductions – now made clear in the line below his (14c) – that the rightmost term in (36) is $\langle BC\,|\,E\rangle$. To that end (in anticipation and protected by our $\stackrel{?}{=}$), $\langle BC\,|\,E\rangle$ is already there, the rightmost term in RHS (36). With all terms in (33)-(36) known to us from (17), we can readily decipher and test each reduction. Testing particles $2n+i$ to $3n$ over the polarizer-pairing $(\hat{\mathbf{b}}, \hat{\mathbf{c}})$, we confirm $\langle BC\,|\,E\rangle = -\hat{\mathbf{b}}\cdot\hat{\mathbf{c}}$; as predicted from (17).

#4.6. Consistent with *Bell's dictum* (#4.0), and given (17), (35) is decisive. Under our assumption that the likes of λ and μ are random variables (multivectors incorporating spin s) in 3-space, it is probability zero that any particle-pairs are the same in our analysis. If this assumption is relaxed to satisfy (35) – that crucial test that Bell's theorem fails – then Bell's theorem goes through under

$$\lambda_i = \lambda_{n+1} = \lambda_{2n+1}. \tag{37}$$

#4.7. Alas for Bell, (37) – unrealistic, in our terms, and against the spirit of EPR – crashes at the next hurdle. For (37) delivers Bell's 1964:(15), which falls at (39); or (42) if you prefer. In that (37) appears to be the basis for many variants of Bell's theorem, they fall too.

#4.8. Many combinations of direction-vectors in 3-space yield interesting truisms; honoring an old friend and tutor, we call them Bourne relations. They relate to 'scalar-angles' (in 3-space for now), where the scalar-angle β between two direction-vectors $\hat{\mathbf{a}}, \hat{\mathbf{b}}$ is $\beta = \hat{\mathbf{a}} \cdot \hat{\mathbf{b}}$). Thus, given three direction vectors $(\hat{\mathbf{a}}, \hat{\mathbf{b}}, \hat{\mathbf{c}})$ in 3-space, here are two such Bourne relations with their consequents:

$$|\,(\hat{\mathbf{a}} \cdot \hat{\mathbf{b}}) - (\hat{\mathbf{a}} \cdot \hat{\mathbf{c}})\,| + (\hat{\mathbf{a}} \cdot \hat{\mathbf{b}})(\hat{\mathbf{a}} \cdot \hat{\mathbf{c}}) \leq 1. \;\therefore\; |\,(\hat{\mathbf{a}} \cdot \hat{\mathbf{b}}) - (\hat{\mathbf{a}} \cdot \hat{\mathbf{c}})\,| \leq 1 - (\hat{\mathbf{a}} \cdot \hat{\mathbf{b}})(\hat{\mathbf{a}} \cdot \hat{\mathbf{c}}). \tag{38}$$

$$|\,(\hat{\mathbf{a}} \cdot \hat{\mathbf{b}}) - (\hat{\mathbf{a}} \cdot \hat{\mathbf{c}})\,| + (\hat{\mathbf{b}} \cdot \hat{\mathbf{c}}) \leq \tfrac{3}{2}. \;\therefore\; |\,(\hat{\mathbf{a}} \cdot \hat{\mathbf{b}}) - (\hat{\mathbf{a}} \cdot \hat{\mathbf{c}})\,| \not\leq 1 - (\hat{\mathbf{b}} \cdot \hat{\mathbf{c}}). \blacksquare \tag{39}$$

#4.9. RHS (38) is a consequent truism: confirming the result we derived at LHS (36). RHS (39) is a consequent truism: the shortest refutation of Bell's 1964:(15) – and hence of his theorem – that we know.

#4.10. For those who prefer more conventional tests as proof that Bell's work here is in error, let our polarizer direction-angles be such that $(\hat{\mathbf{a}},\hat{\mathbf{b}}) = (\hat{\mathbf{b}},\hat{\mathbf{c}})$, $(\hat{\mathbf{a}},\hat{\mathbf{c}}) = 2(\hat{\mathbf{a}},\hat{\mathbf{b}})$, with an inanity-index $\mathbf{I}$ ($\mathbf{I} > 0$ revealing absurdity; denoted $\blacktriangle$). Then, using (17) and suppressing our now-proven doubt ($\overset{?}{=}$) in favor of two Bellian claims — (i) for equality ($=$) in (36); (ii) that LHS Bell 1964:(15) $\geq$ RHS Bell 1964:(15) — we have:

$$\text{(i)}\quad \mathbf{I}_{\text{Bell 1964:(15)=LHS(36)}} \equiv \frac{\text{LHS(36)}}{\text{RHS(36)}} - 1 = \frac{|1 - (\hat{\mathbf{a}}\cdot\hat{\mathbf{b}})(\hat{\mathbf{a}} \cdot \hat{\mathbf{c}})|}{1 - \hat{\mathbf{b}} \cdot \hat{\mathbf{c}}} - 1; \tag{40}$$

$$\text{whence, for } -\tfrac{\pi}{2} < (\hat{\mathbf{a}},\hat{\mathbf{b}}) < +\tfrac{\pi}{2} : \mathbf{I}_{\text{Bell 1964:(15)=LHS(36)}} > 0. \blacktriangle \; \underset{(\hat{\mathbf{a}},\hat{\mathbf{b}})\to 0}{\text{Lim}} \mathbf{I}_{\text{Bell 1964:(15)=LHS(36)}} = 4. \blacktriangle \tag{41}$$

$$\text{(ii)}\quad \mathbf{I}_{\text{Bell 1964:(15)=LHS(36)}} \equiv \frac{\text{RHS Bell 1964(15)}}{\text{LHS Bell 1964(15)}} - 1 = \frac{|(\hat{\mathbf{a}}\cdot\hat{\mathbf{b}}) - (\hat{\mathbf{a}} \cdot \hat{\mathbf{c}})|}{1 - \hat{\mathbf{b}} \cdot \hat{\mathbf{c}}} - 1; \tag{42}$$

$$\text{whence, for } -\tfrac{\pi}{2} < (\hat{\mathbf{a}},\hat{\mathbf{b}}) < +\tfrac{\pi}{2} : \mathbf{I}_{\text{Bell 1964:(15)}} > 0. \blacktriangle \; \underset{(\hat{\mathbf{a}},\hat{\mathbf{b}})\to 0}{\text{Lim}} \mathbf{I}_{\text{Bell 1964:(15)}} = 2. \blacktriangle \tag{43}$$

#4.11. We thus identify the tacit inequality in Bell's (14a)-(14b) to reveal Bell's consequent error:

$$\text{Bell's error: Bell 1964:(14a)} \neq \text{Bell 1964:(14b)} \Rightarrow \text{LHS 1964:(15)} \ngeq \text{RHS 1964:(15)}. \tag{44}$$

#4.12. With Bell's theorem thus doubly refuted, similar anomalies attach to Bell-inequalities in general. Peres' version of the CHSH (1969) inequality is an example. In our terms Peres (1995:164) has, "If several [particle-pairs] are tested, we have for the j-th pair"

$$A_j(B_j - D_j) + C_j(B_j + D_j) \equiv \pm 2. \tag{45}$$

#4.13. So, by observation, the average of (45) over many pairs should not exceed two; a result known as the CHSH (1969) inequality. To prove that Peres and CHSH are in error: let our polarizer direction-angles be such that $(\mathbf{a},\mathbf{b}) = (\mathbf{b},\mathbf{c}) = (\mathbf{c},\mathbf{d})$, $(\mathbf{a},\mathbf{d}) = 3(\mathbf{a},\mathbf{b})$; let $\{k\} = \{i = 1, 2, ..., n\}$, similar to #4.3; and let $\mathbf{I}$ be an appropriate inanity-index. Then Peres 1995:(6.30) leads to this:

$$|\,\langle AB\rangle_{\{k\}} + \langle BC\rangle_{\{n+k\}} + \langle CD\rangle_{\{2n+k\}} - \langle DA\rangle_{\{3n+k\}}\,| \leq 2. \tag{46}$$

$$\mathbf{I}_{\text{Peres 1995:(6.30)}} \equiv \frac{\text{LHS (46)}}{\text{RHS (46)}} - 1 = \frac{|\,(\hat{\mathbf{a}} \cdot \hat{\mathbf{b}}) + (\hat{\mathbf{b}} \cdot \hat{\mathbf{c}}) + (\hat{\mathbf{c}} \cdot \hat{\mathbf{d}}) - (\hat{\mathbf{d}} \cdot \hat{\mathbf{a}})\,|}{2} - 1; \tag{47}$$

$$\text{whence, for } -\pi < (\hat{\mathbf{a}},\hat{\mathbf{b}}) < +\pi : \mathbf{I}_{\text{Peres 1995:(6.30)}} > 0 \text{ for more than 75\% of that range.} \blacktriangle \tag{48}$$

#4.14. So, thanks to the $q(\lambda_{wn+i}) \subseteq Q$ particle-family – with no two particles necessarily the same – most Bell inequalities fall to the same analysis. Indeed, for us – but against Bell himself, Goldstein, Maudlin, Mermin, Peres, and many others – it's worth repeating Bell's dictum:

> "We are not at all concerned with sequences of measurements on a given particle, or of pairs of measurements on a given particle. We are concerned with experiments in which for each pair the 'spin' of each particle is measured once only," Bell (2004:65); from 1975.

#4.15. So where do we differ? We who operate under EPR, versus Bell with his stated aim to do the same? We who never once doubted that every one of Einstein's demands [#1.1.] were true.

#4.16. For now, with our math still doing the talking, we rest our case. Confident that WM meets EPR's belief (#3.0), we endorse Bell's (1990:9) hope – so similar to EPR's – and move to conclude:

> "Now, it's my feeling that all this [AAD] and no [AAD] business will go the same way [eg, as the ether]. But someone will come up with the answer, with a reasonable way of looking at these things. If we are lucky it will be to some big new development like the theory of relativity. Maybe someone will just point out that we were being rather silly, and it won't lead to a big new development. But anyway, I believe the questions will be resolved."

5 Conclusions

#5.1. Under CLR – and endorsing Einstein-separability – we've here advanced many missions: Einstein's, EPR's, John Bell's, our own. That is, relying on EPR – ie, using parameters λ, and accepting locality without question – our more complete specification of EPRB's physics has succeeded; Bell's rejection of Einstein's worldview is quashed; Bell's ambivalence re AAD is resolved.

#5.2. Given no hint and finding no evidence that Bell (1964) is based on the likes of a d'Espagnat-style inference to classicality, we still find that Bell misses his target, EPR. For – as shown at (33)-(36) – Bell's theorem and its variants are based on an error: Bell's mathematical-reduction of his (14a) to (14b) is inappropriate in the circumstances. Thus, under EPR and EPRB, Bell's famous duo –1964:(3) and 1964:(15), with many variants – are false and therefore inadmissible as critiques of EPR.

#5.3. We remain open to evidence that supports an alternative proposal: that Bell (1964), contrary to initially targeting EPR and Einstein's views, began with a d'Espagnat-style inference. Expecting none, Bells oversight – 1964:(14a) $\neq$ 1964:(14b) – remains for us an error of judgment; but an error nevertheless. And, with so little Bellian discussion about the nature of λ, what then of his claim, #1.3, "In a theory in which parameters [unrelated to EPR] are added ... ."

#5.4. We began on the right track: starting with (1), an ironclad fact, then adding a function to Bell's 1964:(1) to give (8)-(9). We thus arrived at (17), (20), (31), etc, via facts associated with equivalence relations and probability theory. In that these equations yield the same result as QM, we deliver Einstein's hope that EPR-style correlations might be understood in a classical way.

#5.5. Under CLR – and contrary to Bell's (1980) view at #7.3 – we were right to allow that polarizer/particle interaction may perturb a particle. Bell – in bypassing such perturbation in line with d'Espagnat's analysis – limits the validity of his theorem to systems consistent with this error. Under CLR, the consequent strong classicality in Bell's theorem is replaced by the weaker reality of equivalence relations. WM thus reaches beyond the classical.

#5.6. Based on the rightness of CLR and equivalence relations, WM readily refutes the important all-or-nothing test of Bell's ideas in Mermin (1990a). And WM is Lorentz invariant, for Bell (1964:199) – at #1.3 – missed the following fact and its association with the broad reach of equivalence relations: similar tests on similar things produce similar results, and correlated test on correlated things produce correlated results, without mystery.

#5.7. We are thus able to correctly analyze multi-particle experiments via real operators in 3-space; without recourse to AAD, Hilbert-space, non-locality; nor the impossible requirement to fully specify a hidden variable in a given spacetime region, per Bell (2004:242); etc.

#5.8. Given our successful endorsement of EPR, further considerations for a wholesale reinterpretation of QM remain: 'collapse' as the Bayesian updating of an equivalence class via prior correlations; 'states' as multivectors in 3-space; 'measurements' as tests; 'wave-particle duality' as an equivalence relation?

#5.9. Finally, though Bell (1990:13) insists *'you cannot get away with locality'* — we do just that: thanks to EPR and John Bell.

6 Acknowledgments

#6.1. To be done elsewhere if this has merit.

7 Notes

#7.1. In the spirit of Einstein/EPR, we come to QM and Bell bearing gifts: we bring realistic parameters that bypass Bell's (1964:199) *strange mechanisms*; and 'we determine the results of individual measurements, without changing the statistical predictions'. As we'll show, we succeed because the poorly-defined variables that Bell tests are unrealistic under both EPR and EPRB. Thus, in that we too add parameters to QM – though more akin to EPR than Bell – so too are we licensed to test our 'added-variable theory' against Bell and his theorem. Let's return to #1.3 and see.

#7.2. Under an ironclad conservation law, (1) immediately resolves Bell's dilemma re action-at-a-distance (AAD) and locality. Edited excerpts from Bell (1990) follow, *with emphasis added*:

> I cannot say that AAD is required in physics. *I can say that you cannot get away with no AAD.* You cannot separate off what happens in one place and what happens in another. *Somehow they have to be described and explained jointly.* That's the fact of the situation; Einstein's program fails, that's too bad for Einstein, but should we worry about that? *Maybe we have to learn to accept not so much AAD, but the inadequacy of no AAD. That's the dilemma.* We are led by analyzing this situation to admit that, somehow, distant things are connected, or at least not disconnected. I don't know any conception of locality that works with QM. *So I think we're stuck with nonlocality.* There's no energy transfer and there's no information transfer either. That's why I'm always embarrassed by the word action; so I step back from asserting that there is AAD and *I say only that you cannot get away with localit*y. You cannot explain things by events in their neighbourhood. But, I am careful not to assert that there is AAD.
>
> Others, following Bell and not helping, vary from surprised to certain: eg, Goldstein *et al.* (2011), "In light of Bell's theorem, [many] experiments ... establish that our world is non-local. This conclusion is very surprising, since non-locality is normally taken to be prohibited by the theory of relativity." Maudlin (2014), "Non-locality is here to stay ... the world we live in is non-local." [Please return to #2.2.]

#7.3. #7.2 shows our early departure from Bellian ways of thinking. Another follows; ie, in our terms *and* in the context of EPR/EPRB – though seemingly motivated by a shared desire for a return to classicality – Bell invokes an over-strength and unfit equivalence-relation (that of equality):

> Here's Bell (1980:7): "To explain this dénouement [of Bell's theorem] without mathematics I cannot do better than follow d'Espagnat (1979; 1979a)."
>
> And here's d'Espagnat (1979:166), recast for EPRB *with added emphasis*: 'A physicist can infer that in every pair, one particle has the property A^+ and the other has the property A^-. Similarly, he can conclude that in every pair one particle has the property B^+ and one B^-, and one has property C^+ and one C^-. *These conclusions require a subtle but important extension of the meaning assigned to our notation* A^+. Whereas previously A^+ was merely one possible outcome of a measurement made on a particle, it is converted by this argument into an attribute of the particle itself. To be explicit, if some unmeasured particle has the property that a measurement along the axis A would give the definite result A^+, then that particle is said to have the property A^+. *In other words, the physicist has been led to the conclusion that both particles in each pair have definite spin components at all times.* ... This view is contrary to the conventional interpretation of QM, but it is not contradicted by any fact that has yet been introduced.'
>
> Against Bell's view we have Bohr's insight to bolster our case: "... the result of a 'measurement' does not in general reveal some preexisting property of the 'system', but is a product of both 'system' and 'apparatus'," Bell (2004: xi-xii). CLR's physical-realism – *some physical properties change interactively* – is consistent with Bohr's insight: "It seems to me that full appreciation of [Bohr's insight] would have aborted most of the 'impossibility proofs' [[like Bell's *impossibility* theorem]], and most of 'quantum logic'," Bell (2004: xi-xii). We agree. [Return to #F2.5.]

#7.4. Early ideas on equivalence relations, classes, dynamics:

$$[q(\hat{\mathbf{a}}^+)] \equiv \{q(\lambda) \in Q : q(\lambda) \sim q(\hat{\mathbf{a}}^+)\},\ [q(\hat{\mathbf{a}}^-) \equiv \{q(\lambda) \in Q : q(\lambda) \sim q(\hat{\mathbf{a}}^-)\}; \tag{49}$$

where $q(\hat{\mathbf{a}}^\pm)$ denotes a dynamic equivalence class (DEC): termed *dynamic* because subject to dynamic transformations like $\delta^\pm_{\hat{\mathbf{b}}} q(\hat{\mathbf{a}}^\pm) \rightarrow q(\hat{\mathbf{b}}^\pm)$, *with relevant probabilities* given by Malus' Law. (49) thus shows that Q is partitioned dyadically under the mapping $\delta^\pm_{\hat{\mathbf{a}}} q(\lambda) \rightarrow q(\hat{\mathbf{a}}^\pm)$. So $\sim$ on the elements of $\delta^\pm_{\hat{\mathbf{a}}}$'s domain denotes: *has the same output/image under* $\delta^\pm_{\hat{\mathbf{a}}}$; etc. The quotient set S is thus a set of two diametrically-opposed extremes: $S/\sim = \{q(\hat{\mathbf{a}}^+), q(\hat{\mathbf{a}}^-)\}$, a maximal antipodean discrimination; a powerful dynamic push-pull duo, consistent with our trigonometric arguments being dynamical processes. Note that direction-vectors like $\hat{\mathbf{a}}$, vectors under geometry, are operators under WM when representing the physical action of polarizers in the arguments of trig-functions. [Return to #F2.6.]

#7.5. The *slightest correlation* calls forth that never-can-be-false probability relation in #2.6. And Bell recognizes the centrality of *correlation* (which is by no means slight) in EPRB:

> Recasting Bell (2004:208) in line with experiment E: "There are no 'messages' in one system from the other. The inexplicable [sic] correlations of quantum mechanics do not give rise to signalling between noninteracting systems. Of course, however, there may be correlations (eg, those of EPRB) and if something about the second system is given (eg, that it is the other side of an EPRB setup) and something about the overall state (eg, that it is the EPRB singlet state) then inferences from events in one system [eg, A^+ from Alice's detector] to events in the other [eg, B^+ from Bob's detector] are possible."

#7.6. Further, when outcomes are highly correlated – as in EPRB, via (1) – stochastic independence is no proxy for local-causality: a view most gardeners with adjoining crops accept. So via our return to fundamentals, the following issue is resolved.

"One general issue raised by the debates over locality is to understand the connection between stochastic independence (probabilities multiply) [ie, $P(XY) = P(X)P(Y)$] and genuine physical independence (no mutual influence) [ie, there is no mutual influence between $A_i^+(\hat{\mathbf{a}}, \lambda_i)$ and $B_i^+(\hat{\mathbf{b}}, \mu_i)$]. It is the latter that is at issue in 'locality,' but it is the former that goes proxy for it in the Bell-like calculations. We need to press harder and deeper in our analysis here," Arthur Fine, in Schlosshauer (2011:45).

The later Bell's fares no better. For he begins to rely "for example, on a full specification of local beables in a given space-time region," Bell (2004:240). How would such be supplied for $q(\lambda_j)$, yet to be tested by $\delta_{\hat{\mathbf{x}}}^{\pm}$ or $\delta_{\hat{\mathbf{y}}}^{\pm}$?

Factorizability is often "taken as the starting point of the analysis. Here we preferred to see it not as the *formulation* of 'local causality', but as a consequence thereof," Bell (2014:243). Our (10)-(13) show the way through Bell's factorization dilemma; thereby confirming Bell's (2004:239) 'utmost suspicion' – he did throw the baby out with the bathwater. [Return to #2.7.]

#7.7. See Aspect (2002:5-7) for a troubled (but, in our case, helpful) discussion of Malus' Law in the context of QM and photons ($s = 1$). Then read it in our terms, trouble-free dynamically and consistent with Aspect's experimental results. Further, though not developed here, our trigonometric arguments are dynamical processes. For example, let the dynamics relate to the interaction in $\delta_{\hat{\mathbf{a}}}^{\pm} q(\lambda_i) \to q(\hat{\mathbf{a}}^+)$. This may be viewed as the operator composition $\Delta_{\hat{\mathbf{a}}}^{\pm} \cdot \delta(\hat{\mathbf{a}}) \circ q(\lambda_i) \to \Delta_{\hat{\mathbf{a}}}^{\pm} \cdot q(\hat{\mathbf{a}}^+) \simeq \hat{\mathbf{a}} \cdot \hat{\mathbf{a}}^+ = +1 = A_i^+$ — now the output of the [now integrated] analyzer-function; a green light, say. [Return to #2.9.]

#7.8. Knowing from prior study that Bell's error starts with (14b) – but cautious here re the totality of Bell's poor or unstated assumptions in the context of EPR/EPRB – we defer discussion of (14a)-(14b) until he commits at his (15). Our results are the same under either option. [Return to #4.3.]

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