**Khoshnevisan,M PhD**

## Discovery of an Unintended Mathematical Error in Equation (7a) on Page 81 in " INVESTIGATIONS ON THE THEORY OF THE BROWNIAN MOVEMENT" by Albert Einstein in 1926 Perhaps After 88 Years

Professor Albert Einstein in 1926 published his book entitled "INVESTIGATIONS ON THE THEORY OF THE BROWNIAN MOVEMENT", during the time that he was teaching at the University of Berlin. This book was edited by Professor Reinhold Heinrich (Henry) Furth in 1926.He was co-author with Professor Albert Einstein of the Theory of Brownian Movement.

Einstein investigated the process of **diffusion in an undissociated dilute solution**. The diffusion process is subject to **Brownian motion**.

In this book, $\frac{v_2 - v_1}{\Delta} = \frac{dv}{dx}$, (where $v_1$ and $v_2$ are the concentrations of **two cross sections** that are separated by a **very small distance** Δ), $\int_{-\infty}^{+\infty} \varphi(\Delta)\mathrm{d}\Delta = \mathrm{I}$ and $\frac{I}{\tau}\int_{-\infty}^{+\infty}\frac{\Delta^2}{2}\varphi(\Delta)d\Delta = D.$ I found an unintended mathematical error (printing error) in equation (7a) in the 1926 publication which is given by $D=\sqrt{2D}\sqrt{\tau}$ on page 81. $D=\sqrt{2D}\sqrt{\tau}$ must be changed to $\Delta^2 = 2D\tau$ or $\Delta=\sqrt{2D}\sqrt{\tau}$. $\Delta=\sqrt{2D}\sqrt{\tau}$ simply follows from $D=\frac{\mathrm{I}}{2}\frac{\Delta^2}{\tau}$. In fact, Einstein has stated "for an undissociated dissolved substance, whose coefficient of diffusion is known, we can calculate Δ from the equation (7a). For sugar at room temperature $D=\frac{0.33}{24.60.60}$ .Hence we calculate from equation (7a) for τ=I

Δ=27.6μ."

**I note that the unintended mathematical error in equation (7a) in this book has the following consequences**:

(I) On page 81, it is stated "we can calculate the displacement of the molecule resulting on an average from the irregular molecular motion: by means of (7a) from the coefficient of diffusion, by means of equation (8) from the resistance which is offered to a forced motion of velocity $\nu =$ I". But "by means of (7a) from the coefficient of diffusion", one cannot "calculate the displacement of the molecule resulting on an average from the irregular molecular motion". Because there is no $\Delta$ in equation(7a).

(II) On page 83, It is stated "…we can calculate $\Delta$ from the equation (7a)". Again, there is no $\Delta$ in equation (7a).

(III) On page 83, it is stated “For sugar at room temperature

$D=\frac{0.33}{24.60.60}$

“Hence we calculate from equation (7a) for $\tau =$I

$\Delta = 27.6\mu$.” There is no $\Delta$ in equation (7a). It is not possible to calculate $\Delta$ from $D=\sqrt{2D}\sqrt{\tau}$ and obtain $27.6\,\mu$.

(IV) D is the coefficient of diffusion, and it has appeared in various parts of the book. For example,

$\frac{I}{\tau}\int_{-\infty}^{+\infty}\frac{\Delta^2}{2}\phi(\Delta)d\Delta = D$ and $\frac{\partial f}{\partial t} = D\frac{\partial^2 f}{\partial x^2}$ on page 15.Einstein has referred to equation (7a) three times in this book i.e once on page 81 and twice on page 83. Therefore, (7a), $D=\sqrt{2D}\sqrt{\tau}$ must be corrected to $\Delta = \sqrt{2D}\sqrt{\tau}$ in this book.

It appears that equation (7a) has not been amended in this book (1926, first edition), in the 1956 edition by Dover publication and in the 2014 edition by Dover publication. This unintended mathematical error has remained in this book for perhaps more than 88 years. “EINSTEIN INVESTIGATION ON THE THEORY OF THE BROWNIAN MOVEMENT” was written by Professor Albert Einstein, and it was published in 1926 when was teaching at the University of Berlin. That said, this unintended mathematical error should not be attributed to Professor Albert Einstein.

Note: Investigations on the theory of the Brownian movement, by Albert Einstein edited with notes by R. Fürth, translated by A. D. Cowper...with 3 diagrams. London, Methuen & co. ltd. [1926] was first published in 1926 by London, Methuen & co. ltd, and later in 1956 and 2014 by Dover publications, Inc.