Explanation of Special Relativity Theory using Photon Clocks

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Abstract: In this short note it is shown that the Special Relativity Theory (SRT) is a theory describing the reality correctly providing that certain conditions, which are encapsulated in the theory's assumptions, are satisfied. This is demonstrated by using simple photon clocks. This approach avoids many difficulties of a relatively complex mathematics that leads many physics hobby enthusiasts to a wrong conclusion claiming that the theory cannot be correct and needs to be changed.

Introduction: In the previously published paper [1] the author has used the Maxwell's EM Field theory to explain the SRT. The reason was that many electronic gadgets that have been developed during the past decades and are being successfully used by many people; such as the cell phones, TV sets, GPS navigation devices in cars, Laptop computers, etc., are all based on Maxwell's EM Field Equations that are used in describing some of the details of workings of these devices and in their designs. Electronic engineers are very happy with these equations and believe in their correctness. It is therefore puzzling that there are still some engineers that do not believe that SRT is correct, because SRT clearly follows from these equations. Nevertheless, they still demand deriving SRT equations without using the EM Field theory. In order to also avoid the complexity of EM Field theory mathematics, the derivation described below will do just that and will be performed using only simple photon clocks.

Derivation of time dilation and length contraction formulas: In many amateur scientists' comments and discussions that can be found online it is claimed that the Maxwell EM Field theory should not be used in the derivations. This is probably due to the lack of knowledge of this theory and the somewhat complex mathematics that is required. In this paper, therefore, another way of derivation is presented that is much simpler and the assumptions of SR Theory are very succinctly defined. The derivations will be based on the so called "photon clocks" that are in a very simplified form shown below in Fig.1.

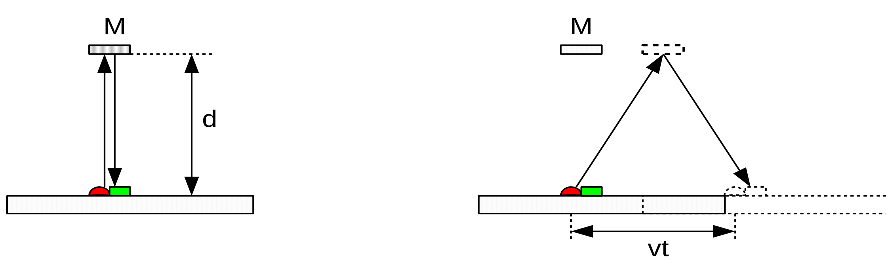

Fig.1 shows simplified details of photon clocks where the photon emitted from the LED bounces from a mirror M, placed at a distance d above the LED, back to a detector located near the light source.

The photon clocks are driven by the photon pulses sent from the red LED light source to a small mirror *M* located at a distance *"d"* above the plane of the source and back to the image sensor detector located in the close vicinity of LED light source. It is well known in the electronic industry that the time of flight

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of photons is relatively easily measured today with a high precision and used to construct 3D images of scenes. It is thus simple to imagine that such an electronic device can drive a number dial of a clock or any digital time display. The time of flight when the setup is in rest is then given by the simple formula:

$$t_{o\perp} = \frac{2d_o}{c} \tag{1}$$

The parameters that are stationary in reference to the laboratory coordinate system have a suffix zero. In the next step it will be considered that the setup is moving with the velocity *v* in the direction parallel to the base plane of the setup as also shown in Fig.1. The small mirror will move some distance in the direction of the velocity vector before the photons reach it. Only the photons that are emitted at a certain angle from the LED will reach the mirror and will be reflected back to the detector. The time of flight is thus simple to calculate as follows:

$$\left(c\frac{t_\perp}{2}\right)^2 = d_o^2 + \left(v\frac{t_\perp}{2}\right)^2 \tag{2}$$

After the formula rearrangement and substitution from Eq.1 for the term $2d_o/c$, the result is:

$$t_\perp = \frac{t_{o\perp}}{\sqrt{1-v^2/c^2}} \tag{3}$$

This is the standard time dilation formula that can be found in many publications, for example online: http://vixra.org/abs/1605.0230. It is important to realize that only the two assumptions were used in its derivation. The first one is that there is a medium, æther, or a transparent dark matter supporting the photon wave propagation with a constant velocity *c.* The second assumption is that the photons did not pick up the velocity of the source. In other words the æther is stationary and in an absolute rest with respect to the laboratory coordinate system. The photon waves propagate in it in a spherical wave fashion from the source, but were restricted to a relatively narrow beam to hit the mirror.

However, an astute reader might have noticed one strange phenomenon. When the mirror has moved into a new position the light beam from the LED has followed it. How does the LED know where to aim the photons? This is very perplexing and mysterious effect, but it can easily be explained using the conservation of photon linear momentum. The photons did not pick up the velocity of the LED from the moving base, they still propagate in the æther with the same velocity *c* as expected, but they have picked up some of the momentum from it, changed their direction of propagation, and their energy. It is thus necessary that the photon momentum conservation law is satisfied. This can be written as follows:

$$\left(\frac{1-v^2/c^2}{1+v^2/c^2}\right)\left(\left(\frac{hf_{o\perp}}{c}\right)^2 + \left(\frac{hf_{o\perp}}{c^2}\right)^2 v^2\right) = \left(\frac{hf_\perp}{c}\right)^2 \tag{4}$$

The first parenthesis term in Eq.4 represents the well-known transversal Doppler Effect frequency shift. The photon frequency can be expressed in terms of the clock ticking time as: $f_\perp = \alpha / t_\perp$. The photon energy *E* equals to: $E = hf = m_i c^2$ as is well known and the photon momentum is equal to: $p_i = hf / c$. After a simple algebra rearrangement the formula in Eq. 4 simplifies to read:

$$f_{\perp} = f_{o\perp} \sqrt{1 - v^2 / c^2} \tag{5}$$

By realizing that the frequency is the reciprocal of time interval, it is clear that Eq.5 is equivalent to Eq.3. This means that the conservation of photon linear momentum as described by Eq.4 is consistent with SRT, the transversal Doppler Effect, and time dilation. The LED thus precisely knows where to aim the photons. This also helps to understand what the time actually is. It is defined here as an interval needed for light to traverse a certain distance *d*. The principal physical parameter is thus the velocity, not time. The time is thus a derived parameter. Our perception or measurement of time is only the comparison of speed of light to the speed of process, the causal event occurrence delay, which we might be observing.

For the length contraction formula derivation the photon clocks can also be used. In this case the photons propagate in parallel along the length of the moving rod of length *"L"*. The LED and the mirror are arranged as shown in Fig.2.

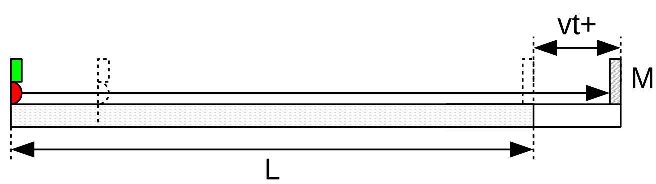

Fig.2 shows the propagation of photons in the direction of rod motion. Photons were emitted from the red LED. The mirror moves the distance equal to *vt+* before the photons reach it. For the return path the detector moves the distance *vt-* before the reflected photons from the mirror M reach it.

From the drawing it is clear that the time the photons emitted from the LED need to reach the mirror is as follows:

$$ct_{+\parallel} = L + vt_{+\parallel} \qquad\qquad t_{+\parallel} = \frac{L}{c - v} \tag{6}$$

For the photons on the returning path from the mirror M the time to reach the detector is similarly equal to:

$$ct_{-\parallel} = L - vt_{-\parallel} \qquad\qquad t_{-\parallel} = \frac{L}{c + v} \tag{7}$$

For the total round trip time of photons: $t_{\parallel} = t_{+\parallel} + t_{-\parallel}$ including the expression for the time dilation obtained from the formula in Eq.3 that is also, of course, applicable here, thus holds that:

$$t_{\parallel} = \frac{t_{o\parallel}}{\sqrt{1 - v^2 / c^2}} = \frac{2L}{c} \frac{1}{1 - v^2 / c^2} \tag{8}$$

However, we also know that when the rod is stationary it holds that: $t_{o\|} = 2L_o / c$. It is thus clear that for the length of the moving rod *"L"* it must hold that:

$$L = L_o \sqrt{1 - v^2 / c^2} \tag{9}$$

This is the famous length contraction formula. It is thus clear that the EM Field theory is not necessary for the derivation of SRT time dilation and length contraction formulas and that only the two simple and obviously true assumptions are necessary for their derivation as is again stated below:

- There is an æther medium that supports the propagation of photons in it as waves.
- The velocity of light in that medium is not affected by the velocity of light source; the moving light source changes only the energy and the linear momentum of emitted photons.

Having thus derived the length contraction and the time dilation formulas it is now easy to understand the Lorentz coordinate transformation equations from moving to stationary coordinate systems:

$$x_o = \frac{x - vt}{\sqrt{1 - v^2 / c^2}} \qquad t_o = \frac{t - vx / c^2}{\sqrt{1 - v^2 / c^2}} \tag{10}$$

and from them derive the well-known and important coordinate transformation invariant:

$$s^2 = (ct)^2 - x^2 = (ct_o)^2 - x_o{}^2 \tag{11}$$

or in a differential form:

$$ds^2 = (cdt)^2 - dx^2 = (cdt_o)^2 - dx_o^2 \tag{12}$$

The time *t* and the distance *x* used in these formulas have a slightly different meaning than in the previous derivations. These symbols now represent coordinates referenced to origins of coordinate systems for stationary and moving coordinate frames, not the time and the distance differences as they were used before. This is a subtle point that is emphasized more clearly in the next section where the time and the length differences have the symbol Δ in front of them. The imprecise notation is sometimes a source of misunderstanding in SRT.

Another important point that is included in the above derivation of Lorentz coordinate transformation is that it is not necessary to consider the laboratory reference frame as being an absolute reference frame. The transformations form a Group if a correct formula is used for adding the velocities. This can be shown by considering that the variables (x_o, t_o) are related to another reference coordinate system (x_1, t_1) that is moving relative to the laboratory coordinate system with a velocity v_1 :

$$x_o = \frac{x_1 - v_1 t_1}{\sqrt{1 - v_1^2 / c^2}} \qquad t_o = \frac{t_1 - v_1 x_1 / c^2}{\sqrt{1 - v_1^2 / c^2}} \tag{13}$$

By eliminating variables with the suffix zero from formulas in Eq.10 and Eq.13 one can obtain the Lorentz coordinate transformation formulas between the variables (x,t) and (x_1,t_1). The derivation steps are shown below:

$$\frac{x-vt}{\sqrt{1-v^2/c^2}}=\frac{x_1-v_1t_1}{\sqrt{1-v_1^2/c^2}} \qquad \frac{t-vx/c^2}{\sqrt{1-v^2/c^2}}=\frac{t_1-v_1x_1/c^2}{\sqrt{1-v_1^2/c^2}} \tag{14}$$

The derivations can be simplified by introducing the relation for addition of velocities applied to velocity v_1. This is referencing the new velocity to the velocity *v*. The formula is well-known as the relativistic velocity addition formula, which for small velocities approaches the classical form $v_1 = v+u$:

$$v_1=\frac{v+u}{1+vu/c^2} \tag{15}$$

Using Eq.15 leads to the following useful relations:

$$\sqrt{1-v_1^2/c^2}=\frac{\sqrt{1-v^2/c^2}\sqrt{1-u^2/c^2}}{\left(1+vu/c^2\right)} \tag{16}$$

$$t=\frac{t_1-x_1u/c^2}{\sqrt{1-u^2/c^2}}+\frac{v}{c^2}\left(x-\frac{x_1-t_1u}{\sqrt{1-u^2/c^2}}\right) \qquad x=\frac{x_1-t_1u}{\sqrt{1-u^2/c^2}}+v\left(t-\frac{t_1-x_1u/c^2}{\sqrt{1-u^2/c^2}}\right) \tag{17}$$

Equations 17 must be satisfied for any arbitrary velocity *v*. This will be true when the following relations hold:

$$t=\frac{t_1-x_1u/c^2}{\sqrt{1-u^2/c^2}} \qquad x=\frac{x_1-t_1u}{\sqrt{1-u^2/c^2}} \tag{18}$$

These are the new Lorentz coordinate transformation equations that are referenced to another coordinate system, which is moving relative to the original laboratory coordinate system with a velocity v_1. This means that any inertial coordinate system moving with a constant velocity in reference to the absolute reference frame, which is for example in rest with respect to the dark matter of the Universe can be used and its motion cannot be detected using these transformations. However, the author of this paper has published a solution to the Twin Paradox problem, which can be used for the detection of absolute motion. http://vixra.org/abs/1410.0192

A comment on the typical source of misunderstanding in the Lorentz coordinate transformation: The main source of confusion and misunderstanding of the SRT and its Lorentz coordinate transformation is the fact that each variable (t_o, x_o) is a function of the two variables (t, x). Many armature SRT enthusiasts fail to comprehend this fact and as a result claim that SRT is inconsistent or completely wrong. It is thus always necessary to specify the second variable when the transformation for the first variable is being looked for. This should be reflected in the notation when, for example, the reference clock is located in the moving coordinate system as follows:

$$\Delta t_o(x_o = const) = \Delta t\sqrt{1-v^2/c^2} \tag{19}$$

This is the famous time dilation equation as derived above in Eq.3. For the reference clock located in the stationary, laboratory, coordinate system the result is as follows:

$$\Delta t_o(x = const) = \frac{\Delta t}{\sqrt{1-v^2/c^2}} \tag{20}$$

It is thus clear that the two completely different dependencies between the time increment in the stationary laboratory coordinate system and the time increment in the moving coordinate system are obtained depending on location of the reference time measuring device. The above notation is typically omitted from the corresponding equations, the location of the time measuring device is implicitly assumed to be understood and this, unfortunately, leads to confusion. Similar relations are also valid for the distances.

Discussion: Several important observations can be made in the above derivations as follows:

1. The first observation is that only the two assumptions are actually needed for the derivation of time dilation and length contraction SRT formulas.

2. The second observation and the assumption confirmation is that photons do not pick up the velocity of the light source, but pick up a part of its momentum. This leads to a change of their energy and depending on the test setup may lead to a change of their propagation direction.

3. Finally, the typical source of confusion and misunderstanding of SRT and its Lorentz coordinate transformation was identified and explained by an inadequate notation typically used in Lorentz coordinate transformation equations.

Conclusions: It is clear that SRT is a correct theory that is valid for any uniform inertial motion and that is closely describing the reality. Therefore, it also follows that the Maxwell's EM Field theory is correct. It is thus perplexing that there are still some hobby scientists claiming that SRT is wrong. When the paradoxes are encountered in SRT it is necessary to always examine assumptions and the details of how the theory is applied. The author hopes that this short note may be helpful to such SRT deniers. Additional references related to this topic can be found online [2, 3, 4, and 5].

References:

[1] http://vixra.org/abs/1605.0230

[2] http://vixra.org/author/jaroslav_hynecek

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