Belief Reliability Analysis and its application

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Abstract

In reliability analysis, Fault Tree Analysis based on evidential networks is an important research topic. However, the existing EN approaches still remain two issues: one is the final results are expressed with interval numbers, which has a relatively high uncertainty to make a final decision. The other is the combination rule is not used to fuse uncertain information. These issues will greatly decrease the efficiency of EN to handle uncertain information. To address these open issues, a new methodology, called Belief Reliability Analysis, is presented in this paper. The combination methods to deal with series system, parallel system, series-parallel system as well as parallel-series system are proposed for reliability evaluation. Numerical examples and the real application in servo-actuation system are used to show the efficiency of the proposed Belief Reliability Analysis methodology.

Keywords: Belief reliability analysis, Fault Tree Analysis, Dempster-Shafer evidence theory, evidential networks, servo-actuation system.

☆Belief Reliability Analysis and its application

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1. Introduction

Probabilistic structural analysis [1] is the art of establishing mathematical models, which can obtain the probability from a structure that behaves in a specified way, when given that one or more of its material properties or geometric dimensions. The properties are of a random or incompletely known nature. Through analyzing different known data, the result can be got to predict the future behaviour and the possible outcomes [2]. Probabilistic analysis can be classified into different types. One of that is Fault Tree Analysis (FTA).

Fault Tree Analysis [3] was firstly developed in 1962 at Bell Laboratories by H.A. Watson. It is a kind of logic causality diagram, which displays the state of the system according to the component state. FTA attracts a large number of researchers to further develop and is widely used in aerospace, nuclear power, chemical and process, pharmaceutical and other areas. In 2002, Reay and Andrews [4] proposed an analytic strategy to increase the likelihood of obtaining a Binary Decision Diagrams (BDD) for any given fault trees. Contini and Matuzas [5] described a new method to analyze large coherent fault trees, which can be advantageously applied when the working memory is not sufficient to construct the BDD. Ferdous *et al.* [6] presented a revised methodology for computer-aided Fault Tree Analysis. Ejlali and Miremadi [7] presented Time-to-Failure tree, which can be used to accelerate the Monte Carlo simulation of fault trees.

However, it is inevitable to handle uncertain information in FTA [8]. Therefore, some math tools, such as Dempster-Shafer (D-S) evidence theory and fuzzy sets theory, are adopted in FTA. For example, In [9], Sun *et al.* used printed circuit board assembly (PCBA) to obtain the PCBA fault-tree, fault-tree nodes, and directly computed the intuitive fuzzy fault-tree interval. Yang *et al.* [10], combined with Dempster-Shafer (D-S) evidence theory [11] and FTA to evaluate different experts' opinions, and generate basic belief assignment (BBA) to present the failure rate of components.

It should be pointed out that evidence theory has some open issues, such as conflicting management [12, 13, 14], generating basic probability assignment [15, 16, 17] and dependence evidence combination[18, 19, 20]. However, evidence theory plays a promising role in FTA and is paid more and more attention. Compared to fuzzy set theory, it can not only handle uncertain information, but also provide Dempster rule to combine uncertain information from different sources. A number of methods based on evidence theory were applied to to many real applications such as decision making under uncertain environment[21, 22, 23, 24], pattern recognition [25, 26], failure analysis [27, 28, 29] and sensor data fusion [30, 31]. Using basic belief assignment to express uncertain information, instead of probability, in the reliability analysis of complex system gradually develops a new research topic: evidential network [8].

Some evidential networks are presented. For example, Simon *et al.* use bayesian networks inference algorithms to compute complex system reliability, and extend it to evidential networks. In addition, Simon and Weber use the evidential networks to solve multi-state system [32] and compared the result with fuzzy fault tree [8]. Xu and Philippe use conditional belief functions to deduce evidential networks [33]. Boukhris *et al.* introduced the belief causal networks [34], which represents dependencies as uncertain causal links and represents the uncertainty as belief masses. Yang *et al.* proposed the calculation method with Exclusive Or gate and Exclusive Nor gate in evidential networks [35]. Qiu *et al.* [36] applied evidential network to handle hazardous material transportation problem and compared with beyesian network to show the efficiency of the evidential network. Benavoli *et al.* constructed an evidential network model for threat assessment [37]. A dynamic evidential network was proposed by Aguilar *et al.* [38] and Laâmari [39].Jiang [40], Yaghlane and Mellouli [41], Laâmari *et al.* [42] proposed the reasoning algorithms to solve problem in evidential networks. Yang *et al.* [43] indicated that the mass belief table in series, parallel, series-parallel and parallel-series systems could be expressed by formula and proposed their EN approach, which expresses the relationship among components and sub-systems in servo-actuation system.

However, the existing EN approaches still remain two issues: one is that the final results are expressed with interval numbers, which has a relatively high uncertainty to make a final decision. The other is that the combination rule is not used to fuse uncertain information. These issues will greatly decrease the efficiency of EN to handle uncertain information. To address these open issues, a new methodology, called Belief Reliability Analysis (RBA), is presented in this paper. The combination methods to deal with series system, parallel system, series-parallel system as well as parallel-series system are proposed for reliability evaluation.

This paper is organized as follows: In section 2, some preliminaries are briefly introduced, including evidence theory, FTA and existing EN. In section 3, the proposed method is detailed. In section 4, some numerical examples are used illustrate the efficiency of the proposed method. In section 5, the BRA methodology was applied in servo-actuation system. The conclusion is given in Section 6 to end the paper.

2. Preliminaries

In this section, some preliminaries are briefly introduced, including evidence theory, FTA and existing EN.

2.1. D-S Evidence Theory

Dempster-Shafer theory of evidence, is used to deal with uncertain information. Some basic concepts of this theory are introduced as follows.

2.1.1. The frame of discernment

The frame of discernment (FD) is proposed to describe the whole circumstances in the event. Θ is used to describe a set of mutually exclusive and collectively exhaustive elements E_i, which is indicated by

$$\Theta = \{E_1, E_2, \cdots, E_i, \cdots, E_N\} \quad (1)$$

Set Θ is called FD. The power set of Θ is denoted by 2^Θ, and

$$2^\Theta = \{\emptyset, \{E_1\}, \cdots, \{E_N\}, \{E_1, E_2\}, \cdots, \{E_1, E_2, \cdots, E_i\}, \cdots, \Theta\} \tag{2}$$

where $\emptyset$ is an empty set.

2.1.2. The basic mass assignment

A mass function m is a mapping from 2^Θ to a probability interval $[0, 1]$, formally defined by:

$$m: \quad 2^\Theta \to [0, 1] \tag{3}$$

which satisfies the following conditions:

$$m(\emptyset) = 0 \qquad \sum_{A \in 2^\Theta} m(A) = 1 \quad 0 \leq m(A) \leq 1 \quad A \in 2^\Theta \tag{4}$$

2.1.3. Belief function and plausibility function

For an elementary proposition $A \subseteq \Theta$, the belief function Bel is a mapping: $2^\Theta \to [0, 1]$ is defined as

$$Bel(A) = \sum_{B \subseteq A} m(B) \tag{5}$$

and satisfies the following conditions:

$$Bel(\emptyset) = 0 \qquad Bel(\Theta) = 1.$$

The plausibility function $Pl: \; 2^\Theta \to [0, 1]$ is defined as

$$Pl(A) = 1 - Bel(\bar{A}) = \sum_{B \cap A \neq \emptyset} m(B) \tag{6}$$

In Fig.1, it is obvious that $Pl(A) \geq Bel(A)$, and functions Bel and Pl represent the lower and upper limit mass functions of proposition A, respectively.

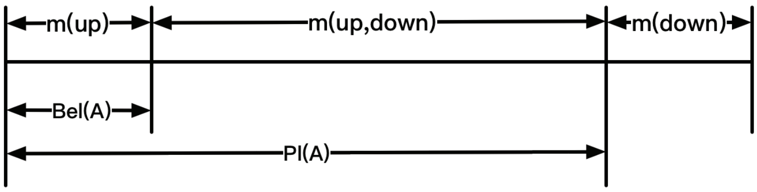

Figure 1: Relationship between the belief and plausibility function

2.1.4. *Dempster rule of combination*

If $m(A) > 0$, A is a focal element, and the set of some focal elements is named a body of evidence (BOE). When multiplying BOEs is available, the Dempster's combination rule can be used to obtain the combined evidence:

$$m(A) = \frac{\sum\limits_{B \bigcap C = A} m_1(B)m_2(C)}{1 - K} \tag{7}$$

where $K = \sum\limits_{B \bigcap C = \varnothing} m_1(B)m_2(C)$ is a normalization constant, which called conflict. The combination rule establish if and only if when $m(\varnothing) \neq 1$.

2.2. *The Evaluation of Fault Tree*

2.2.1. *Probability interval in Fault Tree*

Since the occurrence rate can not be measured exactly, it is hard to express the probability weather event will occur. Combining D-S theory, the occurrence rate of event expresses as:

$$\underline{P(A)} = Bel(A) \qquad \overline{P(A)} = Bel(A) \tag{8}$$

Where $\underline{P(A)}$ represents the upper limit of the rate of occurrence, and $\overline{P(A)}$ means the lower limit of the rate of occurrence.

In the traditional FT, the event contains two basic states: $\{up\}$ and $\{down\}$. $\{up\}$ means the occurrence of the event, while $\{down\}$ means the non-occurrence of the event. Combining FT with EN, the corresponding event contains four states: $\varnothing$, $\{up\}$, $\{down\}$ and $\{up, down\}$.

2.2.2. The evaluation information

In FTA, it is difficult to measure the occurrence probability of basic event exactly at first, because the data is uncertain. In order to compensate for the lack of data caused by occurrence probability inaccuracy, D-S evidence theory is adopted to aggregate different experts' evaluation information, which may be uncertain and imprecise. At the same time, the corresponding model is constructed gradually.

Suppose there are L experts and K basic events, which are expressed as:

$$E = (E_1, E_2, ..., E_L) \qquad C = (C_1, C_2, ..., C_K) \tag{9}$$

Evaluation information set from i experts ($1 \leq i \leq K$) on the occurrence probability event can be defined as follows:

$$P_i = \{[\underline{P_1^i(\{up\})}, \overline{P_1^i(\{up\})}], ..., [\underline{P_L^i(\{up\})}, \overline{[P_L^i(\{up\})}]\} \tag{10}$$

2.2.3. The evaluation opinion

According to Eq.(5), Eq.(6) and Fig.1, for the evaluation opinion of k ($1 \leq k \leq L$) to every single event i ($1 \leq i \leq K$), the formula can be deduced:

$$\begin{aligned} m_k^i(\{up\}) &= Bel_k^i(\{up\}) \\ m_k^i(\{down\}) &= 1 - Pl_k^i(\{up\}) \\ m_k^i(\{up,\ down\}) &= Pl_k^i(\{up\}) - Bel_k^i(\{up\}) \end{aligned} \tag{11}$$

Applying Eq.(11) to Eq.(10), the result can be expressed as:

$$P_i = \{[Bel_1^i(\{up\}), Pl_1^i(\{up\})], ..., [Bel_L^i(\{up\}), [Pl_L^i(\{up\})]\} \tag{12}$$

2.2.4. The vector of evaluation opinion

Construct the whole states mentioned in Eq.(11), the collection:

$$\begin{aligned} m_k^i &= (m_k^i(\{\emptyset\}), m_k^i(\{up\}), m_k^i(\{down\}), m_k^i(\{up,\ down\})) \\ &= (0, Bel_k^i(\{up\}), 1 - Pl_k^i(\{up\}), Pl_k^i(\{up\}) - Bel_k^i(\{up\})) \end{aligned} \tag{13}$$

In CWA, the empty set $\emptyset$ is always satisfied with m($\emptyset$) = 0. Therefore we ignore $\emptyset$:

$$\begin{aligned} m_k^i &= (m_k^i(\{up\}), m_k^i(\{down\}), m_k^i(\{up,\ down\})) \\ &= (Bel_k^i(\{up\}), 1 - Pl_k^i(\{up\}), Pl_k^i(\{up\}) - Bel_k^i(\{up\})) \end{aligned} \tag{14}$$

2.3. The Fusion Model Using Evidence Theory with Evidence Theory

2.3.1. The directed acyclic graph in evidential network

Evidential network, which is also called belief network, is a directed acyclic graph (DAG) from probabilistic reasoning [32]. Probabilistic reasoning is a process getting other probabilistic information through some variables. With probabilistic reasoning, evidential network, could use for solving uncertain and incomplete problems from network, such as events contact and associated state. In these paper, evidential network could represent the relationship among source, actuator and controller.

Evidential network contains variable nodes and directed edges, which connect these nodes [44]. The nodes represent random variables, and the directed edges between nodes represent the relationships among inter nodes. Node variables can be used for solving abstract problems, such as testing values, conducting probability. Through the relation among nodes and edges, evidential networks could express and analyse uncertainty and probability of the event, and finally derive results from incomplete, imprecise or uncertain information.

2.3.2. Series system

The sub-system or components connecting end to end constructs series system. A series system, corresponding to 'AND' gate in logic gate circuit, fails if any of the subsystems or components fails. A typical series system configuration and corresponding to equivalent evidential are shown in Fig.2.

To compute the marginal belief mass of connecting node of series system in EN, Simon's conditional belief mass table is adopted. Ignore state $\{\emptyset\}$, two components or

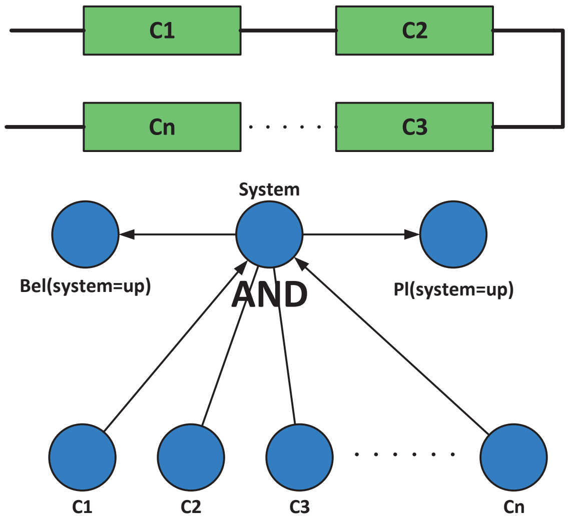

Figure 2: Series system

sub-systems make up nine different state:

$$\begin{array}{ccc} \{\{up\},\{up\}\} & \{\{up\},\{down\}\} & \{\{up\},\{up,down\}\} \\ \{\{down\},\{up\}\} & \{\{down\},\{down\}\} & \{\{down\},\{up,down\}\} \\ \{\{up,down\},\{up\}\} & \{\{up,down\},\{down\}\} & \{\{up,down\},\{up,down\}\} \end{array}$$

Eq.(15) expresses the way to calculate the states after connecting two components or sub-systems in series:

$$\begin{aligned} & m_{ij}(system = C) = (m_i \odot m_j)(C) = \\ & \begin{cases} m_i(\{C\}) \cdot m_j(\{C\}) & C = \{up\} \\ m_i(\{C\}) + m_j(\{C\}) - m_i(\{C\}) \cdot m_j(\{C\}) & C = \{down\} \\ m_i(\{X\}) \cdot m_j(\{C\}) + m_i(\{C\}) \cdot m_j(\{X\}) & +m_i(\{C\}) \cdot m_j(\{C\}) \\ & X = \{up\}, C = \{up, down\} \end{cases} \end{aligned} \tag{15}$$

where m_i and m_j express the belief mass of the two components i and j in series system, and m_{ij} expresses the marginal belief mass in the series system.

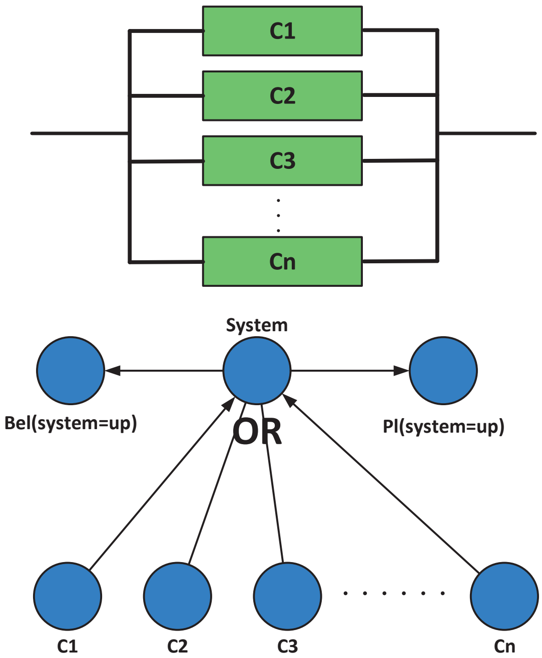

Figure 3: Parallel system

When there are n ($n \geq 2$) components or sub-systems connecting in series end to end, the marginal belief mass in series system is generalized to Eq.(16):

$$m(system) = m_1 \odot m_2 \odot m_3 \odot ... \odot m_n = ((((m_1 \odot m_2) \odot m_3) \odot ...) \odot m_n). \qquad (16)$$

where, m_i(i = 1, 2, ... , n) represents the belief mass of every single components or sub-systems. m is the belief mass of the series system.

2.3.3. Parallel system

Parallel connection, which is another way to connect components and corresponds to 'OR' gate, fails if and only if all the units in the system fail. A typical series system configuration corresponding to EN is shown in Fig.3. The detailed processing conversion can be referred to paper [32].

Similar to the series system, according to the conditional belief mass table, the states is shown in Eq.(17):

$$m_{ij}(system = C) = (m_i \otimes m_j)(C) =$$

$$\begin{cases} m_i(\{C\}) + m_j(\{C\}) - m_i(\{C\}) \cdot m_j(\{C\}) & C = \{up\} \\ m_i(\{C\}) \cdot m_j(\{C\}) & C = \{down\} \\ m_i(\{C\}) + m_j(\{C\}) - m_i(\{X\}) \cdot m_j(\{C\}) & -m_i(\{C\}) \cdot m_j(\{X\}) - m_i(\{C\}) \cdot m_j(\{C\}) \\ & X = \{up\}, C = \{up, down\} \end{cases} \tag{17}$$

Also, when there are more than two components or sub-systems in parallel system, the marginal belief mass of system or the top node of parallel system is shown as following:

$$m(system) = m_1 \otimes m_2 \otimes m_3 \otimes ... \otimes m_n = ((((m_1 \otimes m_2) \otimes m_3) \otimes ...) \otimes m_n) \tag{18}$$

2.3.4. Series-parallel system and parallel-series system

Series-parallel system (shown in Fig.4) and parallel-series system (shown in Fig.5) indicate sub-systems in which several components are connected in parallel, and then in series or sub-systems that several components are connected in series, and then in parallel.

In traditional series-parallel system, Chern [45] indicated that it is very difficult to find out an optimal solution under multiple constraint conditions for the framework of series-parallel system. Misra and Sharma's algorithm [46], solved problems by integer programming, which serves as an algorithm searching for nearby boundary of the domain of feasible solution. Prasad and Kuo [47] pointed out that Misra algorithm sometimes cannot yield an optimal solution, and suggested a method of searching for the upper limit of reliability's objective function.

Simon and Weber [32] then formalized the evidential networks for the handling of imprecise probabilities, and proposed belief mass table to express the states for components and sub-systems. According to Eq.(16) and Eq.(18), equations for calculating series-parallel system and parallel-series system were also put forward by Yang:

When there are m sub-systems and n components in the i^{th} sub-system, the marginal belief mass of system is indicated in Eq.(20). It is both commutative and associative:

$$\begin{aligned} m(system) =& m_{C1} \otimes m_{C2} \otimes m_{C3} \otimes ... \otimes m_{Cn} \\ =& (m_{11} \odot m_{12} \odot m_{13} \odot ... \odot m_{1n}) \otimes (m_{21} \odot m_{22} \odot m_{23} \odot ... \odot m_{2n}) \\ & \otimes (m_{31} \odot m_{32} \odot m_{33} \odot ... \odot m_{3n}) \otimes ... \otimes (m_{m1} \odot m_{m2} \odot m_{m3} \odot ... \odot m_{mn}) \end{aligned} \tag{19}$$

where m_{Ci} (i = 1, 2, ... , m) represents the belief mass of the child nodes in each of the sub-system. C_{ij} (i = 1, 2, ... , m; j = 1, 2, ... , n) represents the belief mass of the ith sub-system. m is the belief mass of the whole system.

The same as the series-parallel system, the marginal belief mass of parallel-series system can be generalized in:

$$\begin{aligned} m(system) =& m_{C1} \odot m_{C2} \odot m_{C3} \odot ... \odot m_{Cn} \\ =& (m_{11} \otimes m_{12} \otimes m_{13} \otimes ... \otimes m_{1n}) \odot (m_{21} \otimes m_{22} \otimes m_{23} \otimes ... \otimes m_{2n}) \\ & \odot (m_{31} \otimes m_{32} \otimes m_{33} \otimes ... \otimes m_{3n}) \odot ... \odot (m_{m1} \otimes m_{m2} \otimes m_{m3} \otimes ... \otimes m_{mn}) \end{aligned} \tag{20}$$

Also, m_{Ci} (i = 1, 2, ... , m) is the belief mass of the sub-system. C_{ij} (i = 1, 2, ... , m; j = 1, 2, ... , n) represents the belief mass of the i^{th} sub-system. m is the belief mass of the whole system.

3. A New Evidential Network

Yang *et al.*'s EN approach [43] remains the uncertain circumstance $m_{ij}(\{up, down\})$. The result is expressed by a probability interval numbers (the upper limit is *Pl(system=up)*,

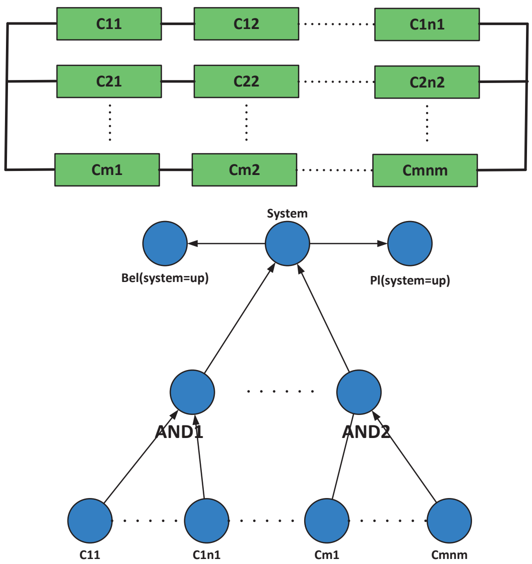

Figure 4: Series-parallel system

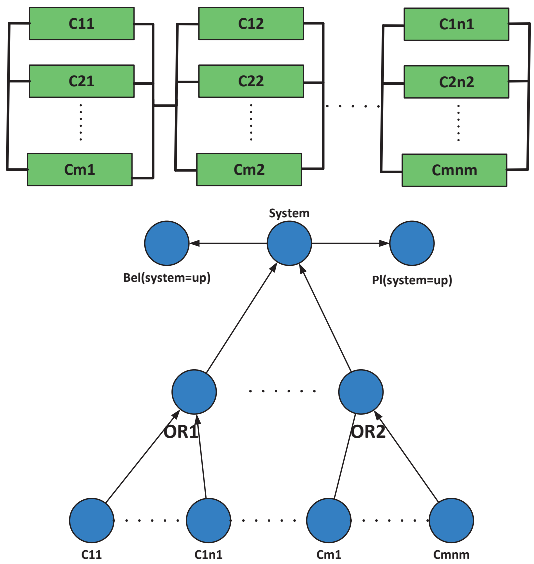

Figure 5: Parallel-series system

and the lower limit is *Bel(system=up)*. Thus, the new methodology is produced for belief reliability analysis.

3.1. *Belief Relief Analysis under series system*

In Eq.(15), $m_{ij}(C) = m_i(X) \cdot m_j(C) + m_i(C) \cdot m_j(X) + m_i(C) \cdot m_j(C)$, $(X = \{up\}, C = \{up, down\})$ can not accurately represent neither system is up nor system is down. It can be regarded as useless information. On the contrary,

$$m_{ij} = m_i(C) \cdot m_j(C), (C = \{up\})$$

$$m_{ij} = m_i(C) + m_j(C) - m_i(C) \cdot m_j(C), (C = \{down\})$$

can determine the state of components. Combining Eq.(11) and Eq.(15), the equation:

$$m_{ij}(system = C) = (m_i \circ m_i)(C) = \begin{cases} \dfrac{Bel_k(\{C\}) \cdot Bel_j(\{C\})}{1-K} \\[2em] \dfrac{1 - Pl_k(\{C\}) \cdot Pl_j(\{C\})}{1-K} \end{cases} \tag{21}$$

where $K = Pl_k(\{C\}) \cdot Pl_j(\{C\}) - Bel_k(\{C\}) \cdot Bel_j(\{C\})$

Eq.(21) can also be generalized to n ($n \geq 2$) components connected in series:

$$m(system) = m_1 \circ m_2 \circ m_3 \circ ... \circ m_n = (((m_1 \circ m_2) \circ m_3) \circ ... \circ m_n) \tag{22}$$

where m$_i$ (i = 1, 2, 3, ... ,n) expresses the belief mass of the series system.

3.2. *Belief Reliability Analysis under parallel system*

Similarly, we could get the rate of occurrence in system by Eq.(11) and Eq.(17):

$$m_{ij}(system = C) = (m_i \bullet m_i)(C) = \begin{cases} \dfrac{Bel_k(\{C\}) + Bel_j(\{C\}) - Bel_k(\{C\}) \cdot Bel_j(\{C\})}{1-K} \\ \\ \dfrac{[1 - Pl_k(\{C\})] \cdot [1 - Pl_j(\{C\})]}{1-K} \end{cases} \tag{23}$$

where $K = [1 - Bel_k(\{C\})] \cdot [1 - Bel_j(\{C\})] - [1 - Pl_k(\{C\})] \cdot [1 - Pl_j(\{C\})]$

Similarly, when more than two (n) components connect in parallel:

$$m(system) = m_1 \bullet m_2 \bullet m_3 \bullet ... \bullet m_n = (((m_1 \bullet m_2) \bullet m_3) \bullet ... \bullet m_n) \tag{24}$$

3.3. *Series-parallel system and parallel-series system under evidential network*

The method to calculate different components connect in serial and parallel is similar to the Yang *et al.*'s EN approach [43]. In Fig.4, the result can be expressed as:

$$\begin{aligned} m_{series-parallel}(system) =& m_{C1} \bullet m_{C2} \bullet m_{C3} \bullet ... \bullet m_{Cn} \\ =& (m_{11} \circ m_{12} \circ m_{13} \circ ... \circ m_{1n}) \bullet (m_{21} \circ m_{22} \circ m_{23} \circ ... \circ m_{2n}) \\ & \bullet (m_{31} \circ m_{32} \circ m_{33} \circ ... \circ m_{3n}) \bullet ... \bullet (m_{m1} \circ m_{m2} \circ m_{m3} \circ ... \circ m_{mn}) \end{aligned}$$

Figure 6: series system calculation

At the same time, the parallel-series system, shown in Fig.5 can be represented as:

$$
\begin{aligned}
m_{parallel-series}(system) &= m_{C1} \circ m_{C2} \circ m_{C3} \circ \ldots \circ m_{Cn} \\
&= (m_{11} \bullet m_{12} \bullet m_{13} \bullet \ldots \bullet m_{1n}) \circ (m_{21} \bullet m_{22} \bullet m_{23} \bullet \ldots \bullet m_{2n}) \\
&\circ (m_{31} \bullet m_{32} \bullet m_{33} \bullet \ldots \bullet m_{3n}) \circ \ldots \circ (m_{m1} \bullet m_{m2} \bullet m_{m3} \bullet \ldots \bullet m_{mn})
\end{aligned}
$$

It can be seen from the equation above, although there are some differences in calculation method between the new method and the Yang *et al.*'s EN approach [43], they have the similar characteristics, which means that many characters in the Yang *et al.*'s EN approach [43] can be inherited by this new combining method.

3.4. Other special circumstances

It should be noted when $Bel_i(system) = Pl_i(system)$, the BRA methodology, both in series and in parallel, degenerates into classical probability calculation in series and in parallel. The way to calculate is the same as Eq.(15) and Eq.(17).

4. Numerical Examples

4.1. Probability networks

As can be seen in Fig.6. When $Bel_i(system)$=$Pl_i(system)$, two components connect in series. Suppose: and the corresponding probability calculation degenerates from com-

	$\{up\}$	$\{down\}$
C_1	70.00%	30.00%
C_2	80.00%	20.00%

bining belief function to classical probability calculation.

$\{up\}$	56.00%
$\{down\}$	44.00%

4.2. Series networks

Fig.6 is also suitable for basic belief assignment calculation, and their corresponding belief mass distribution is: Through BRA methodology under series, the reliability of

	$\{up\}$	$\{up\},\{down\}$	$\{down\}$
C_1	70.00%	20.00%	10.00%
C_2	80.00%	10.00%	10.00%

system is shown below, and the result from traditional EN approach is also displayed:

	Traditional EN approach	BRA methodology
$\{up\}$	56.00%	66.67%
$\{up\},\{down\}$	19.00%	0
$\{down\}$	25.00%	33.33%

4.3. Parallel networks

As is shown in Fig.7, the priori belief mass distribution of the each node is:

	$\{up\}$	$\{up\},\{down\}$	$\{down\}$
C_1	70.00%	20.00%	10.00%
C_2	80.00%	10.00%	10.00%

The compared results is shown below:

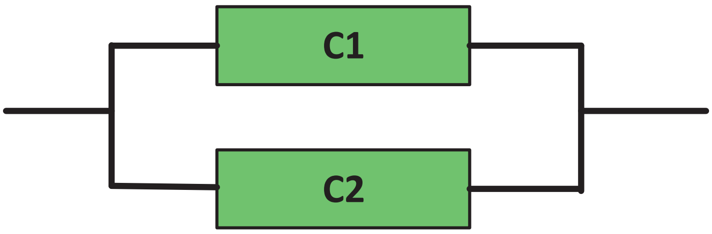

Figure 7: parallel system calculation

	Traditional EN approach	BRA methodology
$\{up\}$	94.00%	98.95%
$\{up\},\{down\}$	1.00%	0
$\{down\}$	5.00%	1.05%

5. Application in Three-redundancy servo-actuation system

Servo-actuation technology is one of the key technologies of computational numerical control (CNC), industrial robots and other industrial machines [48]. This technology has received high attention all over the world. With the development of the servo-actuation technology, higher speed, precision and efficiency of the servo-actuation system are in need. However, servo-actuation system, which is long-running and lack in reliability but important to operate normally, needs to estimate its rate of occurrence.

The basic reliability diagram of three-redundancy servo-actuation system and the corresponding mission reliability diagram of system is shown in Fig.8. Assuming there is no external factors to interfere with the system. The uncertain data of the servo controller is also shown in Fig.8.

According to Yang *et al.*'s EN approach [43], the belief function and the plausibility function can be calculated:

the failure rate of the servo controller is adopted as an interval:[10.61%,28.22%], and the reliability of the servo system is: [71.78%,89.39%].

$Bel(system = up)$	71.78%
$Pl(system = up)$	89.39%

The new combining approach gets the result:

$Bel(system = up)$	87.34%
$Pl(system = up)$	12.66%

Meanwhile, the result is also shown in Fig.8: Compared with Yang *et al.*'s EN ap-

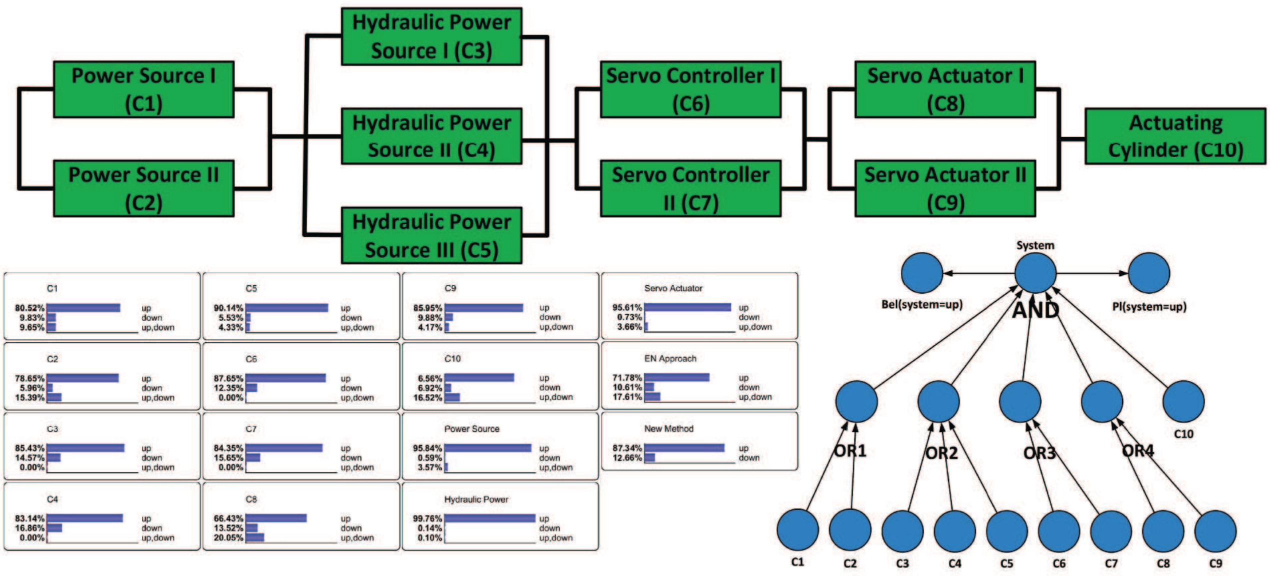

Figure 8: Parallel-series system diagram

proach [43], the proposed BRA methodology decrease the uncertainty of the results significantly. The main reason is that Dempster rule is applied to efficiently combine the data in the proposed BRA methodology. What's more, the support degree of status up is 88.40%, while the result is from 71.78% to 89.39% in Yang *et al.*'s EN approach [43].

Besides, if we change C_1's value, the failure rate of this servo-actuation will also change simultaneously. For instance, if we firstly set the rate to the state $\{down\}$ in C_1 is

10%, the state $\{up\}$ increase from 90% to 0. Correspondingly, the state $\{up, down\}$ change from 0 to 90%. Some part of corresponding rate of occurrence is shown in Tab.1, Fig.9 and 10 can display the state of change more intuitively.

Table 1: Comparison about Rate of Occurrence

Rate of {up} in C_1	Traditional EN Approach			BRA Methodology	
	$\{up\}$	$\{up\},\{down\}$	$\{down\}$	$\{up\}$	$\{down\}$
1%	56.28%	35.30%	8.41%	12.81%	87.19%
10%	57.65%	33.91%	8.44%	12.79%	87.21%
20%	59.18%	32.36%	8.46%	12.77%	87.23%
30%	60.70%	30.81%	8.49%	12.75%	87.25%
40%	62.22%	29.26%	8.52%	12.73%	87.27%
50%	63.75%	27.71%	8.54%	12.71%	87.29%
60%	65.27%	26.16%	8.57%	12.70%	87.30%
70%	66.79%	24.61%	8.59%	12.68%	87.32%
80%	68.32%	23.06%	8.62%	12.67%	87.33%
90%	69.84%	21.51%	8.65%	12.65%	87.35%

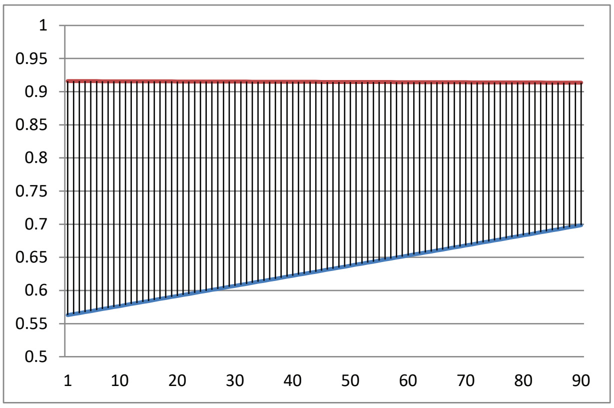

Figure 9: Result of EN Approach

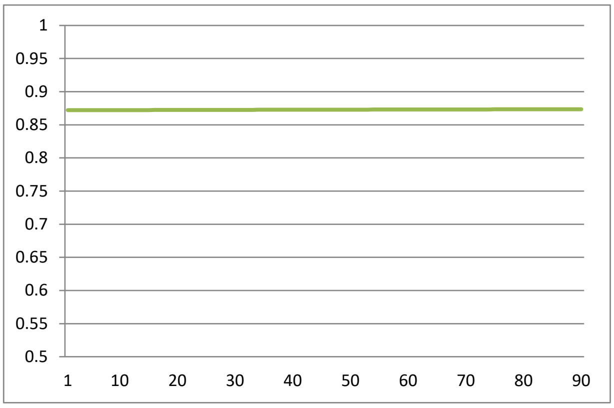

Figure 10: Result of BRA Methodology

This experiment also indicate the traditional EN approach sometimes show low-precision result, especially when we set the rate of $\{up\}$ to 1%, the confidence interval is [56.29%,91.59%] which will increase difficulty to follow-up operation. On the contrary, the result from BRA methodology is simpler and less uncertain. It means that the proposed BRA methodology provides a higher confidence in the reliability evaluation of the servo-actuation system than Yang *et al.*'s EN approach [43] in the final result.

6. Conclusion

Evidential network has a promising aspect in reliability analysis. However, existing methods still remain some issues. For example, the data fusion technology is not fully used. In this paper, a new methodology, called RBA, is proposed. The combination methods to deal with series system, parallel system, series-parallel system as well as parallel-series system are developed for reliability evaluation. The real application in servo-actuation system is illustrated to show the efficiency of the proposed Belief Reliability Analysis methodology.

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