A SYMMETRIC APPROACH ELUCIDATES MULTISENSORY INFORMATION INTEGRATION

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ABSTRACT

Recent advances in neuronal multisensory integration suggest that the five senses do not exist in isolation of each other. Perception, cognition and action are integrated at very early levels of central processing, in a densely coupled system equipped with multisensory interactions occurring at all temporal and spatial stages. In such a novel framework, a concept from the far-flung branch of topology, namely the Borsuk-Ulam theorem, comes into play. The theorem states that when two opposite points on a sphere are projected onto a circumference, they give rise to a single point containing their matching description. Here we show that the theorem applies also to multisensory integration: two environmental stimuli from different sensory modalities display similar features when mapped into cortical neurons. Topological tools not only shed new light on questions concerning the functional architecture of mind and the nature of mental states, but also provide a general methodology which as the advantage to be assessed empirically. We argue that the Borsuk-Ulam theorem is a general principle underlying nervous multisensory integration, resulting in a framework that has the potential to be operationalized.

KEY WORDS: Multimodal, heteromodal, integration, neuron, brain, Borsuk-Ulam theorem, antipodal.

"A color is a physical object when we consider its dependence upon its luminous source; regarding, however, its dependence upon the retina, it becomes a psychological object, a sensation. Not the subject, but the direction of our investigation, is different in the two domains" (Mach 1885).

Current advances in human neurosciences shed new light on questions concerning the status of the mental and its relation to the physical. Multisensory neurons (we will also term them "heteromodal" or "multimodal") receiving convergent inputs from multiple sensory modalities (independent sources of information called "cues") integrate information from the five different senses (Krueger et al. 2009). When cues are available, combining them facilitates the detection of salient events and reduces perceptual uncertainty, to improve reactions to immediate dangers and to rapidly varying events (Fetsch et al. 2010). Multisensory neurons are able as well to carry outcomplex brain activities, such as object categorization (Werner and Noppeney 2010). Heteromodal integration displays complicated temporal patterns: absent in the superior colliculus of newborn's brain, it arises in the earlier weeks/months of postnatal life (Stein and Rowland 2011, Royal et al. 2010). As time goes on, neurons develop their capacity to engage in multisensory integration, which determines whether stimuli are to be integrated or treated as independent events. Multisensory integration's development is due to early sensory experience, extensive experience with heteromodal cues and, above all, maturation of cooperative interactions between superior colliculus and cortex (Burnett et al. 2007, Stein et al. 2009). The ability to engage in multisensory integration specifically requires cortical influences (Stein 2014; Johnson et al 2003): without the help of cortical activity, neurons become responsive to multiple sensory modalities, but are unable to integrate different sensory inputs (Xu et al. 2012). To make the picture more intricate, multisensory experience is a plastic capacity depending on dynamical brain/environment interactions (Porcu 2014): neurons retain indeed sensitivity to heteromodal experience well past the normal developmental period, in order that the brain learns a multisensory principle based on the specifics of experience and is able to apply it to novel incoming stimuli (Yu et al. 2010).

In such a multifaceted framework, the Borsuk-Ulam theorem (BUT) from topology, comes into play. This theorem tells us that two opposite points on a sphere, when projected on a one-dimension lower circumference,

give rise to a single point displaying a matching description (Borsuk 1933). In this review, we will elucidate the underrated role of topology in multimodal integration, illustrating how simple concepts may be applied to brain processes of cues integration. This paper comprises four sections and an appendix. In the first sections, we will compare the traditional hierarchical view of multisensory integration with the very last developments: the latter talk about the nervous system as a sum of multisensory neurons that form extensive forward, backward and lateral connections at the very first steps of integration in the primary sensory areas. The second section provides a description of the Borsuk-Ulam theorem and its role in multisensory integration. Section three will go through the BUT's theoretical and experimental consequences in theory of knowledge and in the explanation of mental states' nature. The last section will elucidate what the BUT brings on the table, when applied to multimodal neurons. We also will provide a technical appendix as supporting information: this material will show how the BUT theory for brain can be operationalized, unpacking topological concepts afforded by a mathematical point of view.

CLASSICAL VS CURRENT VIEW

Traditional research on the basic science of sensation asks what types of information the brain receives from the external world. To elucidate the classical view, as an example we will go through the visual system, the best known and the most relevant among sensory systems in Primates. The retinal receptors are sensitive to simple signals related to the external world. The message is sent to the primary visual cortex V1, where specific aspects of vision such as form, motion or color are segregated in different parallel pathways. V1 projects to the associative areas termed "unimodal", since they are influenced by a sole sensory modality (in this case, the vision) (Nieuwenhuys et al 2008). The visual unimodal associative cortex (secondary visual cortex) is arranged in two streams, devoted to the control of action and objects perception. Processing channels, each serving simultaneously specialized functions, are also present in the central auditory and somatosensory systems. The message is then conveyed from unimodal to associative areas termed "heteromodal", since they are influenced by more than one sensory modality (visual, auditory, somatosensory, and so on) (Pollmann 2014). Ahigh-order heteromodalarea, the prefrontal cortex, collects the highly processed inputs conveyed by other associative areas. In this classical view, the sequential processing of information is hierarchical, such that the initial, low-level inputs are transformed into representations and multisensory integration emerges at multiple processing cortical stages (Werner and Noppeney 2010). The hierarchical view is also embraced by the most successful model of cognitive architecture, the connectome, where hubs/nodes are characterized by preferential railways of information flows (Van den Heuvel and Sporns 2011).

In the last decade, neuroscience has witnessed major advances, emphasizing the potentially vast underestimation of multisensory processing in many brain areas. New mechanisms, such as the "unimodal multisensory neurons" (Allman et al. 2009), have been demonstrated. In addition, multisensory interactions have been reported at system levels traditionally classified as strictly unimodal: both primary and secondary sensory areas receive substantial inputs from sources that carry information about events impacting other modalities (Hackett and Schroeder 2009; Shinder 2014; Reig 2014; Kuang 2014; Kim 2015). In particular, this can now be considered established in the case of the human V1, which is inherently multisensory (Murray 2015). In adult mice, vision loss through one eye leads whiskers to become a dominant nonvisual input source which attains extensive visual cortical reactivation (Nys 2015). In blind individuals, when visual inputs are absent, occipital ("visual") brain regions respond to sound and spoken language (Bedny 2015). There is a growing number of studies reporting the presence of connectivity between V1 and primary auditory cortex (as well as other higher-level visual and auditory cortices). Non-visual stimuli have been shown to enhance the excitability of low-level visual cortices within the occipital pole. Research into crossmodal perception has also linked senses other than vision, such as taste with audition (Yan 2015). Escanilla et al (2015) demonstrated odor-taste convergence in the nucleus of the solitary tract of the awake freely licking rat. A multisensory network for olfactory processing, via primary gustatory cortex connections to primary olfactory cortex, once again suggests that sensory processing may be more intrinsically integrative than previously thought (Maier 2015).

In sum, the current broad consensus is that the multimodal model is widely diffused in the brain and that most, if not all, higher, as well as lower level neural processes are in some form multisensory. Information from multiple senses is integrated already at very early levels of processing, leading to the concept of the whole neocortex as multisensory in function. Forward, backward and lateral extensive connections support communication in a densely coupled system, where multisensory interactions occur at all temporal and spatial stages (Klemen and Chambers 2012). The content of the single neuronal outputs turns out to be increasingly complex. The functional wheel has come full circle when the highest polymodal areas send feedback messages to the primary sensory cortices. From now on, a ceaseless pathway of feedbacks and feedforwards takes place among cortical areas. The functional importance of the so called "backward" connections is worth of mention. The effects of the projections from higher areas to multimodal neurons play an important role both in multisensory's development and in complex brain functions, providing meaningful adjustments to the ongoing activity of a

given area. It has been argued that backward connections might embody the causal structure of the world, while forward connections just provide feedback about prediction errors to higher areas (Bastos 2015). That is, anatomical forward connections are functional feedback connections. For backward connections to effectively modulate lower level processing, higher order areas would need to begin their processing at approximately the same time as lower areas. While this was originally thought not to be the case, we know now that the cross-modal interactions' processing latencies are highly similar to the latencies at which initial sensory processing occurs in the respective primary cortices (Klemen and Chambers 2012).

A TOPOLOGICAL MODEL OF MULTISENSORY INTEGRATION

The Borsuk-Ulam Theorem (BUT) is a remarkable finding by K. Borsuk (Borsuk 1933; Dodson 1997; Matoušek, 2003) about the topological concepts of Euclidean n-spheres and antipodal points. Put simply, BUT states that a sphere displays two antipodal points that emit matching signals. When they are projected on a circumference, they give rise to a single point which contains the features of both antipodal points (**Figure 1A**). To say that points on the sphere are *antipodal* means that they are diametrically opposite (Weisstein). Examples of antipodal points are the endpoints of a line segment, or the opposite points along the circumference of a circle, or the poles of a sphere (Moura 1996). Here "opposite points" means two points on the surface of a three-dimensional sphere (the surface of a beach ball is a good example) which share some characteristics in common and are at the same distance from the center of the beach ball (Marsaglia 1972). To make an example, on the earth's surface there always exist two opposite points with the same pressure and temperature. For a mathematical treatment of BUT, see Tozzi and Peters (2016a) and Tozzi and Peters (2016b). Two opposite points embedded in a sphere project onto a single point on a circumference, and vice versa: it means that the projection from a higher dimension (equipped with two antipodal points) to a lower one gives rise to a single point (equipped with the characteristics of both the antipodal points). It is worth to be mentioned again that the two antipodal points display similar features: we will go through this central issue in the very next paragraph.
What does BUT mean, in a physical/biological context? How could BUT be helpful in the evaluation of multisensory integration? The answer is positive. Here follows the key to solve the question: the two antipodal points can be used not just for the description of simple topological points, but also for the description of broad-spectrum phenomena: *i.e.*, either two antipodal shapes, or functions, or signals, or vectors. For example, a "point" may be described as a collections of signals or a surface shape, where every shape maps to another antipodal one. This expanded view of signals has interest, since every connected couple of antipodal points has a shape. This means that signal shapes can be compared. If you evaluate physical and biological phenomena instead of "signals", BUT leads naturally to the possibility of a region-based, not simply point-based, geometry, with many applications. In terms of activity, two antipodal points model the description of a signal. For a given signal on a sphere, we can expect to find an antipodal point that describes a signal with matching description. In sum, the mapping from the sphere to the circumference is defined by a rule which tells us how to find a single point (Willard 1970; Krantz 2009). The region-based view of the manifold arises naturally in terms of a comparison of shapes produced by different mappings from the sphere to the circumference (Peters and Tozzi, 2016). An interest in continuous mappings from object spaces to feature spaces leads into homotopy theory and the study of shapes: it means that antipodal points on a sphere have the same shape (Manetti 2015, Cohen 1973). It was Borsuk who first associated the geometric notion of antipodal shapes and mappings called homotopies (Borsuk 1958-59, Borsuk 1969, Borsuk 1980). This leads into the geometry of shapes and shapes of space (Collins 2000; Weeks 2002).
BUT can be used not just for the description of topological abstract spaces, but also for a general description of biological and physical systems. In our case, multisensory integration could easily be evaluated in terms of antipodal points. **Figure 1B** depicts an example of multisensory integration in the BUT framework. An observer stands in front of the surrounding environment. A guitar player is embedded in the environment. The observer perceives, through his different sense organs such as ears and eyes, the sounds and the movements produced by the guitar player. According to the BUT dictates, the guitar player stands for an object embedded in a three-dimensional sphere. The two different sensory modalities produced by the guitar player (sounds and movements) stand for the antipodal points on the sphere's circumference. Even if objects belonging to antipodal regions can either be different or similar, depending on the features of objects (Peters 2016), however the two antipodal points must share the same features. Both sounds and movements come indeed from the same object embedded in the sphere, i.e. the guitar player. The two antipodal points are not true points, but instead shapes, or functions, which can also be portrayed as vectors containing all the features of the stimulus: in the case, for example, of the visual inputs, the vector contains information about the colours, the shape, the movements of the guitar player. The two antipodal points at first project to a two-dimensional layer, the brain cortex - where multisensory neurons lie -, then are integrated into a single multimodal signal, which takes into account the features of both. Cerebral hemispheres can be indeed unfolded and flattened into a two-dimensional

reconstruction by computerized procedures (Van Essen 2005). We may think to a single vector, embedded in the two-dimensional brain, which contains the features of all the information from the two inputs.

A variant of the BUT theorem, called RE-BUT (Peters and Tozzi, 2016), states that the two points (or regions, or vectors or whatsoever) with similar description do not need necessarily to be antipodal, in order to be described together (**Figure 2A**). It means that different inputs from different modalities do not need to be perfectly antipodal on the n-sphere (the environment). Sets of signals not only will have similar descriptions, but also dynamic character; moveover, the deformation of one signal shape into another occurs when they are descriptively near (Peters 2014). It elucidates another important feature of multisensory integration. Although multisensory interactions' simulations at the neuronal level clearly show that the simple effect of heteromodal convergence is sufficient to generate multimodal properties (Lim 2011), nonetheless the multisensory integration' occurrence almost never could be predicted on the basis of a simple linear addition of their unisensory response profiles: the interactions may indeed be superadditive, subadditive, or display inverse effectiveness and temporal changes (Krueger 2009, Royal 2009; Perrault et al. 2011; Benson 2014). In the BUT's topological framework, this apparently bizarre behaviour is explained by the dynamical changes occurring at the antipodal points (Peters 2016). There is indeed the possibility that two inputs from different cues are not perfectly integrated. It means that the mapping of the two cues on the S^{n-1} sphere does not match perfectly, and the sensation is doubtful (**Figure 2B**). A superimposition of two spheres with slightly different center occurs. The brain thus might "move" its spheres, in order to restore the coherence between the two signals. It means, by an operational point of view, that we might be able to evaluate the distance between the central points of the two spheres, in order to achieve the value of discrepancy between the two multisensory signals projected from the environment to the brain.
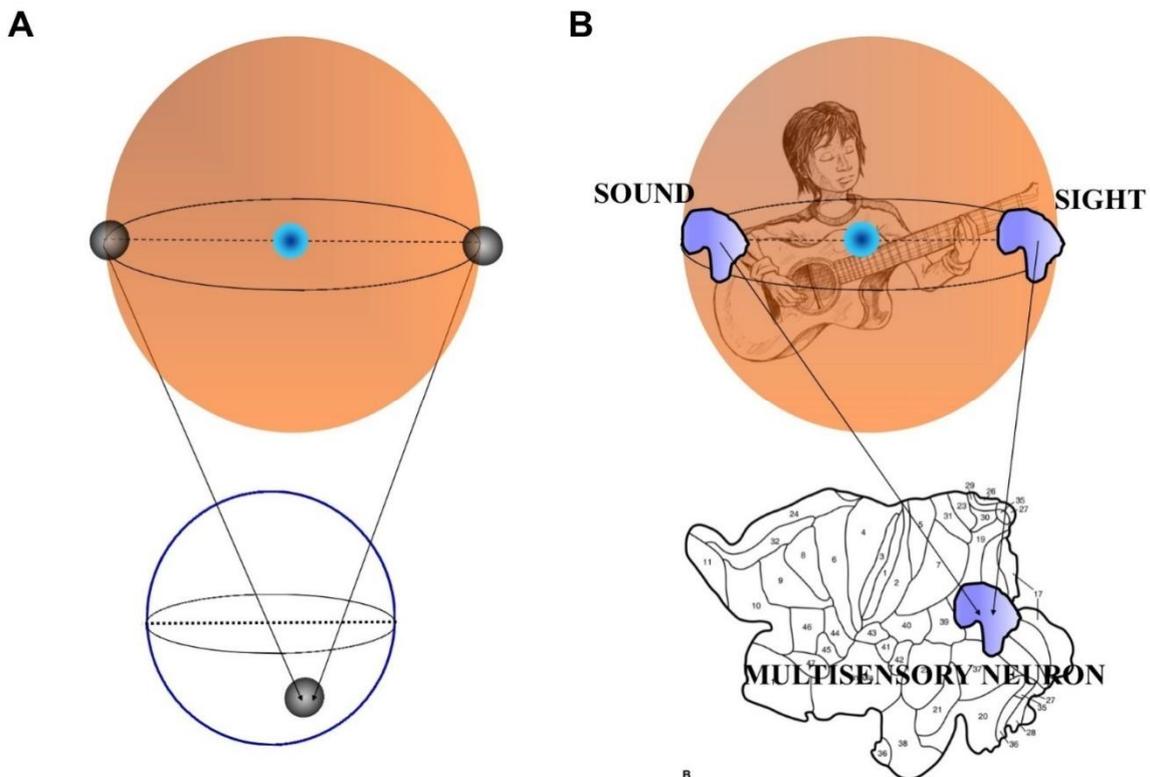

Figure 1A. A simplified sketch of the Borsuk-Ulam theorem: two antipodal points (black spheres) on a three-dimensional sphere project to a single point on a two-dimensional circumference.
Figure 1B shows what happens when an observer is in front of a guitar player. The environmental inputs from different sensory modalities (in this case sound and sight are depicted, in guise of shapes instead of points) converge on a single group of multisensory neurons in the cortical layers of brain's observer. Note that the brain is depicted as a two-dimensional layer. According to the dictates of the Borsuk-Ulam theorem, the single shape contains a "melted" messages from the two modalities. Modified from Tozzi and Peters (2016b).
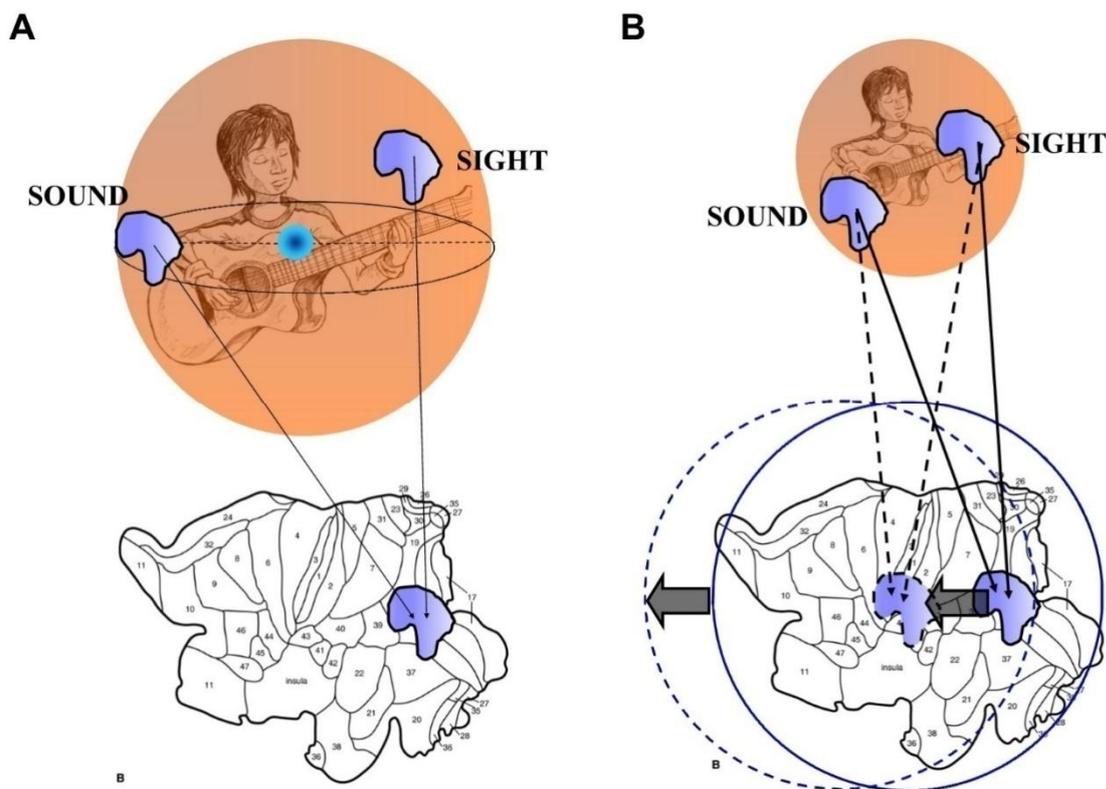
Figure 2A. A simplified sketch of the RE-BUT. The functions with matching description do not need inevitably to be antipodal. Thus, the two inputs from different sensory modalities, if equipped with a signal matching, may come from every point of the environment.

Figure 2B. When the two inputs from different modalities do not perfectly match on the 2-D brain surface, it occurs a discrepancy between two brain spheres which can be quantified, by evaluating the distance between their two centers. Modified from Tozzi and Peters (2016b).

AN ANSWER FOR CURRENT ISSUES

In this section we will go through the issues of the nature of mental states and theory of knowledge, through the lenses of topology. The BUT model takes into account the content-bearing properties of physical neural networks,by reducing representational states to construct of observables and by providing a mechanistic account of the neural processes which underlie the mental processes of knowledge. BUT integration works not just at the low-level processes - such as perception, motor control and simple mnemonic processing - (Sutter 2014), but also at the high-levels - such as planning and reasoning - (Drugowitsch 2014). In the above-described recent model of multisensory integration, sensory modalities outline a broadly interconnected network with multiple forward, backward and lateral connections, rather than a hyerarchical system of parallel sensory streams that converge at higher levels. The key difference between the traditional and the novel view is whether the hierarchy comprises distinct and parallel streams that converge at higher levels, or whether there are dense interactions between all sensory streams, even at lower levels (Klemen and Chambers 2012). The BUT approach supports a model that assume rich cross-modal interconnectedness at all processing levels, suggesting that functional interplay between the senses occurs at all anatomical levels and across a varied time course. To make an example, the connections between the two secondary cortex's visual pathways (the "where" and the "what" systems) providesevidence for a common and simultaneous neural substrate for action and perception.

The BUT allows us to establish the "metaphor" in a general cognitive theory. If a metaphor is a linguistic expression in which at least one part of the expression is transferred from one domain of application (source domain), where it is common, to another (target domain) in which it is unusual (Bailer-Jones 2002), there are striking formal analogies with the topological concept of antipodal points. We hypothesize that the metaphor is produced by the convergence on a multisensory neurons' assembly of both literal and figurative messages (corresponding to the antipodal points located in the environment). The heteromodal integration, aiming to gain insight through the metaphor that no literal paraphrase could ever capture, links together in a linguistic expression the domain of intuitive common-sense understanding of the "literal", and the domain of application

of the "figurative". The peculiar nature of multisensory integration leads to a radical rethinking of mental representations. It takes into account the "cognitive" needs of theoretical representational states and provides a neuroanatomical frame to cognitive semantics. Can a single neuronal output contain more information than each of its countless inputs? The answer to this crucial question implies cognitive theoretical conclusions, concerned both with information and knowledge.

Multimodal neurons utilize the salient, meaningful information from different sensations and "melt" them into a novel message, which holds a "semantic" content. Consider the classical "leg of the table". The leg "appears to us at first a single, indivisible whole. One name designates the whole". But "the apparently indivisible thing is separated into parts" (Mach 1885): the world "leg" is not strictly a literal source domain, because it already contains the "merging" and integration of several features, such as the color, the shape, the temperature, the evoked feelings. In the framework of BUT, mental processes are not just cortical networks' computations operating on the "syntax" of representation (i.e., boolean logic), but they also throw a bridge between the latter and the "semantic" notion of mental content. The countless inputs become thus able to solve not-computable functions and to "fabricate" a cognitive process. BUT mechanisms underlying multisensory integration let us hypothesize that our "naïve" thoughts are semantic (van Ackeren 2014), rather than syntactic, and that, during mental processes, the metaphor goes before the axiomatization and the formal syntax rulesof thefirst-order predicate logic. Scientific data confirm heteromodal integration as "semantic" and show which cortical networks are most likely to mediate these effects, with the temporal area being more responsive to semantically congruent, and the frontal one to semantically incongruent audio-visual stimulation (Doehrmann and Naumer 2008). It has been suggested that multisensory facilitation is associated with posterior parietal activity as early as 100 ms after stimulus onset; as participants are required to classify heteromodal stimuli into semantic categories, multimodal processes extend in cingulate, temporal and prefrontal cortices 400 ms after stimulus onset (Diaconescu et al. 2011).

The BUT model displays astonishing connections with the Gärdenfors' (2011) "cognitive semantics", which states that meanings are in the head, but are not independent of perception and bodily experience. Topology provides a conceptual bridge between "realistic" semantics (the meaning of an expression is something out there in the world) and "cognitive" semantics (meanings are expressions of mental entities independent of truth). Gardenfors states that semantic elements are based on spatial or topological rules (Gärdenfors 2000, Kuhn 2003), not on symbols: in touch also with BUT multisensory integration, cognitive models are primarily image-schemas transformed by metaphoric and metonymic operations (Gardenfors2011). Experimental data provide evidence for a complex interdependency between spatial location and temporal structure in determining the ultimate behavioral and perceptual outcome associated with a paired multisensory stimulus (Stevenson 2012).This experience of time and two- or three-dimensional spatial configurations, together with the expressions pertaining to higher level of conceptual organization, can be easily explained by a topological mapping of the components onto the multisensory. The blending, a standpoint of cognitive semantics (Langacker 2008), refers to a "mental" space configuration in which elements of two input spaces are projected into a third space, the blend, which thus contains elements of both (Fauconnier 1997). If we figure this space not as "mental" but as "biological", the blend turns out to be the BUT. Rather than just combining predicates, the semantic model/BUT structure blends behaviors from multiple (mental or anatomical) spaces and explains "apparently" emergent phenomena (Kim 1999).

Inferring which signals have a hypothetical common underlying cause - and hence should be integrated -is a primary challenge for a perceptual system dealing with multiple sensory inputs. The brain is able to efficiently infer the causes underlying our sensory events. Experiments demonstrate that we use the similarity in the temporal structure of multisensory cue combinations to infer from correlation the causal structure as well as the location of causes (Parise and Spence 2012). This capacity is not limited to the high-level cognition; it is performed recurrently and effortlessly in perception too (Körding 2007).In case of abnormal multisensory experience (visual-auditory spatial disparity), an atypical multisensory integration takes place (Stein 2009). We hypothesize that inferences form a sort of "niche construction", which shapes the common sense of a neural machinery operating in embodied/embedded interaction with the environment (Friston 2010). Our brain is also able of abstraction, i.e. to create its own representational endowment or potential, without recourse to external objects or states of affairs, such as causal antecedents or evolutionary history (Grush 2002). Truth conditions are not crucial for such a kind of representational content. How could multisensory neurons' outputs abstract both causal and not causal events, in order to introduce new multifaceted elements into the thinking process? Perhaps heteromodal neurons are able to re-evoke complex data also in absence of the original external stimulus, by comparing and integrating the countless novel semantic messages included in the single bi-dimensional point predicted by BUT, in order that the brain is able to "look without seeing, listen without hearing" (Leonardo da Vinci) (McLean 2006). The role of the highest cortical areas could be crucial in activating cortical multisensory neurons of lower levels, also in absence of external stimuli. Data from literature suggest this possibility. Two important temporal epochs have been described in the visual, auditory and multimodal response to stimuli: an early phase during which there is weak or no unisensory responses yet a defined multisensory response, and a late period after which the unisensory responses have ended and the multisensory response remains (Royal 2010). In sum, in view of the BUT principles, the higher brain activities may be explained by a sole factor: the

presence of a three-dimensional structure equipped with antipodal points. It astonishingly resembles Godel's suggestion of abstract terms more and more converging to the infinity in the sphere of our understanding.

WHAT DOES TOPOLOGY BRINGTO THE TABLE?

In conclusion, we provided a general topological mechanism which explains the elusive phenomenon of multisensory integration. The model is casted in a physical/biological fashion which has the potential of being operationalized and experimentally tested. A shift in conceptualizations is evident in a theory of knowledge based on BUT multisensory integration: it is no longer the default position to assume that perception, cognition and motor control are unisensory processes (Méndez 2014). In agreement with Ernst Mach (1885) objects are "combinations or complex of sensations". Our senses do not exist in isolation of each other and, in order to fully answer to epistemological queries, the processes in question must be studied in a multimodal context (Klemen and Chambers 2012). The question here is: what for? What does a topologic reformulation add in the evaluation of multimodal integration? The invaluable opportunity to treat elusive mental phenomena as topological structures takes us into the realm of algebraic topology, allowing us to describe brain function in the language of powerful analytical tools, such as combinatorics, hereditary set systems (Matoušek), homology theory and functional analysis. BUT and its extensions provide a methodological approach which makes it possible for us to study multisensory integration in terms of projections from real to abstract phase spaces. The importance of projections between environmental spaces, where objects lie, and brain phase spaces, where multimodal operations take place, is also suggested by Sengupta et al. (2016), who provide a way of measuring distance on concave neural manifolds. Such a methodological approach has been proved useful in the evaluation of brain symmetries and to assess the relationships and affinities among BOLD activated areas (Tozzi and Peters, 2016b) and in the evaluation of cortical histological images (Peters et al., 2016). BUT and its variants are not just a *methodological* approach, but also display a *physical* meaning. Based on the antipodal cortical zones with co-occuring BOLD activation, it has been recently suggested that brain trajectories might display donut-like trajectories (Tozzi and Peters 2016a).

The different animals' senses capture only bits and selected aspects of reality and display the world from limited points of view (Holldobler and Wilson 2002). "The development of the human mind has practically extinguished all feelings, except a few sporadic kinds, sound, colours, smells, warmth, etc." (Peirce 1892). However, a realistic fraction, albeit incomplete, reaches our brain from outside and gives rise, thanks to the BUT, to an isomorphism between sensations and perceptions. As stated above, the BUT perspective allows a symmetry property located in the real space (the environment) to be translated to an abstract space, and *vice-versa,* enabling us to achieve a map from one dynamical system to another. "A truth is like a map, which does not copy the ground, but uses signs to tell us where to find the hill, the stream and the village" (Goodman 2005). If symmetry transformations (antipodal points) can be evaluated, we are allowed to use the pure mathematical toolsof topological group theory. Symmetry transformations therefore furnish us with a topologic family of models able to explain the data. In the study of multisensory integration, promising empiric advances are forthcoming: cutting edge methods have been recently proposed, from oscillatory phase coherence, to multi-voxel pattern analysis as applied to multisensory research at different system levels,to the TMS-adaptation paradigm (for a review, see Klemen and Chambers 2012). The last but not the least, two computational frameworks that account for these multisensory integration have been recently established (Magosso et al. 2008, Ohshiro et al. 2012). In the framework of BUT theorem, such advances might provide a considerable experimental support and a unifying computational account of multisensory neurons' features.

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