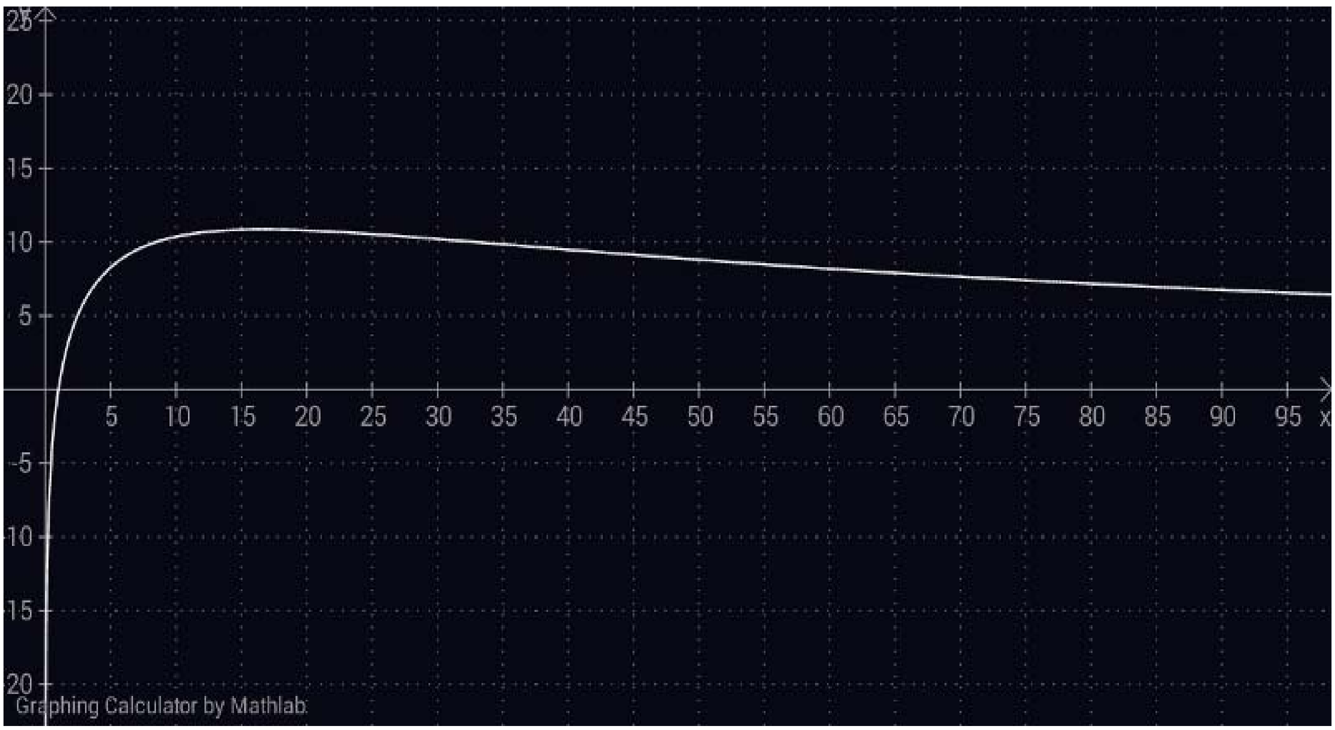
Nuclear Binding Energy Curve

Abstract: An equation for nuclear binding energy per nucleon is found that agree well with observations. The interpretation indicates that we need to explore physical variables involved in the nuclear force or nuclear binding energy.

The equations for nuclear binding energy curve are of the form

f(x)=180logx/(x+30)

The graph of f(x) versus x is shown below.



This smooth curve is similar to the nuclear binding energy curve except for atomic numbers in the range from Z=1 to Z=13. In the nuclear binding energy curve it is not smooth in the range Z=1 to Z=13.

It is assumed that nuclear binding energy per nucleon is distributed equally among the nucleons. We can express the equation for nuclear binding energy curve from atomic number Z=14 as

E=klogx/(ax+c)

x is the mass number. Here a and c are dimensionless constants and k is a physical constant and has dimensions of energy..

For mass number x=1, then E=0. As mass number x→∞ then nuclear binding energy per nucleon E→0

We can express the equation as E=(k/a)logx/(x+c/a)

For atomic number Z=14, the element is 14Si28

The iron has high nuclear binding energy and stability is also high. To find the constants k/a and c/a, we consider the element iron 26Fe56 and the last stable element lead 82Pb208.

The nuclear binding energy per nucleon for iron of mass number x1=56 is E1=8.790323 MeV and for lead of mass number x2=208 is E2=7.86745 MeV.

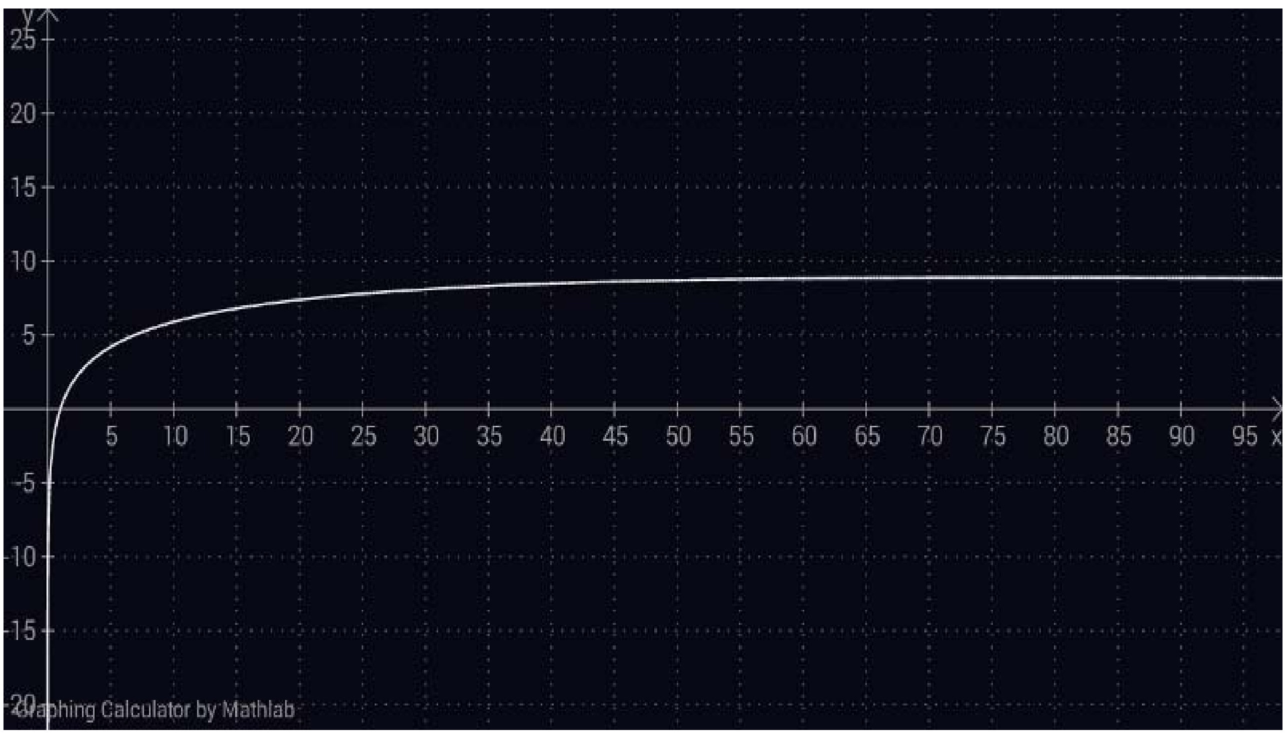
E1/E2 yields c/a=259.665982

Now E1E2 yields k/a=689.332549 MeV

Therefore the equation for nuclear binding energy curve is

E=(689.332549)logx/(x+259.665982)

The graph of binding energy per nucleon versus mass number x is shown below.



This equation fits well in the atomic number ranges from Z=14 ie 14Si28.

It is found by calculations that the variation of binding energy per nucleon obtained by the empirical formula with the actual binding energy per nucleon is between -4% to 3% Here -4% means the binding energy per nucleon obtained by the empirical formula is 4% less than the actual binding energy per nucleon and 3% means the binding energy per nucleon obtained by the empirical formula is 3% more than the actual binding energy per nucleon.

The range of atomic numbers from Z=1 to Z=13 the empirical formula fits if and only if we consider the multiplication factor that varies between 1.18 to 1.9

The above interpretation indicates that the discrepancy between the empirical formula and the actual nuclear binding energy curve.Therefore we need to explore the physical variables that are involved in the nuclear force or nuclear binding energy.

Second version:

It is found that the nuclear binding energy per nucleon equation is given by

$E=k\ e^{(g/x)}\ \log x/(ax+c)$

Where a and c are dimensionless constants (or maybe physical constants) and k is a physical constant. x is a mass number.

Here $g=df$

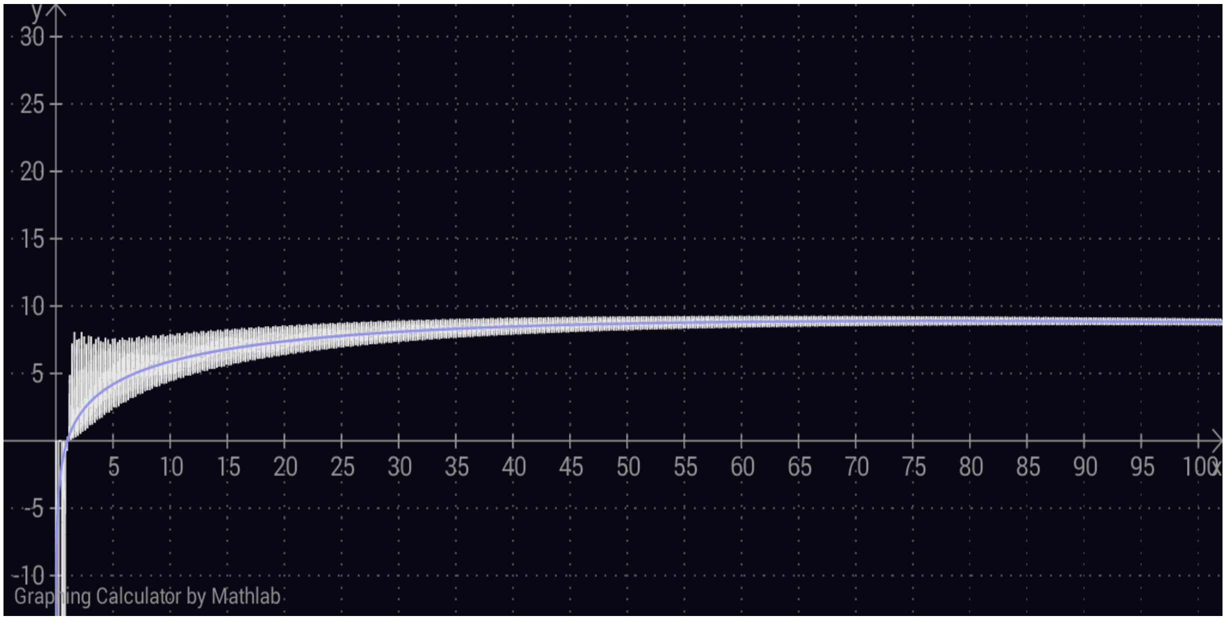
Where d is a constant and f is an independent physical quantity. We don't know the nature of the physical quantity f, because we don't know clearly the exact nature of interaction of nucleons.

We can express a mathematical (not physical) formula for the nuclear binding energy curve for all elements with different isotopes that fits well with observed nuclear binding energy curve for all elements by choosing values such that both curves should be similar.

We choose $g=-3\sin 30x$, $k/a=690$,. $c/a=260$

$E= 690\ e^{(-3\sin 30x/x)}\ \log x/(x+260)$

The graph of E versus x is shown below.



This mathematical curve is similar to the observed nuclear binding energy curve for all elements with different isotopes. The colour curve in the graph indicates that it is a curve of the approximate nuclear binding energy per nucleon equation of the second graph.

This mathematical curve indicates that the binding energy per nucleon in the range from $x=1$ to $x=40$, there are isotopes with different values of binding energy per nucleon and it appears like a thick band. For mass numbers $x>40$ there are different isotopes, but they have binding energy per nucleon almost equal. Therefore it appears like a very thin band.

From the mathematical curve we can infer that the function $g=df$, here the physical quantity f varies between 0 and 1. Because there are similarities in nature in both the curves.

The mathematical function $g=-3\sin30x$, here the sine function can have any value between 0 and 1. There are infinite numbers between 0 and 1. From the mathematical curve we find that, for $x<40$, the sine function makes the curve thick band, and for $x>40$ the sine function has very less effect, therefore the curve has thin band. This means that for $x<40$, the values of f has significant effect, therefore the nuclear binding energy curve has thick band and for $x>40$ the values of f has very less effect, therefore it has thin band. The reason is that the ratio g/x decreases as mass number x increases.

Therefore we conclude that the nuclear binding energy per nucleon depends on mass number x and the independent physical quantity f. We need to explore the nature of the physical quantity f. Also we need to study the variation of f with mass number x.

References:

1) Wikipedia...