Laws of orbital semimajor axes and periods, and their application in 5-planet systems

1. Introduction

Though exoplanets in 7-planet and 6-planet systems do not follow the Bode's law because they are much closer to their parent star than ours to the Sun, they do follow another way of the Bode's law, that is, the law of orbital semimajor axes: $a_n/a_{n-1}=r$. In our solar sysem, there is r=1.73±0.35 and it works very well for the planets as far as Neptune. In HD 10180' s system with 6 planets, there is r=2.3606±0.5345(Zhang 2016). And because of $a_n^3/P_n^2=(GM_*)/(2\pi)^2$, if planets follow the law of orbital semimajor axes, they follow the law of orbital periods too: $P_n/P_{n-1}=r^{3/2}$. Exoplanets in Kepler-20 and Kepler-102's 5-planet systems also follow laws of orbital semimajor axes and periods and their unknown orbital semimajor axes can be caculated using $a_n=a_m(P_n/P_m)^{2/3}$. When the semimajor axis of planet m is known, the unknown or incorrect semimajor axis of planet n can be calculated or checked, such as those of Kepler-20 e and f in Table 1(Gautier et al. 2011;Fressin et al. 2011). When all semimajor axes are unknown, a pseudo-planet orbiting its parent star at its surface with Keplerian velocity can be assumed with $a_0=R_*$, $v_{*\text{kep}0}=(GM_*/R_*)^{1/2}$, and $P_0=2\pi R_*/v_{*\text{kep}0}$, and semimajor axes of all planets in the system can be calculated using $a_n=a_0(P_n/P_0)^{2/3}$, such as those of Kepler-102's 5 planets in Table 2(Marcy et al. 2014).

Table 1 Data of Kepler-20's system with 5 planets

Kepler-20's	n=0	n=1,b	n=2,e	n=3,c	n=4,f	n=5,d
a_n[AU]	0.004389	0.04537	*0.06334*	0.0930	*0.1378*	0.3453
a_n/a_{n-1}		10.34	*1.396*	*1.468*	*1.482*	*2.506*
P_n[days]	0.112	3.6961219	6.098493	10.854092	19.57706	77.61184
P_n/P_{n-1}		33.0	1.650	1.780	1.804	3.964

Table 2 Data of Kepler-102's system with 5 planets

Kepler-102's	n=0	n=1,b	n=2,c	n=3,d	n=4,e	n=5,f
a_n[AU]	0.0035	*0.055*	*0.067*	*0.086*	*0.12*	*0.16*
a_n/a_{n-1}		*16*	*1.2*	*1.3*	*1.4*	*1.3*
P_n[days]	0.086	5.28696	7.07142	10.3117	16.1457	27.4536
P_n/P_{n-1}		60	1.338	1.458	1.566	1.700

There are 20 systems with 5 planets. Are the laws of orbital semimajor axes and periods suitable and appliable in the other 18 systems? Let us take a close look.

2. Laws of orbital semimajor axes and periods in 5-planet systems

First, all planets in Kepler-292, Kepler-296, Kepler-33, Kepler-444, Kepler-80 and HD 40307's system follow the laws of orbital semimajor axes and periods without any exception, as they are shown in Table 3(Rowe et al. 2014), Table 4(Barclay et al. 2015), Table 5(Lissauer et al. 2012), Table 6(Campante et al. 2015), Table 7(MacDonald et al. 2016), and Table 8(Tuomi et al. 2013).

Table 3 Data of Kepler-292's system with 5 planets

Kepler-292's	n=0	n=1,b	n=2,c	n=3,d	n=4,e	n=5,f
a_n[AU]	0.00384	0.035	0.045	0.068	0.097	0.141
a_n/a_{n-1}		9.1	1.3	1.5	1.4	1.5
P_n[days]	0.09293	2.580827	3.715335	7.055679	11.979010	20.834237
P_n/P_{n-1}		27.77	1.440	1.899	1.700	1.739

Table 4 Data of Kepler-296's system with 5 planets

Kepler-296's	n=0	n=1,c	n=2,b	n=3,d	n=4,e	n=5,f
a_n[AU]	0.00223	0.0521	0.079	0.118	0.169	0.255
a_n/a_{n-1}		23.4	1.5	1.5	1.43	1.51
P_n[days]	0.05469	5.8416366	10.864384	19.850291	34.14211	63.33627
P_n/P_{n-1}		106.8	1.860	1.827	1.720	1.855

Table 5 Data of Kepler-33's system with 5 planets

Kepler-33's	n=0	n=1,c	n=2,b	n=3,d	n=4,e	n=5,f
a_n[AU]	0.00846	0.0677	0.1189	0.1662	0.2138	0.2535
a_n/a_{n-1}		8.00	1.76	1.40	1.29	1.19
P_n[days]	0.2509	5.66793	13.17562	21.77596	31.78440	41.02902
P_n/P_{n-1}		22.59	2.325	1.653	1.460	1.291

Table 6 Data of Kepler-444's system with 5 planets

Kepler-444's	n=0	n=1,b	n=2,c	n=3,d	n=4,e	n=5,f
a_n[AU]	0.00350	0.04178	0.04881	0.0600	0.0696	0.0811
a_n/a_{n-1}		11.9	1.168	1.23	1.16	1.17
P_n[days]	0.08696	3.6001053	4.5458841	6.189392	7.743493	9.740486
P_n/P_{n-1}		38.74	1.263	1.362	1.251	1.258

Table 7 Data of Kepler-80's system with 5 planets

Kepler-80's	n=0	n=1,f	n=2,d	n=3,e	n=4,b	n=5,c
a_n[AU]	0.00315	0.0175	0.0372	0.0491	0.0648	0.0792
a_n/a_{n-1}		5.56	2.13	1.32	1.32	1.22
P_n[days]	0.07583	0.98678730	3.07222	4.64489	7.05246	9.52355
P_n/P_{n-1}		13.01	3.113	1.512	1.518	1.350

Table 8 Data of HD 40307's system with 5 planets

HD 40307's	n=0	n=1,b	n=2,c	n=3,d	n=4,f	n=5,g
a_n[AU]	unknown	0.0468	0.0799	0.1321	0.247	0.600
a_n/a_{n-1}			1.71	1.65	1.87	2.43
P_n[days]		4.3123	9.6184	20.432	51.76	197.8
P_n/P_{n-1}			2.230	2.124	2.533	3.821

Second, planets in Kepler-169 and Kepler-62's system follow the laws of orbital semimajor axes and periods with one exception respectively, as they are shown in Table 9(Rowe et al. 2014) and Table 10(Borucki et al. 2013). Is there a planet or an asteroid belt between Kepler-169 e and f, and between Kepler-62 d and e? Further astronomical observations will tell us the answer.

Table 9 Data of Kepler-169's system with 5 planets

Kepler-169's	n=0	n=1,b	n=2,c	n=3,d	n=4,e	n=5,f
a_n[AU]	0.00355	0.040	0.062	0.075	0.105	0.359
a_n/a_{n-1}		11	1.6	1.2	1.4	3.42
P_n[days]	0.0834	3.250619	6.195469	8.348125	13.767102	87.090195
P_n/P_{n-1}		38.98	1.906	1.347	1.649	6.326

Table 10 Data of Kepler-62's system with 5 planets

Kepler-62's	n=0	n=1,b	n=2,c	n=3,d	n=4,e	n=5,f
a_n[AU]	0.00300	0.0553	0.0929	0.120	0.427	0.718
a_n/a_{n-1}		18.4	1.68	1.29	3.56	1.68
P_n[days]	0.07158	5.714932	12.4417	18.16406	122.3874	267.291
P_n/P_{n-1}		79.84	2.177	1.460	6.738	2.184

Third, planets in GJ667 C's system follow the laws of orbital semimajor axes and periods with two exceptions, as they are shown in Table 11(Anglada et al. 2013). Is there a planet or an asteroid belt between GJ667 C b and c, and e and f? Further astronomical observations will tell us the answer.

Table 11 Data of GJ667 C's system with 5 planets

GJ667 C's	n=0	n=1,b	n=2,c	n=3,f	n=4,e	n=5,f
a_n[AU]	unknown	0.0505	0.125	0.156	0.213	0.549
a_n/a_{n-1}			2.48	1.25	1.37	2.58
P_n[days]		7.2004	28.140	39.026	62.24	256.2
P_n/P_{n-1}			3.908	1.387	1.595	4.116

Fourth, planets in Kepler-186's system follow the laws of orbital semimajor axes and periods with one exception, as it is shown in Table 12(Quintana et al. 2014;Torres et al. 2015). Is there a planet or an asteroid belt between Kepler-186 e and f? Further astronomical observations will tell us the answer. Another question of the stellar mass and radius. If there are M_*=0.544$M_\odot$ and R_*=0.523$R_\odot$ as they are in Torres et al. 2015, we have a_0=0.00243 AU and P_0=0.05953 days, but then we have $(a_1/a_0)^3/(P_1/P_0)^2$=2803/4262=0.66, quite different from the expected 1. Is the stellar mass wrong? Or are the orbital semimajor axes wrong? There are severl other problems about Kepler-186's system, as they are shown in Table 12 with red words.

Table 12 Data of Kepler-186's system with 5 planets

Kepler-186's	n=0	n=1,b	n=2,c	n=3,d	n=4,e	n=5,f	or n=5,f
a_n[AU]	0.00243	0.0343	0.0520	0.0781	0.110	0.356 (Quintana et al.)	0.432 (Torres et al.)
a_n/a_{n-1}		14.1	1.52	1.73	1.41	3.24	3.93

$(a_n/a_{n-1})^3$		2803	3.51	5.18	2.8	33.9	60.7
P_n[days]	0.05953	3.8867907	7.267302	13.342996	22.407704	129.9441	129.9441
P_n/P_{n-1}		65.29	1.870	1.836	1.679	5.799	5.799
$(P_n/P_{n-1})^2$		4262	3.50	3.37	2.8	33.6	33.6
$(a_n/a_{n-1})^3/(P_n/P_{n-1})^2$		0.66	1.00	1.54	1.00	1.01	1.81

Fifth, planets in Kepler-32's system follow the laws of orbital semimajor axes and periods with no exception, as it is shown in Table 13(Fabrycky et al. 2012;Swift et al. 2013). There was a tyco of a_4=0.09(Fabrycky et al. 2012), as it should be a_4=0.069. Later it was updated to be a_4=0.067(Swift et al. 2013).

Table 13 Data of Kepler-32's system with 5 planets

Kepler-32's	n=0	n=1,f	n=2,e	n=3,b	n=4,c	n=5,d
a_n[AU]	0.00246	0.0130	0.0323	0.0519	0.067	0.128
a_n/a_{n-1}		5.28	2.49	1.61	1.29	1.91
$(a_n/a_{n-1})^3$		147.2	15.4	4.17	2.15	6.97
P_n[days]	0.06096	0.74296	2.8960	5.9012	8.7522	22.7802
P_n/P_{n-1}		12.17	3.8979	2.0377	1.4831	2.6028
$(P_n/P_{n-1})^2$		148.5	15.194	4.1523	2.1996	6.7746

3. Application of the laws of orbital semimajor axes and periods in 5-planet systems

Some data of orbital semimajor axes are missing at exoplanetarchive.ipac.caltech.edu and the related literatures. They can be calculated using $a_n=a_m(P_n/P_m)^{2/3}$.

First, the orbital semimajor axis of Kepler-122 f is missing. It can be calculated from a_4, as it is shown in Table 14(Rowe et al. 2014). It can be seen that planets in Kepler-122's system follow the laws of orbital semimajor axes and periods without any exception.

Table 14 Data of Kepler-122's system with 5 planets

Kepler-122's	n=0	n=1,b	n=2,c	n=3,d	n=4,e	n=5,f
a_n[AU]	0.00565	0.064	0.108	0.155	0.227	*0.295*
a_n/a_{n-1}		11.3	1.69	1.44	1.46	*1.30*
P_n[days]	0.1565	5.766193	12.465988	21.587475	37.993273	56.268
P_n/P_{n-1}		36.84	2.162	1.732	1.760	1.481

Second, the orbital semimajor axes of Kepler-154 e, f, and d are missing. They can be calculated from a_5, as it is shown in Table 15(Rowe et al. 2014;Morton et al. 2016). It can be seen that planets in Kepler-154's system follow the laws of orbital semimajor axes and periods without any exception.

Table 15 Data of Kepler-154's system with 5 planets

Kepler-154's	n=0	n=1,e	n=2,f	n=3,d	n=4,b	n=5,c
a_n[AU]	0.00465	*0.0480*	*0.0890*	*0.145*	0.198	0.303
a_n/a_{n-1}		*10.3*	*1.85*	*1.63*	1.37	1.53
P_n[days]	0.1233	3.93276465	9.91935684	20.54981883	33.040532	62.303276
P_n/P_{n-1}		31.90	2.522	2.072	1.608	1.886

Third, the orbital semimajor axes of Kepler-238 e, and f are missing. They can be calculated from a_3, as it is shown in Table 16(Rowe et al. 2014;Xie 2014). It can be seen that planets in Kepler-238's system follow the laws of orbital semimajor axes and periods without any exception.

Table 16 Data of Kepler-238's system with 5 planets

Kepler-238's	n=0	n=1,b	n=2,c	n=3,d	n=4,e	n=5,f
a_n[AU]	0.00665	0.034	0.069	0.115	*0.169*	*0.281*
a_n/a_{n-1}		5.1	2.0	1.7	*1.47*	*1.66*
P_n[days]	0.1928	2.090876	6.155557	13.233549	23.654	77.61184
P_n/P_{n-1}		10.84	2.944	2.150	1.787	2.133

Fourth, the orbital semimajor axes of Kepler-55 b, and c are missing. They can be calculated from a_3, as it is shown in Table 17(Rowe et al. 2014;Steffen et al. 2013). It can be seen that planets in Kepler-55's system follow the laws of orbital semimajor axes and periods without any exception.

Table 17 Data of Kepler-55's system with 5 planets

Kepler-55's	n=0	n=1,d	n=2,e	n=3,f	n=4,b	n=5,c
a_n[AU]	0.00288	0.029	0.048	0.081	*0.159*	*0.209*
$a_n/a_{n\text{-}1}$		10.1	1.7	1.7	*1.96*	*1.31*
P_n[days]	0.07179	2.211099	4.617534	10.198545	27.9481449	42.1516418
$P_n/P_{n\text{-}1}$		30.80	2.088	2.209	2.740	1.508

Fifth, the orbital semimajor axes of Kepler-84 b, and c are missing. They can be calculated from a_5, as it is shown in Table 18(Rowe et al. 2014;Xie 2013). It can be seen that planets in Kepler-84's system follow the laws of orbital semimajor axes and periods without any exception.

Table 18 Data of Kepler-84's system with 5 planets

Kepler-84's	n=0	n=1,d	n=2,b	n=3,c	n=4,e	n=5,f
a_n[AU]	0.00543	0.052	*0.093*	*0.109*	0.181	0.250
$a_n/a_{n\text{-}1}$		9.6	*1.79*	*1.17*	*1.66*	1.38
P_n[days]	0.1467	4.224537	8.726	12.883	27.434389	44.552169
$P_n/P_{n\text{-}1}$		28.80	2.066	1.476	2.130	1.624

Sixth, the orbital semimajor axes of all planets in HD 41378's sytem are missing. They can be calculated from a_0, as it is shown in Table 19(Vanderburg et al. 2016). It can be seen that planets in HD 41378's system follow the laws of orbital semimajor axes and periods without any exception.

Table 19 Data of HD 41378's system with 5 planets

HP 41378's	n=0	n=1,b	n=2,c	n=3,e	n=4,d	n=5,f
a_n[AU]	0.00651	*0.128*	*0.205*	*0.528*	*0.593*	*0.966*
$a_n/a_{n\text{-}1}$		*19.6*	*1.60*	*2.58*	*1.12*	*1.63*
P_n[days]	0.1793	15.5712	31.6978	131	156	324
$P_n/P_{n\text{-}1}$		86.84	2.036	4.133	1.191	2.077

4. The only exceptional system

Among the 20 systems with 5 planets, only one system, 55 Cnc's system, does not follow the laws of orbital semimajor axes and periods, as it is shown in Table 19(Baluev 2015;Demory et al. 2016).

55 Cnc's	n=0	n=1,e	n=2,b	n=3,c	n=4,f	n=5,d
a_n[AU]	0.00438	0.01544	0.11522725	0.241376	0.7880	5.503
$a_n/a_{n\text{-}1}$		3.53	7.463	2.095	3.265	6.984
P_n[days]	0.1118	0.736539	14.65152	44.4175	262.00	4825
$P_n/P_{n\text{-}1}$		6.588	19.89	3.032	5.899	18.42

5. Conclusion

Most planets in sysems with 8, 7, 6, or 5 planets follow the laws of orbital semimajor axes and periods. The original reason is that planets were born from periodic bursts of stars in their youth, as it was suggested at http://vixra.org/abs/1609.0158(Zhang 2016).

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