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Special Relativity Direct Refutation with Mathematical Justification

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Abstract

It is revealed with mathematical justification that the Lorentz transformation is actually limited to events with coordinates satisfying the light speed space-time relation (i.e., $x = ct$). It is therefore shown that applying the LT on certain events while maintaining the above limitation, leads to mathematical contradictions.

Key words: Special relativity; Lorentz transformation

Introduction

The Lorentz transformation (LT) equations constitute the basis of the Special Relativity (SR) theory in which their interpretations lead to the peculiar prediction of the space-time distortion characterized by the length contraction and time dilation. The LT was derived by Einstein[1,2] on the basis of the relativity principle and the constancy of the speed of light postulate.

A straight forward method is used in this study to derive and reveal the innate contradictions in the Lorentz transformation.

Straight forward LT derivation exposing its contradiction

Consider two inertial reference frames, $K(x, y, z, t)$ and $K'(x', y', z', t')$, in relative uniform motion along the overlapped x- and x'-axes, at speed v. The transformation equations relating the space and time coordinates of K to those of K' are to be determined under the constancy of the speed of light assumption. Let the spatial transformation have the following linear form;

$$x' = \gamma x + \beta t, \qquad (1)$$

where γ and β are real terms to be determined— y and z remain invariant.

The origin of K' is traveling at speed v with respect to K origin. Therefore, the coordinate $x = vt$ is transformed to $x' = 0$. Hence, plugging the particular conversion $x = vt$; $x' = 0$ in Eq. (1) yields $0 = \gamma vt + \beta t$, or $\beta = -\gamma v$ (for $t \neq 0$), leading to the spatial transformation equation

$$x' = \gamma(x - vt), \ t \neq 0 \qquad (2)$$

Using the particular constraint emerging from the basic form of Einstein's constancy of the speed of light

$$x = ct; \quad x' = ct' \qquad (3)$$

in Eq.(2), leads to the time transformation equation

$$t' = \gamma\left(t - \frac{vt}{c}\right), \ t \neq 0. \qquad (4)$$

Equation (4) infers that for $t' = 0$ (while $t \neq 0$), $v = c$; i.e., any time duration in K is transformed to zero duration relative to K' when $v = c$, which means the time in K stops with respect to K', when $v = c$.

So far, Eq. (4) represents the time transformation between our two reference frames, without any limiting conditions other than $t \neq 0$. However, forcing Einstein's assumption that t' must be a function of t and x, we use $x = ct$, or $t = x/c$, in the term vt/c of Eq. (4), to get

$$t' = \gamma\left(t - \frac{vx}{c^2}\right), \ t \neq 0. \qquad (5)$$

Therefore, Eq. (5) is now limited to the condition $x = ct$, with the above restriction $t \neq 0$ being maintained, leading to the additional restriction of $x \neq 0$.

The limitation of the LT time equation to events with coordinates satisfying the relation $x = ct$ has been demonstrated[3] using Einstein's own derivation of the LT in his 1905 paper[1].

Now, owing to the fact that the reference frame K is traveling at a speed of $-v$ with respect to K', and to Einstein's relativity principle (the laws of physics−hence its governing equations−are the same with respect to all inertial frames; particularly, the coordinate transformation equations), the inverse of the general transformation given by Eq. (2) can be written as

$$x = \gamma(x' + vt'), \qquad (6)$$

which must be as well restricted—by symmetry—to $t' \neq 0$.

Similarly, using the basic principle of the constancy of the speed of light, and forcing the dependency of t on t' and x', the general transformation Eq. (6) leads to the particular equation

$$t = \gamma\left(t' + \frac{vx'}{c^2}\right), \ t' \neq 0, \qquad (7)$$

limited to the condition $x' = ct'$, and equally maintaining the above restriction $t' \neq 0$, leading to $x' \neq 0$.

Substituting Eqs. (2) and (5) in Eq. (7), leads after simplification to

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} . \qquad (8)$$

It follows that Eqs. (2), (5) - (8) constitute the Lorentz transformation—and its inverse—although Eqs. (5) and (7) are shown to be merely particular equations limited to the special conditions of $x = ct$ and $x' = ct'$. In addition, as demonstrated above, the LT Eqs.(2), (5) - (8) are restricted to values of x, t, x', and t' being different from zero.

Mathematical Justification

Consider the following function;

$$z = a(x + y), \qquad (9)$$

where the parameter "a" is defined by the requirement that the function shall satisfy the following particular condition:

$$\text{for } y = kx, \quad z = t(y) = t(kx). \qquad (10)$$

Applying the condition given by Eq.(10) to Eq. (9), the following resulting expressions for t will hold for all values of x or y.

$$t = a\left(\frac{1}{k}y + y\right).$$

$$t = a\left(\frac{1}{k}kx + kx\right).$$

However, if we further substituted $y = kx$ in one term of each of the above equations, the obtained expressions

$$t = a\left(\frac{1}{k}y + kx\right),$$

$$t = a\left(x + y\right),$$

will hold only under the condition $y = kx$. i.e., they are limited to $y = kx$.

The above argument is analogous to the case of determining γ by obtaining the time transformation Eq. (4) from the general transformation Eq. (2) when it is applied to the particular condition given in Eq.(3). Hence, Eq.(4) holds for all values of t. However, when a further substitution of the particular condition given by Eq. (3) is applied to the last term of Eq.(4), the resulting LT Eq. (5) will hold only under the latter condition, namely $x = ct$.

Einstein's predictions of time dilation and length contraction are based on applying the LT equations to restricted coordinates

Considering the LT equations

$$x' = \gamma\left(x - vt\right) \qquad (11)$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right), \qquad (12)$$

Einstein's predicted the length contraction by maintaining that the length of a stick fixed along the x'-axis in K', measured in K as l, being the distance between two simultaneous $(t = 0)$ events occurring at its extremities, would be, according to Eq. (11), measured in K' as $l' = \gamma l$.

Hence the length contraction of the stick from the perspective of K':

$$l = \frac{l'}{\gamma}. \qquad (13)$$

On the other hand, Einstein predicted the time dilation by applying the time transformation on the time t' between two colocal events $(x'=0;\ x=vt)$ in K'. The corresponding time t relative to K will be, according to Eq. (12), dilated by the factor γ:

$$t=\gamma t'. \qquad (14)$$

The above length contraction and time dilation Eqs. (13) and (14) are based on applying the LT equations to restricted coordinates $t=0$ and $x'=0$, which will be shown to result in mathematical contradictions.

Application of LT equations to the restricted coordinates leads to mathematical contradictions

The invalid generalization of the particular Eqs.(5) and (7) would result in mathematical conflicts. Indeed, substituting Eq. (5) into Eq. (7), returns

$$t=\gamma\left(\gamma\left(t-\frac{vx}{c^2}\right)+\frac{vx'}{c^2}\right),$$

which can be simplified to

$$t\left(\gamma^2-1\right)=\frac{vx}{c^2}\left(\gamma^2-\frac{\gamma x'}{x}\right). \qquad (15)$$

Since, as shown earlier, the time Eqs. (5) and (7) are limited to coordinates satisfying $x=ct; x'=ct'$, then Eq. (15) can be written as

$$t\left(\gamma^2-1\right)=\frac{vx}{c^2}\left(\gamma^2-\frac{\gamma t'}{t}\right). \qquad (16)$$

If Eqs. (5), (7) and (16) were generalized (i.e. applied to conditions other than $x=ct; x'=ct'$, or $t=x/c; t'=x'/c$), and particularly applied to an event with the restricted time $t'=0,$ then according to Eq. (5), the transformed t-coordinate with respect to K would be $t=vx/c^2$. Consequently, for $t\neq 0$, Eq. (16) would reduce to

$$t\left(\gamma^2-1\right)=t\gamma^2\ , \qquad (17)$$

yielding the contradiction,

$$\gamma^2-1=\gamma^2\ , \text{ or } \quad 0=1\ .$$

It follows that the conversion of the restricted time coordinate $t'=0$ to $t=vx/c^2$, for $x\neq 0$, by LT Eq.(5), is proved to be invalid, since it leads to a contradiction when used in Eq. (16), resulting from the LT time equations for $t\neq 0$.

Furthermore, substituting Eq. (2) into Eq. (6), yields

$$x=\gamma\left(\gamma\left(x-vt\right)+vt'\right)\ ;$$

$$x\left(\gamma^2-1\right)=\gamma v\left(\gamma t-t'\right)\ ;$$

$$x\left(\gamma^2-1\right)=\gamma vt\left(\gamma-\frac{t'}{t}\right). \qquad (18)$$

Since Eqs. (2) and (6), along with Eqs. (5) and (7), are limited to coordinates satisfying the conditions $x=ct; x'=ct'$, Eq. (18) can be written as

$$x\left(\gamma^2-1\right)=\gamma vt\left(\gamma-\frac{x'}{x}\right). \qquad (19)$$

If Eqs. (2), (6) and (19) were generalized (i.e. applied to conditions other than $x=ct; x'=ct'$), and particularly applied to an event with the restricted coordinate $x'=0$, then according to Eq. (2), the transformed x-coordinate with respect to K would be $x=vt$. Consequently, for $x\neq 0$, Eq. (19) would reduce to

$$x\left(\gamma^2-1\right)=x\gamma^2, \qquad (20)$$

$$\gamma^2-1=\gamma^2, \text{ or } \quad 0=1.$$

It follows that the conversion of the restricted space coordinate $x'=0$ of K' origin to $x=vt$, at time $t>0$, with respect to K by LT Eq. (2), is invalid, since it leads to a contradiction when used in Eq. (19), resulting from LT space equations, for $x\neq 0$.

Conclusions

The LT is demonstrated to be limited to events having non-zero time coordinates and non-zero space coordinates along the reference frames axes parallel to the relative motion direction. With such imposed coordinate restrictions, the predictions of the time dilation and length contraction become unfeasible.

In addition, The Lorentz time transformation equations are demonstrated to limit the involved spatial coordinates (in the terms vx/c^2 and vx'/c^2) to the specific values of $x=ct$ and $x'=ct'$, resulting in mathematical contradictions when applied to events having restricted time or space coordinates.

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