Escape velocity and Schwarzschild’s Solution for Black Holes

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Introduction

Scape velocity for every star or planet is different. As a star starts to collapse during Supernova, amount of mass per radius gets extremely high and gravitational fields are proportional to it. Because escape velocity is a result of gravity, so it increases immediately as the star collapses.

So there should be a frontier that if any object passes and get close to the star, it cannot be able to escape from the star unless it moves with the velocity equal to the escape velocity.

A German astronomer, Karl Schwarzschild, on 1963 created a formula for obtaining the radius of sphere which is a boundary around black hole which every object enters into this sphere, the escape velocity will be equal to the velocity of light.

Also Schwarzschild wrote the formula based on line element in space-time in Cartesian coordinates near the black hole in order to show what happens to the space time near the black hole.

Singularity is the center of black hole and an object starts to swirl around singularity.

Abstract

Escape velocity for black holes depends on the region we are at. When we are on the Schwarzschild's sphere, it's equal to the velocity of light and when we are inside the Schwarzschild's sphere, it exceeds the velocity of light. On the other hand, tidal forces act on the object and turn it apart into pieces. Based on angular momentum conservation, as the pieces of object go inside the Schwarzschild's radius, because their distance with singularity decreases, their velocity increases in order to make a balance and the angular momentum stay constant.

The Schwarzschild's solution for schwarzschild's sphere and inside, will show what happens to the space-time; in the Schwarzschild's sphere, the space would be undefined and the time warps, but inside the sphere, "r" and "t" change their place in equation and because of "θ" and "φ" object's geodesic will bend and object swirl around the singularity but "r" behaves like time and "t" behaves like space. Needless to say, $g_{\mu\upsilon}$ will be different.

1- Escape Velocity

For calculating escape velocity for an object located Schwarzschild's radius far from singularity [1]. $R = R_s$

$$V = \sqrt{\frac{2Gm}{r}} \qquad R_s = \frac{2Gm}{C^2}$$

$$V_1 = \sqrt{\frac{2Gm}{\frac{2Gm}{C^2}}} = C$$

So, at this point, the escape velocity is equal to the velocity of light. Now, the question is how much would be the escape velocity if object enters into the Schwarzschild's sphere and get closer to the black hole? $R < R_s$

$$V_2 = \sqrt{\frac{2Gm}{R}}$$

Because R is less thanR_s, so obviously V_2 is more than V_1 and$V_2 > C$.

On the other hand, objects while getting close to the black hole, face with tidal forces that turn them apart and because no torque is exerted to the pieces of object falling into the black hole, so angular momentum conservation exists here. So we have:

$$\tau = \vec{R} \times \vec{F} \times sin\theta = 0 \qquad \vec{F}_{external} = 0$$

$$L = m_i(R_i \times V_i) = constant$$

As the object twirls around the black hole, R_i decreases and V_i increases as a result.

Escape velocity exceeds the velocity of light for interior regions (inside the Schwarzschild's sphere), velocity of object will increase as well.

2- Schwarzschild's solution

Generally, line element in space-time for the object arrives to the Schwarzschild's sphere, Schwarzschild's solution equation for it is [2]:

$$dS^2 = \frac{dr^2}{1-\frac{R_S}{R}} + r^2 d\theta^2 + r^2 (\sin\theta)^2 d\varphi^2 - (1-\frac{R_S}{R}) C^2 dt^2$$

If R = R_s then:

$$dS^2 = \frac{dr^2}{0} + r^2 d\theta^2 + r^2 (sin\,\theta)^2\, d\varphi^2 - (0) C^2 dt^2$$

Based on the equation above, time warps at the Schwarzschild's radius and space is undefined but it goes to infinity. But what happens if object passes the Schwarzschild's sphere?

If R < R_s then:

$$dS^2 = -dr^2 + r^2 d\theta^2 + r^2 (\sin\theta)^2 \, d\varphi^2 + C^2 dt^2$$

The equation above indicates that time and r component change their place in formula of line element in space-time.

The metric of space time also faces with some changes.

$g_{\mu\nu}$ is the normal metric for Riemannian space time but $g'_{\mu\nu}$ is the metric of space time inside the Schwarzschild's sphere.

$$g_{\mu\nu} = \begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & -1 \end{bmatrix}$$

$$g'_{\mu\nu} = \begin{bmatrix} -1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix}$$

Please note that columns of matrix in spherical coordinates it's r,θ, φ *and* t.

So, time would behave like space in this situation and time, θ *and* φ are the main components for the object.

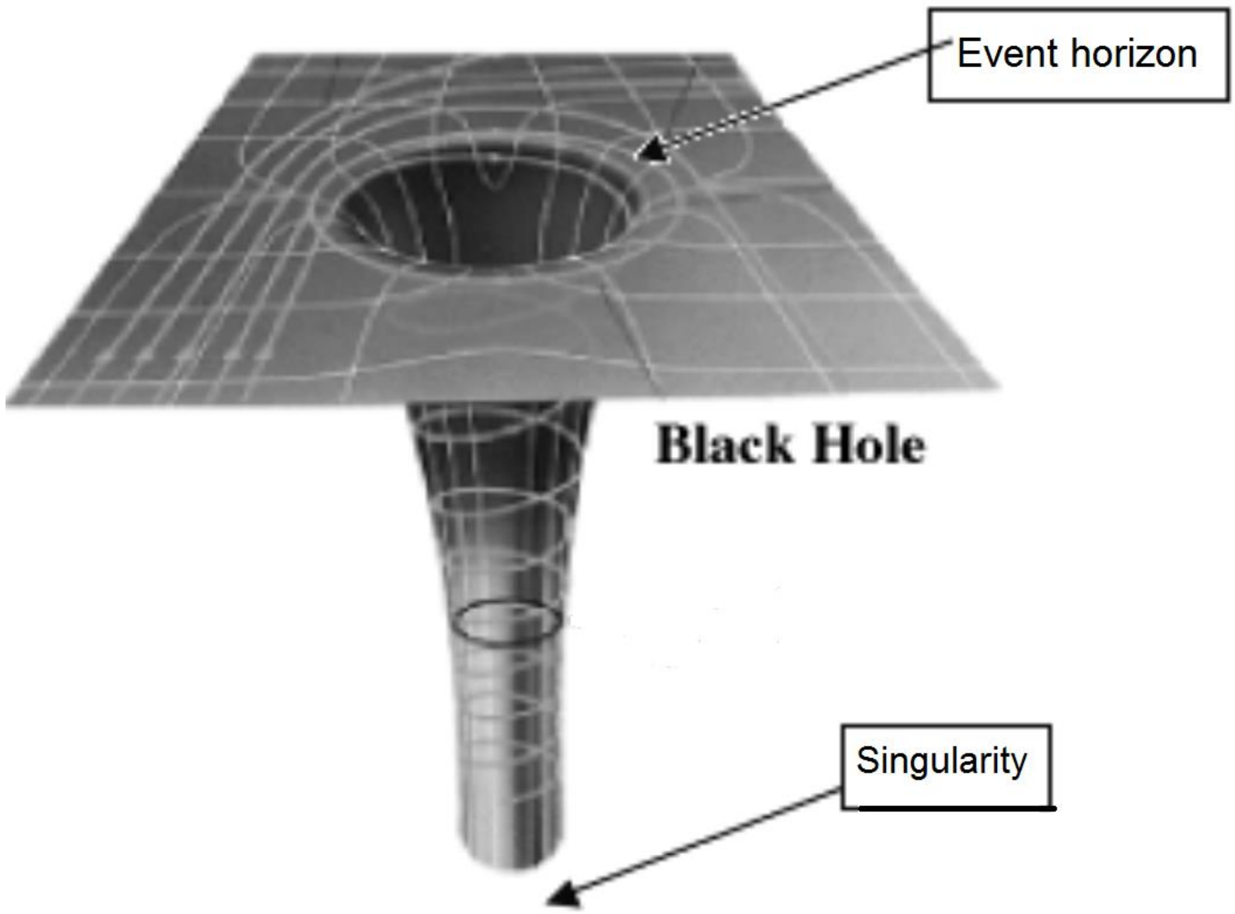
The object is moving toward singularity with the time scales and rotates based on θ *and* φ.

As a result object has a rotational geodesic and it swirls around the singularity of black hole.

Conclusion:

The escape velocity of black holes at the Schwarzschild's sphere is equal to the velocity of light and insight the Schwarzschild's sphere, it exceeds the velocity of light. Pieces of the object will get a higher velocity as they move toward the singularity.

Space-time inside the Schwarzschild's sphere, is absolutely different from outside. Metric of space-time will be changed and time will behave like space and r will behave like time. Object starts to swirl around singularity ($\theta\ and\ \varphi$) and during this time, the r which is radius of circles (geodesic of space time around singularity for the object is like series of circles with different radius and the same center. their radius is decreasing since the schwarzschild's sphere).


Event horizon
Black Hole
Singularity

References:

[1] Ian Morison, Introduction to Astronomy and Cosmology (2008)

[2] Oyvint Gron and Arne Naess, Einstein’s Theory(Springer 2011)

[3]http://www.fysik.su.se/~ingemar/relteori/The%20Angular%20Momentum%20of%20Kerr%20Black%20Holes.pdf