# *Relativistic Dynamics of Gravitational Motions*

*Daniele Sasso**

***Abstract***

This paper has the aim to study the relativistic behaviour of motions that happen in a gravitational field with respect to two reference frames that are in relative motion in the field. Let us consider the two main gravitational motions: the free fall of a body in a gravitational field of first type and the orbital motion in a gravitational field of second type. We will study different physical situations and we will distinguish situations in which the complete physical process happens inside a laboratory (Galilean reference frames) from situations in which physical process happens in open systems (Einsteinian reference frames).

## 1. Introduction

We know in the order of the Theory of Reference Frames, with regard to "Physics of Gravitational Fields" with two bodies, there are three fundamental types of gravitational field while a fourth type regards more than two bodies[1]. We considered those different types of gravitational field in papers "Physics of Gravitational Fields"[1], "Dynamics of Motion in Gravitational Fields of First Type"[2], "Gravitational Field of Second Type, Motions of Precession and the Fourth Law of Orbital Motions"[3], but we didn't consider the relativistic behaviour of motions that happen in a gravitational field with respect to two reference frames in relative motion. To that end we suppose one of the two reference frames is the laboratory, in motion or at rest, that is into the gravitational field generated by mass of a celestial body that represents the other reference frame. We will consider whether Galilean reference frames or Einsteinian reference frames[4][5][6][7] so as to point out prospective differences and to examine all possible situations that may happen in the physical reality. We will examine largely relativistic dynamics that characterizes the gravitational motion of first type, that coincides with the motion of free fall, and relativistic dynamics of the gravitational motion of second type, that coincides with orbital motion. We will search for study besides all possible physical situations that can happen in the reality whether considering Galilean reference frames, that are generally closed and isolated, or Einsteinian reference frames, that are instead open and interactive. To that end we define still the two fundamental principles of the Theory of Reference Frames: the Principle of Reference and the Principle of Relativity.

* e_mail: dgsasso@alice.it

## 2. Principle of Relativity and Principle of Reference

The Principle of Relativity affirms:

***"Laws of physics and mathematical equations that represent them are invariant with respect to all reference frames that are inertial each other".***

We know the absolute resting state doesn't exist because all observable universe is in relative motion, however in the theorization of the physical phenomena it is necessary to assume a convenient reference frame as if it could be at rest. This reference frame represents in our theoretical description the privileged reference frame for the physical situation that we are observing and describing. The meaning of those concepts is clearer if we do a few physical specifications that derive from the "Principle of Reference" that says:

***" In every physical situation a preferred reference frame exists, it coincides with the primary reference frame where the physical phenomenon happens and can be assumed as a resting reference frame".***

This preferred reference frame S[O,x,y,z,t] is characterized by three space coordinates (x,y,z), by a time coordinate t and by an origin O that represents the zero point with space coordinates (0,0,0). That preferred reference frame can be considered for the sake of argument an inertial reference frame, supposed at rest, and this property doesn't have absolute character like in classic physics. Consequently in our physical situation, inertial reference frames are all reference frames that are provided with uniform motion with respect to S, and for the Generalized Principle of Inertia also reference frames with orbital motion with respect to S are inertial[2][3]. In general the inertial characteristic regards all those reference frames in which relative real forces are absent. It is suitable to specify in the present physics there is a big confusion between the concept of invariance and the concept of covariance. The Principle of Relativity affirms in fact the invariance of laws of physics only with respect to inertial reference frames, and in these situations also a few physical magnitudes are invariant (force, acceleration for mechanical systems, etc..) while others are variant (position, speed, etc..). The Principle of Covariance instead is at heart of General Relativity and it affirms the possibility, but not the invariance, to describe laws of physics with respect to every reference frame, inertial and non-inertial.
In the order of inertial reference frames it needs then to distinguish Galilean reference frames from Einsteinian reference frames.

### 2.a Galilean reference frames

Galileo clarified in his main paper[4] reference frames that he considered. In fact his reference frame was a closed structure, non interacting and isolated with respect to the external world, and therefore protected by prospective interferences and noises. He realized like this the first scientific laboratory of history that was precursor with respect to present laboratories and orbital stations.

### 2.b Einsteinian reference frames

Einstein instead considered different reference frames, i.e. open, non isolated and interacting with the external world[5]. Of course these reference frames raise greater problems just because of perturbations and interferences but they have also a few important advantages. For instance the possibility to examine the Doppler effect and all relativistic cosmological phenomena that instead would be excluded considering Galilean reference frames. Anyway whether for Galilean or Einsteinian reference frames fundamental principles of physics are valid.

## 3. Relativistic dynamics of gravitational motions of first type for Galilean reference frames

The gravitational motion of first type consists in the linear motion of fall of a physical system into a field with central symmetry, like the gravitational field[1][2][8].
Let us consider six main cases supposing that the initial condition is $v(t=0)=v(r_o)=0$ and that $m_o$ is the mass of an ordinary body in free fall excluding at the moment massive elementary particles.

### 3.a Free fall of the mass $m_o$ in a still laboratory

Let us suppose that the laboratory with mass m is still on the surface of a celestial body with mass M and radius R, whose atmosphere has a coefficient of resistance of medium that is k. Let us suppose that inside the laboratory a mass $m_o$ is initially at rest at the distance $x_o$ from the barycentre of the pole mass M and the resistance of medium inside the laboratory is $k_o$ (fig.1). Let us suppose besides that $m_o \ll m$ and $m \ll M$.
Let us suppose that S'[O',x',y',z',t'] is the Galilean reference frame of the resting laboratory and S[O,x,y,z,t] is the the reference frame, supposed at rest, where O is the barycentre of the celestial body and besides $x' \equiv x$. Let us suppose at last that at the initial time $t'=t=0$ the mass $m_o$ begins the free fall from the point with coordinate $x=x_o$, $x'=x_o'$. The attractive force on the mass $m_o$ due to the gravitational field of the celestial mass is given by

$$\mathbf{p_o} = m_o\mathbf{g} \qquad (1)$$

and the motion equation of fall of the mass with respect to the reference frame S'[O',x',y',z',t'] of the celestial body is given by the equation

$$\frac{GMm_o}{x^2} = m_o \frac{dv'(t)}{dt} + k_o v'(t) \qquad (2)$$

Because the two reference frames are both at rest, it is certainly t'=t. Considering at the initial time $t'=t=0$ the mass is at distance $x_o$ from the origin O and at distance $x_o'$ from the origin O' with $v(x_o)=v'(x_o')= 0$ , supposing that $k_o=0$, the law of motion with respect to S', before the fall mass reaches the botton of the laboratory, is given by[1][2][4][5]

$$v'(x') = \sqrt{\frac{2\,G\,M}{x'_o+R}\;\frac{x_o' - x'}{x' + R}} \qquad (3)$$

and with respect to S is given by

$$v(x) = \sqrt{\frac{2\,G\,M}{x_o}\;\frac{x_o - x}{x}} \qquad (4)$$

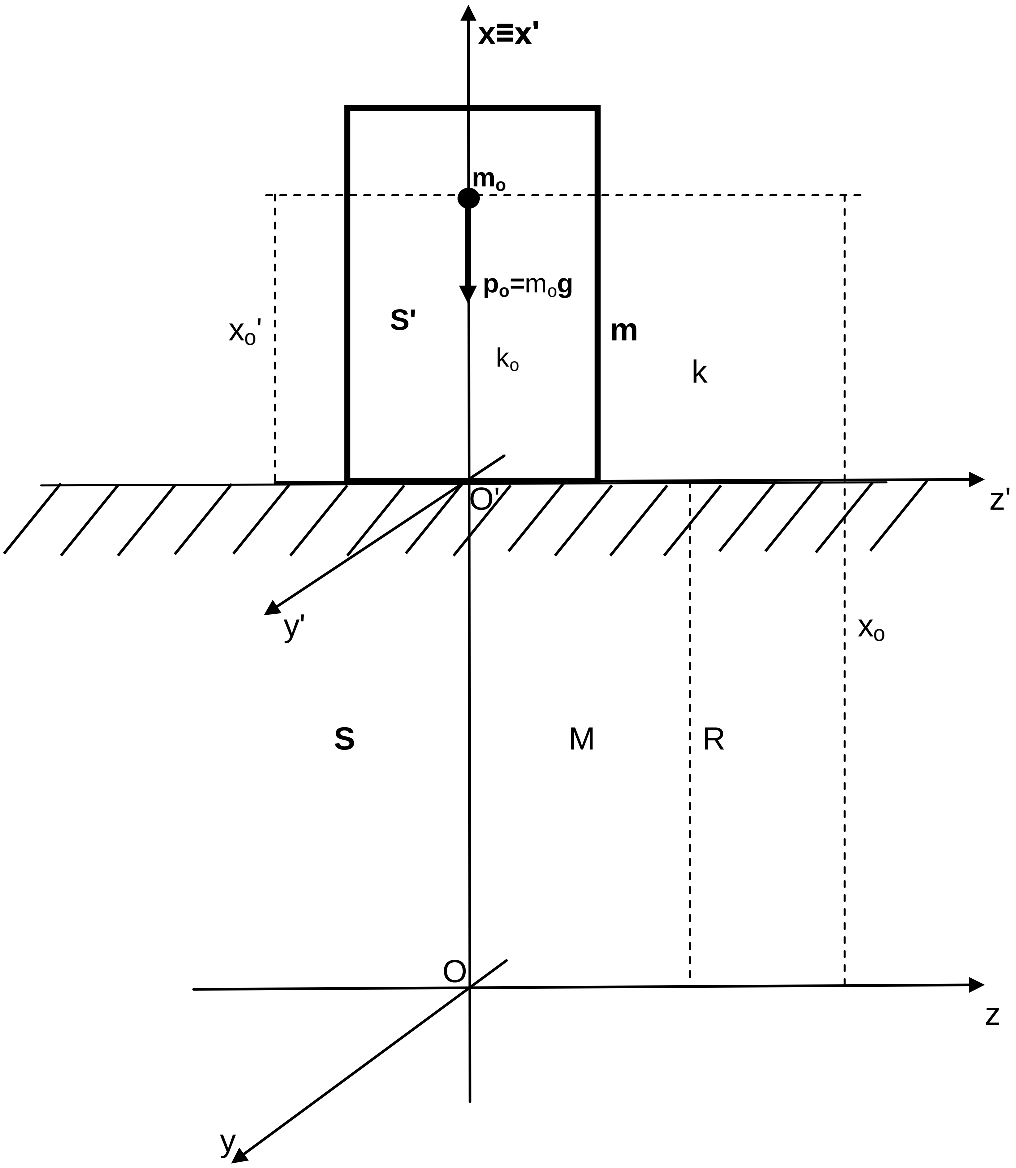


Fig.1 Free fall of an ordinary body with mass $m_o$ in a resting laboratory

Graphs of (3) and (4) have the same trend, even if they are different, and are drawn in fig.2, where $v'(x')=v(x)$ for every $x'=x-R$.
Because both reference frames S and S' are at rest with respect to the celestial body there aren't relativistic effects. It is possible to observe then inside laboratory for $k_o=0$ (absence of atmosphere) fall is independent of mass and therefore it happens similarly for all bodies in fall. It is true also with respect to the reference frame S. It is manifest that in the presence of atmosphere ($k_o\neq0$) fall depends on mass and consequently it is different for every body[2].

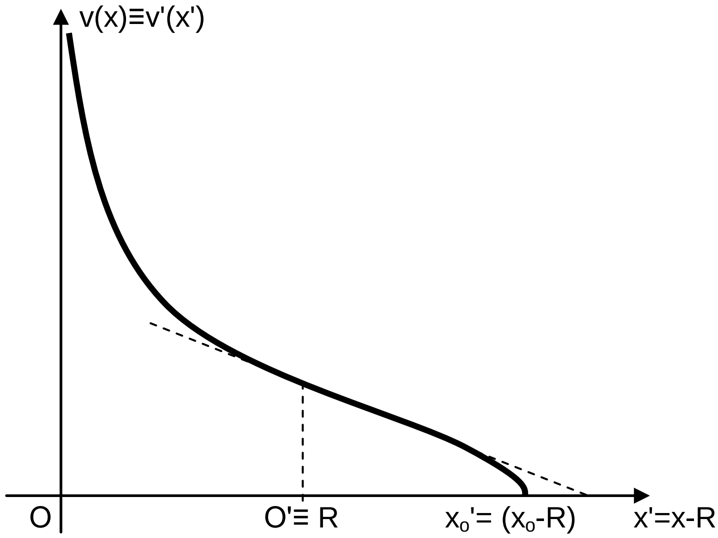


Fig.2 Trend of the speed of a body in free fall in the absence of resistant forces ($k_o=0$).

### 3.b Mass $m_o$ at rest on the bottom of laboratory in free fall

In that case the motion law of the mass $m_o$ is given by the motion law in free fall of the laboratory with mass m where generally $m_o \ll m$ (fig.3).
Consequently the motion equation of m with respect to the reference frame S[O,x,y,z,t] is

$$\frac{GMm}{x^2} = m\frac{dv(t)}{dt} + kv(t) \qquad (5)$$

and the motion law, always supposing that k=0, is

$$v(x) = \sqrt{\frac{2GM}{x_o}\,\frac{x_o - x}{x}} \qquad (6)$$

that is exactly equal to (4) because the fall law is independent of the falling mass when k=0.
With respect to the reference frame S', with origin in O', placed on the surface of the celestial body, the motion law is given by

$$v'(x') = \sqrt{\frac{2\,G\,M}{x_o' + R}\;\frac{x_o' - x'}{x' + R}} \qquad (7)$$

in which still is $v'(x')=v(x)$ for every $x'=x-R$. The(7) is exactly equal to (3).
We observe in the absence of atmosphere and of resistance of medium (k=0), the motion law is still independent of falling mass and therefore it is the same for all falling bodies. Also in that case both reference frames S and S' are at rest with respect to the celestial body and consequently there aren't relativistic effects.
Same conclusions are valid also in the event that mass $m_o$ is at rest in any point inside the laboratory.

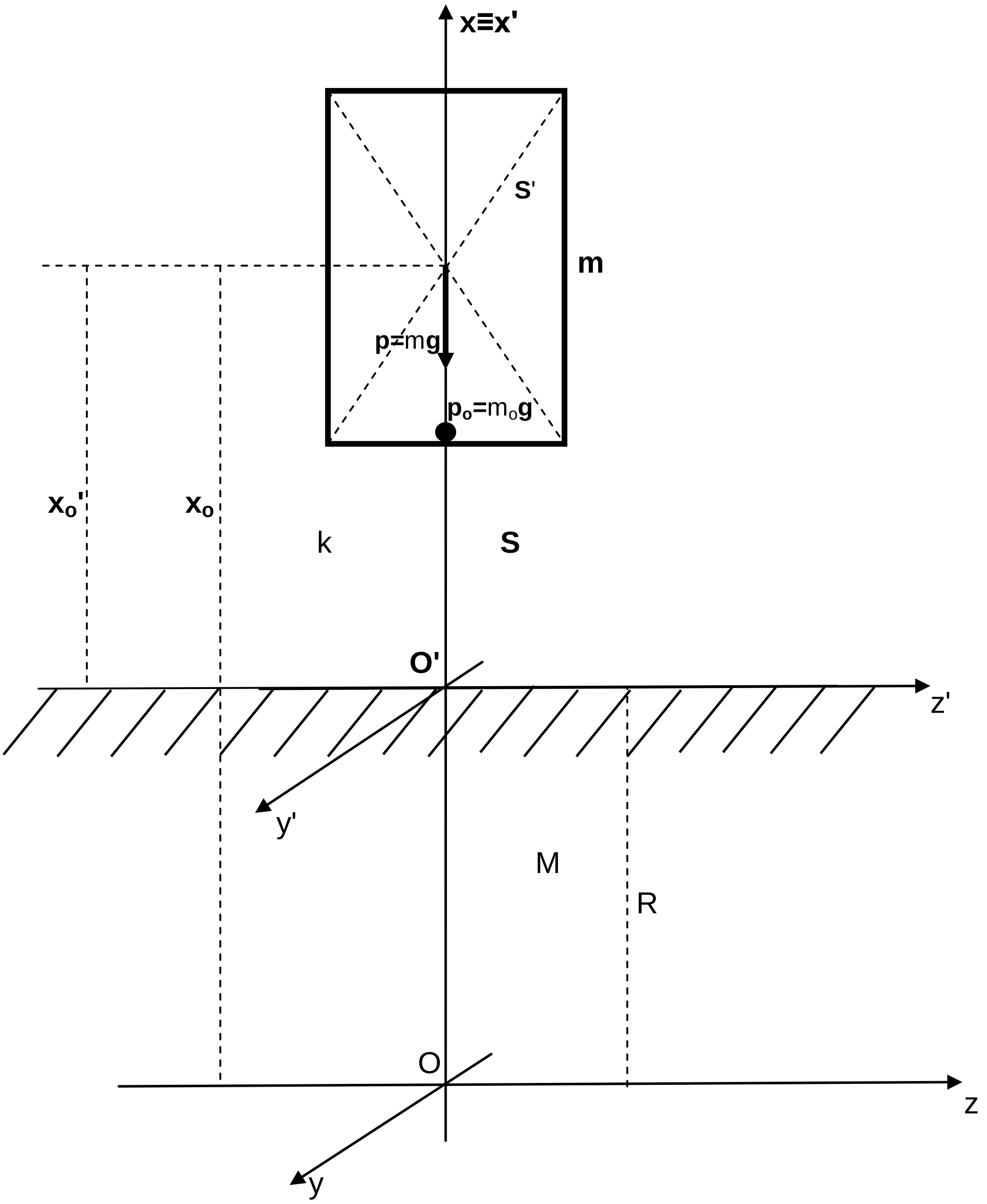


Fig.3 Mass $m_o$ at rest inside the laboratory in free fall

### 3.c Free fall of mass $m_o$ inside the falling laboratory

Considering the fig.4, the motion equation of the laboratory with mass m, including the mass $m_o$, with respect to the reference frame S[O,x,y,z,t] with origin in the barycentre of the celestial body is

$$\frac{GMm}{x^2} = m\,\frac{dv(t)}{dt} + kv(t) \qquad (8)$$

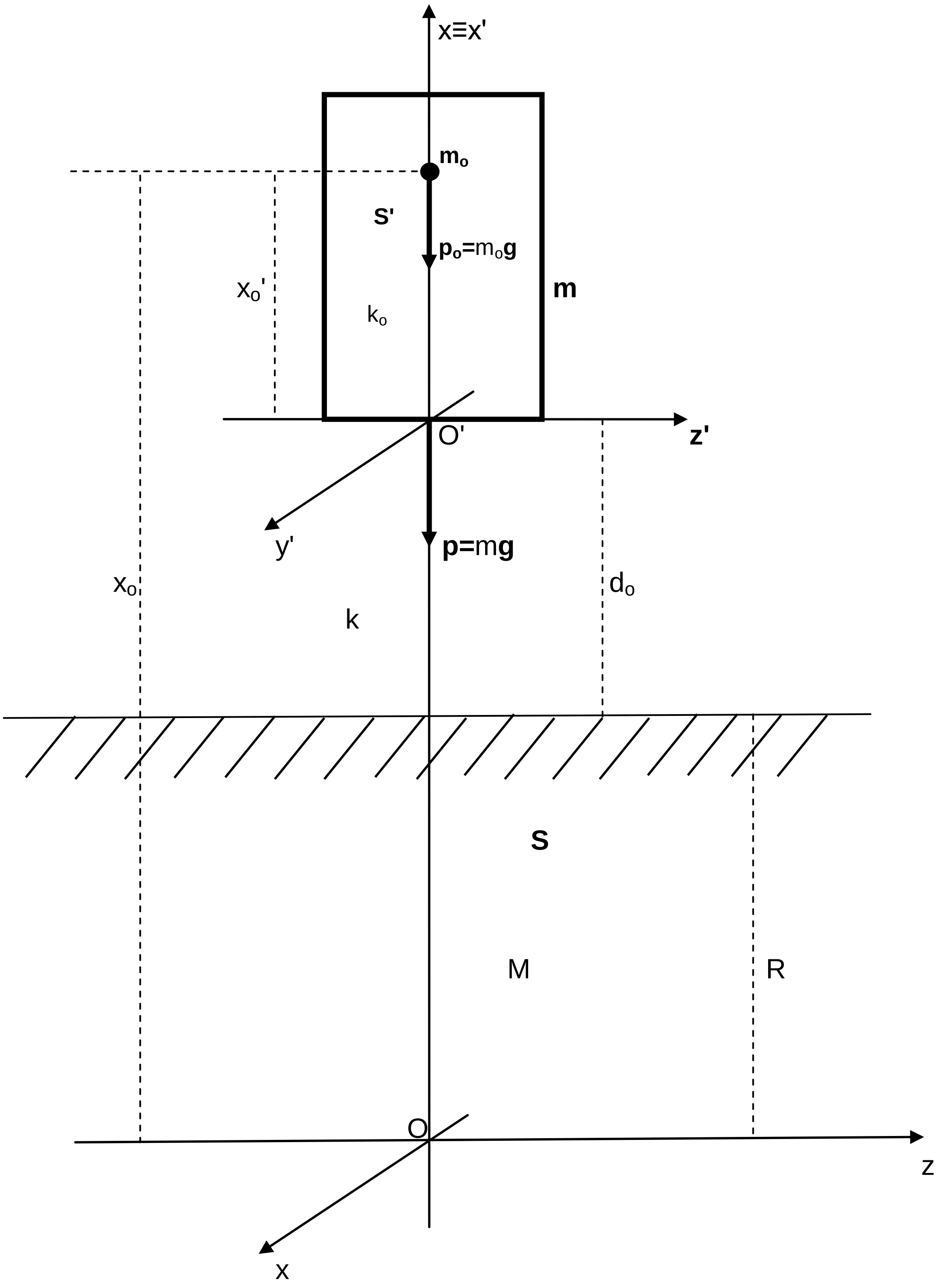


Fig.4 Free fall of mass inside the falling laboratory

The motion law of the laboratory with respect to S, assuming k=0, is as per (6)

$$v_m(x) = \sqrt{\frac{2\,G\,M}{x_o}\;\frac{x_o - x}{x}} \qquad (9)$$

The motion equation of the only mass $m_o$ with respect to S is

$$\frac{GMm_o}{x^2} = m_o \frac{dv(t)}{dt} + k_o v(t) \qquad (10)$$

The motion law of the mass $m_o$ in free fall with respect to S, always supposing $k_o$=k=0, is

$$v_{mo}(x) = \sqrt{\frac{2\,G\,M}{x_o}\;\frac{x_o - x}{x}} \qquad (11)$$

Comparing (11) and (9) we deduce the fall of mass $m_o$, that is inside the laboratory, and the fall of the same laboratory happen similarly with respect to the reference frame S in the important hypothesis $k=k_o=0$. It means the fall speed $v'_{mo}(x')$ of the mass $m_o$ with respect to the reference frame S' of the laboratory, given by $v'_{mo}(x') = v_{mo}(x) - v_m(x)$ is always zero; in other words in the considered physical situation the mass $m_o$ is in a resting state with respect to the reference frame S' of the laboratory and its motion coincides with motion of laboratory.
It is manifest that if two resistant coefficients of the medium inside the laboratory and outside are different from zero ($k\neq0$, $k_o\neq0$), the two falls happen otherwise.

### 3.d Forced fall of the mass $m_o$ inside the laboratory in free fall

In that case we suppose that the mass $m_o$ inside the laboratory in free fall is subject to the force F, before in the same direction and after in the reverse direction with respect to the direction of the gravitational force $\mathbf{p_o}$.

#### 3.d.1 Force with the same direction of gravity

Let us suppose that the force F has the same direction of the gravitational force (fig.5). As per conclusions in 3.c, in the absence of F, the mass $m_o$ is at rest inside the laboratory that is in free fall with respect to the Earth reference frame, having supposed the resistant coefficients of the two mediums are zero ($k_o$=k=0).
With respect to the reference frame S' of the laboratory therefore only the force F acts for which the motion equation of $m_o$ with respect to S' is, because t'=t for ordinary masses,

$$F(t) = m_o \frac{dv'(t)}{dt} + k_o v'(t) \qquad (12)$$

and in the event of constant applied force F, the motion law with respect to S' is[8]

$$v'(t) = \frac{F}{k_o}\left(1 - e^{-k_o t / m_o}\right) \qquad (13)$$

The (13) is graphed in fig.6, where $V_o'=F/k_o$ is the final speed of the mass $m_o$. We observe increasing F also the final speed increases.

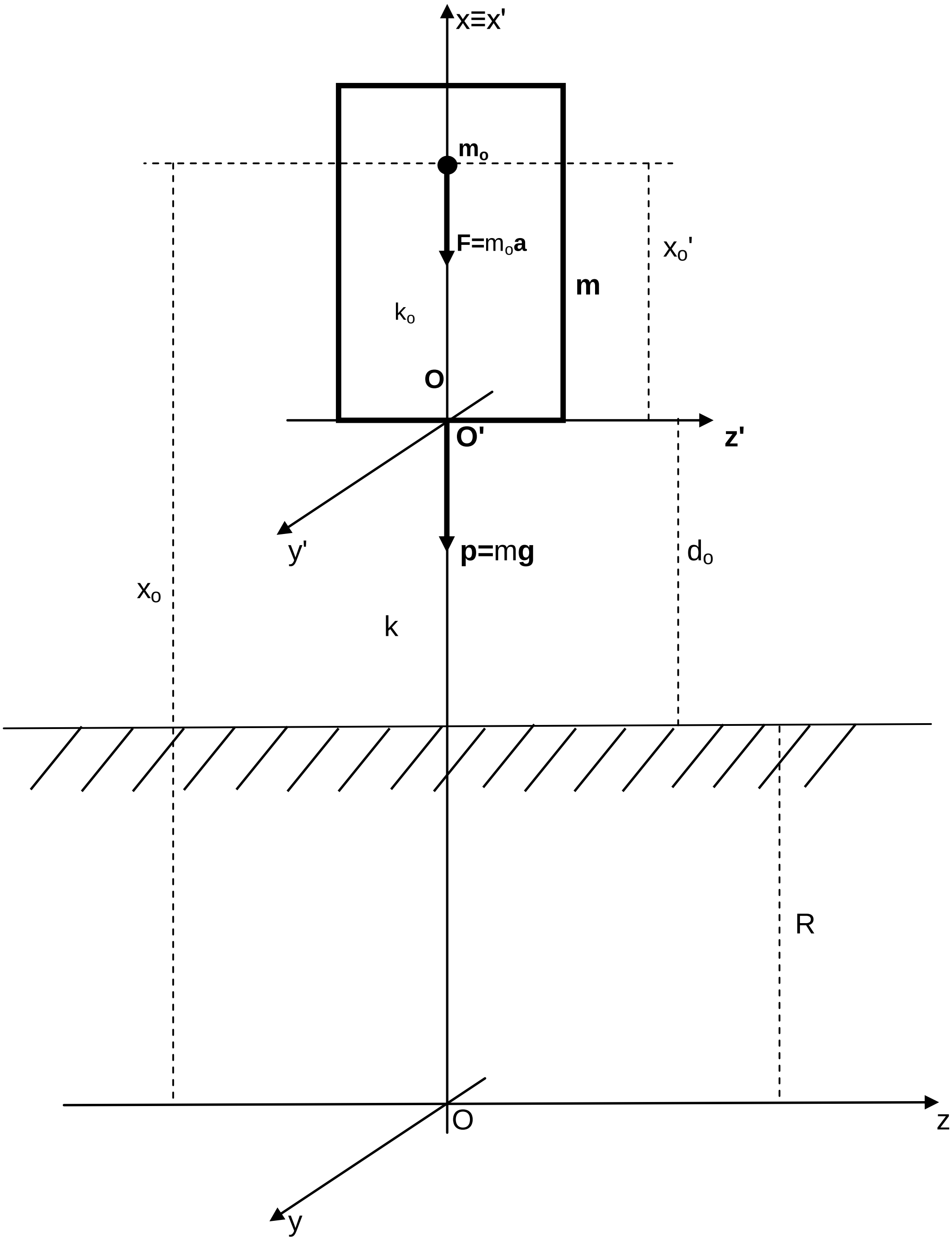


Fig.5 Forced fall of the mass $m_o$ inside the falling laboratory

Supposing that $k_o=0$ the motion law with respect to S' is

$$v'(t) = \frac{F}{m_o} t \qquad (14)$$

and considering

$$\frac{dv'}{dt} = - v' \frac{dv'}{dx'} \qquad (15)$$

the (14) can be written

$$v'(x') = \sqrt{\frac{2F}{m_o} (x_o' - x')} \qquad (16)$$

With respect to the Earth reference frame S, it needs to add the motion gravitational equation of the mass $m_o$, given by the (8) for $m=m_o$, to the motion dynamic equation given by the (12). The comparison between the (8) and the (12) proves the two motion equations with respect to the two reference frames S and S' are different and consequently the Principle of Relativity in this physical situation isn't valid. The motion law of the laboratory is given by the (9), that here we rewrite, always for k=0,

$$v(x) = \sqrt{\frac{2 G M}{x_o} \frac{x_o - x}{x}} \qquad (17)$$

The (17) represents also the motion law in free fall of the mass $m_o$ with respect to S for which the forced motion general law $v_t(x)$ of the mass $m_o$ in free fall with respect to the Earth reference frame S, always supposing that $k_o=k=0$, is in vector shape

$$\mathbf{v_t(x)} = \mathbf{v'(x')} + \mathbf{v(x)} \qquad (18)$$

and in scalar shape

$$v_t(x) = \sqrt{\frac{2 G M}{x_o} \frac{x_o - x}{x}} + \sqrt{\frac{2F}{m_o} (x_o' - x')} \qquad (19)$$

Being $x_o'=x_o-R-d_o$ and $x'=x-R-d$ , the (19) can be written

$$v_t(x) = \sqrt{\frac{2 G M}{x_o} \frac{x_o - x}{x}} + \sqrt{\frac{2F}{m_o} (x_o - x - d_o + d)} \qquad (20)$$

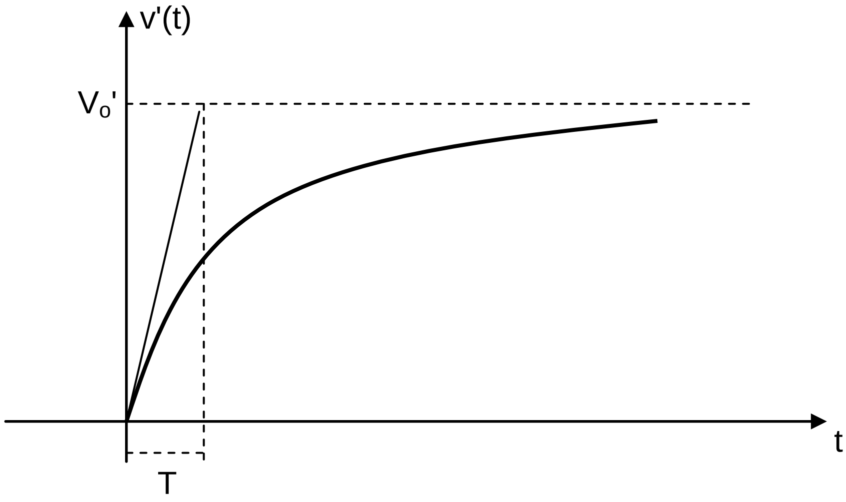


Fig.6 Graph of the motion law of the mass $m_o$ with respect to the reference frame S'.

### 3.d.2 Force with opposite direction towards gravity

If force F has opposite direction towards gravitational force it has the function of antigravity force supposing that F>p (fig.7).
Repeating calculations and considering now

$$\frac{dv'}{dt} = v' \frac{dv'}{dx'} \qquad (21)$$

the (16) can be written

$$v'(x') = \sqrt{\frac{2F}{m_o} (x' - x_o')} \qquad (22)$$

Because the fall law of the laboratory is the same as in the preceding case, the total law $v_t(x)$ of the motion of the mass $m_o$ with respect to the reference frame S of the Earth, always supposing that $k_o$=k=0, is now

$$\mathbf{v_t(x) = v'(x') - v(x)} \qquad (23)$$

with $v'(x')>v(x)$ , for which the motion happens according to the direction of the increasing x, that is

$$v_t(x) = \sqrt{\frac{2\,F}{m_o} (x - x_o + d_o - d)} - \sqrt{\frac{2GM}{x_o} \frac{x - x_o}{x}} \qquad (24)$$

Also in this case the Principle of Relativity isn't respected.

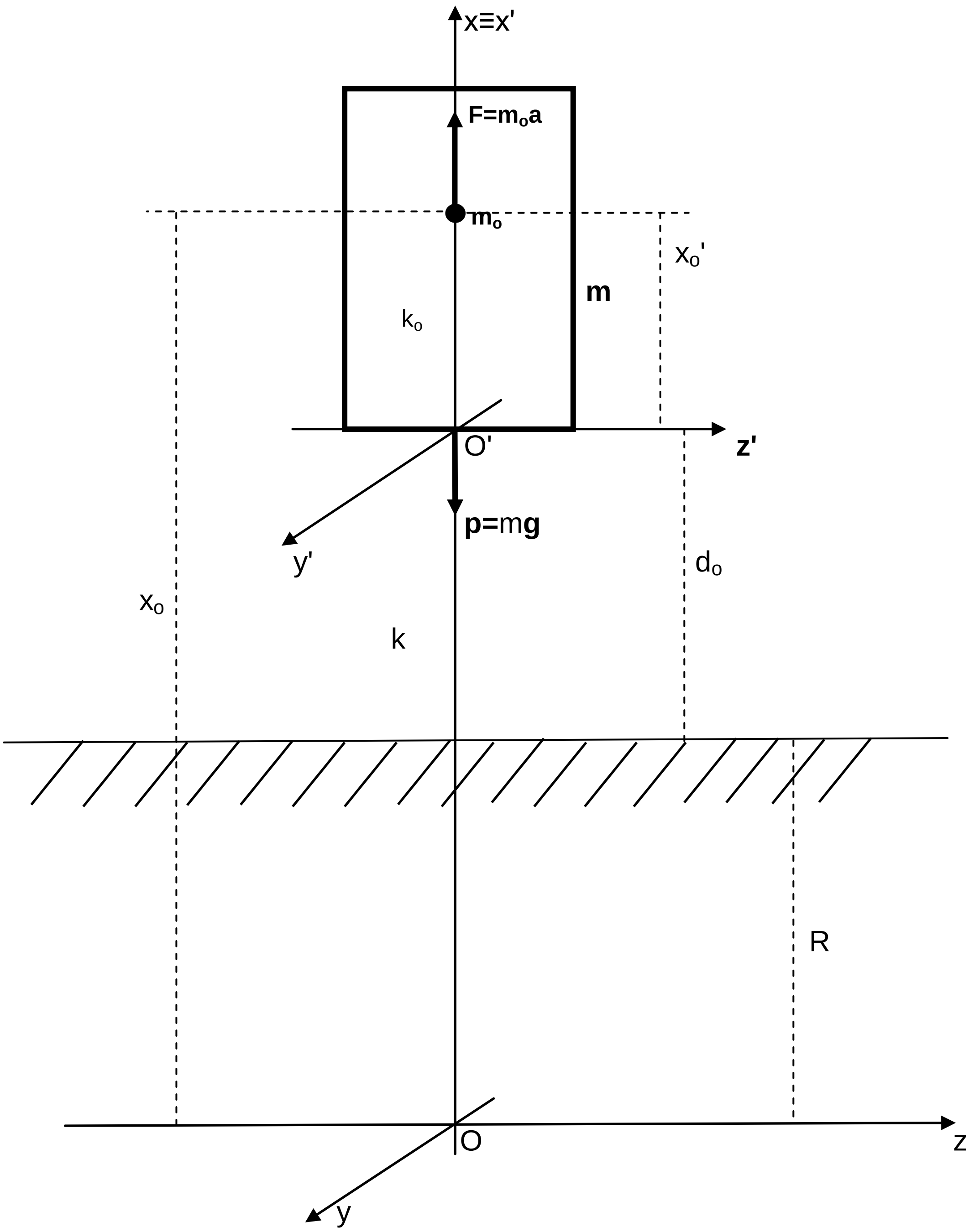


Fig.7 Motion of the mass $m_o$ with opposite force towards gravity inside the falling laboratory

### 3.e Fall of an accelerated mass inside a resting laboratory

Let us suppose that in this new physical situation the force F is perpendicular to the fall direction (fig.8). Also in this event dynamics of motion of the mass $m_o$ is defined by the resultant of the two forces (**F** and $\mathbf{p_o}$). Dynamics can be obtained also by the linear superposition of motions defined apart by the single forces.
The motion equation, due to the force F, with respect to S', is given by

$$F(t) = m_o \frac{dv'(t)}{dt} + k_o v'(t) \qquad (25)$$

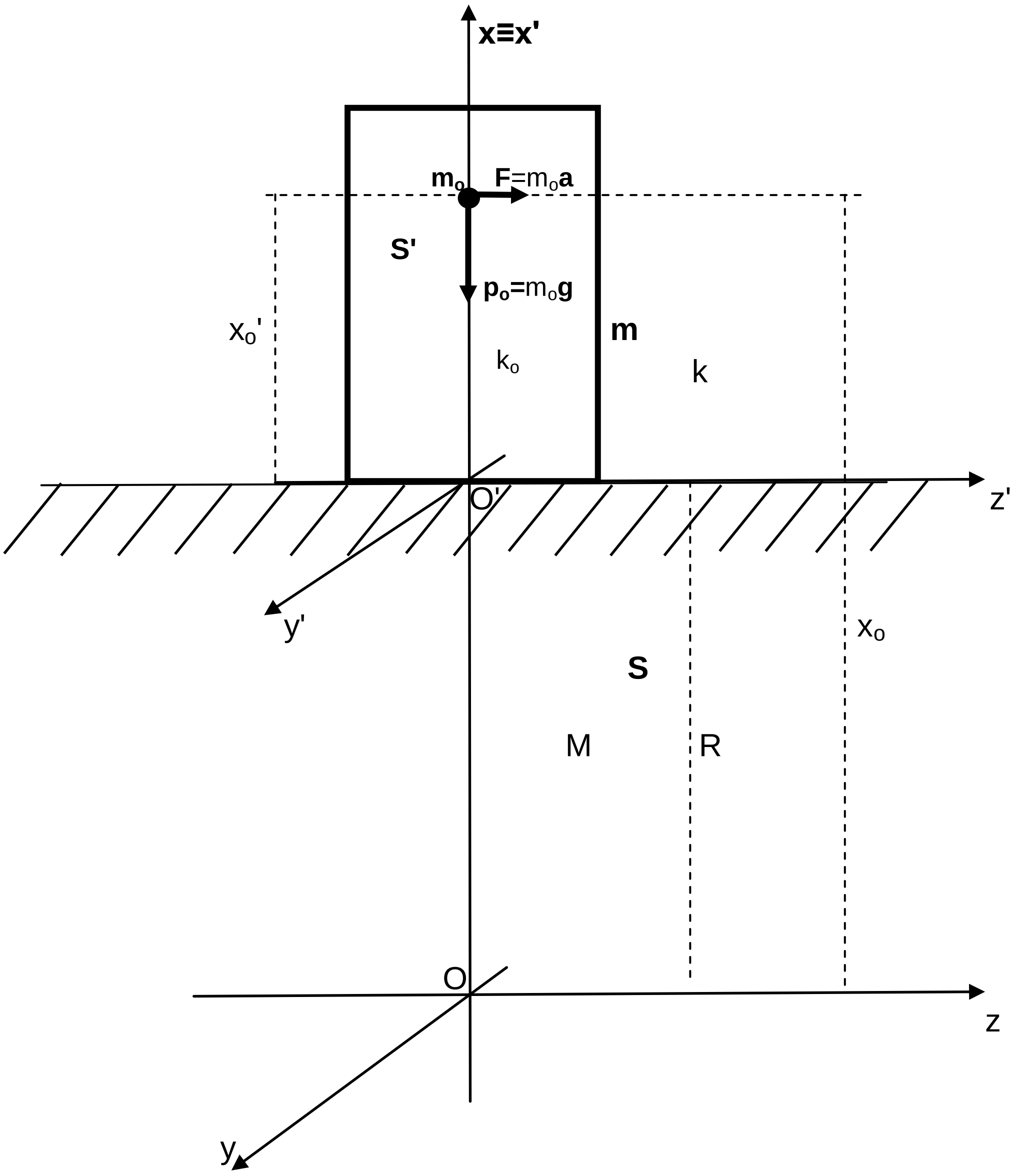


Fig.8 Fall of an accelerated mass inside a resting laboratory

In the event of constant applied force F and $k_o$=0, the motion law, that happens along the axis z' with respect to S', is

$$v'(z') = \sqrt{\frac{2F}{m_o}\, z'} \qquad (26)$$

The motion law due to the gravitational force $p_o$ , always supposing $k_o$=0, is given by

$$v'(x') = \sqrt{\frac{2\,G\,M}{x_o' + R}\;\frac{x_o' - x'}{x' + R}} \qquad (27)$$

The total speed $v_t$ of the mass $m_o$, in vector shape, with respect to S' is

$$\mathbf{v_t'} = \mathbf{v'(x')} + \mathbf{v(z')} \qquad (28)$$

and in scalar shape

$$v_t' = \sqrt{v'^2(x') + v'^2(z')} \qquad (29)$$

Because $x_o = x_o' + R$, $x = x' + R$, $z = z'$, the total law of motion with respect to the reference frame S is given by

$$v_t = \sqrt{\frac{2GM}{x_o} \frac{x_o - x}{x} + \frac{2F}{m_o} z} \qquad (30)$$

The two motion laws with respect to the two reference frames S and S' are equal and because S and S' are both at rest, there aren't relativistic effects in this physical situation.

### 3.f Fall of an accelerated mass inside a laboratory in free fall

We have demonstrated in 3.c the free fall of a mass $m_o$ inside a laboratory in free fall, supposing that $k = k_o = 0$, happens practically with the same law of fall of the laboratory with respect to the reference frame S and consequently with respect to the reference frame S', the mass $m_o$ behaves like a resting system.

It follows that with respect to S' the only effective force is F (fig.9) for which the motion law of $m_o$ with respect to S' happens along the axis z' and, supposing always that $k_o = 0$, it is

$$v'(z') = \sqrt{\frac{2F}{m_o} z'} \qquad (31)$$

In order to calculate the motion law of the mass $m_o$ with respect to the reference frame S, supposed at rest, it needs to add to (31) the motion law of the laboratory and of the mass $m_o$ in free fall that, for k=0, is given by

$$v(x) = \sqrt{\frac{2\,G\,M}{x_o} \frac{x_o - x}{x}} \qquad (32)$$

The total law of motion, being z'=z, is given therefore in scalar shape by

$$v_t = \sqrt{\frac{2GM}{x_o} \frac{x_o - x}{x_o} + \frac{2F}{m_o} z} \qquad (33)$$

Let us observe the motion equation with respect to S, due to both forces **F** and **p,** is different , from the motion equation with respect to S', due only to the force **F**. It follows that the Principle of Relativity in this physical situation isn't valid.

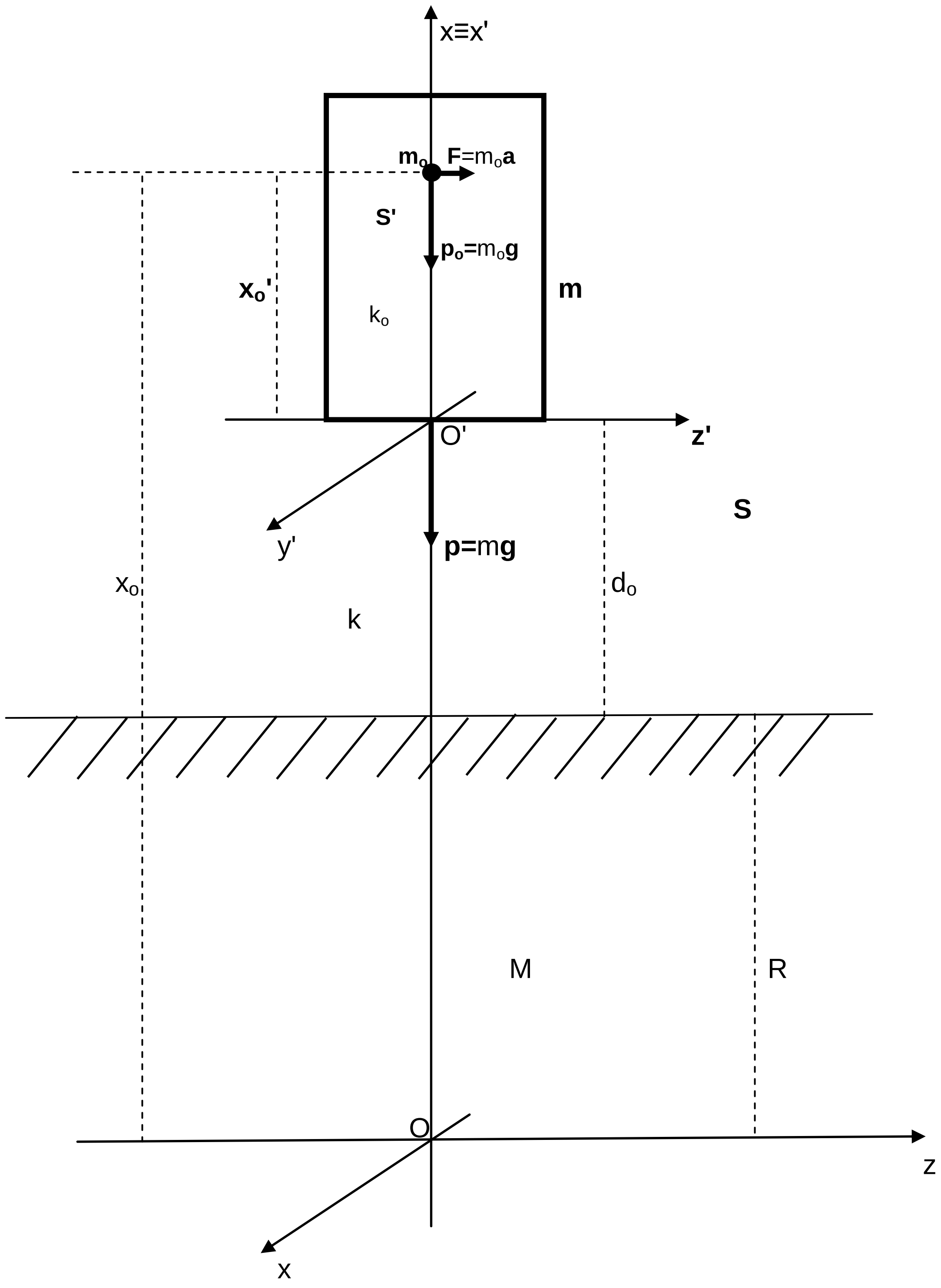


Fig.9 Fall of an accelerated mass inside a laboratory in free fall

## 4. Relativistic dynamics of gravitational motions of first type for Einsteinian reference frames

If the reference frame S' is Einsteinian, that is open and interactive with the rest of the universe, there isn't more the laboratory to define the space of S' and therefore we have a situation like in fig.10, where we have considered for instance the analogous case 3.a regarding the free fall of mass $m_o$ with respect to the resting reference frame S'[O',x',y',z',t'] placed on the surface of the Earth while the reference frame S, itself at rest, has the origin O in the barycentre of mass that generates the gravitational field.
In all these cases in which an Einsteinian reference frame is considered, the diversity of the physical situation consists in the fact that there isn't a closed and insulated laboratory with respect to the rest of the universe.

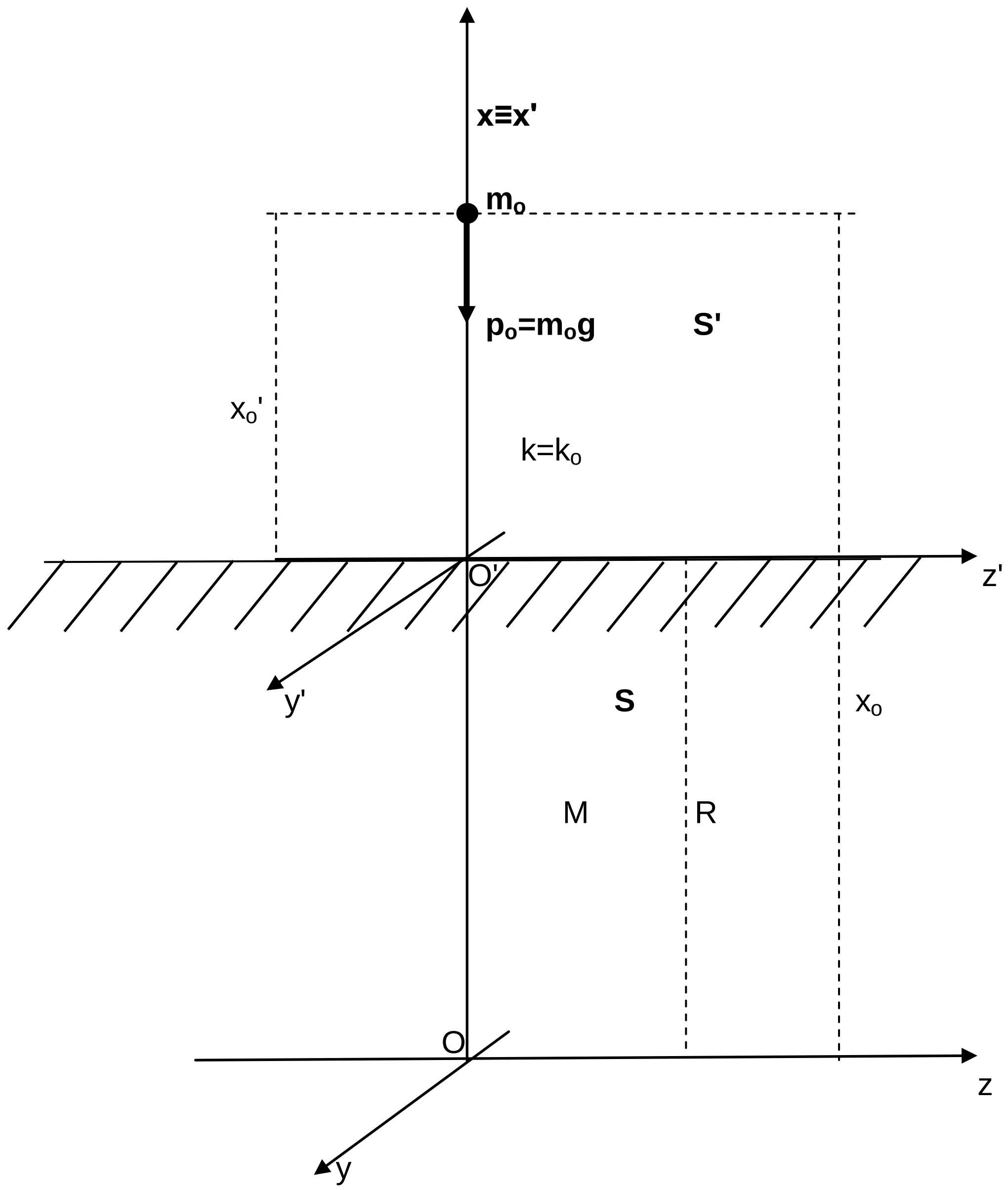


Fig.10 Free fall of an ordinary body with mass $m_o$ in a resting Einsteinian reference frame

It is manifest that for Einsteinian reference frames there is only one resistant coefficient of medium for which is always $k=k_o$. Consequently dynamics of fall happens like in all cases that have been considered in the paragraph 3, when the hypothesis $k=k_o$ is valid. Therefore the analysis of single physical situations for Einstenian reference frames involves the following considerations:

4.a Like in Galilean reference frames for $k=k_o$ (subpar. 3.a)
4.b The physical situation doesn't have sense for Einsteinian reference frames
4.c The physical situation doesn't have sense for Einsteinian reference frames
4.d.1 Like in Galilean reference frames for $k=k_o$ (subpar. 3.d.1)
4.d.2 Like in Galilean reference frames for $k=k_o$ (subpar. 3.d.2)
4.e Like in Galilean reference frames for $k=k_o$ (subpar. 3.e)
3.f The physical situation doesn't have sense for Einsteinian reference frames

## 5. Relativistic dynamics of massive elementary particles in the gravitational field

For charged massive elementary particles the gravitational motion[2] is complicated by the fact that those particles have an electrodynamic mass that changes with the speed according to the relation

$$m' = m_o\left(1 - \frac{1}{2}\,\frac{v^2}{c^2}\right) \qquad (34)$$

where $m_o$ is the resting electrodynamic mass of particle.
Because in the event of electrodynamic particles it is possible to suppose that the effect of medium is practically null (i.e. $k=k_o=0$), then we can do the following conclusions:

**5.a** Free fall of electrodynamic particle in resting laboratory

Because $k_o=0$ the free fall of particle is independent of mass and consequently it happens like in the event of an ordinary body in concordance with the motion law (3) with respect to reference frame S of celestial body and with the motion law (4) with respect to the reference frame S' of the laboratory, whether in the event of Galilean or Einsteinian references.

**5.b** Electrodynamic particle at rest on the bottom of the laboratory in free fall

An electrodynamic particle, supposed at rest on the bottom of the laboratory, like any ordinary body, would respect the same law of motion of the laboratory in free fall with respect to Galilean reference frames, while this physical situation doesn' have particular sense for Einsteinian references.

**5.c** Free fall of electrodynamic particle in the laboratory in fall

The motion law  of electrodynamic particle is the same as for an ordinary body, supposing that  $k=k_o=0$, for Galilean references while this motion doesn't have particular meaning for Einsteinian references.

**5.d**  Forced fall of electrodynamic particle into falling laboratory

In that event  the force F, that is applied to particle, has electric or magnetic nature and it can be whether concordant or discordant  (antigravity) with the gravity force. Because in both cases the motion law with respect to the laboratory depends always on electrodynamic mass of particle[9], also when $k_o=0$, it follows that the motion law of particle in forced fall is always different  from the motion law of an ordinary body in forced fall, whether for Galilean references or Einsteinian.

**5.e**  Like in the preceding case

**5.f**  Like in the preceding case.

## 6.  Relativistic dynamics of gravitational motions of second type

The gravitational motion of second type[1][3]  consists in the orbital motion of a massive physical system into a field with central symmetry. Motion can be circular or elliptic.
Let us consider  two cases:
6.a   orbital motion round the non-rotating barycentre of celestial mass
6.b   orbital motion with respect to the rotating surface of celestial mass

**6.a**   Let us consider the orbital motion of a laboratory with mass m, represented by the reference frame S'[O',x',y',z',t'], around celestial body with mass M, represented by the reference frame S[O,x,y,z,t], that can be considered a resting system and that has the origin in the barycentre O of the celestial mass. Let us suppose then that inside the laboratory there is an ordinary body with mass $m_o$  that is placed in the origin O' of the reference frame S'  (fig.11).
The mass M acts on the mass m of the laboratory through the gravitational force

$$\mathbf{F_g} = - \frac{G\,M\,m}{r^2}\,\mathbf{u'_x} \qquad (35)$$

where  $\mathbf{u'_x}$  is the unitary vector along the direction x'. If the orbital motion of the laboratory is supposed circular, it happens with constant angular speed  $\omega$ and the distance r is practically constant. The centrifugal force  $\mathbf{F_c}$  in every point of the orbit is equal and opposite to the gravitational force that functions in that event as centripetal force  $\mathbf{F_c} = - \mathbf{F_g}$.
Anyway the equilibrium between the two forces generates an orbital motion because of the kinetic energy of the laboratory that is supplied by the initial inertial tangential speed v.

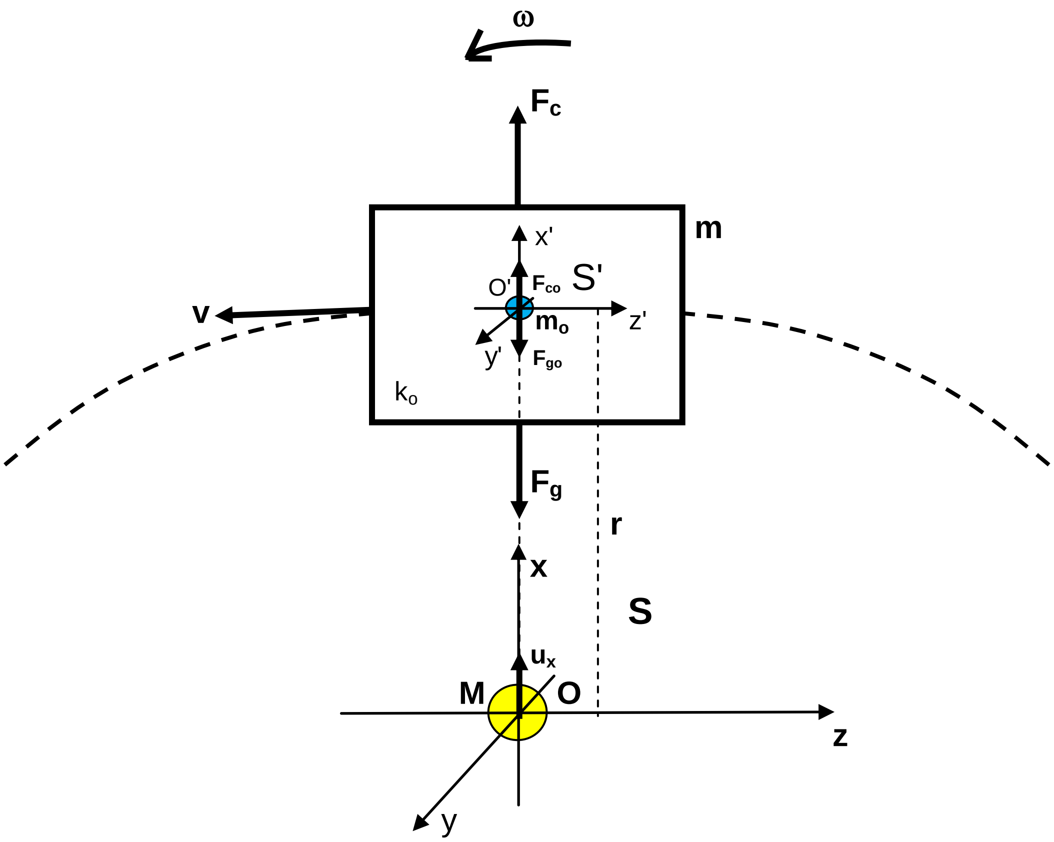


Fig.11 Laboratory orbiting round the resting barycentre O of celestial mass M that generates the gravitational field of second type

The question that we raise now is: what physical behaviour of the body with mass $m_o$ is inside the laboratory?

**6.a.1** If let us suppose that mass $m_o$ is still inside the laboratory, the same forces act on it as on the laboratory with mass m: i.e. the gravitational force or centripetal force $\mathbf{F_{go}}$ and the centrifugal force $\mathbf{F_{co}}$ and these two forces are equal and opposite:

$$\mathbf{F_{co}} = - \mathbf{F_{go}} = \frac{G\, M\, m_o}{r^2}\, \mathbf{u'_x} \qquad (36)$$

Whether the angular speed $\omega$ or the tangential speed **v** are applied to the mass $m_o$, that is integral part of the laboratory. Consequently because the relative speed between the mass $m_o$ and the laboratory is zero, it follows that the mass $m_o$ moves likewise the laboratory with speed **v** along the orbit and the principle of Relativity is satisfied with respect to two reference frames S and S'.

**6.a.2** If instead let us suppose that the body with mass $m_o$ is subject to a force **F** with respect to the reference frame S' of the laboratory (fig.12), then it isn't immobile inside the laboratory but it acquires a motion $\mathbf{v_o'}$ that is defined by the motion equation, in scalar shape, with respect to S'

$$F(t) = m_o \frac{dv_o'(t')}{dt'} + k_o v_o'(t') \qquad (37)$$

The motion law, in the event that the appied force F is constant, is therefore

$$v_o'(t') = \frac{F}{k_o}\left(1 - e^{-k_o t'/m_o}\right) \qquad (38)$$

Supposing that $k_o$=0, the motion law with respect to S' is

$$v_o'(t') = \frac{F}{m_o} t' \qquad (39)$$

The speed $v_o'$ has vector nature, its intensity is given by (39) and it has always a trajectory that is tangential with respect to the orbital trajectory.
Passing from the moving reference frame S' to the reference frame S, supposed at rest, there aren't new other forces the act on the mass $m_o$, but it needs to consider the vector speed **v** of the laboratory with respect to S, for which the vector speed $\mathbf{v_o}$ of mass $m_o$ with respect to S is given by the vector sum

$$\mathbf{v_o} = \mathbf{v_o'} + \mathbf{v} \qquad (40)$$

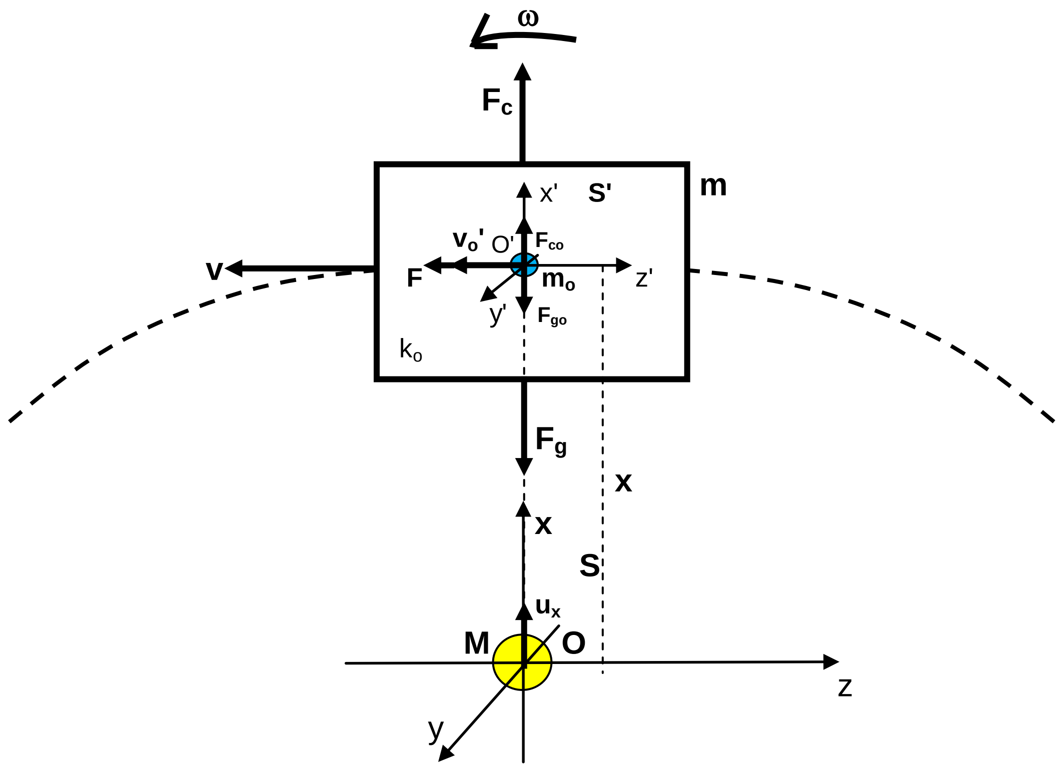


Fig.12 The mass $m_o$ is subject to the action of a force F inside the orbital laboratory

**6.b** Let us suppose now that the reference frame S is placed on the surface of the rotating celestial body (for instance the Earth) with constant angular speed $\Omega$, radius R and tangential speed $v_\Omega$ (fig.13).

**6.b.1** Naturally if the mass $m_o$ is still in the reference frame S', the mass motion with respect to the observer O of the reference frame S coincides with the motion of the laboratory considering the effective relative speed $\mathbf{v_o}$ is given at any time by the vector sum

$$\mathbf{v_o} = \mathbf{v} - \mathbf{v_\Omega} \qquad (41)$$

**6.b.2** If instead the mass $m_o$ is subject to the force **F** then the motion law with respect to S' is given by the equation

$$F(t) = m_o \frac{dv_o'(t)}{dt} + k_o v_o'(t) \qquad (42)$$

in which the speed $v_o'$ is given by the relation (38) or by the relation (39) for $k_o$=0. In that case

$$\mathbf{v_o} = \mathbf{v_o}' + \mathbf{v} - \mathbf{v_\Omega} \qquad (43)$$

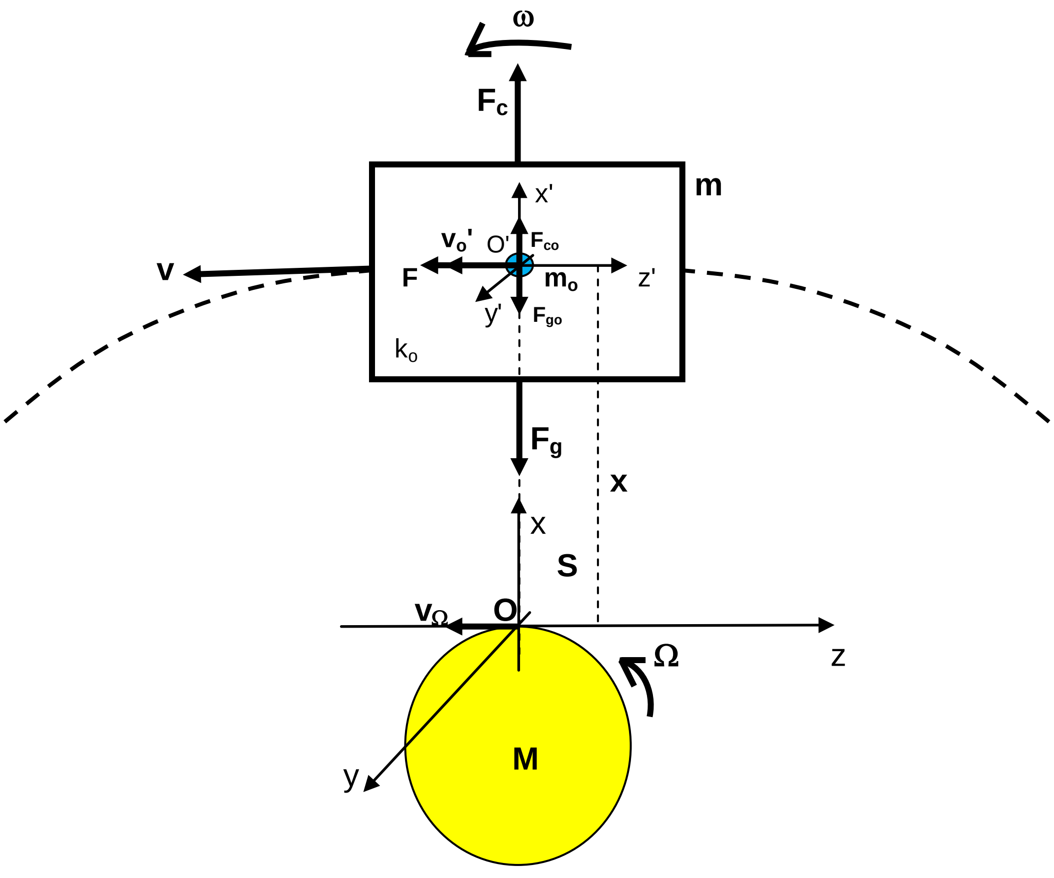


Fig.13 The reference frame S is placed on the surface of the rotating celestial mass

In the event that $m_o$ isn't the mass of an ordinary body but it is the electrodynamic mass of a massive elementary particle, it needs to consider the electrodynamic mass changes with the speed in concordance with the relation (34) and supposing that $k_o$=0, the motion law (39) has to be replaced with the relation

$$v_o'(t') = \frac{F}{m_o}\left(1 - \frac{v^2}{2c^2}\right)t' \qquad (44)$$

where $m_o$ is mass of the resting elementary particle.

## 7. Relativistic effects of free fall of a mass into a laboratory in uniform motion that is perpendicular to the fall direction

Let us want to examine the relativistic behaviour of a mass $m_o$ in fall into a laboratory, that has mass m and provided with uniform motion at constant speed $v_o$ (fig.14). The motion equation is given by (2) and the motion law of fall with respect to S', for $k_o$=0, is

$$v'(x') = \sqrt{\frac{2\,G\,M}{x_o' + R}\;\frac{x_o' - x'}{x' + R}} \qquad (45)$$

The total speed $v_t$ of the mass $m_o$ with respect to S is in vector shape

$$\mathbf{v_t} = \mathbf{v'} + \mathbf{v_o} \qquad (46)$$

and in scalar shape

$$v_t = \sqrt{\frac{2GM\,(x_o - x)}{x_o x} + v_o^2} \qquad (47)$$

With respect to system S' the motion of mass $m_o$ happens according to the law of perpendicular fall along the axis x', but with respect to the reference system S the fall happens along a non-perpendicular direction that is determined by vectors $\mathbf{v'}$ and $\mathbf{v_o}$. The two observers, everyone into the personal reference frame, S or S', see two different falls: the observer in S' sees a perpendicular fall while the observer in S sees an oblique fall. In spite of this, the Principle of Relativity is respected because the motion equation of the mass $m_o$ with respect to S' is

$$\frac{GMm_o}{x^2} = m_o\frac{dv'(t)}{dt} + k_o v'(t) \qquad (48)$$

while the motion equation of the same mass $m_o$ with respect to the reference frame S is

$$\frac{GMm_o}{x^2} = m_o\frac{dv_t(t)}{dt} + k_o v_t(t) \qquad (49)$$

The (49), related to S, is invariant with respect to the (48), that is related to S', and consequently the Principle of Relativity is respected in this physical situation.

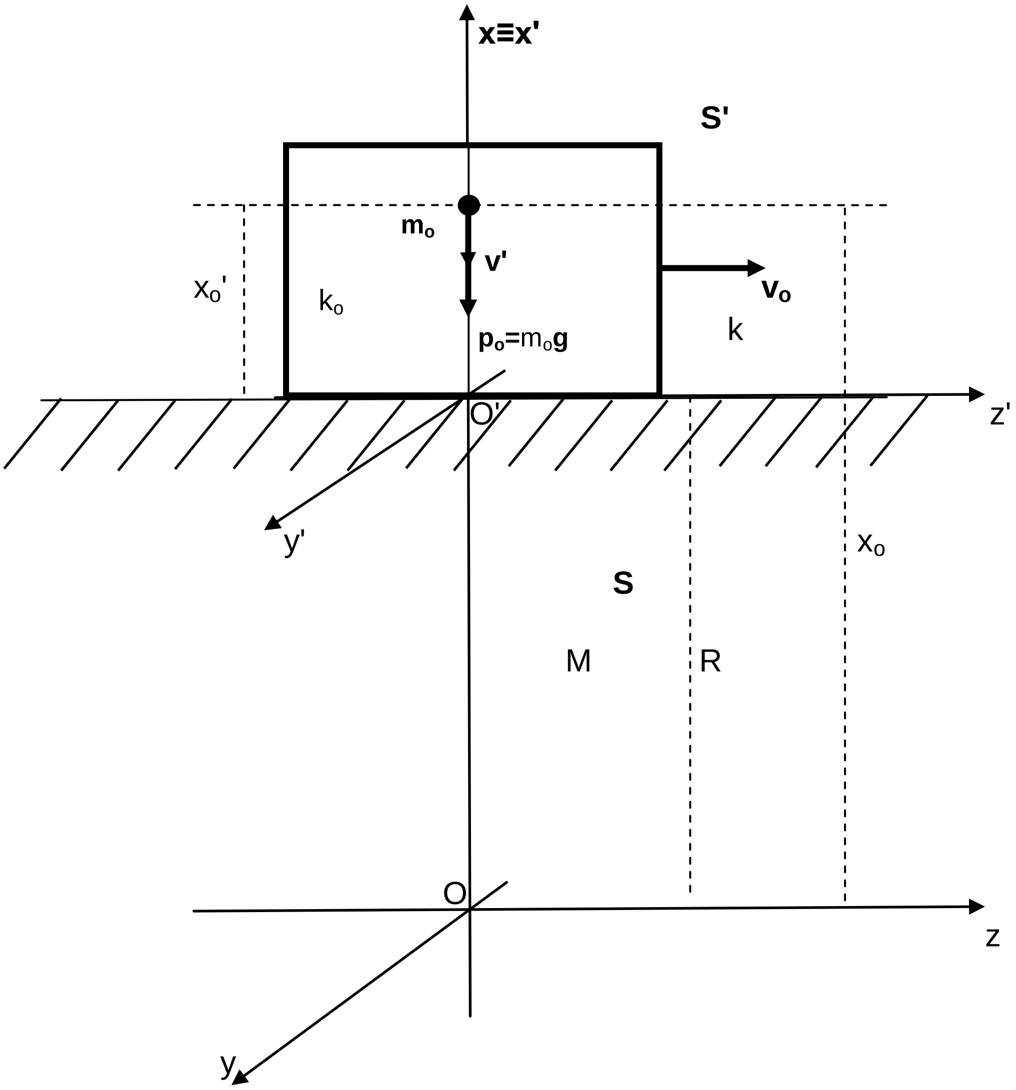


Fig.14 Free fall of an ordinary body with mass $m_o$ into a moving laboratory with uniform speed $v_o$.

## References


[1] D.Sasso, Physics of Gravitational Fields, viXra.org, 2014, id: 1405.0028

[2] D. Sasso, Dynamics of Motion in Gravitational Fields of First Type, viXra.org, 2015, id: 1506.0014

[3] D. Sasso; Gravitational Field of Second Type, Motions of Precession and the Fourth Law of Orbital Motions; viXra.org; 2015; id: 1509.0016

[4] G. Galilei, Dialogue Concerning the Two Chief world Systems, Batista Landini Editor, Florence, 1632

[5] A. Einstein, On Electrodynamics of Moving Bodies, Annales of Physics, 1905

[6] D. Sasso, Relativistic Effects of the Theory of Reference Frames, Physics Essays, 2007, Volume 20, No.1, DOI: 10.4006/1.3073804

[7] D.Sasso, Physico-Mathematical Fundamentals of the Theory of Reference Frames, viXra.org, 2013, id: 1309.0009

[8] D. Sasso, Dynamics and Electrodynamics of Moving Real Systems in the Theory of Reference Frames, arXiv.org, 2010, id: 1001.2382

[9] D. Sasso, On Physical Behavior of Elementary Particles in Force Fields, arXiv.org, 2014, id: 1409.0013