On the Quantum Mechanics

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Abstract

There are two supposed equivalent versions of the quantum mechanics: the matrix mechanics and the wave mechanics. I think that both would be false.

Key words: quantum mechanics, matrix mechanics, wave mechanics.

1. Introduction.

The so-called quantum mechanics (QM) was first developed by Heisenberg in 1925 and was called (soon) the matrix mechanics (MM). Next, but in the same year, Schrödinger developed a second version called the wave mechanics (WM). I think that both would be false.

2. The matrix mechanics.

Heisenberg [1] applied the Fourier series expansion to the position of the (atomic) electron (of the Hydrogen atom), classically

$$x(n,t) = \sum_{\alpha=-\infty}^{\infty} a_\alpha(n)\exp(i\alpha\omega(n)t) \tag{2.1}$$

and quantumly

$$x(n,t) = \sum_{\alpha=-\infty}^{\infty} a(n,n-\alpha)\exp(i\omega(n,n-\alpha)t) \tag{2.2}$$

where x is a spatial coordinate, n the number of the (stationary) state, t the time, α an integer, a the amplitude, ω the angular frequency and i the imaginary unit.

$\omega(n, n - \alpha)$ represents the angular frequency associated with the transition between the states described by the (quantum) numbers n and $n - \alpha$.

(2.2) implies that $\omega(n, n - \alpha) = \alpha\omega(n)$, which is true for very large values of n (Bohr's correspondence principle) [2] (pp. 125-126).

But (2.1) implies that the electron can oscillate with the angular frequencies $\omega(n)$ and its harmonics $\alpha\omega(n)$, which is false because, for a given value of n, it can oscillate and radiate only with the angular frequency $\omega(n)$ without harmonics.

In effect, the centripetal force on the electron is equal to the electric force (Coulomb's force)

$$m\frac{v^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} \tag{2.3}$$

where m, v, r and e, are the moving mass, the speed, the radius of the orbit and the electric charge of the electron, respectively, and ε_0 the electric permittivity of the vacuum. The stationary orbit condition is

$$mvr = n\hbar \tag{2.4}$$

where $\hbar = h/2\pi$ and h is the Planck's constant. And also

$$\omega = \frac{v}{r} \tag{2.5}$$

Putting the harmonic $\alpha\omega(n)$ in the above equations, the following results are obtained: $\alpha\omega = \alpha v/r$, $m(\alpha\omega)r^2 = \alpha n\hbar$ and $m(\alpha\omega)^2 r = \alpha^2 e^2/4\pi\varepsilon_0 r^2$. These imply that the speed of the electron in the orbit, the number of the state and the required electric force would be respectively: αv, αn and $\alpha^2 e^2/4\pi\varepsilon_0 r^2$, all of which are not true.

Therefore, although (2.1) might be mathematically correct, it would not be true from the point of view of the classical physics. Classically, the x-component of the position of the orbit would be

$$x(n,t) = a(n)cos(\omega(n)t + \theta(n)) \tag{2.6}$$

θ being a possible initial angle.

(2.1) and (2.2) are in essence the base for the MM, but as (2.1) is not true physically and (2.2) is obtained from (2.1), then the MM would not be true.

As from (2.1) and (2.2), it is obtained the (canonical) commutation relation [3, 4]

$$[x, p_x] = xp_x - p_x x = i\hbar I \tag{2.7}$$

where x and p_x are the x-components of the position and of the momentum of the electron, respectively, and I the unit (or identity) matrix; and as from (2.7), it is obtained the Heisenberg's uncertainty principle [5] (pp. 276-277)

$$\Delta x \Delta p_x \geq \frac{\hbar}{2} \tag{2.8}$$

where Δx and Δp_x are the uncertainties in the x-components of the position and of the momentum of the electron, respectively; then, both, (2.7) and (2.8), would not be true.

3. The wave mechanics.

The WM is based in the de Broglie equation

$$mv = \frac{h}{\lambda} \tag{3.1}$$

where m, v and λ are, respectively, the moving mass, the speed and the wavelength of the (atomic) electron (and $p = mv$ is its momentum).

Multiplying the Newton's equation of the energy

$$E = T + V = \frac{p^2}{2m_0} + V \tag{3.2}$$

(where E, T, V, p and m_0 are, respectively, the total, kinetic and potential energies, the momentum and the rest mass of the electron) by the so-called wave function Ψ, and substituting E by the operator $i\hbar\partial/\partial t$ and p by $-i\hbar\nabla$ (where $\nabla = (\partial/\partial x, \partial/\partial y, \partial/\partial z)$, and x, y and z are the spatial coordinates), we have the Schrödinger's equation

$$i\hbar\frac{\partial}{\partial t}\Psi(\vec{r},t) = \left[-\frac{\hbar^2}{2m_0}\nabla^2 + V(\vec{r},t)\right]\Psi(\vec{r},t) \tag{3.3}$$

where $\vec{r} = (x, y, z)$.

As a single real wave does not represent correctly a particle, it was introduced the wave packet using the Fourier transform. Then, Ψ is not anymore a real wave function representing a single real wave but a complex wave function representing a wave packet. And, proposed by Born, $|\Psi|^2 = \Psi\Psi^*$ (where Ψ^* is the complex conjugate of Ψ) represents the probability density ρ of finding the particle at the point (x, y, z) at the instant t, which implies that

$$\Psi = \rho^{1/2}e^{i\alpha} \tag{3.4}$$

where α is an argument ($\Psi^* = \rho^{1/2}e^{-i\alpha}$).

But (3.1), (3.3) and (3.4) are based on suppositions, not in real facts, then the WM would not be true.

As the wave function does not exist, there are no waves of matter it is only a supposition, all the equations that use wave functions are incorrect. This invalidates the equations such as: (3.3), Klein-Gordon, Dirac, etc.

4. Conclusion.

There are two supposed equivalent versions of the QM: the MM and the WM. I think that both would be false.

References

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