Quantum and Classical Mechanics of Hamiltonian Systems Having Exponential-Type Potentials

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Abstract

The problem to solve consists of finding explicit solutions for the classical equation of motion of a particle subject to an exponential-type potential and for the quantum version.

1. Consider in this section, according to [1], the general anharmonic potential of the form

$$V(x) = kg(x)e^{\gamma\varphi(x)} \tag{1}$$

where k and γ are arbitrary parameters, $g(x)$ and $\varphi(x)$ are arbitrary functions of x. The corresponding classical equation of motion may then be written as

$$\ddot{x} + [kg'(x) + \gamma kg(x)\varphi'(x)]e^{\gamma\varphi(x)} = 0 \tag{2}$$

where prime means differentiation with respect to x and overdot denotes differentiation with respect to time. A typical example of the potential (1) may be written

$$V(x) = kx^2 e^{\gamma x^2} \tag{3}$$

so that the equation (2) becomes

$$\ddot{x} + 2kx(1 + \gamma x^2)e^{\gamma x^2} = 0 \tag{4}$$

and the associated time-independent Schrödinger equation takes the form

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + kx^2 e^{\gamma x^2}\right]\psi(x) = E\psi(x) \tag{5}$$

that is

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}\left(E - kx^2 e^{\gamma x^2}\right)\psi(x) = 0 \tag{6}$$

where $\psi(x)$ designates the wave function and E the energy. The equation (6) will be investigated as future work. The anharmonic potentials of the form

$$V(x) = f(x)\int x^l e^{\gamma\varphi(x)}dx \tag{7}$$

$$V(x) = ax^l \ln(x_0 - bx^q) \tag{8}$$

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where $\ln$ designates the natural logarithm, $f(x)$ is an arbitrary function of x, a, b, l, q and x_0 are arbitrary parameters, will be also investigated as future works.

Reference

[1] J. Akande , D. K. K. Adjaï, L. H. Koudahoun, Y. J. F. Kpomahou, M. D. Monsia, Schrödinger equation for a system with exponential-type restoring force function, viXra:1609.0142v1.(2016).