Quantum Statistical Theory of Quantum Cognition

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Abstract : In quantum mechanics a wave function is connected to a quantum system. This abstract entity represents a description of the quantum state of this system. The wave function is a complex-valued probability amplitude and the probabilities for the possible results of measurements made on the system can be derived from it . In the present paper it is demonstrated that the tentative often adopted from scholars to attribute a wave function as representation of a mental state in current quantum cognition studies cannot be accepted. A mental state is a so complex structure , depending contextually at each instant from a so complex structure of inner and subjective elements and psychological and neurological components, to exclude its representation by a simple and elementary wave function, The mental state may be represented by a quantum wave function that, owing of the complex structure of the mental state, has an intrinsic indetermination and fluctuation of the basic probability amplitudes to respond to the basic notion of intrinsecally , incompletely specified system that was introduced by P. T. Landsberg in 1961. As consequence a simple quantum wave function cannot be adopted and instead an averaging statistical formulation is absolutely required. Therefore, the subsequent quantum statistical theory is formulated.

Keywords : quantum cognition, quantum statistical theory in quantum cognition, dispersion free ensembles.

1. Introduction.

Starting with 1983, following our other previous and preliminary studies, continuing with the current our results, we prospected and delineate that our consciousness as well as our basic neurological and psychological functions of perception and cognition find in the principles of quantum mechanics the first physical, theoretical and experimental way of foundation. We will not enter here in the details of the previously obtained results, recalled of course shortly in references 1-26.. since the purpose of the present paper is to introduce additional finishing elements in order to allow the scholars to fully understand the nature of the studies that we performed avoiding the possible spread of inaccuracies and approximations that at present it is possible to warn in some literature.

It is well known that in studies of quantum mechanics it is used currently the concept of wave function ,usually indicated by $\psi(t)$. In our previous studies we analyzed the perceptive and cognitive functions of subjects submitted to tasks based in particular on ambiguous figures but also analyzing neurological and psychological processes as the Stroop effect, priming, cognitive anomalies as conjunction fallacy, cognitive - emotive conflicts in children, and still more (1-28). In all such studies we admitted a wave function $\psi(t)$ existing at mental level and characterizing , as function of knowledge, the mental state of the subject at the moment in which the task is posed In this case the subject, in a condition of inner conflict, poses to himself potential alternatives respect to the perception, cognition, identification and choice about two fundamental steps, "what is it ?" or "where is it ?". From information point of view a quantum-bit entity is realized at mental level. In our studies we have evidenced that at the neurological level the following brain structures are involved : the V1(the striate cortex), V2(secondary visual/pre-striate cortex),V3(ventral and dorsal complexes),V4(color center in extra-striate visual cortex/prelunategyrus), and BA20 (Inferior temporal, Fusiform and Parahippocampalgyri). The neurological model was formulated by using the Mach-Zehnder interferometer that is a standard quantum mechanical device used in quantum mechanics to demonstrate the peculiar features of quantum interference. The reason to introduce this quantum mechanical model was to demonstrate that not only experiments executed but tasks given to subjects give results confirming existing quantum interference at the levels of human perception and cognition and that also the basic neurological structures of the brain confirm that this is exactly what really happens. All the details of the model are formulated in [26,27,28] and therefore we will not insist more on it. Previously, by using a Clifford algebraic quantum formulation, we discussed the Mach-Zehnder interferometer [11] on the basis of a pure logic viewpoint outlining the logical origins of quantum mechanics and confirming that basic Clifford elements are quantum-bit representative [29,30,31].

In the present paper we intend to evidence that our position to associate a wave function to a mental state, requires to be understood in detail in its meaning. The considered matter in itself is not so simple as often one could be induced to retain. The distance of this our formulation from the purely traditional physical approach is considerable and therefore one immediately falls into error when thinking on this specific matter of quantum cognition only assuming simplistically an analogy with the corresponding physical situation that we elaborate when we take in consideration physical systems. We are distant light years from a so simple and trivial exemplification. Therefore we will formulate now some theoretical evaluations to elucidate the matter once again.

Let us consider that we give to a subject an ambiguous figure admitting two alternatives. Using quantum mechanics and, in particular the superposition principle, we formulated a quantum model admitting that as consequence of such input, it is realized a contextually installation in the mind of the subject of a mental state that, in quantum mechanical terms, may be represented by a wave function $\psi(t)$ containing the two alternatives

$$\psi(t) = c_1(t)\left|the\ \ first\, alternative\right\rangle + c_2(t)\left|the \sec ond\ alternative\right\rangle$$

being $p_1(t) = \left|c_1(t)\right|^2$ and $p_2(t) = \left|c_2(t)\right|^2$ the probabilities that, as subsequent transition of such initial superposition of alternatives, the subject will finally actualize one and only one of them.

This is precisely what we do in physics when we study a simple quantum mechanical system. In fact we use the wave function $\psi(t)$ as the representation of the quantum state of the system. In this case the wave function is a complex-valued probability amplitude and the probabilities for the possible results of measurements made on the system can be derived from it as described.

Considering now quantum cognition studies, we cannot naively retain to have obviously exhausted the matter and have so exhaustively represented the considerable complexity of a mental state with the use of such an elementary instrument as the scheme previously indicated. It should be clear that it was used by us only with the finality to give an indicative representation as in each model that an author formulates. Each model is a simple bridge that is formulated between reality and its representation . Soon after it must be intended and deepened. In this case, we have to consider what really happens by application of such Conte theoretical model in principle and in the experimental case of the previously mentioned experiments. Some observations are important. As is known, tasks are given to groups of subjects. Consider the experimental case conducted on a large number of respondents as it was in the different experimental studies that were conducted. Having given the task to a group of subjects, each of them will bring into it the uniqueness of his(her) mind, each one with a specific conceptual network forming his(her) inner memory structure, learning, cognition, emotion. Each subject will have his/her wave function as complex-valued probability amplitude and probabilities for the possible results . This is expected since participants in the experiment will have di erent and individual psychological patterns that will engage the individual perceptive, cognitive and emotional inner dynamics when performing their selection of the possible alternatives choosing his (her) answers among those that are available to be selected. Also from a physical point of view differences will be evident. As example, the timed answers of each subject cannot be accurately defined as in an ordinary experiment of physics , depending instead upon the personal and subjective Reaction Time (R.T). RT is a physical valuable variable that is a psychological and subjective variable that literature has shown to be linked to a great many psycho-physiologic and neurological factors. These are only small but enough indications to highlight what is the real macroscopic and complex status of the matter when we consider Conte model with the considered quantum mechanical wave function $\psi(t)$ as representative of the mental state of the subject . We have to consider the basic features and the intrinsic complexity of each mental state of each subject and the increased complexity relating experiments that involve a group of subjects. This is the first point that should be carefully considered when starting to study quantum cognition. Of course, this is a situation that is well known in physics. Technically it is called as the question of a macro-observer M that analyzes a large system. In principle one may think of the observer and the system to be fully defined, but there may be an incompleteness in the specification of the wave function . For example the previously mentioned question of the time cannot really be known accurately for macroscopic measurements which always extend over time intervals. This is the case of our quantum cognition studies. Each subject responds with his/her time . Consequently for each subject we have to consider contextually the time of measurement t . The same features arise when considering a group of subjects. Consequently, one should average over an interval which includes t, and the weighted influence of the other wave functions $\psi(t \pm \delta)$ that consequently lead to a probability distribution for this quantity . Another circumstance is connected to general conditions of the considered system so that an incomplete knowledge of $\psi(t)$ may be due to insufficient information being available concerning the initial states $\psi(0)$. Each subject at any time starts contextually with an initial state $\psi(0)$. Mental states as well as mental experimental contexts are all marked from some specific and peculiar subjective features. The conceptual consequence of such a situation is at an ontological level , and it represents what P.T. Landsberg called a condition of an incompletely specified system [29,30,31, 32,33,34]. The substance of such considerations is that when we use quantum mechanics applied to analyze the case of mental conditions we must not consider the wave function $\psi(t)$ simplistically but intending correctly that $\psi(t)$ is subjected to a class of averaging processes and a probability distribution . Therefore in order to characterize in detail quantum cognition studies, we will to formulate the theory of such averaging process.

2. The Theoretical Elaboration

Suppose a Macro-Observer studies a so much large and complex system.

Considering von Neumann we may use in analogy the notion to decompose the energy shell of phase space into a maximum number, N, of macroscopically distinguishable phase cells. The $\nu - th$

phase cell contains $s(\nu)$ quantum states . If we indicate by $\psi(t)$ the wave function of this system at time t, we have scalar products

$a_{t,\nu j}=(\psi(t),\omega(\nu,j))$

that must be considered. The class of the previously mentioned averaging process is defined in terms of these.

The basis $\{\omega(\nu,j)\}$ ($\nu=1,2,.....,N$; $j=1,2,......,s(\nu)$; $\sum_{1}^{N} s(\nu)=S$)

decomposes the energy shell of phase space into a maximum number , N , of distinguishable phase cells.

Mental states are admitted in analogy as lying in an energy shell in phase space. In quantum mechanical terms identify a dimensionality L_0. We have to consider that the wave function $\psi(t)$ may lie also in an appropriate subspace having dimensionality L_1 of L_0. Similarly , we may have a wave function lying in a subspace of dimensionality L_2 This hierarchy may be continued until a sub-space of dimensionality L_r is reached so that $L_r \geq 1, L_{r-1}=0$.

This is the way to characterize the complex dynamics in the attribution of a wave function $\psi(t)$ to mental states. Owing to their complexity, they are grouped in subspaces and for each subspace we have specified dimensionality with the following relation holding

$L_0 \geq L_1 \geq L_2 \geq ....L_{r-1} \geq L_r \geq L_{r-1}=0$

The orthonormal vectors $\omega_1,\omega_2,.....,\omega_r$ are a basis for L_r and this set is completed to give a basis for L_{r-1} and then it is again completed to give a basis for L_{r-2} until the basis of L_0 is reached.

The wave function of the mental state at time t is

$\psi(t)=U_t\psi(0)$

where U_t is the unitary time evolution operator.

The initial mental state may be admitted in L_0 as possible and the probability to lie in L_1 is

$$p_1(t) = \sum_{j=1}^{L_1} \left| a_{t,j} \right|^2$$

and depends on the initial mental conditions of the subject. It is certainly to consider that we have different mental conditions in the subject at the initial time as due to the intrinsic complexity of the nature of a mental state and consequently, subsisting so much and complex initial conditions of the mental state of each subject, it becomes necessary to average over such conditions. Consequently in quantum cognition studies we must estimate the average probability relating the mental state and we have

$$< p_1(t) >= \sum_{j=1}^{L_1} < \left| a_{t,j} \right|^2 >$$

still depending on the above mentioned initial mental conditions of the subject.

The spread of the individual values of $p_1(t)$ due to the different initial mental conditions will be given by the second moment

$$Z_1 = \frac{< [p_1(t) - < p_1(t) >]^2 >}{[< p_1(t) >]^2} = \frac{< p_1^2(t) >}{< p_1(t) >^2} - 1$$

In the same manner we may of course estimate $Z_2, Z_3, ...$ and so on . Such indices give estimation of our quantum cognition performance .

3. An Example of Application.

Consider the case of our experiments. Always we used a dichotomic experimental situation with a posed task admitting only two possible alternatives, or Yes or Not.

Consequently, we have to consider a two-dimensional space, S = 2, and the projections of a unit vector on the two Cartesian basis vectors ω_1, ω_2 so that s(l) = s(2) = 1. Only two variables occur: to exemplify let us admit $a_1 = \cos\alpha, a_2 = sen\alpha$. The situation in mental state must be structured as it follows. The condition, say y, which completes the specification of the mental situation in its specificity, inner subjective structure, for each subject in its contexuality, is then represented by the angle α that cannot be considered to have a single and definite value, valid always for each subject and for all the participants, but it extends from 0 to 2π . The sense of this sentence is that the inner mental state fluctuates in the α values for each subject in his /her mental status at the moment with a probability of being an angle lying between α and $\alpha + d\alpha$ that we have to estimate . To this purpose, we shall assume that, with b a constant,

$$\int f(\alpha)d\alpha = 1 \ ; \ f(\alpha)d\alpha = Asen^{b}(2\alpha)d\alpha \ ; \ A = \frac{2}{\sqrt{\pi}}\left[\Gamma(1/2(b+2)/\Gamma(1/2(b+1)\right]\left[1+(-1)^{b}\right]^{-2}$$

assuming a weakly uniform distribution $f(\alpha)$ for α .

On this basis we may estimate

$<|a_i|^2>$, $<|a_i|^4>$, and $<|a_i a_j|^2>$, $i = 1,2$, $j = 1,2, i \neq j$

as required for the estimation of the second moment as previously expressed by Z .

4 . Conclusion

A rule of quantum mechanics is that we connect to a quantum system a wave function that represents a description of the quantum state of this system. The wave function is a complex-valued probability amplitude and the probabilities for the possible results of measurements made on the system can be derived from it . In the present paper it is demonstrated that the tentative often adopted from scholars to attribute a wave function as representation of a mental state in current quantum cognition studies, cannot be accepted. A mental state is a so complex structure, depending contextually at each instant from a so complex structure of inner and subjective elements and psychological and neurological components, to exclude its representation by a simple and elementary wave function, The mental state may be represented by a quantum wave function that, owing of the complex structure of the mental state, has an intrinsic indetermination and fluctuation of the basic probability amplitudes to respond to the basic notion of intrinsecally , incompletely specified system that was introduced by P. T. Landsberg in 1961. As consequence a simple quantum wave function cannot be adopted and instead an averaging statistical formulation is absolutely required. A detailed description of the basic features of a mental state, leading to an intrinsecally, incompletely specified system is given in the paper and the subsequent quantum statistical theory is given.

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