Search for physical origin of intelligence

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I can calculate the motion of heavenly bodies, but not the madness of people.

Isaac Newton

What I cannot create, I do not understand.
Richard Feynman.

Abstract.
The challenge of this work is to connect physics with the concept of intelligence. By intelligence we understand a capability to move from disorder to order without external resources, i.e. in violation of the second law of thermodynamics. The objective is to find such a mathematical object described by ODE that possesses such a capability. The proposed approach is based upon modification of the Madelung version of the Schrodinger equation by replacing the force following from quantum potential with non-conservative forces that link to the concept of information. A mathematical formalism suggests that a hypothetical intelligent particle, besides the capability to move against the second law of thermodynamics, acquires such properties like self-image, self-awareness, self- supervision, etc. that are typical for Livings. However since this particle being a quantum-classical hybrid acquires non-Newtonian and non-quantum properties, it does not belong to the physics matter as we know it: the modern physics should be complemented with the concept of the *information force* that represents a bridge to intelligent particle. As a follow-up of the proposed concept, the following question is addressed: can artificial intelligence (AI) system composed only of physical components compete with a human? The answer is proven to be negative if the AI system is based only on simulations, and positive if digital devices are included. It has been demonstrated that there exists such a quantum neural net that performs simulations combined with digital punctuations. The universality of this quantum-classical hybrid is in capability to violate the second law of thermodynamics by moving from disorder to order without external resources. This advanced capability is illustrated by examples. In conclusion, a mathematical machinery of the ***perception*** that is the fundamental part of a cognition process as well as intelligence is introduced and discussed.

The discovery of isolated dynamical systems that can decrease entropy in violation of the second law of thermodynamics, and resemblances of these systems to livings implies that Life can slow down heat death of the Universe, and that can be associated with the purpose of Life.

1.Introduction.
The recent statement about completeness of the physical picture of our Universe made in Geneva raised many questions, and one of them is the ability to create Life and Intelligence out of physical matter without any additional entities. The main difference between living and non-living matter is in directions of their evolution: it has been recently recognized that the evolution of livings is progressive in a sense that it is directed to the highest levels of complexity. Such a property is not consistent with the behavior of *isolated* Newtonian systems that cannot increase their complexity without external forces. That difference created so called Schrödinger paradox: in a world governed by the second law of thermodynamics, all isolated systems are expected to approach a state of maximum *disorder;* since life approaches and maintains a highly *ordered* state – one can argue that this violates the Second Law implicating a paradox,[1]

But livings are not isolated due to such processes as metabolism and reproduction: the increase of order inside an organism is compensated by an increase in disorder outside this organism, and that removes the paradox. Nevertheless it is still tempting to find a mechanism that drives livings from disorder to order. The purpose of this paper is to demonstrate that moving from a disorder to order is not a prerogative of open systems: an isolated system can do it without help from outside. However such system cannot belong to the world of the modern physics: it belongs to the world of living matter, and that lead us to the concept of an intelligent particle – the first step to physics of livings. In order to introduce such a particle, we start with an idealized mathematical model of livings by addressing only one aspect of Life: a ***biosignature,*** i.e. ***mechanical*** invariants of Life, and in particular, the ***geometry and kinematics of intelligent behavior*** disregarding other aspects of Life such as metabolism and reproduction. By narrowing the problem in this way, we are able to extend the mathematical formalism of physics' First Principles to include description of intelligent behavior. At the same time, by ignoring metabolism and reproduction, we can make the system isolated, and it will be a challenge to show that it still can move from disorder to order.

2. Starting with quantum mechanics.

The starting point of our approach is the Madelung equation that is a hydrodynamics version of the Schrödinger equation

$$\frac{\partial \rho}{\partial t} + \nabla \bullet \left(\frac{\rho}{m}\nabla S\right) = 0 \tag{1}$$

$$\frac{\partial S}{\partial t} + (\nabla S)^2 + F - \frac{\hbar^2 \nabla^2 \sqrt{\rho}}{2m\sqrt{\rho}} = 0 \tag{2}$$

Here ρ and S are the components of the wave function $\psi = \sqrt{\rho} e^{iS/\hbar}$, and $\hbar$ is the Planck constant divided by 2π. The last term in Eq. (2) is known as quantum potential. From the viewpoint of Newtonian mechanics, Eq. (1) expresses continuity of the flow of probability density, and Eq. (2) is the Hamilton-Jacobi equation for the action S of the particle. Actually the quantum potential in Eq. (2), as a feedback from Eq. (1) to Eq. (2), represents the difference between the Newtonian and quantum mechanics, and therefore, it is solely responsible for fundamental quantum properties.

The Madelung equations (1), and (2) can be converted to the Schrödinger equation using the ansatz

$$\sqrt{\rho} = \Psi \exp(-iS/h) \tag{3}$$

where ρ and S being real function.

Our approach is based upon a modification of the Madelung equation, and in particular, upon replacing the quantum potential with a different Liouville feedback, Fig.1

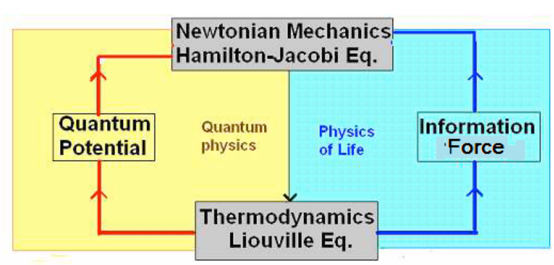

Figure 1. Classic Physics, Quantum Physics and Physics of Life.

In Newtonian physics, the concept of probability ρ is introduced via the Liouville equation

$$\frac{\partial \rho}{\partial t} + \nabla \bullet (\rho \mathbf{F}) = 0 \tag{4}$$

generated by the system of ODE

$$\frac{d\mathbf{v}}{dt} = \mathbf{F}[\mathbf{v}_1(t),...\mathbf{v}_n(t),t] \tag{5}$$

where **v** is velocity vector.
It describes the continuity of the probability density flow originated by the error distribution

$$\rho_0 = \rho(t=0) \tag{6}$$

in the initial condition of ODE (6).

Let us rewrite Eq. (2) in the following form

$$\frac{d\mathbf{v}}{dt} = \mathbf{F}[\rho(\mathbf{v})] \tag{7}$$

where **v** is a velocity of a hypothetical particle.

This is a fundamental step in our approach: in Newtonian dynamics, the probability never explicitly enters the equation of motion. In addition to that, the Liouville equation generated by Eq. (7) is nonlinear with respect to the probability density ρ

$$\frac{\partial \rho}{\partial t} + \nabla \bullet \{\rho \mathbf{F}[\rho(\mathbf{V})]\} = 0 \tag{8}$$

and therefore, the system (7),(8) departs from Newtonian dynamics. However although it has the same topology as quantum mechanics (since now the equation of motion is coupled with the equation of continuity of probability density), it does not belong to it either. Indeed Eq. (7) is more general than the Hamilton-Jacoby equation (2): it is not necessarily conservative, and **F** is not necessarily the quantum potential although further we will impose some restriction upon it that links **F** to the concept of information. The relation of the system (7), (8) to Newtonian and quantum physics is illustrated in Fig.1.

Remark. Here and below we make distinction between the random *variable v(t)* and its *values V* in probability space.

3. Information force instead of quantum potential.

In this section we propose the structure of the force **F** that plays the role of a feedback from the Liouville equation (8) to the equation of motion (7). Turning to one-dimensional case, let us specify this feedback as

$$F = c_0 + \frac{1}{2}c_1\rho - \frac{c_2}{\rho}\frac{\partial \rho}{\partial v} + \frac{c_3}{\rho}\frac{\partial^2 \rho}{\partial v^2} \tag{9}$$

$$c_0 > 0, c_1 > 0, c_3 > 0 \tag{10}$$

Then Eq.(9) can be reduced to the following:

$$\dot{v} = c_0 + \frac{1}{2}c_1\rho - \frac{c_2}{\rho}\frac{\partial \rho}{\partial v} + \frac{c_3}{\rho}\frac{\partial^2 \rho}{\partial v^2} \tag{11}$$

and the corresponding Liouville equation will turn into the nonlinear PDE

$$\frac{\partial\rho}{\partial t}+(c_0+c_1\rho)\frac{\partial\rho}{\partial V}-c_2\frac{\partial^2\rho}{\partial V^2}+c_3\frac{\partial^3\rho}{\partial V^3}=0 \quad (12)$$

This equation is known as the KdV-Bergers' PDE. The mathematical theory behind the KdV equation became rich and interesting, and, in the broad sense, it is a topic of active mathematical research. A homogeneous version of this equation that illustrates its distinguished properties is nonlinear PDE of parabolic type. But a fundamental difference between the standard KdV-Bergers equation and Eq. (12) is that Eq. (12) ***dwells in the probability space***, and therefore, it must satisfy the normalization constraint

$$\int_{-\infty}^{\infty}\rho\, dV=1 \quad (13)$$

However as shown in [2], this constraint is satisfied: in physical space it expresses conservation of mass, and it can be easily scale-down to the constraint (13) in probability space. That allows one to apply all the known results directly to Eq. (12). However it should be noticed that all the conservation invariants have different physical meaning: they are not related to conservation of momentum and energy, but rather impose constraints upon the Shannon information.

In physical space, Eq. (12) has many applications from shallow waves to shock waves and solitons. However, application of solutions of the same equations in probability space is fundamentally different. In the following sections we will present a phenomena that exist neither in Newtonian nor in quantum physics.

4. Emergence of randomness.

In this section we discuss a fundamentally new phenomenon: transition from determinism to randomness in ODE that coupled with their Liouville PDE.

In order to complete the solution of the system (11), (12), one has to substitute the solution of Eq. (12):

$$\rho=\rho(V,t) \quad at \quad V=v \quad (14)$$

into Eq.(11). Since the transition from determinism to randomness occurs at $t\to 0$, let us turn to Eq. (12) with sharp initial condition

$$\rho_0(V)=\delta(V)\, at \qquad t=0, \quad (15)$$

Then applying one of the standard analytical approximations of the delta-function, one obtains the asymptotic solution

$$\rho=\frac{1}{t\sqrt{\pi}}e^{-\frac{V^2}{t^2}} \quad at \quad t\to 0 \quad (16)$$

Substitution this solution into Eq. (11) shows that

$$O(c_0+\frac{1}{2}c_1\rho)=\frac{1}{t}, \quad O(\frac{c_2}{\rho}\frac{\partial\rho}{\partial v})=\frac{1}{t^2},$$
$$and \quad O(\frac{c_3}{\rho}\frac{\partial^2\rho}{\partial v^2})=\frac{1}{t^4} \quad at \quad t\to 0, \quad v\neq 0 \quad (17)$$

i.e.

$$c_0+\frac{1}{2}c_1\rho<<\frac{c_2}{\rho}\frac{\partial\rho}{\partial v}<<\frac{c_3}{\rho}\frac{\partial^2\rho}{\partial v^2} \quad at \quad t\to 0, \quad v\neq 0 \quad (18)$$

and therefore, the first three terms in Eq. (11) can be ignored

$$\dot{v}=\frac{c_3}{\rho}\frac{\partial^2\rho}{\partial v^2} \quad at \quad t\to 0, \quad v\neq 0 \quad (19)$$

or after substitution of Eq. (16)

$$\dot{v} = \frac{4c_3 v^2}{t^4} \quad at \quad t \to 0, \quad v \neq 0 \tag{20}$$

Eq. (20) has the following solution (see Fig. 2)

$$v = \frac{t^3}{4c_3 + Ct^3} \quad at \quad t \to 0, \quad v \neq 0 \tag{21}$$

where C is an arbitrary constant.

This solution has the following property: the Lipchitz condition at $t \to 0$ fails

$$\frac{\partial \dot{v}}{\partial v} = \frac{8c_3 v}{t^4} = \frac{8c_3 t^3}{t^4(4c_3 + Ct^3)} \to \infty \quad at \quad t \to 0, \quad v \neq 0 \tag{22}$$

and as a result of that, the uniqueness of the solution is lost. Indeed, as follows from Eq. (21), for any value of the arbitrary constant C, the solutions are different, but they satisfy the same initial condition

$$v \to 0 \quad at \quad t \to 0 \tag{23}$$

Due to violation of the Lipchitz condition (22), the solution becomes unstable. That kind of instability when infinitesimal errors lead to finite deviations from basic motion (the Lipchitz instability) has been discussed in [3]. This instability leads to unpredictable shift of solution from one value of *C* to another. It means that appearance of any specified solution out of the whole family is random, and that randomness is controlled by the feedback (9) from the Liouville equation (12). Indeed if the solution (21) runs independently many times with the same initial conditions, and the statistics is collected, the probability density will satisfy the Liouville equation (12), Fig.3.

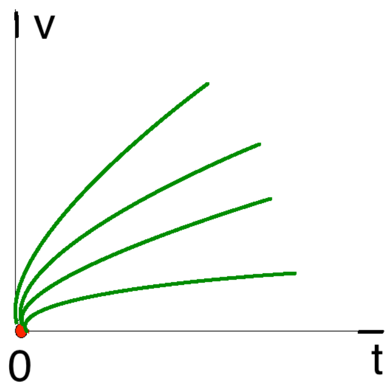

Figure 2. Family of random solutions describing transition from determinism to stochastisity.

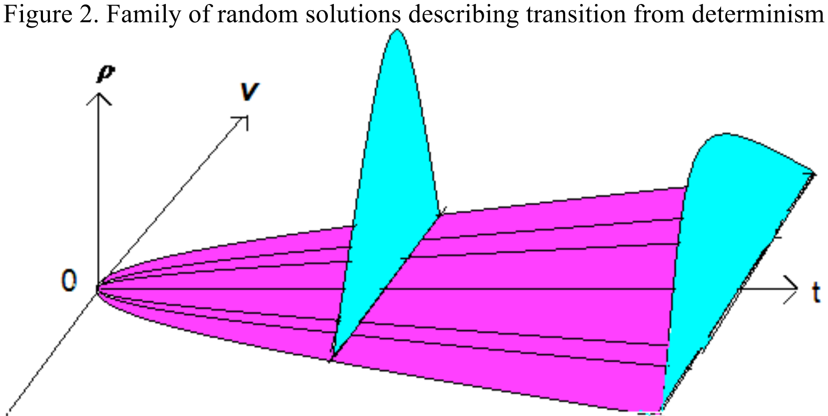

Figure 3. Stochastic process and probability density.

5. Departure from Newtonian and quantum physics.

In this section we will derive a distinguished property of the system (16),(17) that is associated with violation of the second law of thermodynamics i.e. with the capability of moving from disorder to order without help from outside. That property can be predicted qualitatively even prior to analytical proof: due to the nonlinear term in Eq. (17), the solution form shock waves and solitons in probability space, and that can be interpreted as "concentrations" of probability density, i.e. departure from disorder. In order to demonstrate it analytically, let us turn to Eq. (17) at

$$c_1 >> |c_2|, c_3 \tag{24}$$

and find the change of entropy H

$$\frac{\partial H}{\partial t} = -\frac{\partial}{\partial t}\int_{-\infty}^{\infty} \rho \ln \rho \, dV = -\int_{-\infty}^{\infty} \dot{\rho}(\ln \rho + 1)\, dV = \int_{-\infty}^{\infty} c_1 \frac{\partial}{\partial V}(\rho^2)(\ln \rho + 1) dV$$
$$= c_1 [\, \Big|_{-\infty}^{\infty} \rho^2 (\ln \rho + 1) - \int_{-\infty}^{\infty} \rho \, dV] = -c_1 < 0 \tag{25}$$

At the same time, the original system (11), (12) is isolated: it has no external interactions. Indeed the information force Eq. (9) is generated by the Liouville equation that, in turn, is generated by the equation of motion (11). In addition to that, the particle described by ODE (11) is in equilibrium $\dot{v} = 0$ prior to activation of the feedback (9). Therefore the solution of Eqs. (11), and (12) can violate the second law of thermodynamics, and that means that this class of dynamical systems does not belong to physics as we know it. This conclusion triggers the following question: are there any phenomena in Nature that can be linked to dynamical systems (11), (12)? The answer will be discussed bellow.

Thus despite the mathematical similarity between Eq.(12) and the KdV-Bergers equation, the physical interpretation of Eq.(12) is fundamentally different: it is a part of the dynamical system (11),(12) in which Eq. (12) plays the role of the Liouville equation generated by Eq. (11). As follows from Eq. (25), this system, being isolated and being in equilibrium, has the capability to decrease entropy, i.e. to move from disorder to order without external resources. In addition to that, the system displays transition from deterministic state to randomness (see Eq. (22)).

This property represents departure from classical and quantum physics, and, as shown in [2,3], provides a link to behavior of livings. That suggests that this kind of dynamics requires extension of modern physics to include physics of life.

The process of violation of the second law of thermodynamics is illustrated in Fig. 4: the higher values of ρ propagate faster than lower ones. As a result, the moving front becomes steeper and steeper, and that leads to formation of solitons (c_3>0), or shock waves (c_3=0) ***in probability space.*** This process is accompanied by decrease of entropy.

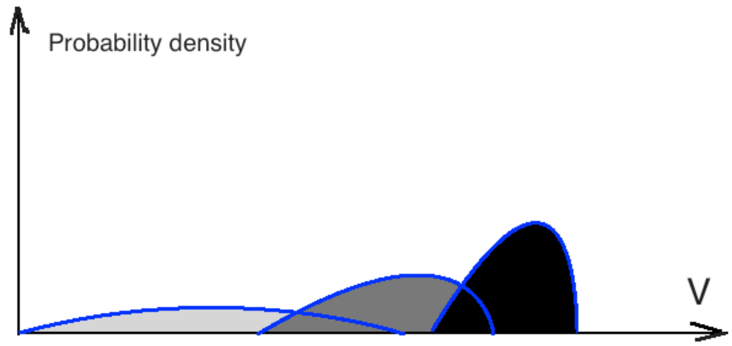

Figure 4. Formation of shock waves in probability space.

Remark. The system (11), (12) displays transition from deterministic state to randomness (see Eq. (22)), and this property can be linked to the similar property of the Madelung equation, although strictly speaking, Eq.(1) is a "truncated" version of the Liouvile equation: it does not include the contribution of the quantum potential.

6. Comparison with quantum mechanics.

a. Mathematical Viewpoint. The model of intelligent particle is represented by a nonlinear ODE (7) and a nonlinear parabolic PDE (8) coupled in a master-slave fashion: Eq. (8) is to be solved independently, prior to solving Eq. ((7). The coupling is implemented by a feedback that includes the probability density and its space derivatives, and that converts the first order PDE (the Liouville equation) to the second or higher order nonlinear PDE. As a result of the nonlinearity, the solutions to PDE can have attractors (static, periodic, or chaotic) in probability space. The solution of ODE (7) represents another major departure from classical ODE: due to violation of Lipchitz conditions at states where the probability density has a sharp value, the solution loses its uniqueness and becomes random. However, this randomness is controlled by the PDE (8) in such a way that each random sample occurs with the corresponding probability, Fig.3.

b. Physical Viewpoint. The model of intelligent particle represents a fundamental departure from both Newtonian and quantum mechanics. The fundamental departure of all the modern physics is the violation of the second laws of thermodynamics,(see Eq.(25), and Fig. 4). However a more detailed analysis, [3], shows that due to similar dynamics topology to quantum mechanics,(see Fig.1) the model preserves some quantum properties such as entanglement and interference of probabilities.

c. Biological Viewpoint. The L_1 model illuminates the "border line" between living and non-living systems. The model introduces a biological particle that, in addition to Newtonian properties, possesses the ability to process information. The probability density can be associated with the self-image of the biological particle as a member of the class to which this particle belongs, while its ability to convert the density into the information force - with the self-awareness (both these concepts are adopted from psychology). Continuing this line of associations, the equation of motion (such as Eqs (3.11)) can be identified with a motor dynamics, while the evolution of density (see Eqs. (3.12) –with a mental dynamics. Actually the mental dynamics plays the role of the Maxwell sorting demon: it rearranges the probability distribution by creating the information potential and converting it into a force that is applied to the particle. One should notice that mental dynamics describes evolution of the whole class of state variables (differed from each other only by initial conditions), and that can be associated with the ability to generalize it as a privilege of living systems. Continuing our biologically inspired interpretation, it should be recalled that the second law of thermodynamics states that the entropy of an isolated system can only increase. This law has a clear probabilistic interpretation: increase of entropy corresponds to the passage of the system from less probable to more probable states, while the highest probability of the most disordered state (that is the state with the highest entropy) follows from a simple combinatorial analysis. However, this statement is correct only if there is no Maxwell' sorting demon, i.e., nobody inside the system is rearranging the probability distributions. But this is precisely what the Liouville feedback is doing: it takes the probability density ρ from Equation (3.12), creates functionals and functions of this density, converts them into a force and applies this force to the equation of motion (3.11). As already mentioned above, because of that property of the model, the evolution of the probability density becomes nonlinear, and the entropy may decrease "against the second law of thermodynamics", Fig.6. Obviously the last statement should not be taken literary; indeed, the proposed model captures only those aspects of the living systems that are associated with their behavior, and in particular, with their motor-mental dynamics, since other properties are beyond the dynamical formalism. Therefore, such physiological processes that are needed for the metabolism are not included into the model. That is why this model is in a formal disagreement with the second law of thermodynamics while the living systems are not. In order to further illustrate the connection between the life-nonlife discrimination and the second law of thermodynamics, consider a small physical particle in a state of random migration due to thermal energy, and compare its diffusion i.e. physical random walk, with a biological random walk performed by a bacterium. The fundamental difference between these two types of motions (that may be indistinguishable in physical space) can be detected in probability space: the probability density evolution of the physical particle is always linear and it has only one attractor: a stationary stochastic process where the motion is trapped. On the contrary, a typical probability density evolution of a biological particle is nonlinear: it can have many dif- ferent attractors, but eventually each attractor can be departed from without any "help" from outside.

That is how H. Berg, [11], describes the random walk of an E. coli bacterium:" If a cell can diffuse this well by working at the limit imposed by rotational Brownian movement, why does it bother to tumble? The answer is that the tumble provides the cell with a mechanism for biasing its random walk. When it swims in a spatial gradient of a chemical attractant or repellent and it happens to run in a favorable direction, the probability of tumbling is reduced. As a result, favorable runs are extended, and the cell diffuses with drift". Berg argues that the cell analyzes its sensory cue and generates the bias internally, by changing the way in which it rotates its flagella. This description demonstrates that actually a bacterium interacts with the medium, i.e., it is not isolated, and that reconciles its behavior with the second law of thermodynamics. However, since these interactions are beyond the dynamical world, they are incorporated into the proposed model via the self- supervised forces that result from the interactions of a biological particle with "itself," and that formally "violates" the second law of thermodynamics. Thus, the L_1 model offers a unified description of the progressive evolution of living systems. Based upon this model, one can formulate and implement the principle of maximum increase of complexity that governs the large-time-scale evolution of living systems. It should be noticed that at this stage, our interpretation is based upon logical extension of the proposed mathematical formalism, and is not yet corroborated by experiments.

7. Origin of intelligence.

a. Relevance to model of intelligent particle. The proposed model illuminates the "border line" between living and non-living systems. The model introduces an intelligent particle that, in addition to Newtonian properties, possesses the ability to process information. The probability density can be associated with the *self-image* of the intelligent particle as a member of the class to which this particle belongs, while its ability to convert the density into the information force - with the *self-awareness* (both these concepts are adopted from psychology). Continuing this line of associations, the equation of motion (such as Eq (11)) can be identified with a motor dynamics, while the evolution of density (see Eq. (12)) –with a mental dynamics. Actually the mental dynamics plays the role of the Maxwell sorting demon: it rearranges the probability distribution by creating the information potential and converting it into a force that is applied to the particle. One should notice that mental dynamics describes evolution of the whole class of state variables (differed from each other only by initial conditions), and that can be associated with the ability to generalize that is a privilege of intelligent systems. Continuing our biologically inspired interpretation, it should be recalled that the second law of thermodynamics states that the entropy of an isolated system can only increase. This law has a clear probabilistic interpretation: increase of entropy corresponds to the passage of the system from less probable to more probable states, while the highest probability of the most disordered state (that is the state with the highest entropy) follows from a simple combinatorial analysis. However, this statement is correct only if there is no Maxwell' sorting demon, i.e., nobody inside the system is rearranging the probability distributions. But this is precisely what the Liouville feedback is doing: it takes the probability density ρ from Equation (12), creates functions of this density, converts them into the information force and applies this force to the equation of motion (11). As already mentioned above, because of that property of the model, the evolution of the probability density can become nonlinear, and the entropy may decrease "against the second law of thermodynamics". Actually the proposed model represents governing equations for interactions of intelligent agents. In order to emphasize the autonomy of the agents' decision-making process, we will associate the proposed models with ***self-supervised (SS) active systems.*** By an active system we will understand here a set of interacting intelligent agents capable of processing information, while an intelligent agent is an autonomous entity, which observes and acts upon an environment and directs its activity towards achieving goals. The active system is not derivable from the Lagrange or Hamilton principles, but it is rather created for information processing. One of specific differences between active and physical systems is that the former are supposed to act in uncertainties originated from incompleteness of information. Indeed, an intelligent agent almost never has access to the whole truth of its environment. Uncertainty can also arise because of incompleteness and incorrectness in the agent's understanding of the properties of the environment. That is why *quantum-inspired SS* systems represented by the particles under consideration are well suited for representation of active systems, and the hypothetical particle introduced above can be associated with the term "intelligent" particle. It is important to emphasize that self-supervision is implemented by the feedback from mental dynamics, i.e. by internal force, since the mental dynamics is generated by intelligent particle itself.

b. Comparison with control systems. In this sub-section we will establish a link between the concepts of intelligent control and phenomenology of behavior of intelligent particle.
*Example**.*** One of the limitations of classical dynamics, and in particular, neural networks, is inability to change their structure without an external input. As will be shown below, an intelligent particle can change the locations and even the type of the attractors being triggered only by information forces i.e. by an internal effort. We will start with a simple dynamical system

$$\dot{v} = 0, \quad v = 0 \quad at\; t = 0 \tag{26}$$

and than apply the following control

$$F = -k\overline{v} + a\overline{\overline{v}} - \sigma\frac{\partial}{\partial v}\ln\rho\,, \tag{27}$$

where $$\overline{\overline{V}} = \int_{-\infty}^{\infty}\rho(V-\overline{V})^2\,dV, \qquad \overline{V} = \int_{-\infty}^{\infty}\rho V dV\,, \tag{28}$$

and k, a, σ are constant coefficients.

Then the controlled version of the motor dynamics (26) is changed to

$$\dot{v} = -k\overline{v} + a\overline{\overline{v}} - \sigma\frac{\partial}{\partial v}\ln\rho \tag{29}$$

while F represents the information forces that play the role of ***internal*** actuator.

Let us notice that the internal actuator (27) is a particular case of the information force (9) at

$$c_0 = -k\overline{v} + a\overline{\overline{v}}, \quad c_1 = 0, \quad c_2 = \sigma, \quad c_3 = 0 \tag{30}$$

For a closure, Eq. (29) is complemented by the corresponding Liouville equation

$$\frac{\partial \rho}{\partial t} = k\overline{V}\frac{\partial \rho}{\partial V} - a\overline{\overline{V}}\frac{\partial \rho}{\partial V} + \sigma\frac{\partial^2 \rho}{\partial V^2}, \qquad (31)$$

to be solved subject to sharp initial condition

$$\rho_0(V) = \delta(V)\, at \qquad t = 0, \qquad (32)$$

As shown above, the solution of Eq.(29) is random, (see Eq. (21) and Fig. 2) while this randomness is controlled by Eq. (31). Therefore in order to describe it, we have to transfer to the mean values $\overline{v}$ and $\overline{\overline{v}}$. For that purpose, let us multiply Eq.(31) by V .Then integrating it with respect to V over the whole space, one arrives at ODE for the expectation $\overline{v}(t)$

$$\dot{\overline{v}} = -k\overline{v} + a\overline{\overline{v}} \qquad \mathbf{(33)}$$

Multiplying Eq.(31) by V^2, then integrating it with respect to V over the whole space, one arrives at ODE for the variance $\overline{\overline{v}}(t)$

$$\dot{\overline{\overline{v}}} = -2k\overline{\overline{v}} + 2a\overline{v}\,\overline{\overline{v}} + 2\sigma \qquad (34)$$

Let us find fixed points of the system (33) and (34) by solving the system of algebraic equations:

$$0 = -k\overline{v} + a\overline{\overline{v}} \qquad (35)$$

$$0 = -2k\overline{\overline{v}} + 2a\overline{v}\,\overline{\overline{v}} + 2\sigma \qquad (36)$$

By selecting

$$\sigma = \frac{k^3}{2a^2} \qquad (37)$$

we arrive at the following single fixed point

$$\overline{v}^* = \frac{k}{2a}, \quad \overline{\overline{v}}^* = \frac{k^2}{2a^2} \qquad (38)$$

In order to establish whether this fixed point is an attractor or a repeller, we have to analyze stability of the homogeneous version of the system (33), (34) linearized with respect to the fixed point (38)

$$\dot{\overline{v}} = -k\overline{v} + a\overline{\overline{v}} \qquad (39)$$

$$\dot{\overline{\overline{v}}} = -k\overline{\overline{v}} + \frac{k^2}{a}\overline{v} \qquad (40)$$

Analysis of its characteristic equation shows that it has non-positive roots:

$$\lambda_1 = 0, \quad \lambda_2 = -2k < 0$$

and therefore, the fixed point (38) is a stochastic attractor with stationary mean and variance. However the higher moments of the probability density are not necessarily stationary: they can be found from the original PDE (31).

Thus as a result of a ***mental*** control, an ***isolated*** dynamical system (26) that prior to control was at rest, moves to the stochastic attractor (38) having the expectation $\overline{v}^{\,*}$ and the variance $\overline{\overline{v}}^{\,*}$.

The distinguished property of the particle introduced above definitely fits into the concept of intelligence. Indeed, the evolution of intelligent living systems is directed toward the highest levels of complexity if the complexity is measured by an irreducible number of different parts that interact in a well-regulated fashion. At the same time, the solutions to the models based upon dissipative Newtonian dynamics eventually approach attractors where the evolution stops while these attractors dwell on the subspaces of lower dimensionality, and therefore, of the lower complexity (until a "master" reprograms the model). Therefore, such models fail to provide an autonomous progressive evolution of intelligent systems (i.e. evolution leading to increase of complexity). At the same time, a self-controlled particle can create its own complexity based only upon an *internal* effort.

Thus the actual source of intelligent behavior of the particle introduced above is a new type of force - the information force - that contributes its work into the Law of conservation of energy. However this force is internal: it is generated by the particle itself with help of the Liouvile equation. The machinery of the intelligence is similar to that of control system with the only difference that control systems are driven by external actuators while the intelligent particle is driven by a feedback from the Liouvile equation without any external resources. New modification of intelligent particle that lead to modeling decisions based upon intuition and utilizing interference of probabilities are introduced in [12].

8. Quantum recurrent neural nets.

a. Background.

In the previous sections, we presented a ***mathematical*** answer to the ancient philosophical question "How mind is related to matter". We proved that in mathematical world, the bridge from matter to mind requires extension and modification of quantum physics. In this context, we will comment on the recent statement made by Stephen Hawking on December 2, 2014, in which he warns that artificial intelligence could end mankind. Based upon our work, part of which is presented in the previous sections, it can be stated that machines composed only out of physical components and ***without any digital devices*** being included, cannot, in principle, overperform a human in creativity, regardless of the level of technology. But what happens if a machine does include digital devices? The answer to this question is the subject of the following sections. In these sections we propose a quantum version of recurrent neural nets (QRN) that along with classical performance, possess the capability to move from disorder to order without external recourses, and that makes their intelligence comparable with that of a human. The QRN incorporate classical feedback loops into conventional quantum networks. It is shown, [4], that dynamical evolution of such networks, which interleave quantum evolution with measurement and reset operations, exhibits novel dynamical properties. Moreover, decoherence in quantum recurrent networks is less problematic than in conventional quantum network architectures due to the modest phase coherence times needed for network operation. It is proven that a hypothetical quantum computer can implement an exponentially larger number of the degrees of freedom within the same size.

It should be emphasized that the QRN presented below as an example of possible implementation of ODE model of intelligent particle, has a ***phenomenological*** rather than physical origin: it is driven by properties of recurrent neural nets that have quantum-like features, but dwell in the world of Newtonian scale. In other words, the QRN model belongs to quantum ***technology*** rather than to quantum ***theory***, and the clear illustration of this statement is exploitation of quantum collapse as a sigmoid function of QRN.

b. Quantum model of evolution.

A state of a quantum system is described by a special kind of time dependent vector $|\psi >$ with complex components called amplitudes:

$$\{a_0 a_1 \ldots a_n\} = |\psi > \qquad (41)$$

If unobserved, the state evolution is governed by the Schrödinger equation:

$$i\hbar \frac{da_k}{dt} = \sum_l H_{kl} a_l \qquad (42)$$

which is linear and reversible.

Here H_{kl} is the Hamiltonian of the system, $i = \sqrt{-1}$, $\hbar = 1.0545 \times 10^{-34} JS$.

The solution of Eq. (42) can be written in the following form:

$$\{a_0(t),..a_n(t)\} = \{a_0(0),..a_n(0)\} U^* \qquad (43)$$

where *U* is a unitary matrix uniquely defined by the Hamiltonian:

$$U = e^{-iHt/\hbar}, \qquad UU^* = I \qquad (44)$$

After *m* equal time steps Δt

$$\{a_0(m\Delta t),..a_n(m\Delta t)\} = \{a_0(0),..a_n(0)\} U^{*m} \qquad (45)$$

the transformation of the amplitudes formally looks like those of the transition probabilities in Markov chains. However, there is a fundamental difference between these two processes: in Eq. (45) the probabilities are represented not by the amplitudes, but by squares of their modules:

$$p = \{|a_0|^2, \ldots |a_n|^2\} \qquad (46)$$

and therefore, the unitary matrix *U* is not a transition probability matrix.

It turns out that this difference is the source of so called quantum interference, which makes quantum computing so attractive. Indeed, due to interference of quantum probabilities:

$$p = |a_1 + a_2|^2 \neq p_1 + p_2 \qquad (47)$$

each element of a new vector $a_i(m\Delta t)$ in Eq. (45) will appear with the probability $|a_i|^2$ that includes all the combinations of the amplitudes of the previous vector.

c. Quantum Collapse and Sigmoid Function.

As well known, neural nets have two universal features: dissipativity and nonlinearity. Due to dissipativity, a neural net can converge to an attractor and this convergence is accompanied by a loss of information. But such a loss is healthy: because of it, a neural net filters out insignificant features of a pattern vector while preserving only the invariants which characterizes it's belonging to a certain class of patterns. These invariants are stored in the attractor, and therefore, the process of convergence performs generalization: two different patterns that have the same invariants will converge to the same attractor. Obviously, this convergence is irreversible. The nonlinearity increases the neural net capacity: it provides many different attractors including static, periodic, chaotic and erogdic, and that allows one to store simultaneously many different patterns. Both dissipativity and nonlinearity are implemented in neural nets by the sigmoid (or squashing) function. It is important to emphasize that the only qualitative properties of the sigmoid function are those, which are important for the neural net performance, but not any specific forms of this function**.** Can we find a qualitative analog of a sigmoid function in quantum mechanics? Fortunately, yes: it is so called quantum collapse that occurs as a result of quantum measurements. Indeed, the result of any quantum measurement is always one of the eigenvalues of the operator corresponding to the observable being measured. In other words, a measurement maps a state vector of the amplitudes (41) into an eigenstate vector

$$\{a_0 a_1 .. a_n\} \to \{00 \ldots \underset{\uparrow_i}{1} \ldots 00\} \qquad (48)$$

while the probability that this will be the i^{th} eigenvector is:

$$p_i = |a_i|^2 \qquad (49)$$

The operation (49) is nonlinear, dissipative, and irreversible, and it can play the role of a natural "quantum" sigmoid function.

10. QRN Architectures.

Let us introduce the following sequence of transformations for the state vector (41):

$$|\psi(0)\rangle \to U|\psi(0)\rangle \to \sigma_1\{U|\psi(0)|\} = |\psi(t+1)\rangle \qquad (50)$$

which is a formal representation of Eq.(48)) with σ_1 denoting a "quantum" sigmoid function.
In order to continue this sequence, we have to reset the quantum device considering the resulting eigenstate as a new input. Then one arrives at the following neural net:

$$a_i(t+1) = \sigma_1\{\sum U_{ij} a_j(t)\}, \qquad i = 1.2...n \qquad ((51)$$

The curly brackets are intended to emphasize that σ_1 is to be taken as a measurement operation with the effect similar to those of a sigmoid function in classical neural networks (Fig. 5).

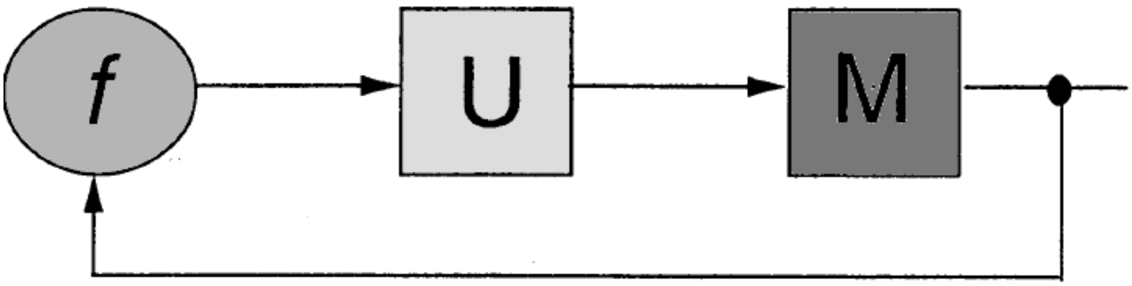

Figure 5. The simplest architecture of quantum neural net.

However, there are two significant differences between the quantum (51) and classical neural nets. Firstly, in Eq. (51) the randomness appears in the form of quantum measurements as a result of the probabilistic nature of the quantum mechanics, while in neural network a special device generating random numbers is required. Secondly, if the dimension of the classical matrix T_{ij} is $N \times N$, then within the same space one can arrange the unitary matrix U (or the Hamiltonian H) of dimension $2^N \times 2^N$ exploiting the quantum entanglement and direct product decomposability of the Schrödinger equation. One should notice that each non-diagonal element of the matrix H might consist of two independent components: real and imaginary. The only constraint imposed upon these elements is that H is the Hermitian matrix, i.e.,

$$H_{ij} = \overline{H}_{ji} \qquad (52)$$

and therefore, the $n \times n$ Hermitian matrix has n^2 independent components.
So far the architecture of the neural net (51) was based upon one measurement per each run of the quantum device. However, in general, one can repeat each run for l times $l \leq n$ collecting l independent measurements. Then, instead of the mapping (48), one arrives at the following best estimate of the new state vector:

$$\{a_0...a_n\} \to \{0...\underset{\uparrow_{i_1}}{\frac{1}{\sqrt{l}}}...0...\underset{\uparrow_{i_l}}{\frac{1}{\sqrt{l}}}...\} \qquad (53)$$

while the probability that the new state vector has non-zero i_k^{th} component is

$$p_{ik} = | a_{ik} |^2 \qquad (54)$$

Denoting the sigmoid function corresponding to the mapping (53) as σ_l, one can rewrite Eq. (51) in the following form:

$$a_i(t+1) = \sigma_l\{\sum U_{ij} a_j(t)\}, \qquad i = 1.2...n \qquad (55)$$

The next step in complexity of the ORN architecture can be obtained if one introduces several quantum devices with synchronized measurements and resets:

$$a_i^{(1)}(t+1) = \sigma_{l_1 l_2}\{\sum U_{ij}^{(1)} a_j^{(1)}(t)\}, \quad i = 1.2...n_1 \qquad (56)$$

$$a_i^{(2)}(t+1) = \sigma_{l_2 l_1}\{\sum U_{ij}^{(2)} a_j^{(2)}(t)\}, \qquad i \neq 1.2...n_2 \qquad (57)$$

Here the sigmoid functions $\sigma_{l_1 l_2}$ and $\sigma_{l_2 l_1}$, map the state vectors into a weighted mixtures of the measurements:

$$\{a_1^{(1)} \ldots a_n^{(1)}\} \to \frac{a_{11} a_{l_1}^{(1)} + a_{12} a_{l_2}^{(2)}}{|a_{11} a_{l_1}^{(1)} + a_{12} a_{l_2}^{(2)}|} \tag{58}$$

$$\{a_1^{(2)} \ldots a_n^{(2)}\} \to \frac{a_{21} a_{l_1}^{(1)} + a_{22} a_{l_2}^{(2)}}{|a_{21} a_{l_1}^{(1)} + a_{22} a_{l_2}^{(2)}|} \tag{59}$$

where $a_{l_1}^{(1)}$ and $a_{l_2}^{(2)}$ are the result of measurements presented in the form (53), and a_{11}, a_{12}, a_{21} and a_{22} are constants.

Thus, Eqs. (56) and (57) evolve independently during the quantum regime, i.e., in between two consequtive measurements; however, during the measurements and resets they are coupled via the Eqs. (58) and (59). It is easy to calculate that the neural nets (51), (55) and (56), (57) operate with patterns whose dimensions are $n, n(n-1)(n-l), n_1(n_1-1)(n_1-l), n_2(n_2-1)(n_2-l)$, respectively.

In a more general architecture, one can have K-parallel quantum devices U_i with l_i consequtive measurements M_i for each of them *(i=1,2...k)*, see Fig. 6.

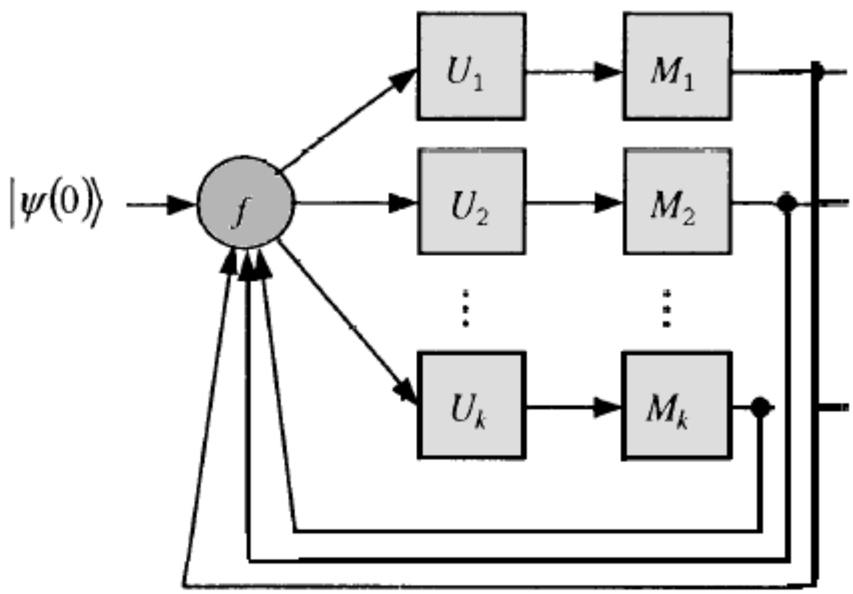

Figure 6*. The k-Parallel Quantum Neural Network Architecture*

Recall that one is free to record, duplicate or even monitor the sequence of measurement outcomes, as they are all merely bits and hence constitute classical information. Moreover, one is free to choose the function used during the reset phase, including the possibility of adding no offset state whatsoever. Such flexibility makes the QRN architecture remarkably versatile. To simulate a Markov process, it is sufficient to return just the last output state to the next input at each iteration.

11. Evolution of probabilities.

Let us take another look at Eq. (51). Actually it performs a mapping of an i^{th} eigenvector into an j^{th} eigenvector:

$$\{00...010...0\} \to \{00...010...0\} \qquad (60)$$
$$\quad\uparrow_i \qquad\qquad\qquad \uparrow_j$$

The probability of the transition (60) is uniquely defined by the unitary matrix *U*:

$$p_{ij} = |U_{ji}|^2, \qquad \sum_{i=1}^{n} p_{ij} = 1 \qquad (61)$$

and therefore the matrix $\| p_{ij} \|$ plays the role of the transition matrix in a generalized random walk which is represented by the chain of mapping (60).

Thus, the probabilistic performance of Eq. (51) has remarkable features: it is quantum (in a sense of the interference of probabilities) in between two consecutive measurements, and it is classical in description of the sequence of mapping (1). Applying the transition probability matrix (61) and starting, for example, with eigenstate $\{10...0\}$, one obtains the following sequence of the probability vectors:

$$\pi_0 = \{10...0\}; \quad \pi_1 = \{10...0\}\begin{pmatrix} p_{11}\cdots p_{11} \\ \dots\dots\dots\dots \\ p_{n1}\cdots p_{nn} \end{pmatrix} = \{\pi_1{}^1 ... \pi_n{}^1\}; etc \qquad (62)$$

An i^{th} component of the vector π_m, *i.e* $\pi_m{}^i$ expresses the probability that the system is in the i^{th} eigenstate after m steps. As follows from Eqs. (62), the evolution of probabilities is a linear stochastic process, although each particular realization of the solution to Eq. (51) evolves nonlinearly, and one of such realization is the maximum likelihood solution. In this context, the probability distribution over different particular realizations can be taken as a measure of possible deviations from the best estimate solution. However, the stochastic process (62) as an ensemble of particular realizations, has its own invariant characteristics which can be expressed independently on these realizations. One of such characteristics is the probability $f_{ij}{}^{(m)}$ that the transition from the eigenstate *i* to the eigenstate *j* is performed in *m* steps. This characteristic is expressed via the following recursive relationships, [5]:

$$\begin{aligned} &f_{ij}{}^{(1)} = p_{ij}{}^{(1)} = p_{ij}, f_{ij}{}^{(2)} = p_{ij}{}^{(2)} - f_{ij}{}^{(1)} p_{ij} \\ &f_{ij}{}^{(m)} = p_{ij}{}^{(m)} - f_{ij}{}^{(1)} p_{jj}{}^{(n-1)} - f_{ij}{}^{(3)} p_{jj}{}^{(n-2)} ... - f_{ij}{}^{(n-1)} p_{jj}. \end{aligned} \qquad (63)$$

If

$$\sum_{m=1}^{\infty} f_{ij}{}^{(m)} < 1 \qquad (64)$$

then the process initially in the eigenstate *i* may never reach the eigenstate *j*.

If $\sum_{m=1}^{\infty} f_{ij}{}^{(m)} = 1$ (65)

then the i^{th} eigenstate is a recurrent state, i.e., it can be visited more than once. In partiicular, if

$$p_{ii} = 1 \qquad (66)$$

this recurrent state is an absorbing one: the process will never leave it once it enters.

From the viewpoint of neural net performance, any absorbing state represents a deterministic static attractor without a possibility of "leaks." In this context, a recurrent, but not absorbing state can be associated with a periodic or an aperiodic (chaotic) attractor. To be more precise, an eigenstate *I* has a period τ $(\tau > 1)$ *if* $p_{ii}{}^{(m)} = 0$ whenever *m* is not divisible by τ, and τ is the largest integer with this property. The eigenstate is aperiodic

if $\tau = 1$ (67)

Another invariant characteristic which can be exploited for categorization and generalization is reducibility, i.e., partitioning of the states of a Markov chain into several disjoint classes in which motion is trapped. Indeed, each hierarchy of such classes can be used as a set of filters, which are passed by a pattern before it arrives at the smallest irreducible class whose all states are recurrent. For the purpose of evaluation of deviations (or "leaks") from the maximum likelihood solution, long-run properties of the evolution of probabilities (62) are important. Some of these properties are known from theory of Markov chains, namely: for any irreducible erogodic Markov chain the limit $p_{ij}^{(m)}$ exists and it is independent of *I*, .e.,

$$\lim p_{ij}^{(m)} = \pi_i \qquad at \qquad m \to \infty \qquad (68)$$

while

$$\pi_j > 0, \quad \pi_j = \sum_{i=0}^{k} \pi_i p_{ij}, \quad j = 0,1,..k, \qquad \sum_{j=}^{k} \pi_j = 1, \qquad \pi_j = \frac{1}{\mu_{ii}} \qquad (69)$$

Here μ_{ii} is the expected recurrence time

$$\mu_{ii} = 1 + \sum_{l \neq j} p_{ij} \mu_{li} < \infty \qquad (70)$$

The definition of ergodicity of a Markov chain is based upon the conditions for aperiodicity (67) and positive recurrence (68), while the condition for irreducibility requires existence of a value of *m* not dependent upon *i* and *j* for which $p_{ij}^{(m)}$ >0 for *i* and *j*. The convergence of the evolution (62) to a stationary stochastic process suggests additional tools for information processing. Indeed, such a process for *n*-dimensional eigenstates can be uniquely defined by *n* statistical invariants (for instance, by first *n* moments) which are calculated by summations over time rather than over the ensemble, and for that a single run of the quantum net (51) is sufficient. Hence, triggered by a simple eigenstate, a prescribed by *n*-invariants stochastic process can be retrieved and displayed for the purposes of Monte-Carlo computations, for modelling and prediction of behavior of stochastic systems, etc.

Continuing analysis of evolution of probability, let us introduce the following difference equation

$$\pi_i(t+\tau) = \sum_{j=1}^{n} \pi_j(t) p_{ij}, \qquad \sum_{i=1}^{n} \pi_i = 1, \qquad \pi_i \geq 0, \quad i = 1,2,..n \qquad (71)$$

It should be noticed that the vector $\pi = (\pi_1,..\pi_n)$ as well as the stochastic matrix p_{ij} exist only in an abstract Euclidean space: they never appear explicitly in physical space. The evolution (71) is also irreversible, but it is linear and deterministic.

The only way to reconstruct the probability vector $\pi(t)$ is to utilize the measurement results for the vector *a(t).* In general case, many different realizations of Eq. (60) are required for that purpose. However, if the condition (64) holds, the ergodic attractor $\pi = \pi^{\infty}$ can be found from the only one realization due to the ergodicity of the stochastic process. The ergodic attractor π^{∞} can be found analytically from the steady-state equations:

$$\pi_i^{\infty} = \sum_{j=1}^{n} p_{ij} \pi_j^{\infty}, \quad \sum_{i=1}^{n} \pi_i^{\infty} = 1, \qquad \sum_{j=1}^{n} p_{ij} = 1, \qquad \pi_i = 1, \quad p_{ij} = 0 \qquad (72)$$

This system of *n+1* equations with respect to *n* unknowns $\pi_i^{\infty} = 1,2,...n$ has a unique solution.

As an example, consider a two-state case (*n=2):*

$$p_{11}\pi_1^{\infty} + p_{21}\pi_2^{\infty} = \pi_1^{\infty}, \qquad p_{12}\pi_1^{\infty} + p_{22}\pi_2^{\infty} = \pi_2^{\infty} \qquad (73)$$

Utilizing the constraints in Eqs. (72) one obtains:

$$\pi_1^{\infty} = \frac{1-p_{22}}{2-(p_{11}+p_{22})}, \qquad \pi_2^{\infty} = \frac{1-p_{11}}{2-(p_{11}+p_{22})} \tag{74}$$

Hence on the first sight, there are infinite numbers of unitary matrices u_{ij}, which provide the same ergodic attractor. However, such a redundancy is illusive since the fact that the stochastic matrix p_{ij} has been derived from the unitary matrix u_{ij} imposes a very severe restriction upon p_{ij}: not only the sum of each row, but also the sum of each column is equal to one, i.e., now in addition to the constrain in Eqs. (72), an additional constraint

$$\sum_{i=1}^{n} p_{ij} = 1 \tag{75}$$

is imposed upon the stochastic matrix. This makes the matrix p_{ij} doubly stochastic that always leads to an ergodic attractor with uniform distribution of probabilities. Obviously such a property significantly reduces the usefulness of the Quantum recurrent net (QRN). However, as will be shown below, by slight change of the QRN architecture, the restriction (75) can be removed.

12. Multivariate ONR.

In the previous section we have analyzed the simplest quantum neural net whose probabilistic performance was represented by a single-variable stochastic process equivalent to generalized random walk. In this section we will turn to multi-variable stochastic process and start with the two-measurement architecture. Instead of Eq.(60) now we have the following mapping:

$$\frac{1}{\sqrt{2}}\{00...1_{i_1}0...1_{i_2}0...0\} \rightarrow \frac{1}{\sqrt{2}}\{00...1_{j_1}0...1_{j_2}0...0\} \tag{76}$$

i.e., $$I_1 + I_2 \rightarrow J_1 + J_2 \tag{77}$$

where I_1, I_2, J_1 and J_2 are the eigenstates where the unit *1* is at the $i_1^{th}, i_2^{th}, j_1^{th}$ and j_2^{th} places, respectively. Then the transitional probability of the mappings:

$$p_{i_1 i_2}^{\ j_1}(I_1 + I_2 \rightarrow J_1) = \frac{1}{2}|U_{j_1 i_1} + U_{j_1 i_2}|^2 \tag{78}$$

$$p_{i_1 i_2}^{\ j_2}(I_1 + I_2 \rightarrow J_2) = \frac{1}{2}|U_{j_1 i_1} + U_{j_1 i_2}|^2 \tag{79}$$

Since these mapping result from two independent measurements, the joint transitional probability for the mapping (76) is

$$p_{i_1 i_2}^{\ j_1 j_2}(I_1 + I_2 \rightarrow J_1 + J_2) = \frac{1}{2}|U_{j_1 i_1} + U_{j_1 i_2}|^2 |U_{j_2 i_1} + U_{j_2 i_2}|^2 \tag{80}$$

One can verify that

$$\sum_{j=1}^{n} p_{i_1 i_2}^{\ j} = 1, \qquad \sum_{j_1 j_2 = j}^{n} p_{i_1 i_2}^{\ j_1 j_2} = 1, \tag{81}$$

It should be emphasized that the input patterns *I* interfere, i.e., their probabilities are added according to the quantum laws since they are subjected to a unitary transformation in the quantum device. On the contrary, the output patterns *J* do not interfere because they are obtained as a result of two independent measurements. As mentioned above, Eq. (80) expresses the joint transition probabilities for two stochastic processes

$$I_1 \rightarrow J_1 \text{ and } I_2 \rightarrow J_2 \tag{82}$$

which are coupled via the quantum interference. At the same time, each of the stochastic processes (80) considered separately has the transition probabilities following from Eq. (61), and by comparing Eqs. (61) and Eq. (80), one can see the effect of quantum interference for input patterns.

It is interesting to notice that although the probabilities in Eqs. (80) have a tensor structure, strictly speaking they are not tensors. Indeed, if one refers the Hamiltonian *H*, and therefore the unitary matrix *U* to a different coordinate system, the transformations of the probabilities (80) will be different from those required for tensors. Nevertheless, one can still formally apply the chain rule for evolution of transitional probabilities, for instance:

$$p_{i_1 i_2 q_1 q_2}(I_1 + I_2 \to J_1 + J_2 \to Q_1 + Q_2) = p_{i_1 i_2 j_1 j_2} p_{j_1 j_2 q_1 q_2} etc \qquad (83)$$

Eqs. (80) is easily generalized to the case of *l* measurements $l \le n$:

$$p_{i_1 \ldots i_l q_1 \ldots q_l} = p_{i_1 \ldots i_l j_1 \ldots j_l} p_{j_1 \ldots j_l q_1 \ldots q_l} \quad etc \qquad (84)$$

$$p_{i_1 \ldots i_l j_1 \ldots j_l} = \frac{1}{l^l} \prod_{\alpha=1}^{l} | \sum_{\beta=1}^{l} U_{j_\alpha i_\beta} |^2 \qquad (85)$$

Now the evolution in physical space, instead of Eq. (51)), is described by the following:

$$a_i(t+\tau) = \sigma_l \{ \sum U_{ij} a_j(t) \}, \qquad i = 1.2 \ldots n \qquad (86)$$

where σ_l is the *l*-measurements operator.

Obviously, the evolution of the state vector a_i is more "random" than those of Eq. (51) since the corresponding probability distribution depends upon *l* variables.

Eq. (86) can be included in a system with interference inputs and independent outputs as a generalization of the system (56),(57).

13. QRN with input interference.

In order to remove the restriction (75), let us turn to the architecture shown in Fig. 5 and assume that the result of the measurement, i.e., a unit vector $a_m(t) = \{00 \ldots 010 \ldots 0\}$ is combined with an arbitrary complex (interference) vector, Fig. 7.

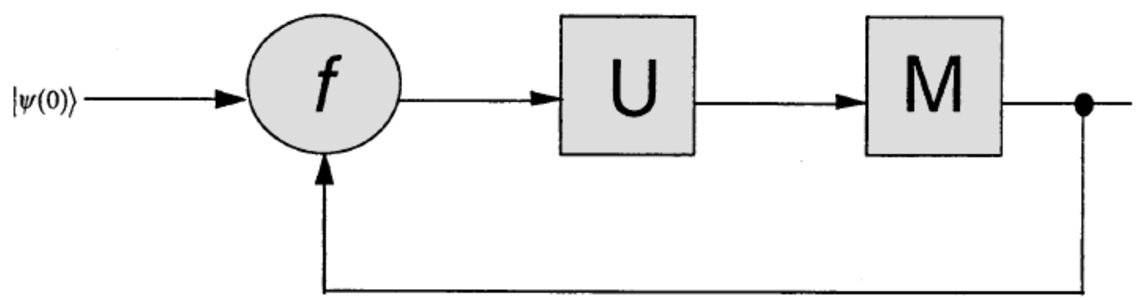

Figure 7. QRN with input interference.

$$m = \{m_1, \ldots m_n\} \qquad (87)$$

such that

$$a(t) = [a_m(t) + m]c, \; c = \frac{1}{m_1^2 + \ldots (m_i + 1)^2 \ldots m_n^2} \qquad (88)$$

Then the transition probability matrix becomes

$$p_{ij} = \frac{| U_{j1} m_1 + \ldots U_{ji}(m_i + 1) \ldots U_{jn} m_n |^2}{| m_1^2 + \ldots (m_i + 1)^2 + \ldots m_n^2 |} \qquad (89)$$

Thus, now the structure of the transition probability matrix p_{ij} can be controlled by the interference vector *m*.

Eq. (89) is derived for a one-dimensional stochastic process, but its generalization to *l*-dimensional case is straightforward.

This architecture produces several interesting algorithms, and one of them is quantum model of emerging grammar, [6]. But in this paper our goal is different: we wish to demonstrate that QRN possesses a distinguished property to violate the second law of thermodynamics and to move from disorder to order without external recourses. That would make the QRN universal in terms of its intelligence capability. In other words, we expect that QRN would implement the model described by Eqs (11) and (12) introduced and analyzed in sections 3,4,5,6, and 7.

14. QRN with nonlinear evolution of probabilities.

So far we were dealing with linear evolution of probabilities (see Eqs. (62) and (71) while evolution of the state vector was always nonlinear(see Eqs. (51),(56) and (57)). Now let us assume that along with the Eq. (51) that is implemented by quantum device, we implement (in a classical way) the associated probability equation (71). At this point these two equations are not coupled yet. Now turning to Eqs. (87), (88), and (89), assume that the role of the interference vector m is played by the probability vector π. Then Eqs. (51) and (71) take the form:

$$a_i(t+1) = \sigma_1\{\sum U_{ij} a_j(t)\}, \qquad i = 1.2...n \qquad (90)$$

$$\pi_i(t+\tau) = \sum p_{ij}\pi_j(t), \quad i=1,2,...n \qquad (91)$$

where $a_i(t) = [\{00...010...0\} + \{\pi_1\pi_2..\pi_n\}]C$ (92)

$$C = \frac{1}{\pi_1^2 + ...\pi_i^2 + ... + \pi_n^2} \qquad (93)$$

$$p_{ij} = \frac{|U_{j1}\pi_1 + ...U_{ji}\pi_i + ...U_{jn}\pi_n|^2}{|\pi_1^2 + ...\pi_i^2 + ... + \pi_n^2|} \qquad (94)$$

and they are coupled. Moreover, the probability evolution (91) becomes nonlinear since the matrix p_{ij} depends upon the probability vector π.

15. Comparison QNR and intelligent particle.

One can associate Eq. (90) with the equation of motion in physical space,(see Eq. (11)) and Eq. (91) – with the Liouville equation describing the evolution of an initial randomness in a probability (virtual) space, (see Eq. (12)). In QNR architecture, Eq. (90) is always nonlinear (due to quantum collapse), while Eq. (91) is linear unless it is coupled with Eq. (90) via the feedback (92). Therefore, one arrives at two fundamentally different dynamical topologies of QRN: the first one is linked to Newtonian physics where equation of motion is never coupled with the corresponding Liouvil(95)
then the evolution (73) becomes nonlinear, and it may have many different le equation, and the second one can be linked to quantum physics (in the Madelung version of the Schrödinger equation) where the Hamilton-Jacobi equation is coupled with the corresponding Liouville equation by the quantum potential. It is interesting to note that the randomness in Eqs. (11) and (12) is caused by the failure of the Lipchitz conditions accompanied by the blow-up type of instability, (see section 4), while the randomness in QNR is a fundamental quantum phenomenon associated with the quantum collapse as a result of measurement.

Now the following question could be asked: why Eqs.(11) and (12) that describe performance of intelligent particle cannot be simulated directly, and instead, they have to be implemented in the form of QRN prior to simulation? The answer to this question is similar to that given by R. Feynman who explained why quantum phenomena couldn't be simulated with only Newtonian recourses: these phenomena do not belong to the Newtonian world. At the same time, the Schrödinger equation can be computed using a classical computer, and therefore the restriction is imposed only upon simulations, but not upon computations. For a similar reason, Eqs. (11) and (12) cannot be simulated by physical resources without biological parts included since phenomena described by these equations belong neither to Newtonian nor to quantum world: they belong to the world of livings since they violate the second law of thermodynamics. In this context, it is interesting to take a closer look into the architecture of QRN: an initial state, $|\psi(0)>$, is fed into the network, transformed under the action of a unitary operator, *U*, subjected to a measurement indicated by the measurement operator M{ }, and the result of the measurement is used to control the new state fed back into the network at the next iteration. One is free to record, duplicate or even monitor the sequence of measurement outcomes, as they are all merely bits and hence constitute classical information. Moreover, one is free to choose the function used during the reset phase, including the possibility of adding no offset state whatsoever.

As follows from this description, QRN is a hybrid of simulation and computation: the period of action of the unitary operator obviously belongs to quantum simulation. However building a probability vector and feeding it into the net is the element of digital computing. Therefore the hybrid nature of QRN is the reason why QRN could become a universal tool for modeling intelligence. As the last step, we have to prove that QRN can violate the second law of thermodynamics, and that will be the subject of the next section.

16. Spontaneous self-organization.

In this section we will demonstrate a relation of non-linear QRN considered above to a concept of a spontaneous self-organization as a component of life and intelligence. As shown in section 11, a linear QRN eventually approaches an attractor in probability space (see Eq. (74)) that represents a stationary stochastic process, and this attractor does not depends upon initial conditions. Therefore, from the viewpoint of information processing, this attractor performs generalization by placing all possible entry patterns in the same class. Let us ask now the following question: can the system (73) change its evolution, and consequently, its limit distribution, without any external "help"? The formal answer is definitely positive. Indeed, if the transition matrix depends upon the current probability distribution

$$p = p(\pi)$$

scenarios depending upon the initial state π^0. In particular case (71), it can "overcome" the second law of thermodynamics decreasing its final entropy by using only the "internal" resources. The last conclusion illuminates the Schrödinger's statement that 'life is to create order in the disordered environment against the second law of thermodynamics". Indeed, suppose that the selected unitary matrix is

$$U = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \tag{96}$$

Then the corresponding transition probability matrix in Eq. (71), according to Eq. (61) will be doubly-stochastic:

$$p = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix} \tag{97}$$

and the stochastic process (71) is already in its thermodynamics limit (97), i.e.,

$$\pi_1 = \pi_2 = 1/2 \tag{98}$$

Let us assume that the objective of the system is to approach the deterministic state

$$\pi_1 = 1, \quad \pi_2 = 0 \tag{99}$$

without help from outside. In order to do that, the system should adapt a feedback (95) in the form:

$$a = (a_1, a_2), \qquad a_1 = -2\pi_1, \qquad a_2 = 1 \tag{100}$$

Then, according to Eqs. (61 89), the new transition probability matrix p_{ij} will be:

$$p_{11} = \frac{\pi_1^2}{2\pi_1^2 - 2\pi_1 + 1} \qquad p_{12} = \frac{(1-\pi_1)^2}{2\pi_1^2 - 2\pi_1 + 1}$$

$$p_{21} = \frac{(1+\pi_1)^2}{2\pi_1{}^2 + 2} \qquad p_{22} = \frac{(1-\pi_1)^2}{2\pi_1{}^2 + 2} \qquad (101)$$

Hence, the evolution of the probability π_1 now can be presented as:

$$\pi_1{}^{(n+1)} = \pi_1{}^{(n)} p_{11} + (1-\pi^{(n)}) p_{21} \qquad (102)$$

in which $p_{11}$and p_{22} are substituted from Eqs. (101).

It is easily verifiable that

$$\pi_1{}^{\infty} = 1, \ \ \pi_2{}^{\infty} = 0 \qquad (103)$$

i.e., the objective is achieved due to the "internal" feedback (100).
The application of QRN-based self-organization model to common sense decision-making process has been introduced in [7].
As follows from Eqs.(99) and (103), due to the built-in feedback (100) and without any external effort, the system moved from the state of maximum entropy to the state of minimum entropy, and that violates the second law of thermodynamics. This means that such a system does not belong to physical world: it is neither a Newtonian nor a quantum one. But to what world does it belong? Let us recall again the Schrödinger statement (Schrödinger. 1945): "life is to create order in the disordered environment *against the second law of thermodynamics*". That gives a hint for exploiting the effect of self-organization for modeling some aspects of life, and that makes QRN a universal tool of AI that can compete with human intelligence.

17. Mathematical machinery of perception.

In this section, we connect the concept of intelligent particle and the phenomenon of perception, i.e. representation and understanding the environment. It should be noticed that our model is not necessarily associated with a violation of the second law of thermodynamics. Indeed, these violations occur only if $c_1 > 0$, (this case corresponds to formation of a shock wave in probability space, Fig. 4), or $c_2 < 0$, (this case that leads to negative diffusion has been analyzed in [10]). Therefore the second law of thermodynamics does not bind study of the perception phenomena.

a. System of intelligent particles. Since perception is a collective phenomenon, as a first step, we have to move from one- to *n*-dimensional case. For illustration, we confine ourselves with a particular case

$$c_2 \neq 0, \quad c_0 = c_1 = c_3 = 0 \qquad (104)$$

Then as a direct generalization of Eqs. (11) and (12), one obtains

$$\dot{v}_i = -\xi \sum_{j=1}^{n} \alpha_{ij} \frac{\partial}{\partial v_j} \ln \rho(v_1, \ldots v_n, t), \quad i = 1, 2, \ldots n \qquad (105)$$

where α_{ij} are function of the correlation moments D_{ks}

$$\alpha_{ij} = \alpha_{ij}(D_{11}, \ldots D_{ks}, \ldots D_{nn}) \qquad (106)$$

and

$$D_{ks} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (v_k - \bar{v}_k)(v_s - \bar{v}_s) \rho \, dv_k dv_s, \quad v_j = \int_{-\infty}^{\infty} v_j \rho \, dv_j \qquad (107)$$

Eqs. (105) have to be complemented by the corresponding Liouville equation

$$\frac{\partial \rho}{\partial t} = \xi \sum_{j=1}^{n} \alpha_{ij} \frac{\partial^2 \rho}{\partial V_j^2} \qquad (108)$$

Here ξ is a positive constant that relates Newtonian and information forces. It is introduces in order to keep the functions α_{ij} dimensionless.

The solution of Eqs. (105) and (108)

$$v_i = v_i(t), \quad \rho = \rho(V_1, \ldots V_n, t), \quad i = 1, 2, \ldots n \qquad (109)$$

must satisfy the constraint that is an n-dimensional generalization of the constraint $D \geq 0$, namely, a non-negativity of the matrix $|D_{ij}|$, i.e. a non-negativity of all the left-corner determinants

$$Det\,|D_{ij}| \geq 0, \quad i = 1, 2, \ldots n \qquad (110)$$

b. Entanglement. In this sub-section, we will show that the system (105) is isolated and entangled. Both of these properties follow from the fact that Eqs.(105) are coupled by information forces derived from the joint probability. The entanglement can be illustrated for two dimensional case:

$$\dot{v}_1 = -a_{11} \frac{\partial}{\partial v_1} \ln \rho - a_{12} \frac{\partial}{\partial v_2} \ln \rho, \qquad (111)$$

$$\dot{v}_2 = -a_{21} \frac{\partial}{\partial v_1} \ln \rho - a_{22} \frac{\partial}{\partial v_2} \ln \rho, \qquad (112)$$

$$\frac{\partial \rho}{\partial t} = a_{11} \frac{\partial^2 \rho}{\partial V_1^2} + (a_{12} + a_{21}) \frac{\partial^2 \rho}{\partial V_1 \partial V_2} + a_{22} \frac{\partial^2 \rho}{\partial V_2^2}, \qquad (113)$$

As in the one- dimensional case, this system describes diffusion without a drift
The solution to Eq. (113) has a closed form

$$\rho = \frac{1}{\sqrt{2\pi \det[\hat{a}_{ij}]t}} \exp(-\frac{1}{4t} b'_{ij} V_i V_j), \qquad i = 1, 2. \qquad (114)$$

Here

$$[b'_{ij}] = [\hat{a}_{ij}]^{-1}, \hat{a}_{11} = a_{11}, \hat{a}_{22} = a_{22}, \hat{a}_{12} = \hat{a}_{21} = a_{12} + a_{21}, \ \hat{a}_{ij} = \hat{a}_{ji}, b'_{ij} = b'_{ji}, \qquad (115)$$

Substituting the solution (114) into Eqs. (111) and (112), one obtains

$$\dot{v}_1 = \frac{b_{11} v_1 + b_{12} v_2}{2t} \qquad \dot{v}_2 = \frac{b_{21} v_1 + b_{22} v_2}{2t}, \qquad b_{ij} = b'_{ij} \hat{a}_{ij} \quad (116)$$

Eliminating t from these equations, one arrives at an ODE in the configuration space

$$\frac{dv_2}{dv_1} = \frac{b_{21} v_1 + b_{22} v_2}{b_{11} v_1 + b_{12} v_2}, \ v_2 \to 0 \qquad at \qquad v_1 \to 0, \qquad (117)$$

This is a classical singular point treated in textbooks on ODE.
Its solution depends upon the roots of the characteristic equation

$$\lambda^2 - 2b_{12}\lambda + b_{12}^2 - b_{11} b_{22} = 0 \qquad (118)$$

Since both the roots are real in our case, let us assume for concreteness that they are of the same sign, for instance, $\lambda_1 = 1$, $\lambda_2 = 1$. Then the solution of Eq. (3.44) is represented by the family of straight lines

$$v_2 = \tilde{C}v_1, \qquad \tilde{C} = const. \tag{119}$$

Substituting this solution into Eq. (3.42)) yields

$$v_1 = Ct^{\frac{1}{2}(b_{11}+\tilde{C}b_{12})} \qquad v_2 = \tilde{C}Ct^{\frac{1}{2}(b_{11}+\tilde{C}b_{12})} \tag{120}$$

Thus, the solutions of Eqs. (111) and (112) are represented by two-parametrical families of random samples, as expected, while the randomness enters through the time-independent parameters C and $\tilde{C}$ that can take any real numbers. Let us now find such a combination of the variables that is deterministic. Obviously, such a combination should not include the random parameters C or $\tilde{C}$. It easily verifiable that

$$\frac{d}{dt}(\ln v_1) = \frac{d}{dt}(\ln v_2) = \frac{b_{11} + \tilde{C}b_{12}}{2t} \tag{121}$$

and therefore,

$$(\frac{d}{dt}\ln v_1) / (\frac{d}{dt}\ln v_2) \equiv 1 \tag{122}$$

Thus, the ratio (122) is deterministic although both the numerator and denominator are random,(see Eq.(121). This is a fundamental non-classical effect representing a global constraint. Indeed, in theory of stochastic processes, two random functions are considered statistically equal if they have the same statistical invariants, but their point-to-point equalities are not required (although it can happen with a vanishingly small probability). As demonstrated above, the *diversion of determinism into randomness via instability (due to a Liouville feedback), and then conversion of randomness to partial determinism (or coordinated randomness) via entanglement* is the fundamental non-classical paradigm that may lead to instantaneous transmission of *conditional* information on remote distance.

c. Measure of survivability. Since we are dealing with ***physical*** systems that are supposed to ***simulate*** behavior of livings, we adopt the principle of survivability of livings to create incentive for complexity of perceptions in artificial systems. We will introduce, as a measure of survivability, the strength of the random force that, being applied to a particle, nullifies the decrease of entropy

$$\frac{\partial H}{\partial t} < 0 \tag{123}$$

For better physical interpretation, it will be more convenient to present this inequality in terms of the variance D

$$\dot{D} < 0 \tag{124}$$

remembering that for normal probability density distribution

$$H = \log_2 \sqrt{2\pi e D^2}$$

while the normal density is the first term in the Gram-Charlier series for representation of an arbitrary probability distribution.

Thus, the ability to survive (in terms of preserving the property (123)) under action of a random force) can be achieved only with help of increased complexity. However, physical complexity is irrelevant: no matter how complex is Newtonian or Langevin dynamics, the second law of thermodynamics will convert the inequality (123) into the opposite one. The only complexity that counts is that associated with mental dynamics. Consequently, increase of complexity of mental dynamics, and therefore, complexity of the information, is the only way to maximize the survivability of Livings. This conclusion will be reinforced by further evidence to be discussed in the following sub-section.

g. Chain of abstractions. In view of importance of mental complexity for survival of Livings, we will take a closer look into cognitive aspects of information forces. It should be recalled that classical methods of information processing are effective in a deterministic and repetative world, but faced with the uncertainties and unpredictabilities, they fail. At the same time, many natural and social phenomena exhibit some degree of regularity only on a higher level of abstraction, i.e.in terms of some invariants. Indeed, it is easier to predict the state of the solar system in a billion years ahead than to predict a price of a stock of a single company tomorrow. In this sub-section we will discuss a new type of attractors and associated with them a new chain of abstraction that is provided by complexity of mental dynamics.

α. *Attractors in mental dynamics***.** Significant expansion of the concept of an attractor as well as associated with it generalization via abstraction is provided by mental dynamics. We will start with the model (105-108) being discussed in the previous sub- section

Let us express Eq. (108) in terms of the correlation moments: multiplying it by V_i^2, then using partial integration, one arrives at an *n*-dimensional analog of Eq. (108)

$$\dot{D}_{ii} = 2\zeta\alpha_{ii}(D_{11},...D_{nn}), \qquad n = 1,2,...n, \tag{125}$$

The next step is to choose such a structure of the functions (106) that would enforce the constraints (110), i.e.

$$D_{ii} \geq 0, \qquad i = 1,2,...n, \tag{126}$$

The simplest (but still sufficiently general) form of the functions (106) is a neural network with terminal attractors, [3],

$$\alpha_{ii} = \frac{1}{2}(w_{ij} \tanh \tilde{D}_{jj} - c_i\sqrt{\tilde{D}_{ii}}), \qquad i = 1,2,...n, \quad \tilde{D}_{ii} = \frac{D_{ii}}{D^0} \tag{127}$$

that reduces Eqs.(125) to the following system

$$\dot{\tilde{D}}_{ii} = \zeta(w_{ij} \tanh \tilde{D}_{jj} - c_i\sqrt{\tilde{D}_{ii}}), \qquad i = 1,2,...n, \tag{128}$$

Here D_0 is a constant scaling coefficient of the same dimensionality as the correlation coefficients D_{ii}, and w_{ij} are dimensionless constants representing the identity of the system.

Let us now analyze the effect of terminal attractor and, turning to Eq.(128), starting with the matrix $\left|\frac{\partial \dot{D}_{ii}}{\partial D_{ii}}\right|$. Its diagonal elements, i.e. eigenvalues, become infinitely negative when the variances vanish since

$$\frac{\partial\sqrt{D_{ii}}}{\partial D_{ii}} = \frac{1}{2\sqrt{D_{ii}}} \to \infty \qquad when \;\; D_{ii} \to 0 \tag{129}$$

while the rest terms are bounded. Therefore, due to the terminal attractor, Eq. (128) linearized with respect to zero variances has infinitely negative characteristic roots, i.e. it is infinitely stable regardless of the parameters w_{ij}. Hence the principal variances cannot overcome zero if their initial values are positive. This provides the well-posedness of the initial value problem.

Now we can present Eq. (105) in the form

$$\dot{v}_i = (-v_{ii}^{1/2} + w_{ij}\tanh v_{jj})\frac{\partial}{\partial v_j}\ln\rho(v_1,...v_n) \tag{130}$$

Here, for further convenience, we have introduced new compressed notations

$$\bar{V}_i = \int_{-\infty}^{\infty} V_i \rho dV_i, \qquad V_{ii} = D_{ii} = \int_{-\infty}^{\infty} (V_i - \bar{V}_i)^2 \rho dV_i,$$
$$V_{iiii} = \int_{-\infty}^{\infty} (V_{ii} - \bar{V}_{ii})^2 \rho dV_{ii} \qquad ...etc \tag{3.131}$$

The corresponding mental dynamics in the new notations follows from Eq. (128)

$$\frac{\partial\rho}{\partial t} = (V_{ii}^{1/2} - w_{ij}\tanh V_{jj})\frac{\partial^2\rho}{\partial V_j^2} \tag{132}$$

In the same way, the mental neural nets can be obtained from Eqs. (105) and (127)

$$\dot{v}_{ii} = (-v_{ii}^{1/2} + w_{ij}\tanh v_{jj}) \tag{133}$$

where the state variables v_{ii} represent variances of ρ.

β. *Hierarchy of higher order mental abstractions***.** Following the same pattern as those discussed in the previous sub-section, and keeping the same notations, one can introduce the next generation of mental neural nets starting with the motor dynamics

$$\dot{v}_i = [(-v_{iiii}^{1/2} + w_{ij}\tanh v_{jjjj})\frac{\partial}{\partial v_{ii}}\ln\rho'(v_{11},...v_{nn})]\frac{\partial}{\partial v_i}\ln\rho(v_1,...v_n)$$

Here, in addition to the original random state variables v_i, new random variables v_{ii} are included into the structure of information forces. They represent invariants (variances) of the original variables that are assumed to be random too, while their randomness is described by the secondary joint probability density $\rho'(v_{11},...v_{nn})$. The corresponding Fokker-Planck equation governing the mental part of the neural net is

$$\frac{\partial\rho}{\partial t} = [(v_{jjjj}^{1/2} - w_{ij}\tanh v_{jjjj})\frac{\partial}{\partial v_{ii}}\ln\rho'(V_{11},...V_{nn})]\frac{\partial^2\rho}{\partial V_{jj}^2}$$

Then, following the same pattern as in Eqs. (130), (132), and (133), one obtains

$$\dot{v}_{ii} = (-v_{jjjj}^{1/2} + w_{ij}\tanh v_{jjjj})\frac{\partial}{\partial v_{ii}}\ln\rho'(v_{11},...v_{nn}) \tag{136}$$

$$\frac{\partial\rho'}{\partial t} = [(V_{jjjj}^{1/2} - w_{ij}\tanh V_{jjjj})\frac{\partial^2\rho'}{\partial V_{jj}^2} \tag{137}$$

$$\dot{v}_{iiii} = (-v_{jjjj}^{1/2} + w_{ij}\tanh v_{jjjj}). \tag{138}$$

Here Eqs. (136) and (138) describe dynamics of the variances v_{ii} and variances of variances v_{iiii} respectively, while Eq. (137) governs the evolution of the secondary joint probability density $\rho'(V_{11},..V_{nn})$. As follows from Eqs. (134)- (138), the only variables that have attractors are the variances of variances; these attractors are controlled by Eq. (138) that has the same structure as Eq. (133). The stationary values of these variables do not depend upon the initial conditions: they depend only upon the basins where the initial conditions belong, and that specifies a particular attractor out of the whole set of possible attractors. On the contrary, no other variables have attractors, and their values depend upon the initial conditions. Thus, the attractors have broad membership in terms of the variables v_{iiii}, and that represents a high level of generalization. At the same time, such "details" as values of v_i and v_{ii} at the attractors are not defined being omitted as insignificant, and that represent a high level of abstraction.

It should be noticed that the chain of abstractions was built upon only principal variances, while co-variances were not included. There are no obstacles to such an inclusion; however, the conditions for preserving the positivity of the tensors v_{ij} and v_{ijkq} are too cumbersome while they do not bring any significant novelty into cognitive aspects of the problem other than increase of the number of attractors.

δ. *Activation of new levels of abstractions***.** A slight modification of the model of motor-mental dynamics discussed above leads to a new phenomenon: the capability to activate new levels of abstraction needed to preserve the inequality (123). The activation is triggered by the growth of variance caused by applied random force. In order to demonstrate this, let us turn to a one-dimensional version of Eqs. (134)- (138) in which the neural net structure is replaced by a linear term and to which noise of the strength q^2 is added

$$\dot{v} = q\Gamma(t) + \lambda \frac{\alpha}{\rho}\frac{\partial \rho}{\partial v} \quad \text{where } \alpha = q^2 \exp\sqrt{D} \tag{139}$$

Then the equations of the mental dynamics are modified to

$$\frac{\partial \rho}{\partial t} = [q^2(-\lambda \exp\sqrt{D})]\frac{\partial^2 \rho}{\partial V^2} \tag{140}$$

$$\dot{D} = [2q^2(-\lambda \exp\sqrt{D})] \tag{141}$$

respectively. Here λ is a new variable defined by the following differential equation

$$\dot{\lambda} = \sqrt{\lambda(1-\lambda)}\dot{D} \tag{142}$$

One can verify that Eq. (142) implements the following logic:

$$\lambda = 0 \quad if \qquad \dot{D} \le 0, and \quad \lambda = 1 \quad if \qquad \dot{D} > 0, \tag{143}$$

Indeed, Eq. (142) has two static attractors: $\lambda = 1 \quad and \quad \lambda = 0$; when $\dot{D} > 0$, the first attractor is stable; when $\dot{D} < 0$, it becomes unstable, and the solution switches to the second one that becomes stable. The transition time is *finite* since the Lipchitz condition at the attractors does not hold, and therefore, the attractors are terminal, [3]. Hence, when there is no random force applied, i.e. $q=0$, the first level of abstraction does not need to be activated, since then $\dot{D} = 0$, and therefore. λ is zero. However, when random force is applied, i.e. $q \ne 0$, the variance D starts growing, i.e. $\dot{D} > 0$. Then the first level of abstraction becomes activated, λ switches to 1, and, according to Eq. (141), the growth of the entropy is eliminated. If the first level of abstraction is not sufficient, the next levels of abstractions can be activated in a similar way.

δ. *Measure of complexity***.** Let us turn to the system of Eqs. (130) and (132). Its solution is represented by n random functions $v_i(t)$, $i=1,2...n$ and a deterministic function $\rho(\{V\},t\}$ representing the density of their joint probability distribution. As a measure of complexity of this system, one can choose a maximum number of independent coefficients of the linear regression β_{ik} that express each variable $v_i(t)$ via the rest $n\text{-}1$ variables while

$$\beta_{ik} = -\frac{\Lambda_{ik}}{\Lambda_{ii}}, \quad \|\Lambda\| = \|\lambda\|^{-1} \qquad (144)$$

where $\|\lambda\|$ is the matrix of variances.

The system (130), (132) that has one mental layer of complexity requires n^2 coefficients (144), and that number characterizes its complexity, i.e. $N_1 = n^2$

The system (136)-(143) has two mental layers of complexity, and its solution is represented by the functions $v_i(t)$ and $v_{ij}(t)$. It is easy to calculate that the number of the coefficients of linear regression in this case, and therefore, the complexity, will be $N_2 = n^2 + n^4$. Now it is clear that complexity of a system with *m* mental layers is

$$N_m = \sum_{k=1}^{m} n^{2k} \qquad (145)$$

In this connection, it is interesting to pose the following problem. What is a more effective way for Livings to promote Life: through a simple multiplication, i.e. through increase of the number of "primitives" *n*, or through individual self-perfection, i.e. through increase of the number *m* of the levels of abstractions ("What do you think I think you think. . . ")? The solution of this problem may have fundamental social, economical and geo-political interpretations. But the answer immediately follows from Eq. (145) demonstrating that the complexity grows *exponentially* with the number of the levels of abstractions *m*, but it grows only *linearly* with the dimensionality *n* of the original system. Thus, in contradistinction to Darwinism, *a more effective way for Livings to promote Life is through higher individual complexity (due to mutually beneficial interactions) rather than trough a simple multiplication of "primitives".* This statement can be associated with recent consensus among biologists that the symbiosis, or collaboration of Livings, is even more powerful factor in their progressive evolution than a natural selection

18. Discussion and conclusion.

The objective of this paper is to relate the concept of ***intelligence*** to the first principles of physics, and, in particular, to answer the following question: can AI system composed only of physical components compete with a human? The first part of the answer has been addressed in the sections 2 trough 7, the second part – in the sections 8 through 16.

The first seven sections introduce and discuss the concept of an intelligent particle. One of many obstacles to developing a mathematical theory of AI is absence of a definition of intelligence that would fit into mathematical formalism in the form of a state variable. Such definition was proposed in the first section: intelligence of an ***isolated*** dynamical system is defined as a capability to move from disorder to order in violation of the second law of thermodynamics. Then, as a result, intelligence is measured by the absolute value of negative time derivative of the system's entropy. This concept was inspired by the discovery of the Higgs boson and the following from it claim of completeness of the physical picture of our Universe. However the ability to create Life and Mind out of physical matter without any additional entities is still a mystery. The primary objective of this paper is to presents a ***mathematical*** answer to the ancient philosophical question, "How mind is related to matter" in connection with this outstanding accomplishment in physics. The paper is inspired by analysis of the Madelung equation and discovery of the origin of randomness in quantum mechanics, [8]. It turns out that replacement of the quantum potential by the information force, while preserving some quantum properties, introduces fundamental changes in the first and the second laws of thermodynamics, and that leads to a mathematical model that captures behavior of livings. The idea of an intelligent particle has been introduced as a first step of physics of life since it does not include such properties as metabolism and reproduction. Instead it concentrates attention to intelligent behavior. At the same time, by ignoring metabolism and reproduction, we can make the system isolated, and it will be a challenge to show that it still can move from a disorder to the order. It has been demonstrated that the model of intelligent particle belongs neither to Newtonian, nor to quantum mechanics. Its departure from Newtonian mechanics is due to a feedback from the underlying Liouville equation to the equations of motion that represents an additional (internal) information force. Topologically this feedback shifts intelligent particles towards quantum mechanics. However since the information force is different from forces produced by quantum potential, the intelligent particle is not quantum, and it can be identified as quantum-classical hybrid. Therefore intelligent particle dwells in an abstract mathematical world rather than in the physical world, as we know it. This means that intelligent particles, in principle, cannot be composed out of physical particles. It also means that it's behavior can be computed, but not simulated using Newtonian or quantum resources.

The next nine sections introduce a model of quantum recurrent nets for implementation of intelligent particles as a challenge to human intelligence.

There are several advantages that can be expected from quantum implementation of recurrent nets. Firstly, since the dimension of the unitary matrix *n* can be exponentially larger within the same space had it been implemented by a quantum device, the capacity of quantum neural nets in terms of the number of patterns stored as well as their dimensions can be exponentially larger too.

Secondly, QRN have a new class of attractors representing different stochastic processes, which in terms of associated memory, can store complex behaviors of biological and engineering systems, or in terms of optimization, to minimize a functional whose formulation includes statistical invariants.

The details of ORN performance in learning, optimization, associative memory, as well as in generation of stochastic processes can be found in [6],[7] and [9]. However in this paper, the attention is focused on the most remarkable property of nonlinear QRN that is associated with the spontaneous self-organization as a possible bridge to model intelligent behavior. It is important to emphasize that the architecture of that ORN includes a built-in feedback from the probability evolution to the evolution of the state vector, and that leads to such a non-Newtonian property as transition from a disorder to the order without any external interference. And this property provides the capability of QRN to compete with human intelligence.

The last section introduces and discusses a mathematical machinery of the ***perception*** that is the fundamental part of a cognition process as well as intelligence. It should be recalled that classical methods of information processing are effective in a deterministic and repetative world, but faced with the uncertainties and unpredictabilities, they fail. At the same time, many natural and social phenomena exhibit some degree of regularity only on a higher level of abstraction, i.e.in terms of some invariants. In this section, a new type of attractors and associated with them a new chain of abstraction that is provided by complexity of mental dynamics of the proposed model is addresssed. It demonstrates that the capability of the proposed model is not limited by violations of the second law of thermodynamics: it is much broader since the model handles many aspect of cognition specific for livings that do not violate this law. In this context, it should be emphasized that the proposed approach to perception is fundamentally different from the idea of quantum collapse after reformulation the Schrödinger equation, [11]: the proposed model is based upon reformulation of the Madelung version of the Schrödinger equation, and after that there is no way back to the Schrödinger equation.

This work has interesting philosophical implications associated with the theory of heat death. The theory of heat death stems from the second law of thermodynamics, of which one version states that entropy tends to increase in an isolated system. From this, the theory infers that if the universe lasts for a sufficient time, it will asymptotically approach a state where all energy is evenly distributed. In other words, according to this theory, in nature there is a tendency to the dissipation (energy loss) of mechanical energy (motion); hence, by extrapolation, there exists the view that the mechanical movement of the universe will run down, as work is converted to heat, in time because of the second law of thermodynamics. The discovery of isolated dynamical systems that can decrease entropy in violation of the second law of thermodynamics, and resemblances of these systems to livings implies that Life can slow down heat death of the Universe, and that can be associated with the purpose of Life.

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