FROM ABSTRACT TOPOLOGY TO REAL THERMODYNAMIC BRAIN ACTIVITY

Arturo Tozzi[1,2] (Corresponding Author), James F. Peters[2]

[1]Computational Intelligence Laboratory, University of Manitoba, Winnipeg, Canada
Winnipeg R3T 5V6 Manitoba

[2]Department of Electrical and Computer Engineering, University of Manitoba
75A Chancellor's Circle
Winnipeg, MB R3T 5V6 Canada
james.peters3@umanitoba.ca

ABSTRACT

Recent approaches to brain phase spaces reinforce the foremost role of symmetries and energy requirements in the assessment of nervous activity. Changes in thermodynamic parameters and dimensions occur in the brain during symmetry breakings and transitions from one functional state to another. Based on topological results and string-like trajectories into nervous energy landscapes, we provide a novel method for the evaluation of energetic features and constraints in different brain functional activities. We show how abstract approaches, namely the Borsuk-Ulam theorem and its variants, display real, energetic physical counterparts. When topology meets the physics of the brain, we arrive at a general model of neuronal activity, in terms of multidimensional manifolds and computational geometry, that has the potential to be operationalized.

INTRODUCTION

The brain is a complex, non-linear system operating at the edge of chaos, formed by inter-dependent components which exhibit spontaneous self-organization and emergent properties (Tognoli and Kelso, 2013; Fraiman and Chialvo, 2012; Zare and Grigolini, 2013). In such a vein, the brain is equipped with phase spaces where particle movements take place (Watanabe et al., 2013; Yan et al., 2013). Such trajectories may display different paths. It has been suggested that the brain is equipped with funnel-like locations in phase space where trajectories converge as time progresses, following the shortest path (Tozzi et al., 2016a; Sengupta et al., 2016). Others proposed that brain function does not exhibit erratic brain dynamics nor attractors, but a stable sequence, the so-called transient heteroclinic channel (Afraimovich et al., 2013) and that a multidimensional functional torus might be displayed during spontaneous brain activity (Tozzi and Peters, 2016a). Furthermore, crucial concepts like communication-through-coherence (Deco and Jirsa, 2012) and *plasma-like* collisionless collective movements (Touboul 2012) must be taken into account. In sum, different functional regimes occurring in the brain phase space have been described, both in central nervous systems and in artificial neural networks, and they have been correlated with different brain functions (Tozzi et al., 2016a).

Despite the large number of possible trajectories, the processes governing brain paths may be unified when we take just into account energetic requirements and constraints. Indeed, the second law of thermodynamics states that e*very process occurring in nature proceeds in the sense in which the sum of the entropies of all bodies taking part in the process is increased* (Planck's formulation). This paper aims to evaluate brain energetic constraints in the framework of algebraic topology, namely the Borsuk-Ulam theorem (BUT) (Borsuk 1933). We will take into account also another important topological ingredient, e.g., the symmetries, widespread at every level of nervous organization. Symmetries may be regarded as the most general feature of biological systems (including the brain), perhaps more general also than energetic requirements, so that giving insights into them might provide a general approach to nervous activities (Tozzi and Peters, 2016b). Here we show how BUT and its variants provide powerful insights into brain functioning, especially if we assess the noteworthy relationships between symmetry breaks, changes of neural dimensions, thermodynamic free-energy and informational entropy.

ABSTRACT **TOPOLOGY COMES INTO PLAY**

The *standard* version of the Borsuk-Ulam theorem (BUT). BUT states that (Dodson 1997):

Every continuous map $f : S^n \rightarrow R^n$ *must identify a pair of antipodal points (on S^n).*

In other words, if a sphere S^n is mapped continuously into a *n*-dimensional Euclidean space R^n, there is at least one pair of antipodal points on S^n which map onto the same point of R^n. For further details, see Tozzi and Peters (2016a).

Examples of antipodal points are the opposite points along the poles of a sphere (Matousek, 2003). The notation S^n denotes an n-sphere of convex curvature, which is embedded in a *n+1* euclidean space (Weeks, 2002; Marsaglia, 1972). BUT variants provide a topological methodology for the evaluation of the most general features of brain activity, cast in an empirical fashion that has the potential to be operationalized. This is a quantitative way to give a physical meaning to the otherwise *abstract* concept of BUT. Many BUT variants, useful for the evaluation of brain functions, have been recently described (Tozzi and Peters, 2016b; Peters and Tozzi, 2016a).

Systems' symmetry breaking (Sym-BUT): changes in brain dimensions. Symmetries are invariances underlining physical and biological systems (Weyl, 1982). A symmetry break occurs when the symmetry is present at one level of observation, but "hidden" at another level (Roldàn et al., 2014). BUT tells us that we can find, on an *n*-dimensional sphere, a pair of opposite points that have the same encoding on an *n*-1 sphere. This means that symmetries can be found when evaluating the system in a proper dimension, while they disappear (are hidden or broken) when we assess the same system in just one dimension lower (Tozzi and Peters, 2016b).

There are two different ways to define and assess brain dimensions. Indeed, the term *dimension* may reflect either functional relationships of brain activities, or anatomical connections between cortical areas. The first approach takes into account the dimensionality of the neural space. Connectivity and complex network analyses of neural signals allow the assessment of the complex dynamics of brain activity, providing a novel insight into the multidimensionality of various neural functions' representations (Kida et al., 2016). From a dynamical system perspective, one would expect that brain activities are represented as, for example, some scalar quantity measured at different brain locations (say N locations) at different points in time. Then one could describe nervous dynamics as trajectories and/or manifolds in a N-dimensional phase space (Lech et al., 2016). Mazzucato et al (2016) demonstrated that stimuli reduce the dimensionality of cortical activity. Clustered networks, such as default mode network, have instead a larger dimensionality, because the latter grows with ensemble size: the more neurons are recruited, the more the dimensions (Mazzucato et al, 2016). Apart from giving insights in neural dynamics in the *canonical* three dimensions (space, time, and frequency), complex network analyses are also able to evaluate other functional dimensions, e.g. categories of neuronal indices such activity magnitude, connectivity, network properties and so on (Kida et al., 2016). It must be taken into account that dimension reduction and symmetry breaking display close relationships, so that symmetries are correlated with changes in functional dimensions in the brain. Indeed, a key feature of dynamical approaches is that the dynamics they predict are characterized by nonequilibrium phase transitions, and therefore breaks of symmetries (Scholz et al., 1987). Many studies emphasized how different levels of behavioral dynamics' organization take place in neural ensembles. To make some examples, Jirsa et al. (1998), focusing on the cortical left-right symmetry, derived a bimodal description of the brain activity that is connected to behavioral dynamics, while Jirsa et al. (1994) demonstrated that, when an acoustic stimulus frequency is changed systematically, a spontaneous transition in coordination occurs at a critical frequency, in both motor behavior and brain signals.

Concerning the second approach to brain dimensionality, it has been recently suggested that brain trajectories, at least during spontaneous activity, might display four spatial dimensions, instead of three (Tozzi and Peters 2016a). Brain symmetric states display dimensions higher than asymmetric ones, so that, in this case, the space of interest does not refer to dynamical neural spaces, but to detectable physical cortical locations. In such a vein, Stemmler et al. (2015) proposed that animals can navigate by reading out a simple population vector of grid cell activity across multiple spatial scales. Combining population vectors at different microscopic dimensions predicts indeed neural and behavioral correlates of multiscale grid cell readout, that transcend the known link between entorhinal grid cells and hippocampal place cells. While the spatial activity of a single grid cell does not constitute a metric, an ensemble of hierarchically organized grid cells does provide instead a distance measure (Stemmler et al., 2015). In our paper, the mapping of trajectories from high dimensional manifold to lower dimensions refers to both the above described definitions of dimensionality.

In sum, the study of changes in brain dimensions is a promising novel methodology. We need to take into account that, despite neural networks modelling complex systems are known to exhibit rich, lower-order connectivity patterns at the level of individual nodes and edges, however higher-order organization remains largely unknown. Benson et al. (2016) recently developed an algorithmic framework for studying how complex networks are organized by higher-order connectivity patterns, revealing unexpected hubs and geographical elements. In such a vein, Kleinberg et al. (2016) demonstrated that real networks are not just random combinations of single networks, but are instead organized in specific ways dictated by hidden geometric correlation between layers. Such correlations allowed the detection of multidimensional communities, e.g., sets of nodes that are simultaneously similar in multiple layers. Crucial for our topological arguments, such multidimensionality also enables accurate trans-layer link prediction, meaning that

connections in one layer can be predicted by observing the hidden geometric space of another layer. For example, when the geometric correlations are sufficiently strong, a multidimensional framework outperforms navigation in the single layers, allowing efficient targeted navigation simply by using local multilayer knowledge (Kleineberg et al., 2016).

General BUT (Gen-BUT). Here we provide also a generalized version of BUT, which allows a topological study of the brain in the context of physics. Gen-BUT states that:
Multiple sets of objects with matching descriptions in a d-dimensional manifold M^d are mapped to a single set of objects in M^{d-1} and vice versa. The sets of objects, which can be mathematical, physical or biological features, do not need to be antipodal and their mappings need not to be continuous. The term *matching description* means the sets of objects display common feature values or symmetries. M stands for a manifold with any kind of curvature, either concave, convex or flat. M^{d-1} may also be a part of M^d. The projection from a sphere to an Euclidean space in not anymore required, just M is required. The notation d stands for a natural, or rational, or irrational number. This means that the need for spatial dimensions of the classical BUT is no longer required. Note that a force, or a group, an operator, an energetic source, is needed, in order to project from one dimension to another. The process is reversible, depending on energetic constraints.

String-BUT: a topological approach to brain paths. Another foremost BUT variant is the string-based BUT (briefly, **strBUT**) (Peters and Tozzi, 2016b). The usual continuous function required by BUT (Peters and Tozzi, 2016a; Peters, 2016) is replaced by a proximally continuous function, which guarantees that, whenever a pair of strings (regions that are called *world lines*) are close (near enough to have common elements), then we always know that their mappings will be also be close. A string is a region of space with either bounded or unbounded length. As a particle moves through space following a world line (Olive and Landsberg, 1989), interactions occur at the junctions of world lines. A string is then a part of a hypersphere surface, over which a particle travels. Put another way, a string is path-connected and its path is defined by a sequence of adjacent surface points. The points can be physical, as opposed to abstract geometric points. In other words, a string is a thin region of space that has describable features such as connectedness, length, open-ended or closed-ended, and shape. Strings are antipodal, provided they are disjoint and yet have the same description (Petty, 1971). In terms of nervous theory, a string is a path which stands for a moving particle into either the functional or anatomical spaces of the brain.

In order to map S^n to S^{n-1}, we need to work with lower dimensional spaces containing regions where each point in S^{n-1} has one less coordinate than a point in S^n. Let X be a topological space equipped with Lodato proximity δ (Peters, 2016). str$A$ δ $\neg$str$A$ reads str$A$ and$\neg$str$A$ are close. Dochviri and Peters (2016) introduce a natural approach in the evaluation of the nearness of sets in topological spaces. The objective is to classify levels of nearness of sets relative to each given set. The main result is a proximity measure of nearness for disjoint sets in an extremely disconnected topological space. Let $\mathrm{int}(\mathrm{str}A)$ be the set of points in the interior of str$A$. Another result is that if strings str$A$, $\neg$str$A$ are nonempty semi-open sets such that str$A$ δ $\neg$str$A$, then $\mathrm{int}(\mathrm{str}A)$ δ $\mathrm{int}(\neg\mathrm{str}A)$.

An important feature is that the manifolds M^d and M^{d-1} are topological spaces equipped with a strong descriptive proximity relation. Recall that in a topological space M, every subset in M and M itself are open sets. A set E in M is open, provided all points sufficiently near E belong to E (Bourbaki, 1966). The description-based functions in genBUT are strongly proximally continuous and their domain can be mathematical, physical or biological features of world line shapes. Let A,B be subsets in the family of sets in M (denoted by 2^M) and let $f : 2^M \to R^n, A \in 2^M, f(A) =$ a feature vector that describes A. That is, $f(A), f(B)$ are descriptions of A and B. Nonempty sets are *strongly near*, provided the sets of have elements in common. The function f is strongly proximally continuous, provided A strongly near B implies $f(A)$ is strongly near $f(B)$. This means that strongly near sets have nonempty intersection. From a genBUT perspective, multiple sets of objects in M^d are mapped to $f(A \cap B)$, which is a description of those objects common to A and B. In other words, the functions in genBUT are set-based embedded in a strong proximity space. In particular, each set is set of contiguous points in a path traced by a moving particle. The path is called a world line. Pairs of world lines have squiggly, twisted shapes opposite each other on the surface of a manifold. Unlike the antipodes in a conventional hypersphere assumed by the BUT, the antipodes are now sets of world lines that are discrete and extremely disconnected. Sets are extremely disconnected, provided: a) the closure of every set is an open set (Dochviri and Peters, 2016), b) the closure of every set is embedded in the discrete space, and c) the intersection of the closure of the intersection of every pair of antipodes is empty. The shapes of the antipodes are separated and belong to a computational geometry. That is, the shapes of the antipodal world lines approximate the shapes in conventional homotopy theory (Peters, 2016). The focus here is on the descriptions (sets of features) of world line shapes. Mappings onsets with matching description, or, in other words, mappings on descriptively strongly proximal sets, here means that such mappings preserve the nearness of pairs of sets. The assumption made here is that antipodal sets live in a descriptive

Lodato proximity (DLP) space. Therefore, antipodal sets satisfy the requirements for a DLP (Peters, 2016). Let δ be a DLP and write $A \, \delta \, B$ to denote the descriptive nearness of antipodes A and B. And let f be a DLP continuous function. This means $A \, \delta \, B$ implies f(A) δ f(B) = f(A) $\cap$ f(B) $\neq \varnothing$.

Example: Assume that antipodes A and B have symmetries (shape, bipolar, overlap, path-connectedness), and f is DLP strongly continuous function, then $A \, \delta \, B \implies$ f(A) δ f(B)

This means that, whenever A and B are descriptively close, then A is mapped to $f(A)$ and B is mapped to $f(B)$ and f(A) δ f(B). If we include in the description of A and B the location of the discrete points in A and B, then the DLP mapping is invertible. That is, $f(A)$ maps to A, $f(B)$ maps to B and $f(A) \, \delta \, f(B)$ implies $A \, \delta \, B$.

To make an example, in nervous functional terms we may state that the paths followed by nervous trajectories in brain phase spaces are closed and can be described in guise of moving strings.

ABSTRACT TOPOLOGY MEETS *REAL* PHYSICS: BRAIN ENERGETIC REQUIREMENTS

A BUT variant, termed **energy-BUT**, is particularly useful in our context. There exists a physical link between the abstract concept of BUT and the energetic features of the system formed by two manifolds M^d and M^{d-1}. We start from a manifold M^d equipped with a pair of antipodal points, standing for a symmetry according to Sym-BUT. When these opposite points map to a n-Euclidean manifold (where M^{d-1} lies), a symmetry break/dimensionality reduction occurs, and a single point is achieved (Tozzi and Peters 2016b). However, it is widely recognized that a decrease in symmetry goes together with a decrease in entropy and free-energy, at least in a closed system. This means that the single mapping function on M^{d-1} displays energy parameters lower than the two corresponding antipodal functions on M^d. Therefore, decreases of dimensions give rise to decreases of energy and energy requirements (**Figure 1**). In such a way, BUT and its variants yield physical quantities, because we achieve a system in which energetic changes do not depend anymore on thermodynamic parameters, rather on affine connections and homotopies.

It must be taken into account that energy-BUT concerns not just energy, but also information. Indeed, two antipodal points contain more information than their single projection in a lower dimension. Dropping down a dimension means that each point in the lower dimensional space is simpler, because each point has one less coordinate. In sum, energy-BUT provides a way to evaluate decreases in energy and information in topological, other than thermodynamic, terms.

An example. Here we provide an example in order to calculate the energy requirements of different functional states in the central nervous system. We start from a nervous closed system, shaped in guise of a M^{d-1} equipped with a single physical function A characterized, say, by a free-energy =1 and an entropy =2. For gen-BUT, when we project the function to M^d, we achieve two antipodal functions B and C with matching description, forming a symmetric system. The question here is: which are the free-energy and the entropy of each one of the two antipodal (symmetric) functions on M^d? And what happens to enthalpy? This question is crucial, because it calls attention to energy conservation and symmetries. In effect, this question leads to the Noether theorem (Noether, 1918), which gives us a physical, testable counterpart to the otherwise *algebraic* topological BUT. Indeed, if we do not take into account the changes in free energy from A to B and C, there is no possibility to translate the abstract BUT to the physics of brain activity, and we have to use the BUT, as we already did, simply as a useful methodological tool (Peters et al., 2016).

The following scenario can be depicted. The projection (mapping) of the description of a pair of physical points (or regions, or functions) on M^d into a point in M^{d-1} occurs because we have found a continuous function between the two manifolds. This is a flexible situation, because we can vary the description of the pair physical points (regions, or functions) and achieve a mapping to various Euclidean spaces, depending on the number of features of the antipodes. From the gen-BUT perspective, the entropy of antipodal regions would be part of the description of the antipodes and would be the same for each antipode. This works for regions, since informational entropy is defined in terms of a set of events. Each regional antipode would be the culmination of a set of random events, leading to each antipode. Similarly, the free energy of each antipode would be the same. In sum, if the region A is characterized by free-energy =1 and an entropy =2, the regions B and C are both characterized by a free-energy =1 and an entropy =2. The total free energy of the system $B+C$ is doubled, compared with A.

BRAIN THERMODYNAMIC PARAMETERS: WHEN BUT ENCOMPASSES A PHYSICAL QUANTITY

In the previous paragraph, our discussion on energetic requirements assumed that the brain is a closed system. Changing the state of system necessarily entails a modification in thermodynamic free-energy, which is equivalent to the work done on the system, and which can be regarded as the average uncertainty, or the information we have about the system's microscopic states (Sengupta et al. 2013a). However, we need to take into account that the brain is an open, non-

equilibrium biological system. As thermodynamic entropy measures the dispersion over microstates of a thermodynamic canonical ensemble, informational entropy plays the same role, but over some phase functions or macroscopic variables that change with time (Sengupta et al. 2013a). It means that symmetry breaking in the brain may occur through the widely described phenomena of critical fluctuations and critical slowing (Scholz et al, 1987). In such a vein, this paragraph aims to correlate classical thermodynamical parameters with brain dynamical features, in order to analyze and quantify them.

Free-energy. The brain represents 2% of the human body mass yet it accounts for about 20% of total energy consumed, a substantial proportion (Attwell and Laughlin, 2001). The metabolic brain activity, influenced by a balance between the energy cost incurred by its operation and the benefits realized by energy expenditure, is therefore high and constant over time (Sengupta et al., 2013a). Why does the brain consume such remarkable amounts of energy, despite the fact that evolution is geared toward minimizing very high metabolic costs? Almost 20–60% of the energy allocated for the brain is used to support the metabolic rate of the cortical grey matter and for synapses and action potentials (Sengupta et al., 2013b). For our purposes, we limit our analysis to spike frequency. It easy to see that the free energy roughly corresponds to the electric spike frequency. See Tozzi et al. (2016b) for a technical explanation. It means that the increase in thermodynamic free-energy during brain activity is mostly due to spikes, and that we are allowed to evaluate variations in thermodynamic free-energy during brain activity just in terms of electric spiking. In the context of ongoing fluctuations with complex properties caused by variations in thermodynamic parameters, a foremost issue is the *free-energy principle* (Friston 2010). A self-organizing system like the brain, at non-equilibrium steady-state with its environment, needs to minimize its free-energy (and associated entropy), in order to resist a tendency towards disorder/entropy. The key thrust is that energy expenditure is balanced by homeostatic mechanisms, in an effort to minimize free-energy, and in an interplay between neuronal structure and activity at many different spatiotemporal scales. A subtle equilibrium takes place among actual sampled sensations, brain's predictions (e.g, the expected energy), expectation (e.g., the best possible guess), surprise (e.g. an improbable outcome caused by unknown quantities), accuracy (e.g., the surprise about the sensations that are expected) and complexity (e.g., the beliefs before and after observing data through sensory inputs). Such different mechanisms tend towards a main goal: minimising entropy production, which corresponds to minimising the so called *variational free-energy*. There is a strict correspondence between concepts in Friston's formalism and that of the thermodynamics. For example, the Gibb's thermodynamical free-energy stands, in Friston's framework, for the entropy of recognition density, or in other words, for the above mentioned actual sensation sampled by the brain. Further, because variational and thermodynamic free-energy share a common minimum, we are also allowed to link information processing (e.g., Bayesian belief updating) to metabolic efficiency (Sengupta et al., 2013b), so that the average time of variational free-energy becomes a proxy for entropy.

Temperature. The cortical temperature is not a stable parameter as currently believed. The brain displays instead thermal gradients observed at many spatiotemporal scales (Wang H. et al., 2014). Local temperature fluctuations may act as a dynamic variable, modulating presynaptic and postsynaptic events, sensory stimuli, behavioral changes, memory encoding and fine-tune activity-dependent processes (Kalmbach and Waters, 2012; Long and Fee, 2008). In terms of symmetry breakings achieved during nervous second order phase-transitions, temperature might stand for one of the critical control parameters which dictates how the brain evolves from one coordinated state to another. Therefore, brain temperature can be used as an order parameter to monitor the dynamics of the nervous collective state and deviations from the symmetrical state. Indeed, in terms of informational entropy, cortical temperatures contain information about how large-scale physiological and pathological outcomes arise from the interactions of many small-scale processes, in order that thermal brain variations may lead to different probability outcomes. In sum, non-stationary thermal cortical fluctuations, an underrated general mechanism of nervous function able to modify the energy of the brain and to influence psychophysical characteristics, can be assessed in topological terms.

Entropies. Entropies are evaluated in fMRI functional studies through different techniques, e.g., pairwise entropy (Schneidman et al., 2006; Watanabe et al., 2014; Wang Z. et al., 2014), Granger causality index, phase slope index, and so on (Kida et al., 2016). Such approaches also make it possible to analyse how the complexity of an adaptive system like the brain is best understood as a dynamic network that aims to decrease its free-energy, for example via entropy transfer. Here we propose a novel topological way to assess, in brain fMRI functional studies, changes in informational entropies. The method, referred here to changes in cortical spatial dimensions, is described in **Figure 2**. The Figure shows how, by knowing just the entropy values for each BOLD-activated brain area, we are allowed to correlate two different brain states, e.g., a state with symmetry breaking and a state with preserved symmetries.
During a symmetry break or vice versa, the brain may use different mechanisms in order to modify thermodynamic parameters. The possible mechanisms are displayed in **Figure 1**. For example, when the system goes from symmetry to symmetry break, the enthalpy must be reduced of the half, via, e.g., a decrease of local blood flow (enthalpy), or a decrease of spike frequency (free-energy). When the system goes from a symmetry break towards a restored symmetry, the brain requires a surplus of *external* energy to *inject* into M^d, and vice versa. Brain spikes, in this framework, could stand for

one of the thermodynamic parameters able to give rise to different attractors, and to supply the required enthalpy, in order to proceed from a symmetry to a symmetry breaking and vice versa.

CONCLUSIONS

We showed how, from an *abstract* topological assessment of brain activity, we may achieve *real* thermodynamic parameters, in order to evaluate and correlate different cortical functions. BUT and its novel variants display very useful general features which help us to explain a wide-range of brain phenomena. By satisfying the requirements for BUT, it is possible for us to quantify increases and decreases of free-energy/entropy/enthalpy when going from one functional brain conformation to another, *e.g.*, from the functional 4D sphere of the default mode network during spontaneous brain activity (Tozzi and Peters, 2016a), to the lower dimensional 3D manifold during evoked brain activity (Papo, 2014). Therefore, the existence of one pair of mappings implies an overall change in thermodynamic and informational parameters. Despite BUT theorem states that it exists *at least a pair* of antipodal point that maps a dimension lower, nevertheless it does not say that *every antipodal pair* will be mapped to a single set, nor that *all* the two sets of objects in higher dimensions display matching descriptions with *all* the single sets of objects in lower dimensions. It means that we would neither specifically assess if a recorded brain state or a set of trajectories is the image of any antipodes, nor the accurate matching of the mapping from higher-dimension manifolds to lower-dimension ones. However, BUT is still very informative about brain dynamics in practice. Indeed, there exist some trajectories, even if we don't know which, that are mapped to a lower-dimensional space and imply a predictable energetic change. Because the brain functional micro-zones are countless, the use of BUT is helpful in achieving a drastic reduction and simplification of the areas to investigate. Instead of looking for a needle in a haystack, BUT makes it possible for neuro-researchers to remove the most of the straw and to increase the number of needles. The BUT approach also overtakes the claim of Simas et al. (2016), who suggested that the algebraic topological approach of embedding a brain network on metric spaces (of different dimensions) may reveal regions that are members of large areas or subsystems, rather than regions with a specific role in information processing. In sum, contrary to the classical averaging of connectivity matrices and to the recent algebraic topological methodologies, the BUT approach reveals brain areas with a specific role in information processing. A shift in conceptualizations is evident in a methodological approach based on BUT. That is, the opportunity to treat brain dynamics as topological structures gives us the invaluable chance to describe them through the tools of functional analysis (Dol'nikov, 1992). The BUT perspective enunciates a symmetry property located in the physical space (the environment and the brain) to be translated to an abstract space and *vice-versa,* enabling us to achieve maps from one system to another. This approach is in touch with recent proposals, which provide a rigorous way of measuring distance on brain manifolds (Sengupta et al., 2016). We might imagine the brain as a manifold, equipped with a high number of symmetries and with an internal, mathematically structured, holistic generative model of the external world. Depending on external stimuli (Tozzi et al., 2016a) and on individual background, symmetry breaks occur, giving rise to phase spaces equipped with a lower number of dimensions. This takes us into the realm of metric algebraic topology (Willard, 1990), where multidimensional manifold describe the structural order of the relationships between nervous anatomical components and their functional paths.

REFERENCES

1) V. Afraimovich, I. Tristan, P. Varona, M. Rabinovich. 2013. Transient Dynamics in Complex Systems: Heteroclinic Sequences with Multidimensional Unstable Manifolds. Discontinuity, Nonlinearity and Complexity 2(1): 21-41.

2) Attwell, D., & Laughlin, S. B. 2001. An energy budget for signaling in the grey matter of the brain. Journal of Cerebral Blood Flow and Metabolism: Official Journal of the International Society of Cerebral Blood Flow and Metabolism, 21(10), 1133–1145. http://doi.org/10.1097/00004647-200110000-00001

3) Benson AR, Gleich DF, Leskovec J. 2016. Higher-order organization of complex networks. Science 353, 6295: 163-166. DOI: 10.1126/science.aad9029

4) Borsuk M. 1933. Drei satze uber die n-dimensionale euklidische sphare. Fundamenta Mathematicae, XX, 177–190.

5) Deco G, Jirsa VK. 2012 Ongoing cortical activity at rest: criticality, multistability, and ghost attractors. J Neurosci. 7;32(10):3366-75. doi: 10.1523/JNEUROSCI.2523-11.2012.

6) DochviriI, Peters JF 2016. Topological sorting of finitely near sets. Math. in Comp. Sci., *in press.*

7) Dodson CTJ. 1997. A user's guide to algebraic topology. Dordrecht, Netherlands: Kluwer Academic Publishers.

8) Dol'nikov VL. 1992. A generalization of the ham sandwich theorem. Mathematical Notes, 52, 771–779.

9) Fraiman D,Chialvo DR. 2012. What kind of noise is brain noise: Anomalous scaling behavior of the resting brain activity fluctuations. Frontiers in Physiology, 3 JUL(July), 1–11. http://doi.org/10.3389/fphys.2012.00307

10) Friston, K. 2010. The free-energy principle: a unified brain theory? Nature Reviews Neuroscience, 11(2), 127–138. http://doi.org/10.1038/nrn2787

11) Jirsa VK, Friedrich R, Haken H, Kelso JAS. 1994. A theoretical model of phase transitions in the human brain. Biol. Cybern. 71: 27. doi:10.1007/BF00198909

12) Jirsa VK, Fuchs A, Kelso JAS. 1998. Connecting Cortical and Behavioral Dynamics: Bimanual Coordination. Neural Computation archive, Volume 10 Issue 8, November 1998, 2019 - 2045

13) Kalmbach, AS, Waters J. 2012. Brain surface temperature under a craniotomy. J Neurophysiol. 108(11), 3138-3146.

14) Kida T, Tanaka E and Kakigi R. 2016. Multi-Dimensional Dynamics of Human Electromagnetic Brain Activity. Front. Hum. Neurosci. 9:713. doi: 10.3389/fnhum.2015.00713

15) Kleineberg K-K, Boguñá M, Serrano MA, Papadopoulos F. 2016. Hidden geometric correlations in real multiplex networks. Nature Physics, in press. doi:10.1038/nphys3812

16) Lech RK, Güntürkün O, Suchan B. 2016. An interplay of fusiform gyrus and hippocampus enables prototype- and exemplar-based category learning. Behavioural Brain Research, doi:10.1016/j.bbr.2016.05.049

17) Long MA, Fee MS. 2008. Using temperature to analyse temporal dynamics in the songbird motor pathway. Nature 456(7219): 189-194.

18) Marsaglia G. 1972. Choosing a Point from the Surface of a Sphere. Annals of Mathematical Statistics, 43(2), 645–646. http://doi.org/10.1214/aoms/1177692644

19) Matoušek J. 2003. Using the Borsuk–Ulam Theorem. Lectures on Topological Methods in Combinatorics and Geometry. Berlin Heidelberg: Springer-Verlag.

20) Mazzucato L, Fontanini A La Camera G. 2016. Stimuli Reduce the Dimensionality of Cortical Activity. Front. Syst. Neurosci, in press http://dx.doi.org/10.3389/fnsys.2016.00011.

21) Noether E.1918. Invariante Variationsprobleme. Nachr. D. König. Gesellsch. D. Wiss. Zu Göttingen, Math-phys. Klasse 1918: 235–257.

22) Olive DI, Landsberg PT. 1989. Introduction to string theory: Its structure and its uses. Philos. Trans. of the Royal Soc. of London, Series A, Mathematical and Physical Sciences 329, 319-328.

23) Papo D. 2014. Functional significance of complex fluctuations in brain activity: from resting state to cognitive neuroscience. Frontiers in Systems Neuroscience, 8(June), 112. http://doi.org/10.3389/fnsys.2014.00112

24) Peters JF. 2016. Computational Proximity. Excursions in the Topology of Digital Images. Edited by Intelligent Systems Reference Library. Berlin: Springer-Verlag. doi:10.1007/978-3-319-30262-1.

25) Peters JF, Tozzi A. 2016a. Region-Based Borsuk-Ulam Theorem. arXiv.1605.02987

26) Peters JF, Tozzi A. 2016b. String-Based Borsuk-Ulam Theorem. arXiv. 1606.04031v1

27) Peters JF, Tozzi A, Ramanna S. 2016. Brain Tissue Tessellation Shows Absence of Canonical Microcircuits. Neuroscience Letters 626: 99–105. doi:10.1016/j.neulet.2016.03.052.

28) Petty CM. 1971. Equivalent sets in Minkowsky spaces. Proc. Amer. Math. Soc. 29(2), 369-374.

29) Roldán É, Martínez I, Parrondo JMR, Petrov D. 2014. Universal features in the energetics of symmetry breaking. Nature Physics, 10(6), 457–461. http://doi.org/10.1038/nphys2940

30) Schneidman E, Berry MJ, Segev R Bialek W. 2006. Weak pairwise correlations imply strongly correlated network states in a neural population. Nature 440:1007-1012.

31) Scholz JP, Kelso JAS, Schöner G. 1987. Nonequilibrium phase transitions in coordinated biological motion: Critical slowing down and switching time. Physics Letters A. Volume 123, 8, 390-394. doi:10.1016/0375-9601(87)90038-7

32) Sengupta B, Stemmler MB, Friston KJ. 2013a. Information and Efficiency in the Nervous System-A Synthesis. PLoS Computational Biology, 9(7). http://doi.org/10.1371/journal.pcbi.1003157

33) Sengupta B, Laughlin SB, Niven JE. 2013b. Balanced Excitatory and Inhibitory Synaptic Currents Promote Efficient Coding and Metabolic Efficiency. PLoS Computational Biology, 9(10). http://doi.org/10.1371/journal.pcbi.1003263

34) Sengupta B, Tozzi A, Cooray GK, Douglas PK, Friston KJ. 2016. Towards a Neuronal Gauge Theory. PLOS Biology, 14(3), e1002400. http://doi.org/10.1371/journal.pbio.1002400

35) Simas T, Chavez M, Rodriguez PR, Diaz-Guilera A. 2015 An algebraic topological method for multimodal brain networks comparisons. Front Psychol. 6;6:904. doi: 10.3389/fpsyg.2015.00904. eCollection 2015.

36) Stemmler M, Mathis A, Herz AVM 2015 Connecting multiple spatial scales to decode the population activity of grid cells. Sci. Adv. 1:e1500816 18 December 2015

37) Tognoli E, Kelso JS. 2013. On the Brain's Dynamical Complexity: Coupling and Causal Influences Across Spatiotemporal Scales. Advances in Cognitive Neurodynamics (III), (Iii), 259–265. http://doi.org/10.1007/978-94-007-4792-0

38) Touboul J. 2012. Mean-field equations for stochastic firing-rate neural fields with delays: Derivation and noise-induced transitions. Physica D: Nonlinear Phenomena. 241 (15):1223–1244. doi:10.1016/j.physd.2012.03.010

39) Tozzi A, Fla T, Peters PJ. 2016a. Building a minimum frustration framework for brain functions in long timescales. J Neurosci Res. DOI: 10.1002/jnr.23748

40) Tozzi A, Zare M, Benasich AA. 2016b. New Perspectives on Spontaneous Brain Activity: Dynamic Networs and Energy Matter. Frontiers in Human Neuroscience. doi:10.3389/fnhum.2016.00247.

41) Tozzi A, Peters JF. 2016a. Towards a Fourth Spatial Dimension of Brain Activity. Cognitive Neurodynamics 10 (3): 189–99. doi:10.1007/s11571-016-9379-z.

42) Tozzi A, Peters JF. 2016b. A Topological Approach Unveils System Invariances and Broken Symmetries in the Brain. Journal of Neuroscience Research 94 (5): 351–65. doi:10.1002/jnr.23720.

43) Van Essen DC. 2005. A Population-Average, Landmark- and Surface-based (PALS) atlas of human cerebral cortex. Neuroimage. 28, 635–666.

44) Yan H, Zhao L, Hu L, Wang X, Wang E, Wang J. 2013. Nonequilibrium landscape theory of neural networks. PNAS 110(45): 4185-4194.

45) Wang H, Wang B, Normoyle KP, Jackson K, Spitler K. 2014. Brain temperature and its fundamental properties: a review for clinical neuroscientists. Front Neurosci. 8;8:307.

46) Wang Z, Li Y, Childress AR, Detre JA. 2014. Brain entropy mapping using fMRI. PLoS ONE, 9(3), 1–8. http://doi.org/10.1371/journal.pone.0089948

47) Watanabe, T., Hirose, S., Wada, H., Imai, Y., Machida, T., Shirouzu, I., Masuda, N. 2013. A pairwise maximum entropy model accurately describes resting-state human brain networks. Nature Communications, 4, 1370. http://doi.org/10.1038/ncomms2388

48) Watanabe, T., Kan, S., Koike, T., Misaki, M., Konishi, S., Miyauchi, S. Masuda, N. 2014. Network-dependent modulation of brain activity during sleep. NeuroImage, 98, 1–10. http://doi.org/10.1016/j.neuroimage.2014.04.079.

49) Weeks JR. 2002. The shape of space, IInd edition. Marcel Dekker, inc. New York-Basel.

50) Weyl H. 1982. Symmetry. Princeton: Princeton University Press. ISBN 0-691-02374-3.

51) Willard S. 1970. General topology. Mineola, New York: Dover Pub., Inc.

52) Zare M, Grigolini P. 2013. Chaos, Solitons & Fractals Criticality and avalanches in neural networks. Chaos, Solitons and Fractals: The Interdisciplinary Journal of Nonlinear Science, and Nonequilibrium and Complex Phenomena, 55, 80–94. http://doi.org/10.1016/j.chaos.2013.05.009.
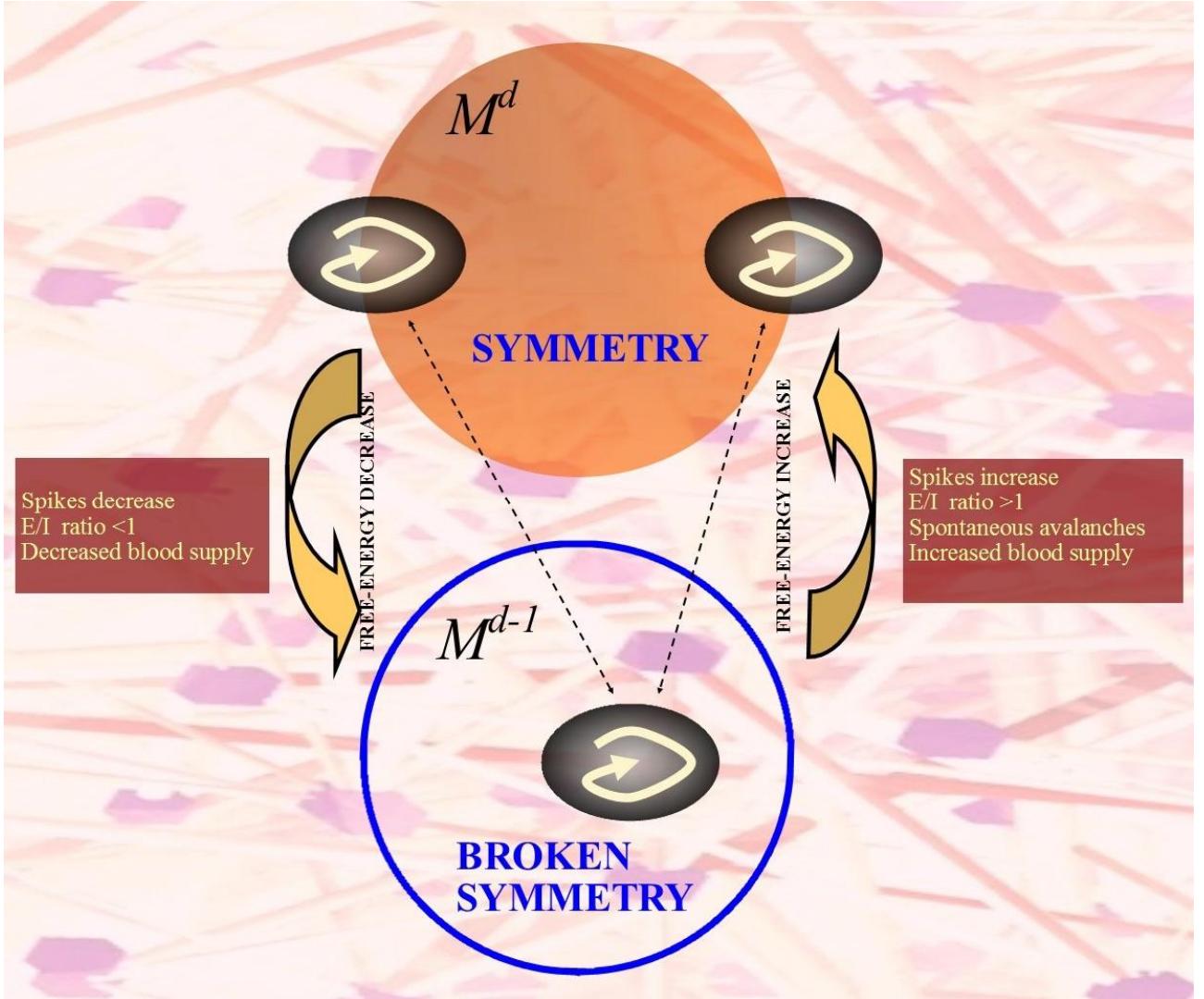
Figure 1. The manifold M^d displays two antipodal points with matching description. It this case, according to strBUT dictates, the antipodal points stand for two symmetric functions equipped with the same energetic conformation (black ovals containing curved arrows). When a symmetry break occurs, the manifold M^{d-1}displays just a single function, equipped with an energetic level lower than the sum of the antipodal functions' ones. Therefore, dimension loss occurs with a decrease of energy. The lateral dark boxes illustrate some hypothetical but plausible conditions which might cause increase or decrease of energy in the brain. In sum, the system displays a configuration with higher energy in M^d, and with lower energy when a symmetry break occurs. The background stands for a schematized structure of the brain phase space.
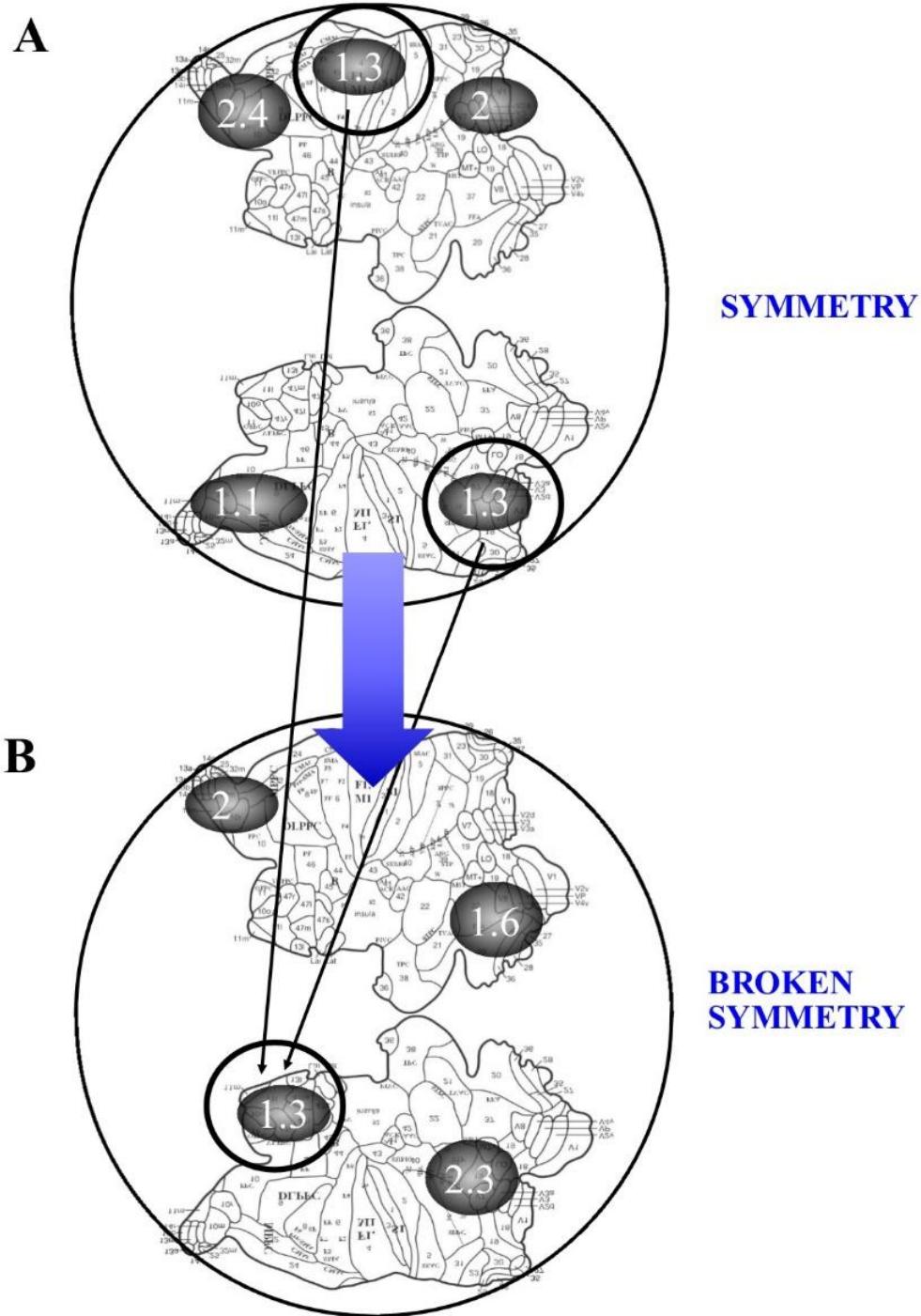
Figure 2. The two brain hemispheres are flattened and displayed in 2D, according to Van Essen (2005). The black circles, depicting hypothetical micro-areas of BOLD signal during fMRI functional studies, contain a number which stands for the corresponding entropy value. **Figure A** displays a functional state with preserved symmetry (e.g., mind wandering), while **Figure B** one with symmetry breaking (e.g., a task-related activity). Note that two micro-areas with the same entropy values in **Figure A** stand for two points with matching description. The latter necessarily project to a single point, in case the brain symmetry is broken, according to the dictates of gen-BUT. It means that in **Figure B** there must be a micro-area with the same entropy value of the two matching points in **Figure A**. It allows us to recognize which zones of the brain could be correlated during symmetry breaks.