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Perhaps this is the proof?

Abstract

This problem is devoted a huge number of articles and books. So it does not make sense to list them. I wrote this note 10 years ago and since then a lot of time I tried to find the error in the reasoning and I can not this to do. I'll be glad if someone will be finds a mistake and even more will be happy if an error will be not found.

Consider the expression

$$a^n + b^n = c^n, \qquad (1)$$

where a, b, c and n - integer numbers, where $a -$ odd, $b -$ even and $c -$ odd number. Then $c = a + d$, where $d -$ even number.
Note that $d < b$ when $n > 1$. This is obvious from (1).

$a^n + b^n = (a+d)^n$ or $b^n = \sum_{k=1}^{n-1} \binom{n}{k} a^{n-k} d^k + d^n$, where $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.

Raising (1) to the power n

We obtain $(a^n + b^n)^n = c^{n^2}$ (2)

Raising (1) to the power n^{n-1}

We obtain $(a^n + b^n)^{n^{n-1}} = c^{n^n}$ (3)

We make subtract (3) and (2).

We obtain $(a^n + b^n)^{n^{n-1}} - (a^n + b^n)^n = c^{n^n} - c^{n^2}$, (4)

and since $c = a + d$, then we write

$$\sum_{k=1}^{n^{n-1}-1} \binom{n^{n-1}}{k} a^{n^n - kn} b^{kn} + b^{n^n} - \sum_{k=1}^{n-1} \binom{n}{k} a^{n^2 - kn} b^{kn} - b^{n^2} =$$

$$= \sum_{k=1}^{n^n - 1} \binom{n^n}{k} a^{n^n - k} d^k + d^{n^n} - \sum_{k=1}^{n^2-1} \binom{n^2}{k} a^{n^2-k} d^k - d^{n^2}. \qquad (5)$$

Rewrite this equation as follows

$$\sum_{k=1}^{n-1} b^{kn} \binom{n}{k} a^{n^2-kn} \left(\frac{(\frac{n^{n-1}}{k})}{(\frac{n}{k})} a^{n^n - n^2} - 1 \right) + b^{n^2} (b^{n^n - n^2} - 1) + \sum_{k=n}^{n^{n-1}-1} \binom{n^{n-1}}{k} a^{n^n - kn} b^{kn} =$$

$$= \sum_{k=1}^{n^2-1} d^k \binom{n^2}{k} a^{n^2-k} \left(\frac{(\frac{n^n}{k})}{(\frac{n^2}{k})} a^{n^n - n^2} - 1 \right) + d^{n^2} (d^{n^n - n^2} - 1) + \sum_{k=n^2}^{n^n-1} \binom{n^{n-1}}{k} a^{n^n - k} d^k. \qquad (6)$$

Each term in the left and right sides of this equation ≥ 0 by $n \geq 1$. Note that the last terms in the left and right sides of this equation is zero by $1 \leq n \leq 2$, since in this case lower summation index is greater than the upper summation index. The left and right sides of this equation, all the terms-even numbers, but varying degrees of parity. The degree of parity we mean the degree of 2. Let be n - an even number. The left side of (6) of smallest has a degree of parity term

$b^n n a^{n^2 - n} (n^{n-2} a^{n^n - n^2} - 1)$, and in the right-hand side of (6) has the least degree of parity term

$dn^2 a^{n^2-1}(n^{n-2}a^{n^n-n^2}-1),$

Comparing only the even factors of these terms we find that the least degree of parity in the left-hand side of (6) will have a number

$b^n,$ (7)

and in the right-hand side

dn . (8)

Since $b > d,$ then by $n \geq 1$ $b^n > nd.$ Let b have even factors $m,$ that is, it can be written as $b = (2l+1)\,2^m,$ where l integer number. And dn have even factors $p,$ that is, it can be written as $dn = (2r+1)\,2^p,$ where r integer number. Then the least degree of parity the left-hand side of (6) will be number

$b^n = (2l+1)^n\,2^{mn},$ (9)

and in the right-hand side of (6) will be number

$dn = (2r+1)\,2^p.$ (10)

If the degree of parity (9) is greater than the degree of parity (10) equal to p, then dividing both sides of (6) on 2^p we get on the left side of the equality is an even number, and the right is odd.

If the degree of parity (9) equal to mn is less than the degree of parity (10), then dividing both sides of (6) on 2^{mn} we get on the left side of the equality is an odd number, and the right-even.

And the last time the degree of parity (9) and (10) are equal $mn = p$ .But in this case, since $b^n > nd$, then $(2l+1)^n > (2r+1)$ and in left-hand of (6) will be located a positive number $((2l+1)^n - (2r+1))\,2^p.$ If the expression in parentheses has a degree of parity $t,$ then in the left side of (6) will be a positive number $(2s+1)\,2^{pt},$ where s – integer number. If this number will have the least degree of parity with respect to the other is positive monomials on the left side of (6), then dividing this equation by $2^{pt},$ we get the left side of this equation is an odd number, and the right even. And this process can in principle be repeated if the necessary several times and in the end to make sure that in one part of right or left of (6) was an odd number, and the other is even number. It follows that (6), and hence (5) is incorrect.

But all these arguments are valid for the case when n -even number.

If n - odd number, is an expression of the degree of parity $n^{n-2}a^{n^n-n^2}-1,$ in relation to other terms in the equation (6) can not be judged. Therefore, we consider the case $k = 2$ in (6).

On the left we have the term

$$b^{2n}\,\frac{n(n-1)}{2}\,a^{n^2-2n}[n^{n-2}\,\frac{n^{n-1}-1}{n-1}\,a^{n^n-n^2}-1],$$

On the right we have the term

$$d^2\,\frac{n^2(n-1)}{2}\,a^{n^2-2}[n^{n-2}\,\frac{n^n-1}{n-1}\,a^{n^n-n^2}-(n+1)],$$

Expression $\frac{n^{n-1}-1}{n-1}$ - is the sum of a geometric progression

$1+n+n^2+n^3+.....+n^{n-2}$ and since n-odd number, then the sum is even number. Similarly, it is obvious that $\frac{n^n-1}{n-1}$ is odd number. Therefore expressions in brackets on the left and right are odd numbers. So when n-an odd number of the least degree of parity the left-hand side of (6) will have a number

$$b^{2n}, \tag{11}$$

and the right-hand of (6) will have a number

$$d^2 \ . \tag{12}$$

Let me remind you that we are comparing only even factors.
And then exactly the same argument that we held above, comparing dependence (7) and (8). The result is that the equations (5) and (6) is not correct for the even and odd n.
Of (4), (5) and (6) in only one case is true, unless it both parts will be equal to zero and then

$$\text{or } c=1, \text{ or } n^n=n^2, \tag{13}$$

From $c=1$, that $a=1$ and $b=0$. From $n^n=n^2$ it is obvious that or $n=1$, or $n=2$. In all the three cases theorem of Fermat unfair.