ALGORITHM OF REPRESENTATION OF PRIME NUMBERS DETERMINANTS OF THE SPECIAL KIND

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Abstract

For a positive integer n I construct an $n \times n$ matrix of special shape, whose determinant equals the n-th prime number, and whose entries are equal to 1,-1 or 0. Specific calculations which I have carried out so far, allowed me to construct such matrices for all n up to 63. These calculations are based on my own method for quick calculations of determinants of special matrices along with a variation on the Sieve of Eratosthenes.

Let's consider a matrix dimension n , looking like $a_{i,i} = 1. i = 1,2,...n.$

$a_{i,i+1} = 0, i = 1,2,...,n-1.$ $a_{i,i+2} = 1, i = 1,2,...,n-2.$

$a_{i,i+k} = 0,\ i = 1,2,...,n-k; k = 3,4,...,n-1.$ $|a_{i,j}| = 1, i = 2,3,...,n;\ j = 1,2,...,i-1.$

For this matrix special factors $E_i^{(n)}, i = 1,2,..,n-1.$ calculation

under the formula [1]. $E_{i+1}^{(n)} = \dfrac{-1}{E_i^{(n)}(1 + \sum\limits_{j=1}^{n-2} a_{i,i-j} \times \prod\limits_{l=1}^{j} E_{i-l}^{(n)})}$ $i = 1,2,..,n-2.$ (*)

In this case fairly following statement which is simple to prove by decomposition of a determinant of the specified matrix on elements last column. $\Delta_{k-1} E_k^{(n)} E_{k-1}^{(n)} = -\Delta_{k-2} + p_{k-1} E_k^{(n)}$, $k = 2,3,....,n-1.$ **(+)**

Here Δ_k -determinant of k dimension. p_k , $k = 2,3,...$ as it will be clear more low, always odd number. Let's consider the following matrix of 3-tx dimension

1 0 1 Determinant of this matrix is equal to 3rd. (third prime number (1,2,3)).
-1 1 0 This fact is checked directly. From (+) at $n = 3$ follows,
-1-1 1 that $E_2^{(3)} E_1^{(3)} = -1.$ (1)

Here is accepted, that $\Delta_0 = 1, p_1 = 0.$ we will make for the specified matrix of 3 rd dimension expression (line) $UX_2 = E_2^{(3)} E_1^{(3)}; E_2^{(3)}.$ Dimension of this line on 1-tsu there is less than dimension of a matrix. According to (1), we write $UX_2 = -1; E_2^{(3)}.$ From [1] follows, that $E_2^{(3)} = \dfrac{1}{\Delta_2}$, where Δ_2 a determinant received from the initial deletion of last line and last column and it. It is equal 1.

Then $UX_2 = \dfrac{1}{\Delta_2}(-1;1).$ We will consider a column $DUX_n = UX_n \Delta_n$ (2).

In particular at n=2 it is had $DUX_2 = -1;1.$ (3)

and $DUX_2(2) = 1$ (4)

More low becomes clear, that $DUX_n(n) = p_n$. According to [2] size determinant of third order it is calculated so

$\Delta_3 = A_3 DUX_2 + \Delta_2,$, (5)

where A_3 -third line in a matrix of third order without an element.

really $\Delta_3 = -1(-1) + (-1)\cdot(-1) + 1 = 3.$ And, as shown in [2] this rule Remains for any determinant of dimension n.

$$\Delta_n = A_n DUX_{n-1} + \Delta_{n-1} \quad , \tag{6}$$

where A_n -n- a line in a matrix of n order without an element $a_{n,n}$.

Let's consider the matrix of fourth order having precisely same structure.

1	0	1	0	According to [1] for this matrix N=4; M=2; L=3.
-1	1	0	1	So it is designated in [1]. Here and more low N=n.
-1	-1	1	0	
$a_{4,1}$	$a_{4,2}$	$a_{4,3}$	1	

$$E_{i+1}^{(4)} = \frac{-a_{i,i+2}}{E_i^{(4)}(a_{i,i} + \sum_{j=1}^{3} a_{i,i-j} \times \prod_{l=1}^{j} E_{i-l}^{(4)}) + a_{i,i+1}} \quad i = 1,2,3 \tag{7}$$

Cases i=1 and i=2 us are already known. According to (+) it

$$E_2^{(4)} E_1^{(4)} = -1$$

$$E_3^{(4)} E_2^{(4)} = -1 - E_3^{(4)}. \tag{8}$$

At i=3 it is had

$$E_4^{(4)} = \frac{-a_{3,5}}{E_3^{(4)}(a_{3,3} + \sum_{j=1}^{3} a_{3,3-j} \times \prod_{l=1}^{j} E_{3-l}^{(4)}) + a_{3,4}} \tag{9}$$

According to [1] it is $a_{3,5}$ believed equal to zero, as such element in matrix 4th order is not present. As the numerator in (9) is equal to zero, and $E_4^{(4)}$ in general case to not equally zero the denominator in (9) is equal to zero. And from here

$$E_2^{(4)} = \frac{1}{1 + E_1^{(4)}} \tag{10}$$

One more parity connecting $E_2^{(4)}$ and $E_1^{(4)}$.

Then $E_1^{(4)} = -\frac{1}{2}; E_2^{(4)} = 2; E_3^{(4)} = -\frac{1}{3}.$

Let's pay attention to that $E_3^{(4)} = -\frac{1}{\Delta_3}$ (11)

Line UX_3 we will write down so

$$E_3^{(4)} E_2^{(4)} E_1^{(4)}; E_3^{(4)} E_2^{(4)}; E_3^{(4)} = -E_3^{(4)}; -1 - E_3^{(4)}; E_3^{(4)} \tag{12}$$

or $UX_3 = \frac{1}{\Delta_3}(1;-2;-1)$ and

$$DUX_3 = 1; -2; -1 \tag{13}$$

$$DUX_3(3) = p_3 = -1 \ . \tag{14}$$

And from (6) at n =4 we will have

$$\Delta_4 = a_{4,1} - 2 \cdot a_{4,2} - a_{4,3} + 3 \tag{15}$$

We while do not know to that are equal $a_{4,1}; a_{4,2}; a_{4,3}$. It is known only, that each of them is equal or (+1), or (-1). Δ_4 – the fourth prime number is not known also.

Let's consider the matrix of fifth order having precisely same structure.

N=5; M=2; L=4

$$E_{i+1}^{(5)} = \frac{-a_{i,i+2}}{E_i^{(5)}(a_{i,i} + \sum_{j=1}^{4} a_{i,i-j} \times \prod_{l=1}^{j} E_{i-l}^{(5)}) + a_{i,i+1}} \quad i = 1,.,4 \tag{16}$$

Cases i=1,2,3 us are already known. According to (+) it

$$E_2^{(5)} E_1^{(5)} = -1$$
$$E_3^{(5)} E_2^{(5)} = -1 - E_3^{(5)}$$
$$3E_4^{(5)} E_3^{(5)} = -1 - E_4^{(5)}. \tag{17}$$

At i=4 it is had

$$E_3^{(5)} = \frac{-1}{y_4 + z_4 E_2^{(5)}}. \tag{18}$$

And then

$$E_4^{(5)} = \frac{p_4}{\Delta_4} = \frac{p_4}{\Delta_3 + x_4}. \tag{19}$$

Here it is designated

$$x_4 = a_{4,1} - 2a_{4,2} - a_{4,3}$$
$$y_4 = -a_{4,1} + a_{4,3}$$
$$z_4 = a_{4,2}$$
$$p_4 = -a_{4,1} - a_{4,2} + a_{4,3}.$$

As all a on the module are equal 1,then x_4 and y_4 - even numbers, and z_4 and p_4 -odd. And, according to (15), and under an obvious condition, that $\Delta_4 > \Delta_3$, x_4 to that still positive number. And, as it has appeared these laws further remain.

Further we will receive

$$UX_4 = E_4^{(5)} E_3^{(5)} E_2^{(5)} E_1^{(5)}; E_4^{(5)} E_3^{(5)} E_2^{(5)}; E_4^{(5)} E_3^{(5)}; E_4^{(5)} = -E_4^{(5)} E_3^{(5)}; E_4^{(5)} E_3^{(5)} E_2^{(5)}; E_4^{(5)} E_3^{(5)}; E_4^{(5)} =$$

$$= -E_4^{(5)} E_3^{(5)}; -E_4^{(5)} - E_4^{(5)} E_3^{(5)}; E_4^{(5)} E_3^{(5)}; E_4^{(5)} = \frac{1}{3}(-3E_4^{(5)} E_3^{(5)}; -3E_4^{(5)} - 3E_4^{(5)} E_3^{(5)}; 3E_4^{(5)} E_3^{(5)}; 3E_4^{(5)}) =$$

$$= \frac{1 + E_4^{(5)}}{3}; \frac{1 - 2E_4^{(5)}}{3}; \frac{-1 - E_4^{(5)}}{3}; E_4^{(5)}.$$

Substituting here (19), we have

$$UX_4 = \frac{1}{\Delta_4}(1 - U_4; 1 - V_4^{(1)}; -1 + U_4; p_4) \quad , \tag{20}$$

Where $U_4 = a_{4,2}$ and $V_4^{(1)} = -a_{4,1} + a_{4,3}$, and in this case they coincide with z and y. Following parities are Thus fair

$$x_4 + y_4 = -2U_4; x_4 + p_4 = -3U_4.$$

Thus we have received 2 linear homogeneous equations with 4 unknown numbers about whom it is known, that they integers, and also or even, either odd, or even and positive.

Let. $x_4 = 2$ .It is rather U_4 known, that it or-1, or +1. But the case $U_4 = 1$ is excluded, so in this case the first and third components vector (20) address in a zero, that as it becomes clearly undesirable more low.

Let's accept, that it is equal $U_4 = -1$. Then $y_4 = 0; p_4 = 1; z_4 = -1; V_4^{(1)} = 0$.

And these all 4 unknown numbers satisfy with all 2 linear homogeneous

to the equations and all specified restrictions. In this case $a_{4,2} = -1; a_{4,3} = a_{4,1}$.
As in a determinant of 3rd order below the main diagonal
costs-1 we will write down definitively $\underline{a_{4,1} = -1; a_{4,2} = -1; a_{4,3} = -1.}$ (21)

$\Delta_4 = 5$ - the fourth prime number (22)

$DUX_4 = 2;1;-2;1$ (23)

$DUX_4(4) = p_4 = 1$ (24)

And from (6) at n =5 we will have

$$\Delta_5 = 2 \cdot a_{5,1} + a_{5,2} - 2 \cdot a_{5,3} + a_{5,4} + 5 \quad (25)$$

We while do not know to that are equal $a_{5,1}; a_{5,2}; a_{5,3}; a_{5,4}$. It is known only, that each of them is equal or (+1), or (-1). $\Delta_5 -$ the fifth prime number is not known also. Let's consider the matrix of sixth order having precisely same structure.
N=6; M=2; L=5

$$E_{i+1}^{(6)} = \frac{-a_{i,i+2}}{E_i^{(6)}(a_{i,i} + \sum_{j=1}^{5} a_{i,i-j} \times \prod_{l=1}^{j} E_{i-l}^{(6)}) + a_{i,i+1}} \quad i = 1,.,5 \quad (26)$$

Cases i=1,2,.,4 us are already known. According to (+) it

$$E_2^{(6)} E_1^{(6)} = -1$$
$$E_3^{(6)} E_2^{(6)} = -1 - E_3^{(6)}$$
$$3E_4^{(6)} E_3^{(6)} = -1 - E_4^{(6)}$$
$$5E_5^{(6)} E_4^{(6)} = -3 + E_5^{(6)}. \quad (27)$$

At i=5 it is had

$$E_4^{(6)} = \frac{-1}{y_5 + z_5 E_3^{(6)}}. \quad (28)$$

And then

$$E_5^{(6)} = \frac{p_5}{\Delta_5} = \frac{p_5}{\Delta_4 + x_5}. \quad (29)$$

Here it is designated

$$x_5 = 2a_{5,1} + a_{5,2} - 2a_{5,3} + a_{5,4}$$
$$y_5 = -a_{5,2} + a_{5,4}$$
$$z_5 = -a_{5,1} - a_{5,2} + a_{5,3}$$
$$p_5 = a_{5,1} - 2a_{5,2} - a_{5,3} + 3a_{5,4}.$$

As all a on the module are equal 1,then x_5 and y_5 - even numbers, and z_5 and p_5 -odd number. Condition, that $\Delta_5 > \Delta_4$, x_5 to that still positive number.
Further we will receive
$x_5 - 2p_5 = 5U_5; x_5 + 2z_5 = -U_5; U_5 = a_{5,2} - a_{5,4} = -y_5$ - even number .
Then we will write down
$UX_5 = E_5^{(6)} E_4^{(6)} E_3^{(6)} E_2^{(6)} E_1^{(6)}; E_5^{(6)} E_4^{(6)} E_3^{(6)} E_2^{(6)}; E_5^{(6)} E_4^{(6)} E_3^{(6)}; E_5^{(6)} E_4^{(6)}; E_5^{(6)} = -E_5^{(6)} E_4^{(6)} E_3^{(6)}; E_5^{(6)} E_4^{(6)} E_3^{(6)} E_2^{(6)};$
$E_5^{(6)} E_4^{(6)} E_3^{(6)}; E_5^{(6)} E_4^{(6)}; E_5^{(6)} = -E_5^{(6)} E_4^{(6)} E_3^{(6)}; -E_5^{(6)} E_4^{(6)} + E_5^{(6)} E_4^{(6)} E_3^{(3)}; E_5^{(6)} E_4^{(6)} E_3^{(6)}; E_5^{(6)} E_4^{(6)}; E_5^{(6)} =$

$$=\frac{1}{3}(-E_5^{(6)}3E_4^{(6)}E_3^{(6)};-3E_5^{(6)}E_4^{(6)}+E_5^{(6)}3E_4^{(6)}E_3^{(6)};E_5^{(6)}3E_4^{(6)}E_3^{(6)};3E_5^{(6)}E_4^{(6)};3E_5^{(6)})=\frac{1}{3}(E_5^{(6)}+E_5^{(6)}E_4^{(6)};$$

$$-2E_5^{(6)}E_4^{(6)}+E_4^{(6)};-E_5^{(6)}-E_5^{(6)}E_4^{(6)};3E_5^{(6)}E_4^{(6)};3E_5^{(6)})=\frac{1}{15}(5E_5^{(6)}+5E_5^{(6)}E_4^{(6)};-2\cdot 5E_5^{(6)}E_4^{(6)}+5E_5^{(6)};$$

$$-5E_5^{(6)}-5E_5^{(6)}E_4^{(6)};3\cdot 5E_5^{(6)}E_4^{(6)};15E_5^{(6)})=\frac{1}{15}(6E_5^{(6)}-3;6+3E_5^{(6)};3-6E_5^{(6)};-9+3E_5^{(6)};15E_5^{(6)})=$$

$$=\frac{-1+2E_5^{(6)}}{5};\frac{2+E_5^{(6)}}{5};\frac{1-2E_5^{(6)}}{5};\frac{-3+E_5^{(6)}}{5};E_5^{(6)}.$$

Let's notice, that it is possible to manage and without this conclusion. Free members in the received expression 1; 2;-1;-3 coincide with factors at a in dependence for p_5 only with a sign a minus, and factors at E_5 2;-1;-2;-1 coincide with factors at a in expression for x_5. And this tendency in the further remains.

Substituting here (29), we have

$$UX_5=\frac{1}{\Delta_5}(-1-U_5;2-V_5^{(1)};1+U_5;-3+z_5;p_5),\qquad (30)$$

where $V_5^{(1)}=-a_{5,1}+a_{5,3}-a_{5,4}$ -odd number.

Thus we have received system of 3 linear homogeneous the equations with 5 unknown numbers about whom it is known, that they integers, and also either even, or odd, or even and positive.

Let. $\underline{x_5=2}$ It is U_5 rather known, that it or 0 or +2, or-2.

But $U_5=\pm 2$ there can not be as in this case p_5-even number, that it is impossible.

So $U_5=0$. Then $y_5=0;p_5=1;z_5=-1;V_5^{(1)}=-1.$ In this case we will write down definitively $\underline{a_{5,1}=1;a_{5,2}=-1;a_{5,3}=-1;a_{5,4}=-1.}$ (31)

$\Delta_5=7$ - fifth prime number (32)

$DUX_5=-1;3;1;-4;1$ (33)

$DUX_5(5)=p_5=1$ (34)

And from (6) at n =6 we will have

$$\Delta_6=-a_{6,1}+3\cdot a_{6,2}+a_{6,3}-4\cdot a_{6,4}+a_{6,5}+7\qquad (35)$$

Let's notice, that z_5 coincides with p_4 only instead a_4 of it is necessary to write a_5. And p_5 similarly in the same sense with x_4. It is necessary to add to it $\Delta_3\cdot a_{5,4}$.

We while do not know to that are equal $a_{6,1};a_{6,2};a_{6,3};a_{6,4};a_{6,5}$. It is known only, that each of them is equal or (+1), or (-1). Δ_6 – the sixth prime number is not known also. Let's consider the matrix of seventh order having precisely same structure.

N=7; M=2; L=6

$$E_{i+1}^{(7)}=\frac{-a_{i,i+2}}{E_i^{(7)}(a_{i,i}+\sum_{j=1}^{6}a_{i,i-j}\times\prod_{l=1}^{j}E_{i-l}^{(7)})+a_{i,i+1}}\quad i=1,.,6\qquad (36)$$

Cases i=1,2,.,5 us are already known. According to (+) it

$$E_2^{(7)}E_1^{(7)} = -1$$
$$E_3^{(7)}E_2^{(7)} = -1 - E_3^{(7)}$$
$$3E_4^{(7)}E_3^{(7)} = -1 - E_4^{(7)}$$
$$5E_5^{(7)}E_4^{(7)} = -3 + E_5^{(7)}$$
$$7E_6^{(7)}E_5^{(7)} = -5 + E_6^{(7)}. \quad (37)$$

At i=6 it is had

$$E_5^{(7)} = \frac{-3}{y_6 + z_6E_4^{(7)}}. \quad (38)$$

And then

$$E_6^{(7)} = \frac{p_6}{x_6 + 7}. \quad (39)$$

Here it is designated

$$x_6 = -a_{6,1} + 3a_{6,2} + a_{6,3} - 4a_{6,4} + a_{6,5}$$
$$y_6 = a_{6,1} + a_{6,2} - a_{6,3} + 3a_{6,5}$$
$$z_6 = a_{6,1} - 2a_{6,2} - a_{6,3} + 3a_{6,4}$$
$$p_6 = 2a_{6,1} + a_{6,2} - 2a_{6,3} + a_{6,4} + 5a_{6,5}.$$

As all a on the module are equal 1,then x_6 and y_6 - even numbers, and z_6 and p_6 -odd number. Condition, that $\Delta_6 > \Delta_5$, x_6 to that still positive number.

Further we will receive

$x_6 + y_6 = 4U_6; x_6 + z_6 = U_6; 2x_6 + p_6 = 7U_6; U_6 = a_{6,2} - a_{6,4} + a_{6,5}$ - odd number.

Then we will write down

$$UX_6 = \frac{-2 - E_6^{(7)}}{7}; \frac{-1 + 3E_6^{(7)}}{7}; \frac{2 + E_6^{(7)}}{7}; \frac{-1 - 4E_6^{(7)}}{7}; \frac{-5 + E_6^{(7)}}{7}; E_6^{(7)}.$$

Substituting here (39), we have

$$UX_6 = \frac{1}{\Delta_6}(-2 - U_6; -1 - V_6^{(1)}; 2 + U_6; -1 - y_6; -5 + z_6; p_6) \quad (40)$$

where $V_6^{(1)} = -a_{6,1} + a_{6,3} - a_{6,4} - 2a_{6,5}$ -odd number.

As well $-x_6 + 3p_6 = -7V_6^{(1)}$; and $x_6 - V_6^{(1)} = 3U_6$.

Thus we have received system of 4th linear homogeneous the equations with 6 unknown numbers about whom it is known, that they integers, and also either even, or odd, or even and positive.

We postulate, that each of components of vector UX represents a rational number, that is fraction, in numerator and a denominator which mutually simple integers.

Let. $\underline{x_6 = 2}$ It is U_6 rather known, that it $U_6^{(i)} = \pm(2i+1), i = 0,1$

but $U_6 = \pm 3$ there can not be as $|y_6| \le 6$. Let $U_6^{(i)} = \pm(2i+1), i = 0$ and

$$y_6^{(i)} = \pm 4(2i+1) - 2; z_6^{(i)} = \pm(2i+1) - 2; p_6^{(i)} = \pm 7(2i+1) - 4; V_{6,i}^{(1)} = 2 \mp 3(2i+1), i = 0$$

But then $\frac{-1 - V_6^{(i)}}{x_6 + \Delta_5} = \frac{-3 \pm 3(2i+1)}{2+7} = \frac{-1 \pm (2i+1)}{3}$, $i = 0$, that is unacceptable.

Let $\underline{x_6 = 4}$. U_6 can accept the same two values ± 1.

$U_6 = -1$ there can not be as $|y_6| \le 6$.

Let $U_6 = 1$. Then $y_6 = 0; z_6 = -3; p_6 = -1; V_6^{(1)} = 1$.

This the variant arranges in every respect. So it is definitive

$$\underline{a_{6,1} = 1; a_{6,2} = 1; a_{6,3} = -1; a_{6,4} = -1; a_{6,5} = -1.} \quad (41)$$

$\Delta_6 = 11$ - sixth prime number (42)

$$DUX_6 = -3; -2; 3; -1; -8; -1 \quad (43)$$

$$DUX_6(6) = p_6 = -1 \quad (44)$$

And from (6) at n =7 we will have

$$\Delta_7 = -3a_{7,1} - 2a_{7,2} + 3a_{7,3} - a_{7,4} - 8a_{7,5} - a_{7,6} + 11 \quad (45)$$

Let's notice, that y_6 coincides with z_5 only instead a_5 of it is necessary to write $-a_6$.

It is necessary to add to it $\Delta_3 \cdot a_{6,5}$.

Let's consider the matrix of eighth order having precisely same structure.

N=8; M=2; L=7

$$E_{i+1}^{(8)} = \frac{-a_{i,i+2}}{E_i^{(8)}(a_{i,i} + \sum_{j=1}^{7} a_{i,i-j} \times \prod_{l=1}^{j} E_{i-l}^{(8)}) + a_{i,i+1}} \quad i = 1,.,7 \quad (46)$$

Cases i=1,2,.,6 us are already known. According to (+) it

$$E_2^{(8)} E_1^{(8)} = -1$$

$$E_3^{(8)} E_2^{(8)} = -1 - E_3^{(8)}$$

$$3E_4^{(8)} E_3^{(8)} = -1 - E_4^{(8)}$$

$$5E_5^{(8)} E_4^{(8)} = -3 + E_5^{(8)}$$

$$7E_6^{(8)} E_5^{(8)} = -5 + E_6^{(8)}$$

$$11E_7^{(8)} E_6^{(8)} = -7 - E_7^{(8)}. \quad (47)$$

At i=7 it is had

$$E_6^{(8)} = \frac{-5}{y_7 + z_7 E_5^{(8)}} \quad (48)$$

And then

$$E_7^{(8)} = \frac{p_7}{x_7 + 11}. \quad (49)$$

Here it is designated

$$x_7 = -3a_{7,1} - 2a_{7,2} + 3a_{7,3} - a_{7,4} - 8a_{7,5} - a_{7,6}$$

$$y_7 = -a_{7,1} + 2a_{7,2} + a_{7,3} - 3a_{7,4} + 5a_{7,6}$$

$$z_7 = 2a_{7,1} + a_{7,2} - 2a_{7,3} + a_{7,4} + 5a_{7,5}$$

$$p_7 = -a_{7,1} + 3a_{7,2} + a_{7,3} - 4a_{7,4} + a_{7,5} + 7a_{7,6}.$$

As all a on the module are equal 1,then x_7 and y_7 - even numbers, and z_7 and p_7 -odd number. Condition, that $\Delta_7 > \Delta_6$, x_7 to that still positive number.

Further we will receive

$x_7 - 3y_7 = -8U_7; 2x_7 + 3z_7 = -U_7; x_7 - 3p_7 = -11U_7; U_7 = a_{7,2} - a_{7,4} + a_{7,5} + 2a_{7,6}$ -

-odd number.

Then we will write down

$$UX_7 = \frac{1-3E_7^{(8)}}{11};\frac{-3-2E_7^{(8)}}{11};\frac{-1+3E_7^{(8)}}{11};\frac{4-E_7^{(8)}}{11};\frac{-1-8E_7^{(8)}}{11};\frac{-7-E_7^{(8)}}{11};E_7^{(8)}.$$

Substituting here (49), we have

$$UX_7 = \frac{1}{\Delta_7}(1-U_7;-3-V_7^{(1)};-1+U_7;4+V_7^{(2)};-1-y_7;-7+z_7;p_7), \text{ where} \quad (50)$$

$$V_7^{(1)} = -a_{7,1}+0a_{7,2}+a_{7,3}-a_{7,4}-2a_{7,5}+a_{7,6} \vdots ev \vdots 3x_7+2p_7 = 11V_7^{(1)}; x_7-3V_7^{(1)} = -2U_7.$$

$$V_7^{(2)} = -a_{7,1}-a_{7,2}+a_{7,3}+0a_{7,4}-3a_{7,5}-a_{7,6} \vdots od \vdots 4x_7-p_7 = 11V_7^{(2)}; x_7-3V_7^{(2)} = U_7.$$

Thus we have received system of 5 linear homogeneous the equations
with 7 unknown numbers about whom it is known, that they integers, and also
either even, or odd, or even and positive.

Let $\underline{x_7 = 2}$. U_7 can accept following values $\pm 1;\pm 3;\pm 5$. But approaches only one value $U_7 = -1$, as in all other cases y_7 not the whole number or $|y_7| > 12$.

Then $y_7 = -2; z_7 = -1; p_7 = -3; V_7^{(1)} = 0; V_7^{(2)} = 1$. This the variant arranges in every respect. So it is definitive

$$\underline{a_{7,1} = -1; a_{7,2} = -1; a_{7,3} = 1; a_{7,4} = -1; a_{7,5} = 1; a_{7,6} = -1.} \quad (51)$$

$\Delta_7 = 13$ - seventh prime number (52)

$$DUX_7 = 2;-3;-2;5;1;-8;-3. \quad (53)$$

$$DUX_7(7) = p_7 = -3. \quad (54)$$

And from (6) at n =8 we will have

$$\Delta_8 = 2a_{8,1}-3a_{8,2}-2a_{8,3}+5a_{8,4}+a_{8,5}-8a_{8,6}-3a_{8,7}+13. \quad (55)$$

Let's consider the matrix of ninth order having precisely same structure.

N=9; M=2; L=8

$$E_{i+1}^{(9)} = \frac{-a_{i,i+2}}{E_i^{(9)}(a_{i,i}+\sum_{j=1}^{8} a_{i,i-j}\times\prod_{l=1}^{j} E_{i-l}^{(9)})+a_{i,i+1}} \quad i=1,.,8 \quad (56)$$

Cases i=1,2,.,7 us are already known. According to (+) it

$$E_2^{(9)}E_1^{(9)} = -1$$
$$E_3^{(9)}E_2^{(9)} = -1-E_3^{(9)}$$
$$3E_4^{(9)}E_3^{(9)} = -1-E_4^{(9)}$$
$$5E_5^{(9)}E_4^{(9)} = -3+E_5^{(9)}$$
$$7E_6^{(9)}E_5^{(9)} = -5+E_6^{(9)}$$
$$11E_7^{(9)}E_6^{(9)} = -7-E_7^{(9)}$$
$$13E_8^{(9)}E_7^{(9)} = -11-3E_8^{(9)}. \quad (57)$$

At i=8 it is had

$$E_7^{(9)} = \frac{-7}{y_8+z_8E_6^{(9)}}. \quad (58)$$

And then

$$E_8^{(9)} = \frac{p_8}{x_8+13}. \quad (59)$$

Here it is designated

$x_8 = 2a_{8,1} - 3a_{8,2} - 2a_{8,3} + 5a_{8,4} + a_{8,5} - 8a_{8,6} - 3a_{8,7}$.

$y_8 = -2a_{8,1} - a_{8,2} + 2a_{8,3} - a_{8,4} - 5a_{8,5} + 7a_{8,7}$.

$z_8 = -a_{8,1} + 3a_{8,2} + a_{8,3} - 4a_{8,4} + a_{8,5} + 7a_{8,6}$.

$p_8 = -3a_{8,1} - 2a_{8,2} + 3a_{8,3} - a_{8,4} - 8a_{8,5} - a_{8,6} + 11a_{8,7}$.

As all a on the module are equal 1,then x_8 and y_8 - even numbers, and z_8 and p_8 -odd number. Condition, that $\Delta_8 > \Delta_7$, x_8 to that still positive number.

Further we will receive

$x_8 + y_8 = -4U_8; x_8 + 2z_8 = 3U_8; 3x_8 + 2p_8 = -13U_8; U_8 = a_{8,2} - a_{8,4} + a_{8,5} + 2a_{8,6} - a_{8,7}$.

- even number.

Then we will write down

$$UX_8 = \\ = \frac{3+2E_8^{(9)}}{13}; \frac{2-3E_8^{(9)}}{13}; \frac{-3-2E_8^{(9)}}{13}; \frac{1+5E_8^{(9)}}{13}; \frac{8+E_8^{(9)}}{13}; \frac{1-8E_8^{(9)}}{13}; \frac{-11-3E_8^{(9)}}{13}; E_8^{(9)}.$$

Substituting here (59), we have

$UX_8 = \frac{1}{\Delta_8}(3-U_8; 2-V_8^{(1)}; -3+U_8; 1+V_8^{(2)}; 8-V_8^{(3)}; 1-y_8; -11+z_8; p_8)$,где (60)

$V_8^{(1)} = -a_{8,1} + 0a_{8,2} + a_{8,3} - a_{8,4} - 2a_{8,5} + a_{8,6} + \vdots od \vdots 2x_8 - 3p_8 = -13V_8^{(1)}; x_8 + 2V_8^{(1)} = -3U_8$.
$+3a_{8,7}$.

$V_8^{(2)} = -a_{8,1} - a_{8,2} + a_{8,3} + 0a_{8,4} - 3a_{8,5} - a_{8,6} + \vdots od \vdots x_8 + 5p_8 = 13V_8^{(2)}; x_8 + 2V_8^{(2)} = -5U_8$.
$+4a_{8,7}$.

$V_8^{(3)} = -a_{8,1} + 2a_{8,2} + a_{8,3} - 3a_{8,4} + 0a_{8,5} + 5a_{8,6} + \vdots od \vdots 8x_8 + p_8 = -13V_8^{(3)}; x_8 + 2V_8^{(3)} = U_8$.
$+a_{8,7}$.

Thus we have received system of 6-th linear homogeneous the equations with 8-th unknown numbers about whom it is known, that they integers, and also either even, or odd, or even and positive.

Let $\underline{x_8 = 2}$. U_8 can accept seven values $U_8^{(i)} = \pm 2i, i = 0,1,2,3.$ But as $|y_8| \le 18$, that remains five values $U_8^{(i)} = \pm 2i, i = 0,1,2.$

Then $y_8^{(i)} = -2 \pm 8i; z_8^{(i)} = -1 \pm 3i. \quad i = 0,1,2.$

But in this case $\frac{-11+z_8^{(i)}}{x_8+\Delta_7} = \frac{-11 \pm 3i - 1}{2+13} = \frac{-4 \pm i}{5}, i = 0,1,2$, that is unacceptable.

Let $\underline{x_8 = 4}$. U_8 can accept the same five values $0; \pm 2; \pm 4$.

At $U_8 = 0; \pm 4$ z_8 will be even number that cannot be.

Let's accept $U_8 = 2$. Then $y_8 = -12; z_8 = 1; p_8 = -19; V_8^{(1)} = -5; V_8^{(2)} = -7; V_8^{(3)} = -1.$

This the variant arranges in every respect. So it is definitive

$a_{8,1} = -1; a_{8,2} = -1; a_{8,3} = -1; a_{8,4} = 1; a_{8,5} = 1; a_{8,6} = 1; a_{8,7} = -1.$ (61)

$\Delta_8 = 17$ - 8-th prime number (62)

$DUX_8 = 1; 7; -1; -6; 9; 13; -10; -19.$ (63)

$DUX_8(8) = p_8 = -19.$ (64)

And from (6) at n =9 we will have

$\Delta_9 = a_{9,1} + 7a_{9,2} - a_{9,3} - 6a_{9,4} + 9a_{9,5} + 13a_{9,6} - 10a_{9,7} - 19a_{9,8} + 17$ (65)

Let's consider the matrix of tenth order having precisely same structure.

N=10; M=2; L=9

$$E_{i+1}^{(10)} = \frac{-a_{i,i+2}}{E_i^{(10)}(a_{i,i} + \sum_{j=1}^{9} a_{i,i-j} \times \prod_{l=1}^{j} E_{i-l}^{(10)}) + a_{i,i+1}} \quad i = 1,.,9 \qquad (66)$$

Cases i=1,2,.,8 us are already known. According to (+) it

$$E_2^{(10)} E_1^{(10)} = -1$$
$$E_3^{(10)} E_2^{(10)} = -1 - E_3^{(10)}$$
$$3E_4^{(10)} E_3^{(10)} = -1 - E_4^{(10)}$$
$$5E_5^{(10)} E_4^{(10)} = -3 + E_5^{(10)}$$
$$7E_6^{(10)} E_5^{(10)} = -5 + E_6^{(10)}$$
$$11E_7^{(10)} E_6^{(10)} = -7 - E_7^{(10)}$$
$$13E_8^{(10)} E_7^{(10)} = -11 - 3E_8^{(10)}$$
$$17E_9^{(10)} E_8^{(10)} = -13 - 19E_9^{(10)}. \qquad (67)$$

At i=9 it is had

$$E_8^{(10)} = \frac{-11}{y_9 + z_9 E_7^{(10)}} . \qquad (68)$$

And then

$$E_9^{(10)} = \frac{p_9}{x_9 + 17}. \qquad (69)$$

Here it is designated

$x_9 = a_{9,1} + 7a_{9,2} - a_{9,3} - 6a_{9,4} + 9a_{9,5} + 13a_{9,6} - 10a_{9,7} - 19a_{9,8}.$

$y_9 = a_{9,1} - 3a_{9,2} - a_{9,3} + 4a_{9,4} - a_{9,5} - 7a_{9,6} + 11a_{9,8}.$

$z_9 = -3a_{9,1} - 2a_{9,2} + 3a_{9,3} - a_{9,4} - 8a_{9,5} - a_{9,6} + 11a_{9,7}.$

$p_9 = 2a_{9,1} - 3a_{9,2} - 2a_{9,3} + 5a_{9,4} + a_{9,5} - 8a_{9,6} - 3a_{9,7} + 13a_{9,8}.$

As all a on the module are equal 1,then x_9 and y_9 - even numbers, and z_9 and p_9 -odd number. Condition, that $\Delta_9 > \Delta_8$, x_9 to that still positive number.

Further we will receive

$x_9 - y_9 = 10U_9; 3x_9 + z_9 = 19U_9; 2x_9 - p_9 = 17U_9; U_9 = a_{9,2} - a_{9,4} + a_{9,5} + 2a_{9,6} - a_{9,7} - 3a_{9,8}.$

- odd number. Then we will write down $UX_9 = \frac{-2 + E_9^{(10)}}{17}; \frac{3 + 7E_9^{(10)}}{17}; \frac{2 - E_9^{(10)}}{17};$

$; \frac{-5 - 6E_9^{(10)}}{17}; \frac{-1 + 9E_9^{(10)}}{17}; \frac{8 + 13E_9^{(10)}}{17}; \frac{3 - 10E_9^{(10)}}{17}; \frac{-13 - 19E_9^{(10)}}{17}; E_9^{(10)}.$

Substituting here (69), we have

$UX_9 = \frac{1}{\Delta_9}(-2-U_9;3-V_9^{(1)};2+U_9;-5+V_9^{(2)};-1-V_9^{(3)};8-V_9^{(4)};3-y_9;-13+z_9;$

$;p_9)$ (70),where

$V_9^{(1)} = -a_{9,1}+0a_{9,2}+a_{9,3}-a_{9,4}-2a_{9,5}+a_{9,6}+\vdots od\vdots 3x_9+7p_9=-17V_9^{(1)};x_9+V_9^{(1)}=7U_9.$

$+3a_{9,7}-2a_{9,8}.$

$V_9^{(2)} = -a_{9,1}-a_{9,2}+a_{9,3}+0a_{9,4}-3a_{9,5}-a_{9,6}+\vdots ev\vdots 5x_9+6p_9=-17V_9^{(2)};x_9+V_9^{(2)}=6U_9.$

$+4a_{9,7}+a_{9,8}.$

$V_9^{(3)} = -a_{9,1}+2a_{9,2}+a_{9,3}-3a_{9,4}+0a_{9,5}+5a_{9,6}+\vdots od\vdots -x_9+9p_9=-17V_9^{(3)};x_9+V_9^{(3)}=9U_9.$

$+a_{9,7}-8a_{9,8}.$

$V_9^{(4)} = -2a_{9,1}-a_{9,2}+2a_{9,3}-a_{9,4}-5a_{9,5}+0a_{9,6}+\vdots od\vdots 8x_9+13p_9=-17V_9^{(4)};2x_9+V_9^{(4)}=13U_9.$

$+7a_{9,7}-a_{9,8}.$

Thus we have received system of 7-th linear homogeneous the equations
with 9-th unknown numbers about whom it is known, that they integers, and also
either even, or odd, or even and positive.

Let $x_9 = 2$ $.U_9$ can accept ten values $\pm(2i+1), i=0,1,2,3,4.$ But as
$\overline{|y_9| \le 28}$ and $|z_9| \le 29$, that remains two values ± 1. Let's accept $U_9 = -1$.then
$y_9 = 12; z_9 = -25; p_9 = 21; V_9^{(1)} = -9; V_9^{(2)} = -8; V_9^{(3)} = -11; V_9^{(4)} = -17.$

This the variant arranges in every respect. So it is definitive

$\underline{a_{9,1}=1;a_{9,2}=-1;a_{9,3}=-1;a_{9,4}=1;a_{9,5}=1;a_{9,6}=1;a_{9,7}=-1;a_{9,8}=1.}$ (71)

$\Delta_9 = 19$ - 9-th prime number (72)

$DUX_9 = -1;12;1;-13;10;25;-9;-38;21.$ (73)

$DUX_9(9) = p_9 = 21.$ (74)

And from (6) at n =10 we will have

$\Delta_{10} = -a_{10,1}+12a_{10,2}+a_{10,3}-13a_{10,4}+10a_{10,5}+25a_{10,6}-9a_{10,7}-38a_{10,8}+21a_{10,9}+19$
(75)

Let's consider the matrix of eleventh order having precisely same structure.
N=11; M=2; L=10

$$E_{i+1}^{(11)} = \frac{-a_{i,i+2}}{E_i^{(11)}(a_{i,i}+\sum_{j=1}^{10} a_{i,i-j}\times\prod_{l=1}^{j} E_{i-l}^{(11)})+a_{i,i+1}} \quad i=1,..,10 \qquad (76)$$

Cases i=1,2,.,9 us are already known. According to (+) it

$E_2^{(11)}E_1^{(11)} = -1$

$E_3^{(11)}E_2^{(11)} = -1-E_3^{(11)}$

$3E_4^{(11)}E_3^{(11)} = -1-E_4^{(11)}$

$5E_5^{(11)}E_4^{(11)} = -3+E_5^{(11)}$

$7E_6^{(11)}E_5^{(11)} = -5+E_6^{(11)}$

$11E_7^{(11)}E_6^{(11)} = -7-E_7^{(11)}$

$13E_8^{(11)}E_7^{(11)} = -11-3E_8^{(11)}$

$$17E_9^{(11)}E_8^{(11)} = -13 - 19E_9^{(11)}$$
$$19E_{10}^{(11)}E_9^{(11)} = -17 + 21E_{10}^{(11)}. \qquad (77)$$

At i=10 it is had

$$E_9^{(11)} = \frac{-13}{y_{10} + z_{10}E_8^{(11)}}. \qquad (78)$$

And then

$$E_{10}^{(11)} = \frac{p_{10}}{x_{10} + 19}. \qquad (79)$$

Here it is designated

$x_{10} = -a_{10,1} + 12a_{10,2} + a_{10,3} - 13a_{10,4} + 10a_{10,5} + 25a_{10,6} - 9a_{10,7} - 38a_{10,8} + 21a_{10,9}$.

$y_{10} = 3a_{10,1} + 2a_{10,2} - 3a_{10,3} + a_{10,4} + 8a_{10,5} + a_{10,6} - 11a_{10,7} + 13a_{10,9}$.

$z_{10} = 2a_{10,1} - 3a_{10,2} - 2a_{10,3} + 5a_{10,4} + a_{10,5} - 8a_{10,6} - 3a_{10,7} + 13a_{10,8}$.

$p_{10} = a_{10,1} + 7a_{10,2} - a_{10,3} - 6a_{10,4} + 9a_{10,5} + 13a_{10,6} - 10a_{10,7} - 19a_{10,8} + 17a_{10,9}$.

As all a on the module are equal 1,then x_{10} and y_{10} - even numbers, and z_{10} and p_{10} -odd number. Condition, that $\Delta_{10} > \Delta_9$, x_{10} to that still positive number.

Further we will receive

$3x_{10} + y_{10} = 38U_{10}; 2x_{10} + z_{10} = 21U_{10}; x_{10} + p_{10} = 19U_{10}; U_{10} = a_{10,2} - a_{10,4} + a_{10,5} + 2a_{10,6} - a_{10,7} -$ $- 3a_{10,8} + 2a_{10,9}$ - odd number. Further $UX_{10} = \frac{-1 - E_{10}^{(11)}}{19}; \frac{-7 + 12E_{10}^{(11)}}{19}; \frac{1 + E_{10}^{(11)}}{19};$ $; \frac{6 - 13E_{10}^{(11)}}{19}; \frac{-9 + 10E_{10}^{(11)}}{19}; \frac{-13 + 25E_{10}^{(11)}}{19}; \frac{10 - 9E_{10}^{(11)}}{19}; \frac{19 - 38E_{10}^{(11)}}{19}; \frac{-17 + 21E_{10}^{(11)}}{19}; E_{10}^{(11)}$.

Substituting here (79), we have

$$UX_{10} = \frac{1}{\Delta_{10}}(-1 - U_{10}; -7 - V_{10}^{(1)}; 1 + U_{10}; 6 + V_{10}^{(2)}; -9 - V_{10}^{(3)}; -13 - V_{10}^{(4)}; 10 + V_{10}^{(5)}; 19 - y_{10}; -17 + z_{10}; ; p_{10}) \qquad (80),$$ where

$V_{10}^{(1)} = -a_{10,1} + 0a_{10,2} + a_{10,3} - a_{10,4} - 2a_{10,5} + a_{10,6} + \vdots ev \vdots 7x_{10} - 12p_{10} = 19V_{10}^{(1)}; x_{10} - V_{10}^{(1)} = 12U_{10}$.
$+ 3a_{10,7} - 2a_{10,8} - 3a_{10,9}$.

$V_{10}^{(2)} = -a_{10,1} - a_{10,2} + a_{10,3} + 0a_{10,4} - 3a_{10,5} - a_{10,6} + \vdots od \vdots 6x_{10} - 13p_{10} = 19V_{10}^{(2)}; x_{10} - V_{10}^{(2)} = 13U_{10}$.
$+ 4a_{10,7} + a_{10,8} - 5a_{10,9}$.

$V_{10}^{(3)} = -a_{10,1} + 2a_{10,2} + a_{10,3} - 3a_{10,4} + 0a_{10,5} + 5a_{10,6} + \vdots ev \vdots 9x_{10} - 10p_{10} = 19V_{10}^{(3)}; x_{10} - V_{10}^{(3)} = 10U_{10}$.
$+ a_{10,7} - 8a_{10,8} + a_{10,9}$.

$V_{10}^{(4)} = -2a_{10,1} - a_{10,2} + 2a_{10,3} - a_{10,4} - 5a_{10,5} + 0a_{10,6} + \vdots od \vdots 13x_{10} - 25p_{10} = 19V_{10}^{(4)}; 2x_{10} - V_{10}^{(4)} = 25U_{10}$.
$+ 7a_{10,7} - a_{10,8} - 8a_{10,9}$.

$V_{10}^{(5)} = -a_{10,1} + 3a_{10,2} + a_{10,3} - 4a_{10,4} + a_{10,5} + 7a_{10,6} + \vdots od \vdots 10x_{10} - 9p_{10} = 19V_{10}^{(5)}; x_{10} - V_{10}^{(5)} = 9U_{10}$.
$+ 0a_{10,7} - 11a_{10,8} + 3a_{10,9}$.

Thus we have received system of 8-th linear homogeneous the equations with 10-th unknown numbers about whom it is known, that they integers, and also either even, or odd, or even and positive.

Let $\underline{x_{10} = 2}$. U_{10} can accept 12 values $\pm(2i+1), i = 0,1,2,..,5$ But as $|z_{10}| \le 37$, that remains two values $\pm(2i+1), i = 0$.

$y_{10}^{(i)} = -6 \pm 38(2i+1); z_{10}^{(i)} = -4 \pm 21(2i+1), i = 0$

But in this case $\frac{-17 + z_{10}^{(i)}}{x_{10} + \Delta_9} = \frac{-17 - 4 \pm 21(2i+1)}{2+19} = \frac{-3 \pm 3(2i+1)}{3} = -1 \pm (2i+1), i = 0$

that is unacceptable.

Let $\underline{x_{10} = 4}$. U_{10} can accept two values $\pm(2i+1), i = 0$.

Let's accept $U_{10} = 1$, then

$y_{10} = 26; z_{10} = 13; p_{10} = 15; V_{10}^{(1)} = -8; V_{10}^{(2)} = -9; V_{10}^{(3)} = -6; V_{10}^{(4)} = -17; V_{10}^{(5)} = -5.$

This the variant arranges in every respect. So it is definitive

$\underline{a_{10,1} = -1; a_{10,2} = -1; a_{10,3} = 1; a_{10,4} = 1; a_{10,5} = 1; a_{10,6} = 1; a_{10,7} = -1; a_{10,8} = 1; a_{10,9} = 1.}$

(81)

$\Delta_{10} = 23$ - 10-th prime number (82)

$DUX_{10} = -2;1;2;-3;-3;4;5;-7;-4;15$ (83)

$DUX_{10}(10) = p_{10} = 15.$ (84)

And from (6) at n =11 we will have

$\Delta_{11} = -2a_{11,1} + a_{11,2} + 2a_{11,3} - 3a_{11,4} - 3a_{11,5} + 4a_{11,6} + 5a_{11,7} - 7a_{11,8} - 4a_{11,9} +$

$+ 15a_{11,10} + 23$ (85)

And now for presentation we will write out a matrix of 10th order with the numerical the values of its determinants constructed in the image specified above.

1	0	1								
-1	1	0	1							
-1	-1	1	0	1						=3
-1	-1	-1	1	0	1					=5
1	-1	-1	-1	1	0	1				=7
1	1	-1	-1	-1	1	0	1			=11
-1	-1	1	-1	1	-1	1	0	1		=13
-1	-1	-1	1	1	1	-1	1	0	1	=17
1	-1	-1	1	1	1	-1	1	1	0	=19
-1	-1	1	1	1	1	-1	1	1	1	=23

This most accept communication between number of prime number and its value. An order of a determinant-it number of prime number, and its numerical value-size of this number.

Earlier similar it was known for numbers Fibonacci [3].

In [2] it is shown, that any set of integers it is possible to present a corresponding set of determinants, where a determinant order corresponds to integer number in this set.

Here, probably for the first time, such representation is received for the prime numbers. We will result still, in my opinion, a number of interesting consequences (77) it is possible to present expression in the form of continuous fraction.

$$E_{10}^{(11)} = \cfrac{-17}{-21-\cfrac{247}{19-\cfrac{187}{3-\cfrac{91}{1-\cfrac{55}{-1-\cfrac{21}{-1-\cfrac{5}{1-\cfrac{3}{1-\cfrac{1}{E_1^{(11)}}}}}}}}}}\ .$$

And still, if in a considered matrix all elements located below the main diagonal are equal 1 such matrix will be individual.
It is easy to prove it, having spread out it on elements of last column.
But unlike its classical individual matrix own values will not be equal among themselves and equal 1, and will be to represent complex numbers.
The resulted recursive parities can be considered as one of variants big screen with that essential difference, that here instead of prime numbers mutual simplicity is used.
Specific calculations which I have carried out so far, allowed me to construct such matrices for all n up to 63.
I appreciative professor Landon Curt Noll for attention this work.

Conclusion

This most accept communication between number of prime number and its value. An order of a determinant-it number of prime number, and its numerical value-size of this number. Earlier similar it was known for numbers Fibonacci. Here, probably for the first time, such representation is received for the prime numbers. The resulted recursive parities can be considered as one of variants big screen with that essential difference, that here instead of prime numbers mutual simplicity is used.

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