The smallest possible counter-example of the even Goldbach conjecture if any, can lie only between two odd numbers that themselves obey the odd Goldbach conjecture

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Abstract: The even Goldbach conjecture states that any even integer greater than four may be expressed as the sum of two odd primes. The odd Goldbach conjecture states that any odd integer greater than seven must be expressible as a sum of three odd primes. These conjectures remain unverified. In this paper we explore the possible constraints that exist on the smallest possible counterexample of the even Goldbach conjecture. We prove that the odd numbers immediately flanking the smallest counterexample of the even Goldbach conjecture are themselves expressible as the sum of three odd primes and are therefore consistent with the odd Golbach conjecture.

Results:

Let 2n be the smallest counter example of the even Goldbach conjecture. Therefore 2n may not be expressed as the sum of two odd primes.

According to Jingrun's theorem which states than every sufficiently large even number may be expressed either as a sum of two primes or as a sum of prime and a semi-prime.

Therefore 2n=p+qr

(where p, q, r are odd primes)

The semi-prime qr is an odd composite which is atmost equal to the number that is 3 less than 2n.

Therefore qr+1 and qr-1 represent even integers which are atmost 2 and 4 less than 2n.

Therefore qr+1 and qr-1 represent even integers which are atmost equal to two even integers that precede 2n. Since 2n is the smallest counter-example of the even Goldbach conjecture, therefore qr+1 and qr-1 both obey the even Goldbach conjecture.

Let us say qr+1=s+t (where s, t are two odd primes)
Let is say qr-1=u+v (where u,v are two odd primes)

$2n=p+qr$

$2n+1=p+(qr+1)$

$2n-1=p+(qr-1)$

$2n+1=p+s+t$ (where p,s,t are odd primes and therefore consistent with Odd Goldbach conjecture)

and $2n-1=p+u+v$ (where p, u, v are odd primes and therefore consistent with Odd Goldbach conjecture).

This gives the conclusion that the smallest counterexample of the even Golbach conjecture if any can only lie between two odd integers that themselves are true for the odd Goldbach conjecture.

It is also possible to prove this result without invoking the Jingrun theorem by replacing "qr" with the term "odd-composite" which will yield multiple sum partitions for the flanking odd integers.