Liénard-Type and Duffing-Type Nonlinear Oscillators Equations with Exponential-Type Restoring Force

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Abstract

This communication consists of additions to a previous work [1]. It presents certain Liénard-type and Duffing-type nonlinear oscillators equations with exponential-type restoring force, according to the recent theory of nonlinear differential equations of position-dependent mass oscillators formulated by Monsia et al. [1].

1. Consider the general class of exactly solvable mixed Liénard-type differential equations [1-4]

$$\ddot{x} - \gamma\varphi'(x)\dot{x}^2 + \mu\dot{x}\exp(\gamma\varphi(x)) + \omega^2 x\exp(2\gamma\varphi(x)) = 0 \qquad (1)$$

where $\varphi(x)$ is an arbitrary function of x, prime stands for differentiation with respect to variable x, and dot over a symbol means differentiation with respect to time. γ, μ, and ω are arbitrary parameters.

1.1 Let $\varphi(x) = x$. Then (1) takes the form

$$\ddot{x} - \gamma\dot{x}^2 + \mu\exp(\gamma x)\dot{x} + \omega^2 x\exp(2\gamma x) = 0 \qquad (2)$$

which admits an exact solution [1, 2]. Equation (2) reduces to the quadratic Liénard-type oscillator equation with exponential-type restoring force for the parametric choice $\mu = 0$

$$\ddot{x} - \gamma\dot{x}^2 + \omega^2 x\exp(2\gamma x) = 0 \qquad (3)$$

which admits an exact analytical trigonometric periodic solution [1-4].

1.2 The choice $\varphi(x) = x^2$, in the case $\mu = 0$, yields the quadratic Liénard-type equation

$$\ddot{x} - 2\gamma x\dot{x}^2 + \omega^2 x\exp(2\gamma x^2) = 0 \qquad (4)$$

admitting also an exact sinusoidal periodic solution [1-4]. It is worth mentioning that inverted versions of these equations may be formulated and studied by using adequate analytical methods for solving nonlinear differential equations.

2. An inverted version of equation (1) may be written as

$$\ddot{x} + \gamma\varphi'(x)\dot{x}^2 + \omega^2 x\exp(2\gamma\varphi(x)) = 0 \qquad (5)$$

or in the form

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$$\ddot{x}-\gamma\varphi'(x)\dot{x}^2-\omega^2 x\exp(2\gamma\varphi(x))=0 \quad (6)$$

where $\mu=0$. The following form of inversion may also be considered

$$\ddot{x}+\gamma\varphi'(x)\dot{x}^2-\omega^2 x\exp(2\gamma\varphi(x))=0 \quad (7)$$

3. These equations suggest that the following generalized Duffing-type equation with exponential-type restoring force

$$\ddot{x}+\omega^2 x\exp(2q\varphi(x))=0 \quad (8)$$

where q is an arbitrary parameter, may be considered. For example, setting $\varphi(x)=\frac{1}{2}x^2$, then the Taylor expansion of $\exp(qx^2)$, as $\exp(qx^2)=1+qx^2$, gives immediately the well-known cubic Duffing nonlinear oscillator equation

$$\ddot{x}+\omega^2 x+q\omega^2 x^3=0 \quad (9)$$

It is worth noting that equation (8) is enough powerful to give also Duffing-type oscillator equations of higher order terms more than three.

References

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