The Proof of Fermat's Last Theorem

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Abstract

We first prove a weak form of Fermat's Last Theorem; this unique lemma is key to the entire proof. A corollary and lemma follow inter-relating Pythagorean and Fermat solutions. Finally, we prove Fermat's Last Theorem.

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We first prove a weak form of Fermat's Last Theorem :

Lemma 1 (Chizmarik, 2011 - 2016)
for $n > 2$, x,y,z Pythagorean: if $x^n + y^n = z^n$ then $xyz = 0$

Proof.
for some $n > 2$, assume $\exists$ an x,y,z Pythagorean triple such that $x^n + y^n = z^n$ with $xyz \neq 0$; we find a contradiction: we re-write $x^n + y^n = z^n$ as $x^{n-2}x^2 + y^{n-2}y^2 = z^{n-2}z^2$ and note that $z^2 = x^2 + y^2$ since x,y,z is a Pythagorean triple; then we have: $x^{n-2}x^2 + y^{n-2}y^2 = z^{n-2}(x^2 + y^2)$ which after multiplying, re-arranging, and grouping becomes: $(x^{n-2} - z^{n-2})x^2 + (y^{n-2} - z^{n-2})y^2 = 0$ [**L1**].
Now since $z > x,\ z > y$ we see that the LHS of this equation is always < 0; a contradiction. Further, we see that at least one of x,y,z must equal zero for the equality to be satisfied; so the product xyz must be zero.

QED

We note the following important contradictory behavior of the Pythagorean triples in this family that is at the heart of this proof :

Corollary 1.1 (Chizmarik, 2016)

1. for every $n > 2$, every x,y,z Pythagorean, there is never a non-trivial solution to $x^n + y^n = z^n$
2. every Pythagorean triple satisfies $x^n + y^n = z^n$ for some $n > 2$

Proof.

1. from Lemma 1, we note that the consequences of the lemma hold for every $n > 2$, if the equality $x^n + y^n = z^n$ obtains when x,y,z are drawn from the full set of all Pythagorean triples; hence, there is never a non-trivial solution.
2. by contradiction: suppose $\exists$ a Pythagorean triple x_p, y_p, z_p which did not satisfy $x^n + y^n = z^n$ for any $n > 2$; that is: $\forall\ n > 2,\ x_p^n + y_p^n \neq z_p^n$ holds. Then we are restricting the applicability of Lemma 1 to a subset of the Pythagorean triples since the lemma does not apply to x_p, y_p, z_p and could not draw the same conclusion as (1) above since the lemma does not speak to the inequality at all. This violates the lemma, prohibits its conclusion, and thereby presents a contradiction. Therefore, such an x_p, y_p, z_p cannot exist. Hence, every Pythagorean triple satisfies $x^n + y^n = z^n$ for some $n > 2$.

QED

Now we offer a specific construction for the set of Pythagorean triples :

Lemma 2 (Chizmarik, 2013 - 2016)

for $n > 2$, x,y,z positive integers such that $x^n + y^n = z^n$, xyz $\neq 0$ with $x^2 + y^2 \neq z^2$ then $\exists!$ construction of the Pythagorean triples where: $\forall\ X_p, Y_p, Z_p$, $X_p = xr_x$, $Y_p = yr_y$, $Z_p = zr_z$ for r_x, r_y, r_z rational.

Proof.

for $n > 2$, let x_f, y_f, z_f be the non-trivial Fermat triple as above; let x_{p_i}, y_{p_i}, z_{p_i} be a Pythagorean triple and 'i' be an index with 'i' an integer 1, 2, 3 … . Now since $x_f \neq 0$, $y_f \neq 0$, $z_f \neq 0$, we have uniquely: $X_{p_i} = x_f \cdot \dfrac{x_{p_i}}{x_f}$, $Y_{p_i} = y_f \cdot \dfrac{y_{p_i}}{y_f}$, $Z_{p_i} = z_f \cdot \dfrac{z_{p_i}}{z_f}$ or $X_{p_i} = x_f\ r_{x_i}$, $Y_{p_i} = y_f\ r_{y_i}$, $Z_{p_i} = z_f\ r_{z_i}$ for i $\in$ index set is a Pythagorean triple. Hence, every Pythagorean triple can be uniquely constructed from the assumed non-trivial Fermat triple.

QED

Finally, we prove Fermat's Last Theorem :

Fermat's Last Theorem (Chizmarik, 2011 - 2016)

for $n > 2$, x,y,z positive integers: if $x^n + y^n = z^n$ then $xyz = 0$

Proof.

for some $n > 2$, assume $\exists$ a Fermat triple x_f, y_f, z_f, $x_f y_f z_f \neq 0$; we will reach a contradiction: now, if this Fermat triple is a Pythagorean triple, by Lemma 1 the theorem is proved. Otherwise, by Lemma 2, we know that $\exists!$ construction of the set of Pythagorean triples such that each triple is of the form : $X_{p_i} = x_f r_{x_i}$, $Y_{p_i} = y_f r_{y_i}$, $Z_{p_i} = z_f r_{z_i}$ for r_x, r_y, r_z rational. Now consider the implications of the weak form of the theorem and its corollary with this construction of the Pythagorean triples to find the contradiction. We see that by Corollary 1.1, the Pythagorean triple X_p, Y_p, Z_p satisfies $X_p^n + Y_p^n = Z_p^n$ for some $n > 2$. Now, we have by Lemma 1, $X_p Y_p Z_p = 0$. Then (wlog) we can say that $X_p = 0$. Now $X_p = x_f \cdot \frac{x_p}{x_f}$; by construction, the fraction cannot equal zero. Therefore, x_f must equal zero. This invalidates our construction, contradicts our assumption, and completes the proof.

QED

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